Reconstruction of aether scalar tensor theory for various cosmological scenarios*

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Abstract: In this paper, we present several explicit reconstructions for the aether scalar tensor (AeST) theory derived from the background of the Friedmann-Lemaître-Robertson-Walker cosmological evolution. It is shown that the Einstein-Hilbert Lagrangian with a positive cosmological constant is the only Lagrangian capable of accurately replicating the exact expansion history of the Λ cold dark matter (ΛCDM) universe filled solely with dust-like matter. However, the ΛCDM-era can be produced within the framework of the AeST theory for some other fluids, including a perfect fluid with $p=-(1/3)\rho$, multifluids, and nonisentropic perfect fluids. Moreover, we demonstrate that the ΛCDM-era can be replicated with no real matter field for the AeST theory. The cosmic evolution resulting from both the power-law and de-Sitter solutions can also be obtained.

Keywords: reconstruction, modified gravity, cosmology

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I. INTRODUCTION

Even after more than a century since its inception, general relativity (GR) remains the most successful fundamental theory for describing gravitational phenomena. Despite the remarkable success of GR, it has faced new challenges in light of cosmic observations, particularly those concerning the enigmatic phenomena of dark energy and dark matter. Over the past decade, one of the most significant advancements in cosmology has been the rigorous comparison of observations with the standard Λ cold dark matter (ΛCDM) model. This phenomenological model fits a wide range of observations, including supernovae type Ia [1], cosmic microwave background (CMB) radiation [2], large-scale structure formation [3], baryon oscillations [4], and weak lensing [5]. However, it also presents significant fine-tuning problems associated with the vacuum energy scale. Therefore, exploring alternative descriptions of the universe becomes crucial to address these issues.

Recently, a novel relativistic theory of modified Newtonian dynamics (MOND), known as the aether scalar tensor (AeST) theory, has been proposed in Ref. [6]. This theory introduces additional fields in the gravitational sector, i.e., a unit timelike vector field A_μ and a noncanonical shift-symmetric scalar field ϕ. It has been argued that the AeST theory maintains consistency with the observations of CMB and matter power spectra. Despite the fact that it does not postulate the existence of dark matter particles, the presence of additional vector and scalar fields in the theory introduces corrections to the standard Friedman equations that can effectively mimic the behavior of cold dark matter.

In Ref. [7], the authors analyzed the linear stability of the AeST theory on a Minkowski background and found that this theory is free of propagating ghost instabilities. The Schwarzschild and nearly-Schwarzschild black holes were investigated as solutions in this theory in Ref. [8]. In Ref. [9], the authors comprehensively explored the general cosmological behavior of AeST using phase space dynamical analysis. In Ref. [10], the authors analyzed the time evolution of the local Newtonian and MOND parameters and presented a gravitational wave polarization analysis in the framework of this theory. The cosmological structure formation over all scales was investigated in

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Ref. [11]. The particular behaviour of the AeST theory in spherically symmetric static situations was investigated in Ref. [12]. In this paper, we mainly focus on the late-time cosmological behavior of the AeST theory and aim to reconstruct some particular cosmological models within this theory.

Cosmological reconstruction plays a significant role in emulating realistic cosmological scenarios within the context of modified gravitational theories. The reconstruction technique operates under the assumption that the expansion history of the universe is known with precision. By employing this technique, one aims to reverse the field equations and determine the specific class of modified theories that can give rise to a given flat Friedmann-Lemaître-Robertson-Walker (FLRW) model. Cosmological reconstruction has been investigated in many modified gravitational theories. For example, the reconstruction scheme in terms of e-folding was introduced to find some realistic models in the $f(R)$ theory in Ref. [13]. Subsequently, this approach was applied to $f(R,G)$ theories [14], where G represents the Gauss-Bonnet term. Furthermore, cosmic evolution based on power-law solutions of the scale factor has been extensively explored in modified theories [15−17]. In Ref. [18], the authors demonstrated the necessity of introducing additional degrees of freedom to the matter components in order to reconstruct the evolution of the ΛCDM model within the context of $f(R)$ gravity. The cosmological reconstruction in $f(R,T)$ gravity was studied in Ref. [19], and the authors demonstrated the ability of a dust fluid to reproduce various cosmological models, including the ΛCDM model, de-Sitter universe, Einstein static universe, phantom and non-phantom eras, and phantom cosmology. Reconstruction of slow-roll inflation was investigated in Refs. [20, 21]. Cosmological reconstruction and stability were explored in Ref. [22−25]. Besides, new strategies for cosmological reconstruction were introduced in Refs. [26, 27]. Please consult Refs. [18, 28−39] for further investigations on cosmological reconstruction.

In this paper, we focus on several explicit reconstructions within the framework of the AeST theory. In particular, we first focus on the reconstruction of the ΛCDM model with different fluids. Then, we consider the possibility of replicating the ΛCDM-era with no real matter content. Finally, we focus on exploring the feasibility of a well-suited reconstruction for simulating the cosmic evolution exhibited by both the power-law and de-Sitter solutions. This paper is organized as follows. In Sec. II, we briefly review the AeST theory and then derive the functional $\mathcal{K}(Q)$ by reversing the field equations to produce the ΛCDM model with different fluids. In Sec. III, we present the reconstructions that can produce the ΛCDM-era with no real matter fluid. The reconstructions of cosmic evolution based on the power-law and de-Sitter solutions are presented in Sec. IV. Section V concludes the paper.

II. RECONSTRUCTION OF AEST BEHAVING AS THE ΛCDM MODEL

First, we briefly review the aether scalar tensor (AeST) theory. The action constructed by a scalar ϕ and a unit-timelike vector A_μ excluding the metric $g_{\mu\nu}$, is given by $S=S_G+S_M$, where S_M is the action of the ordinary matter field explicitly independent of ϕ and A_μ, and S_G is expressed as [6]

$$S_G=\int d^4x\frac{\sqrt{-g}}{2\kappa}\Big[R-\frac{K_B}{2}F_{\mu\nu}F^{\mu\nu}+2(2-K_B)J^\mu\nabla_\mu\phi \\ -(2-K_B)\mathcal{Y}-\mathcal{F}(\mathcal{Y},Q)-\lambda(A^\mu A_\mu+1)\Big], \tag{1}$$

where $\kappa\equiv 8\pi G_*$, with G_* proportional to the measured value of the Newtonian gravitational constant [40], $F_{\mu\nu}=\nabla_\mu A_\nu-\nabla_\nu A_\mu$, $Q=A^\mu\nabla_\mu\phi$, $\mathcal{Y}=(g^{\mu\nu}+A^\mu A^\nu)\nabla_\mu\phi\nabla_\nu\phi$, $J^\mu=A^\nu\nabla_\nu A^\mu$,$\mathcal{F}$ is a free function of $\mathcal{Y}$ and Q, λ is a Lagrange multiplier leading to the unit-timelike constraint $A^\mu A_\mu+1=0$, and K_B is a dimensionless constant.

The gravitational field equations can be derived by varying the action with respect to the metric [9, 10]:

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\mathcal{H}_{\mu\nu}=T_{\mu\nu}, \tag{2}$$

where we have set $\kappa=1$ and

$$\mathcal{H}_{\mu\nu}=-K_B\Big(F_\mu^{\ \alpha}F_{\nu\alpha}-\frac{1}{4}F^{\alpha\beta}F_{\alpha\beta}\Big)-\lambda A_\mu A_\nu \\ -(2-K_B)g_{\mu\nu}J^\alpha\nabla_\alpha\phi+2(2-K_B)\Big(A^\sigma\nabla_{(\mu}\phi\nabla_{\sigma}A_{\nu)} \\ -\frac{1}{2}A_\mu A_\nu\Box\phi+\nabla_\sigma\phi A_{(\mu}F^{\sigma}_{\ \nu)}\Big)+\frac{1}{2}g_{\mu\nu}\big((2-K_B)\mathcal{Y} \\ +\mathcal{F}\big)-\big((2-K_B)+\mathcal{F}_\mathcal{Y}\big)\nabla_\mu\phi\nabla_\nu\phi \\ -\Big(2Q\big((2-K_B)+\mathcal{F}_\mathcal{Y}\big)+\mathcal{F}_Q\Big)A_{(\mu}\nabla_{\nu)}\phi. \tag{3}$$

Here, $\mathcal{F}_\mathcal{Y}\equiv\dfrac{\partial\mathcal{F}}{\partial\mathcal{Y}}$ and $\mathcal{F}_Q\equiv\dfrac{\partial\mathcal{F}}{\partial Q}$. It can be easily shown that Eq. (2) satisfies the energy-momentum conservation $\nabla_\mu T^{\mu\nu}=0$.

Varying the action with respect to A_μ and ϕ, the vector and scalar field equations can be respectively derived as follows:

$$K_B\nabla_\mu F^{\mu\nu}+2(2-K_B)\big(\nabla_\mu\phi\nabla^\nu A^\mu-\nabla_\mu(A^\mu\nabla^\nu\phi)\big) \\ -\lambda A^\nu-\frac{1}{2}\nabla^\nu\phi\Big(2Q\big((2-K_B)+\mathcal{F}_\mathcal{Y}\big)+\mathcal{F}_Q\Big)=0, \tag{4}$$

$$\nabla_\mu(\mathcal{F}_Q A^\mu) - 2(2-K_B)\nabla_\mu J^\mu + 2\nabla_\mu\Big(Q\big((2-K_B) + \mathcal{F}_\mathcal{Y}\big)A^\mu\Big) + 2\nabla_\mu\Big(\big((2-K_B)+\mathcal{F}_\mathcal{Y}\big)\nabla^\mu\phi\Big) = 0. \tag{5}$$

Let us now consider the flat FLRW metric, i.e., $\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2(\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2)$, with $a(t)$ denoting the cosmic scale factor. It can be easily shown that $A_\mu = (-1,0,0,0)$, $\mathcal{Y}=0$, $J^\mu = 0$, and $Q=\dot\phi$, where the dot stands for the derivative with respect to t. After defining a new functional $\mathcal{K}(Q) = -\mathcal{F}(0,Q)/2$, Eq. (5) can be expressed as $\nabla_\mu(\mathcal{K}_Q A^\mu) = 0$, which reduces to $\dot{\mathcal{K}}_Q + 3H\mathcal{K}_Q = 0$ for the FLRW metric. This equation admits the solution $\mathcal{K}_Q = I_0/a^3 = I_0(1+z)^3$ with I_0 being an integration constant and the usual definition of the redshift $1+z = 1/a$. Thus, Eq. (5) can be significantly simplified. Note that, since there is no potential term for the scalar field ϕ, it might not act as an inflaton candidate.

From Eq. (4), Lagrange multiplier λ can be solved as

$$\lambda = (2-K_B)(\dot Q + 3HQ + Q^2) - Q\mathcal{K}_Q. \tag{6}$$

Assuming a perfect fluid with barotropic density ρ and pressure p, and substituting the Lagrange multiplier into Eq. (2), one can obtain

$$H^2 = \frac{\rho}{3} + \frac{1}{3}Q\mathcal{K}_Q - \frac{1}{3}\mathcal{K}, \tag{7}$$

$$\frac{2}{3}\dot H + H^2 = -\frac{p}{3} - \frac{1}{3}\mathcal{K}. \tag{8}$$

Then, subtracting Eq. (7) from Eq. (8) yields

$$2\dot H + p + Q\mathcal{K}_Q + \rho = 0. \tag{9}$$

It is well-known that present cosmological observations suggest that the Hubble parameter in terms of the redshift is described as

$$H(z) = \sqrt{\frac{\rho_0}{3}(1+z)^3 + \frac{\Lambda}{3}}, \tag{10}$$

where ρ_0 is the present matter density and Λ is the cosmological constant. Now, we have four unknown functions to be solved, i.e., H, ρ, p, and $\mathcal{K}(Q)$. In what follows, we aim to reconstruct the functional $\mathcal{K}(Q)$ that can exactly mimic the above expansion history with different matter contents.

A. Reconstruction for dust-like matter

First, we reconstruct the functional $\mathcal{K}(Q)$ that can reproduce the ΛCDM background only with dust-like matter. From the continuity equation $\dot\rho + 3H(\rho+p) = 0$ and equation of state (EoS) $p = w\rho = 0$, one obtains $\rho = \frac{\rho_0}{a^3}$. Then, inserting the density ρ and $\mathcal{K}_Q = I_0(1+z)^3$ into Eq. (9) and transforming it to the z coordinate, the scalar Q in terms of the redshift can be expressed as

$$Q(z) = \frac{2HH' - \rho_0(1+z)^2}{I_0(1+z)^2}, \tag{11}$$

where the prime denotes the derivative with respect to z. However, by inserting Eq. (10) into Eq. (11), we immediately have $Q=0$, i.e., the AeST theory reduces to GR, which indicates that this theory does not admit the ΛCDM solution only with dust-like matter.

B. Reconstruction for perfect fluid with EoS $p = -\rho/3$

In this case, we reconstruct $\mathcal{K}(Q)$ for a perfect fluid with $p = -\rho/3$, which is interesting because it resides on the boundary of the set of matter fields that adhere to the strong energy condition. Then, from the continuity equation, the density and pressure of the perfect fluid can be obtained as $\rho = \rho_0(1+z)^2$ and $p = -\rho_0(1+z)^2/3$, respectively. Inserting ρ and p into Eq. (9), one can obtain

$$Q(z) = -\frac{2\left(-3HH' + \rho_0(1+z)\right)}{3I_0(1+z)^2}. \tag{12}$$

Inserting Eq. (10) into Eq. (12) and inverting it, we have

$$z = \frac{\rho_0 - 3I_0 Q}{3(I_0 Q - \rho_0)}. \tag{13}$$

Then, from Eq. (8), the functional $\mathcal{K}(Q)$ can be calculated as

$$\mathcal{K}(Q) = \frac{1}{3}\rho_0(1+z)^2 = \frac{4\rho_0^3}{27(\rho_0 - I_0 Q)^2} - \Lambda. \tag{14}$$

C. Reconstruction for multifluids

In general, our universe contains multiple matter components. In this case, we consider matter contents that include dust-like matter and also a noninteracting stiff fluid. Their densities are ρ_0 and ρ_s, respectively, and the total matter density is given by

$$\rho = \frac{\rho_0}{a^3} + \frac{\rho_s}{a^6}. \tag{15}$$

With the help of the continuity equation, the EoS parameter can be calculated as

$$w = \frac{\rho_s}{\rho_s + \dfrac{\rho_0}{(1+z)^3}}. \tag{16}$$

Thus, the pressure is $p = w\rho = \rho_s(1+z)^6$. Substituting ρ, p, and $\mathcal{K}_Q$ into Eq. (9), the scalar Q can be derived as

$$Q(z) = \frac{2HH' - (1+z)^2(\rho_0 + 2(1+z)^3\rho_s)}{I_0(1+z)^2}. \tag{17}$$

Assuming the ΛCDM background and inverting the above expression, one obtains

$$z = -\left(\frac{I_0 Q}{2\rho_s}\right)^{1/3} - 1. \tag{18}$$

From Eq. (8), $\mathcal{K}(Q)$ can be solved as

$$\mathcal{K}(Q) = -\frac{I_0^2 Q^2}{4\rho_s} - \Lambda. \tag{19}$$

Therefore, if the universe is filled with noninteracting stiff fluid and dust-like matter, it is impossible to distinguish the AeST theory from GR using the current cosmological observations at the background level, given that this theory accurately replicates an expansion history consistent with the ΛCDM model.

D. Reconstruction for nonisentropic perfect fluids

The EoS of nonisentropic perfect fluids can be expressed as

$$p = h(\rho, a). \tag{20}$$

In this case, the continuity equation becomes

$$\frac{\mathrm{d}\rho}{\mathrm{d}a} + \frac{3}{a}(\rho + h) = 0. \tag{21}$$

Usually, Eq. (21) may not have a solution in closed form. However, if $h(\rho,a)$ can be expressed as a separable function in the form of $p = h(\rho,a) = w(a)\rho$, the calculations become considerably easier, and the solution is

$$\rho(a) = c_1 \exp\left(-3\int \frac{1+w(a)}{a}\mathrm{d}a\right). \tag{22}$$

As an example, let us consider an explicit time-dependent barotropic index given by

$$w(a) = \frac{2\gamma - a^3\nu}{a^3\nu + \gamma}, \tag{23}$$

where γ and ν are constants. Then, we have

$$\rho(a) = c_1\frac{(\gamma + a^3\nu)^3}{a^9}, \tag{24}$$

where $c_1 = \dfrac{\rho_0}{(\gamma+\nu)^3}$. The substitution of Eq. (24) into Eq. (9) yields

$$Q = \frac{2HH'}{I_0(1+z)^2} - \frac{(3c_1\gamma)\left(\nu + \gamma(1+z)^3\right)^2}{I_0}. \tag{25}$$

Considering the ΛCDM background, the above expression admits the following inverse solution:

$$z = -1 + \left(-\frac{\nu}{\gamma} \pm \sqrt{\frac{\rho_0 - I_0 Q}{3c_1\gamma^3}}\right)^{1/3}. \tag{26}$$

From Eq. (8), the particular solution for $\mathcal{K}(Q)$ can be solved as

$$\mathcal{K}(Q) = \frac{(-\rho_0 + I_0 Q)\left(-9\nu \pm 2\sqrt{3}\sqrt{\dfrac{\rho_0 - I_0 Q}{c_1\gamma}}\right)}{9\gamma} - \Lambda. \tag{27}$$

As another form of nonisentropic perfect fluids, we consider that the EoS is given by $p = w\rho + h(a)$. Thus, the solution of Eq. (21) can be expressed as

$$\rho(a) = a^{-3(w+1)}\left(c_2 - \int 3h(a)a^{(3w+2)}\mathrm{d}a\right), \tag{28}$$

As a further specific example, let us consider $h(a) = \dfrac{h_0}{a^{12}}$ and $w = 0$. This suggests that the matter contents within the universe can be approximated as dust, accompanied by a time-dependent cosmological term that diverges at the singularity of the big bang and diminishes rapidly during subsequent epochs. Then, the density $\rho(a)$ is expressed as

$$\rho(a) = \frac{c_2}{a^3} + \frac{h_0}{3a^{12}}, \tag{29}$$

where $c_2 = \rho_0 - \dfrac{h_0}{3}$. From Eq. (9), the scalar Q can be obtained as

$$Q = -\frac{c_2}{I_0} + \frac{2HH'}{I_0(1+z)^2} - \frac{4h_0(1+z)^9}{3I_0}. \tag{30}$$

Assuming the ΛCDM background and inverting the above expression, one obtains

$$z = -1 + \left[\frac{1}{4}\left(1 - \frac{3I_0 Q}{h_0}\right)\right]^{1/9}, \tag{31}$$

where we have inserted $c_2 = \rho_0 - \frac{h_0}{3}$. Solving Eq. (8), we obtain the following solution:

$$\mathcal{K}(Q) = -h_0 \left[\frac{1}{4}\left(1 - \frac{3I_0 Q}{h_0}\right)\right]^{4/3} - \Lambda. \tag{32}$$

The above explicit reconstructions for replicating the ΛCDM-era were also explored in other physical models, such as the k-essence model, and $f(R)$ and $f(Q)$ gravity. Please, consult Refs. [37, 41–43] for further details.

III. COSMOLOGICAL RECONSTRUCTION WITHOUT MATTER

In this section, we are interested in finding the explicit solutions of $\mathcal{K}(Q)$ that can replicate the ΛCDM-era with no real matter, i.e., $\rho = p = 0$. In this case, the scalar Q solved from Eq. (9) reduces to

$$Q = \frac{2HH'}{I_0(1+z)^2}. \tag{33}$$

In the following, we solve this system using some cosmological models. As a first example, we consider the Chaplygin gas model, in which the universe evolves from a dust-matter dominated phase at early times to a cosmological constant dominated phase at late times. The FLRW equation for this model is given by [44]

$$H^2 = \sqrt{A + B(1+z)^6}, \tag{34}$$

where A is a positive constant and B is an integration constant. Then, by inverting Eq. (33), one can obtain

$$z = -1 \pm \frac{\sqrt[6]{I_0^2 A B^2 Q^2 \left(-9B + I_0^2 Q^2\right)^2}}{\sqrt{B\left(9B - I_0^2 Q^2\right)}}. \tag{35}$$

From Eq. (8), the funtional $\mathcal{K}(Q)$ can be solved as

$$\mathcal{K}(Q) = -\sqrt{\frac{A}{B}\left(9B - I_0^2 Q^2\right)}. \tag{36}$$

Thus, we have demonstrated that the AeST theory admits the Chaplygin gas cosmological solution without introducing any real matter.

As another example, let us consider the following phantom-non-phantom model [45]

$$H^2 = \frac{1}{3}\rho_p(1+z)^b + \frac{\rho_q}{3(1+z)^d}, \tag{37}$$

where ρ_p, ρ_q, b, and d are positive constants. The first term in the right hand side of the above equation dominates in the early universe, which behaves as non-phantom matter with EoS parameter $w_p = -1 + b/3 > -1$ in the Einstein gravity, while the second term dominates in the late universe and behaves as a phantom matter with $w_d = -1 - d/3 < -1$. To mimic the late-time behavior of the universe, we set $b = 3$. Then, by inverting Eq. (33), we obtain

$$z = -1 + \left(\frac{3\left(\rho_p - I_0 Q\right)}{d\rho_q}\right)^{-\frac{1}{d+3}}. \tag{38}$$

From Eq. (8), $\mathcal{K}(Q)$ can be solved as

$$\mathcal{K}(Q) = -\frac{1}{3}(d+3)\rho_q \left(\frac{3\rho_p - 3I_0 Q}{d\rho_q}\right)^{\frac{d}{d+3}}. \tag{39}$$

Thus, the AeST theory is capable of reproducing the solution of Eq. (37), where the parameters $b = 3$ and $d \geqslant 0$ represent the ΛCDM model. Moreover, it is plausible that this solution propels the evolution of the universe towards a phantom phase in the foreseeable future.

IV. COSMOLOGICAL SOLUTIONS IN AEST THEORY

In this section, we focus on exploring the feasibility of obtaining a well-suited functional $\mathcal{K}(Q)$ for simulating the cosmic evolution resulting from both the power-law and de-Sitter solutions.

A. Power-law solutions

It is interesting to explore the presence of precise power-law solutions within the framework of AeST gravitational theory for different cosmic evolution stages. These solutions correspond to the decelerated and accelerated cosmic eras, which are characterized by the scale factor

$$a(t) = a_0 t^m, \quad H(t) = \frac{m}{t}, \tag{40}$$

where $m > 0$. The universe undergoes a decelerated phase for $0 < m < 1$ and experiences an accelerated phase for $m > 1$. To solve the system completely, we need another initial condition. Here, we reconstruct the funtional $\mathcal{K}(Q)$ with some given matter contents as the initial conditions.

First, for dust-like matter with $p = 0$ and $\rho = \rho_0/a^3$, Eq. (9) leads to

$$Q = \frac{2a_0^3 m}{I_0} t^{3m-2} - \frac{\rho_0}{I_0}. \tag{41}$$

Inverting the above expression, we obtain

$$t = \left(\frac{\rho_0 + I_0 Q}{2a_0^3 m}\right)^{\frac{1}{3m-2}}. \tag{42}$$

After a straightforward calculation, $\mathcal{K}(Q)$ can be obtained as

$$\mathcal{K}(Q) = (2-3m)m\left(\frac{\rho_0 + I_0 Q}{2a_0^3 m}\right)^{\frac{2}{2-3m}}. \tag{43}$$

Second, for a perfect fluid with pressure $p = -\frac{1}{3}\rho$, the scalar Q can be solved as

$$Q = \frac{6a_0^3 m t^{3m-2} - 2a_0\rho_0 t^m}{3I_0}. \tag{44}$$

However, the above expression does not admit an analytical inverse solution for a general m. Thus, without loss of generality, we take two particular values of m as examples, i.e., $m = \frac{1}{2}$ and $m = 2$, which correspond to a decelerated phase and an accelerated phase, respectively. For $m = \frac{1}{2}$, the inverse solution of Eq. (44) is calculated as

$$t = \frac{9I_0^2 Q^2 + 12a_0^4\rho_0 \pm 3\sqrt{24I_0^2 a_0^4 \rho_0 Q^2 + 9I_0^4 Q^4}}{8a_0^2\rho_0^2}, \tag{45}$$

and then the solution of $\mathcal{K}(Q)$ derived from Eq. (8) is

$$\mathcal{K} = \frac{8\rho_0^3\left(3I_0^2Q^2 + 6a_0^4\rho_0 \pm \sqrt{24I_0^2a_0^4\rho_0Q^2 + 9I_0^4Q^4}\right)}{9\left(\pm 3I_0^2Q^2 \pm 4a_0^4\rho_0 + \sqrt{24I_0^2a_0^4\rho_0Q^2 + 9I_0^4Q^4}\right)^2}. \tag{46}$$

For $m = 2$, the inverse solution of Eq. (44) is derived as

$$t = \frac{\sqrt{\rho_0 \pm \sqrt{\rho_0^2 + 36I_0a_0Q}}}{2\sqrt{3}a_0}, \tag{47}$$

and the functional $\mathcal{K}(Q)$ is

$$\mathcal{K}(Q) = -\frac{48a_0^3\left(\rho_0 \pm 2\sqrt{\rho_0^2 + 36I_0a_0Q}\right)}{\left(\pm\rho_0 + \sqrt{\rho_0^2 + 36I_0a_0Q}\right)^2}. \tag{48}$$

Finally, for a perfect fluid with EoS $p = -\rho$, the scalar Q is solved as

$$Q = \frac{2a_0^3 m}{I_0} t^{3m-2}, \tag{49}$$

with $t = \left(\frac{I_0 Q}{2a_0^3 m}\right)^{\frac{1}{3m-2}}$ being its inverse solution. Then, the functional $\mathcal{K}(Q)$ can be calculated as

$$\mathcal{K}(Q) = \rho_0 + m(2-3m)\left(\frac{I_0 Q}{2a_0^3 m}\right)^{\frac{2}{2-3m}}. \tag{50}$$

Thus, we have explicitly demonstrated that it is possible to reconstruct the pow-law solution within the AeST theory.

B. de-Sitter solutions

The de-Sitter cosmic evolution is a widely recognized model because it effectively represents the expansion of the universe. According to this model, the universe experiences constant expansion during the epoch dominated by dark energy, and the scale factor exhibits exponential growth with a constant Hubble parameter $H(t) = H_0$, expressed as

$$a(t) = a_0 e^{H_0 t}. \tag{51}$$

For the above scale factor and general EoS $p = w\rho$, the scalar Q can be explicitly solved as

$$Q = -\frac{\rho_0(w+1)\left(a_0 e^{H_0 t}\right)^{-3w}}{I_0}, \tag{52}$$

which admits the following analytical inverse solution

$$t = \frac{1}{H_0 w}\ln\left[\frac{1}{a_0 w}\left(\frac{\rho_0(1+w)}{I_0 Q}\right)^{1/3}\right]. \tag{53}$$

Solving Eq. (8) directly, we obtain

$$\mathcal{K}(Q) = -3H_0^2 - w\rho_0\left(-\frac{\rho_0(1+w)}{I_0 Q}\right)^{-(1+w)/w}. \tag{54}$$

It is clear that the de-Sitter solution can be achieved within the framework of the AeST theory.

V. CONCLUSION

In this study, we investigated the possibility of replicating the ΛCDM expansion history of the universe from the aether scalar tensor theory and derived a number of intriguing and explicit reconstructions. In particular, our findings indicate that the Einstein-Hilbert Lagrangian with a positive cosmological constant is the only Lagrangian capable of accurately replicating the exact expansion history of the ΛCDM universe filled with dust-like matter. This does not imply that the AeST theory is inherently incompatible with an exact ΛCDM expansion history. It also suggests that the theory needs to be extended to allow for such a possibility to be realized. For instance, in a universe comprising both a noninteracting stiff fluid and dust-like matter, it is feasible to formulate a gravitational theory that precisely emulates the expansion history of the ΛCDM. Besides, the reconstruction of the ΛCDM expansion history of the universe can also be achieved with nonisentropic perfect fluids.

Furthermore, we found that some types of requested FLRW cosmology in the Einstein gravity can be reconstructed within the AeST theory even with no real matter, such as the Chaplygin gas and phantom-non-phantom models. In addition, the power-law and de-Sitter solutions can also be obtained within the AeST theory. Consequently, it becomes impossible to distinguish this theory from GR solely based on measurements of the background cosmological parameters. Therefore, it is an interesting problem to probe whether the perturbations of the AeST theory can break this degeneracy. In a forthcoming study, we will analyze this issue in detail.

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