Ground states and first radial excitations of vector tetraquark states with explicit *P*-waves via QCD sum rules*

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Abstract: In this study, we chose the diquark-antidiquark type four-quark currents with an explicit *P*-wave between the diquark and antidiquark pairs to study the ground states and first radial excitations of the hidden-charm tetraquark states with quantum numbers $J^{PC}=1^{--}$. We also obtained the lowest vector tetraquark masses and made possible assignments of the existing *Y* states. There indeed exists a hidden-charm tetraquark state with $J^{PC}=1^{--}$ at an energy of approximately 4.75 GeV as the first radial excitation that accounts for the BESIII data.

Keywords: tetraquark state, QCD sum rules

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1. INTRODUCTION

A number of *Y* states, such as $Y(4008)$, have been observed in the $J/\psi\pi^+\pi^-$ channel [1]. $Y(4220)$ and $Y(4320)$ have also been observed in the $J/\psi\pi^+\pi^-$ channel [2], $Y(4230)$ has been observed in the $\omega\chi_{c0}$ channel [3], $Y(4260)$ has been observed in the $J/\psi\pi^+\pi^-$ channel [4, 5], $Y(4320)$ has been observed in the $\psi'\pi^+\pi^-$ channel [6], $Y(4360)$ and $Y(4660)$ have been observed in the $\psi'\pi^+\pi^-$ channel [7, 8], $Y(4390)$ has been observed in the $\pi^+\pi^-h_c$ channel [9], $Y(4469)$ has been observed in the $D^{*0}D^{*-}\pi^+$ channel [10], $Y(4484)$ has been observed in the K^+K^-J/ψ channel [11], $Y(4544)$ has been observed in the $\omega\chi_{c1}$ channel [12], $Y(4630)$ has been observed in the $\Lambda_c^+\Lambda_c^-$ channel [13], $Y(4710)$ has been observed in the K^+K^-J/ψ channel [14], and $Y(4790)$ has been observed in the $D_s^{*+}D_s^{*-}$ channel [15].

Recently, the BESIII collaboration studied the processes $e^+e^-\to\omega X(3872)$ and $\gamma X(3872)$ using data samples with an integrated luminosity of 4.5 fb^{-1} at center-of-mass energies ranging from 4.66 to 4.95 GeV, and observed that the relatively large cross section for the $e^+e^-\to\omega X(3872)$ process is mainly attributed to the enhancement around 4.75 GeV, which may indicate a potential structure in the $e^+e^-\to\omega X(3872)$ cross section [16]. If the enhancement is confirmed in the future by enough experimental data, another *Y* state, $Y(4750)$, may exist.

Even if $Y(4220)$, $Y(4230)$, and $Y(4260)$ are the same particle; $Y(4320)$, $Y(4360)$, and $Y(4390)$ are the same particle; $Y(4469)$, $Y(4484)$, and $Y(4544)$ are the same particle; $Y(4500)$, $Y(4630)$, and $Y(4660)$ are the same particle; and $Y(4710)$ and $Y(4750)$ are the same particle, the *Y* states are beyond the compatibility of the traditional quark models. Thus, we have to introduce states such as tetraquark, molecular, and hybrid to make reasonable assignments [17–26].

In Refs. [27, 28], we chose the scalar, pseudoscalar, axialvector, vector, and tensor (anti)diquarks as the elementary constituents to construct the four-quark currents without introducing explicit *P*-waves and investigated the hidden-charm and hidden-charm-hidden-strange tetraquark states with quantum numbers $J^{PC}=1^{--}$ and 1^{-+} in a comprehensive and consistent manner via the QCD sum rules. We also revisited the assignments of the X/Y states in the hidden-charm tetraquark scenario, as presented in Tables 1–2, where the subscripts S, P, V $(\tilde{V})$, and A $(\tilde{A})$ stand for the scalar, pseudoscalar, vector, and axialvector (anti)diquarks, respectively.

In Refs. [29, 30], we introduced an explicit *P*-wave between the diquark and antidiquark pairs to construct hidden-charm four-quark currents and explored the vector tetraquark states systematically via the QCD sum rules. We obtained the lowest vector tetraquark masses reported to date and revisited the assignments of the *Y* states in the hidden-charm tetraquark scenario, as presented in Table 3, where the angular momenta are $\vec{S}=\vec{S}_{qc}+\vec{S}_{\bar{q}\bar{c}}$ and $\vec{J}=\vec{L}+\vec{S}$.

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Table 1. Possible assignments of the hidden-charm tetraquark states; the isospin limit is implied [27].

Y_c	J^{PC}	M_Y/GeV	Assignments
$[uc]_P[\overline{dc}]_A-[uc]_A[\overline{dc}]_P$	1^{--}	4.66 ± 0.07	? $Y(4660)$
$[uc]_P[\overline{dc}]_A+[uc]_A[\overline{dc}]_P$	1^{-+}	4.61 ± 0.07	
$[uc]_S[\overline{dc}]_V+[uc]_V[\overline{dc}]_S$	1^{--}	4.35 ± 0.08	? $Y(4360/4390)$
$[uc]_S[\overline{dc}]_V-[uc]_V[\overline{dc}]_S$	1^{-+}	4.66 ± 0.09	
$[uc]_{\tilde{V}}[\overline{dc}]_A-[uc]_A[\overline{dc}]_{\tilde{V}}$	1^{--}	4.53 ± 0.07	? $Y(4500)$
$[uc]_{\tilde{V}}[\overline{dc}]_A+[uc]_A[\overline{dc}]_{\tilde{V}}$	1^{-+}	4.65 ± 0.08	
$[uc]_{\tilde{A}}[\overline{dc}]_V+[uc]_V[\overline{dc}]_{\tilde{A}}$	1^{--}	4.48 ± 0.08	? $Y(4500)$
$[uc]_{\tilde{A}}[\overline{dc}]_V-[uc]_V[\overline{dc}]_{\tilde{A}}$	1^{-+}	4.55 ± 0.07	
$[uc]_S[\overline{dc}]_{\tilde{V}}-[uc]_{\tilde{V}}[\overline{dc}]_S$	1^{--}	4.50 ± 0.09	? $Y(4500)$
$[uc]_S[\overline{dc}]_{\tilde{V}}+[uc]_{\tilde{V}}[\overline{dc}]_S$	1^{-+}	4.50 ± 0.09	
$[uc]_P[\overline{dc}]_{\tilde{A}}-[uc]_{\tilde{A}}[\overline{dc}]_P$	1^{--}	4.60 ± 0.07	
$[uc]_P[\overline{dc}]_{\tilde{A}}+[uc]_{\tilde{A}}[\overline{dc}]_P$	1^{-+}	4.61 ± 0.08	
$[uc]_A[\overline{dc}]_A$	1^{--}	4.69 ± 0.08	? $Y(4660)$

Table 2. Possible assignments of the hidden-charm-hidden-strange tetraquark states [28].

Y_c	J^{PC}	M_Y/GeV	Assignments
$[sc]_P[\overline{sc}]_A-[sc]_A[\overline{sc}]_P$	1^{--}	4.80 ± 0.08	? $Y(4790)$
$[sc]_P[\overline{sc}]_A+[sc]_A[\overline{sc}]_P$	1^{-+}	4.75 ± 0.08	
$[sc]_S[\overline{sc}]_V+[sc]_V[\overline{sc}]_S$	1^{--}	4.53 ± 0.08	
$[sc]_S[\overline{sc}]_V-[sc]_V[\overline{sc}]_S$	1^{-+}	4.83 ± 0.09	
$[sc]_{\tilde{V}}[\overline{sc}]_A-[sc]_A[\overline{sc}]_{\tilde{V}}$	1^{--}	4.70 ± 0.08	? $Y(4710)$
$[sc]_{\tilde{V}}[\overline{sc}]_A+[sc]_A[\overline{sc}]_{\tilde{V}}$	1^{-+}	4.81 ± 0.09	
$[sc]_{\tilde{A}}[\overline{sc}]_V+[sc]_V[\overline{sc}]_{\tilde{A}}$	1^{--}	4.65 ± 0.08	? $Y(4660)$
$[sc]_{\tilde{A}}[\overline{sc}]_V-[sc]_V[\overline{sc}]_{\tilde{A}}$	1^{-+}	4.71 ± 0.08	
$[sc]_S[\overline{sc}]_{\tilde{V}}-[sc]_{\tilde{V}}[\overline{sc}]_S$	1^{--}	4.68 ± 0.09	? $Y(4660)$
$[sc]_S[\overline{sc}]_{\tilde{V}}+[sc]_{\tilde{V}}[\overline{sc}]_S$	1^{-+}	4.68 ± 0.09	?? $X(4630)$
$[sc]_P[\overline{sc}]_{\tilde{A}}-[sc]_{\tilde{A}}[\overline{sc}]_P$	1^{--}	4.75 ± 0.08	
$[sc]_P[\overline{sc}]_{\tilde{A}}+[sc]_{\tilde{A}}[\overline{sc}]_P$	1^{-+}	4.75 ± 0.08	
$[sc]_A[\overline{sc}]_A$	1^{--}	4.85 ± 0.09	

Tables 1−3 explicitly show that there is no room for $Y(4008)$ and $Y(4750)$ in the vector hidden-charm tetraquark scenario. If we select $X(3872)$ and $Z_c(3900)$ as the lowest tetraquark states with $J^{PC}=1^{++}$ and 1^{--}, respectively [31–35], $Y(4008)$ cannot be assigned as a vector tetraquark state owing to the small mass splitting $\delta M\approx 100$ MeV. If $Y(4220/4230/4260)$ can be assigned as the ground state vector tetraquark state, the lowest vector hidden-charm tetraquark state can be obtained by the QCD sum rules, and $Y(4750)$ can be assigned as its first radial excitation according to the mass gap $M_{Y(4750)}-M_{Y(4260)}=0.51$ GeV, which happens to be our naive expectation of the mass gap between the ground state and first radial excitation.

Table 3. Possible assignments of the hidden-charm tetraquark states with explicit P-waves; the isospin limit is implied [30].

$\lvert S_{qc},S_{\bar{q}\bar{c}};S,L;J\rangle$	M_Y/GeV	Assignments
$\lvert 0,0;0,1;1\rangle$	4.24 ± 0.10	$Y(4220)$
$\lvert 1,1;0,1;1\rangle$	4.28 ± 0.10	$Y(4220/4320)$
$\frac{1}{\sqrt{2}}(\lvert 1,0;1,1;1\rangle+\lvert 0,1;1,1;1\rangle)$	4.31 ± 0.10	$Y(4320/4390)$
$\lvert 1,1;2,1;1\rangle$	4.33 ± 0.10	$Y(4320/4390)$

If $Y(4750)$ can be assigned as the first radial excitation of $Y(4220/4230/4260)$, there may exist a spectrum for the first radial excited states, which lie at approximately 4.8 GeV. Therefore, it is interesting to explore such a possibility.

It is known that the heavy-light diquarks $\varepsilon^{ijk}q_j^TC\Gamma Q_k$ have five structures, where i, j, and k are color indexes; $C\Gamma=C\gamma_5$, C, $C\gamma_\mu\gamma_5$, $C\gamma_\mu$, and $C\sigma_{\mu\nu}$ for the scalar, pseudoscal ar, vector, axialvector, and tensor diquarks, respectively; and the P-wave is implicitly embodied in the negative parity of the diquarks. We can also introduce an explicit P-wave inside the heavy-light diquarks to obtain $\varepsilon^{ijk}q_j^TC\Gamma\overleftrightarrow{\partial}_\mu Q_k$, where the derivative $\overleftrightarrow{\partial}_\mu=\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu$ embodies the explicit P-wave. Then, the diquarks $\varepsilon^{ijk}q_j^TC\Gamma\overleftrightarrow{\partial}_\mu Q_k$ can be used as the basic building blocks to construct the four-quark currents and study the tetraquark states with $J^{PC}=1^{--}$. We will explore such a possibility in our next study.

In Refs. [27–30], we used the (modified) energy scale formula to obtain suitable energy scales of the QCD spectral densities, enhance the pole contributions, and improve the convergent behaviors of the operator product expansion [36]. This is a unique feature of our research. In this direction, we have also explored the hidden-charm tetraquark states with $J^{PC}=0^{++}$, 0^{-+}, 0^{--}, 1^{+-}, 2^{++} [37, 38], hidden-bottom tetraquark states with $J^{PC}=0^{++}$, 1^{+-}, 2^{++} [39], hidden-charm molecular states with $J^{PC}=0^{++}$, 1^{+-}, 2^{++} [40], doubly-charm tetraquark (molecular) states with $J^P=0^+$, 1^+, 2^+ [41] ([42]), and hidden-charm pentaquark (molecular) states [43]([44]), and we have assigned the existing exotic states consistently.

In the isospin limit, the vector tetraquark states with the symbol valence quarks, expressed as

$$I=1: c\bar{c}u\bar{d},\ c\bar{c}\frac{u\bar{u}-d\bar{d}}{\sqrt{2}},\ c\bar{c}d\bar{u},$$

$$I=0: c\bar{c}\frac{u\bar{u}+d\bar{d}}{\sqrt{2}}, \tag{1}$$

have degenerated masses. We explore the $c\bar{c}u\bar{d}$ tetraquark states for simplicity. In particular, we update the analysis of our previous studies [29, 30] and extend these studies to systematically explore the first radial excitations of the vector hidden-charm tetraquark states with the QCD sum rules. We use the modified energy scale formula to properly represent suitable energy scales of the QCD spectral densities and make possible assignments of the existing Y states as well as predictions for the mass spectrum of the first radial excitations at an energy of approximately 4.8 GeV.

The rest of this paper is organized as follows. We derive the QCD sum rules for the vector tetraquark states in Section II. In Section III, we present the numerical results and discussions. Finally, we present our conclusions in Section IV.

II. QCD SUM RULES FOR VECTOR TETRAQUARK STATES

We first express the two-point correlation functions $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$as

$$\Pi_{\mu\nu}(p) = \mathrm{i}\int \mathrm{d}^4x e^{\mathrm{i}p\cdot x}\langle 0|T\left\{J_\mu(x)J_\nu^\dagger(0)\right\}|0\rangle\,, \tag{2}$$

$$\Pi_{\mu\nu\alpha\beta}(p) = \mathrm{i}\int \mathrm{d}^4x e^{\mathrm{i}p\cdot x}\langle 0|T\left\{J_{\mu\nu}(x)J_{\alpha\beta}^\dagger(0)\right\}|0\rangle\,, \tag{3}$$

where $J_\mu(x)=J^1_\mu(x)$, $J^2_\mu(x)$ and $J^3_\mu(x)$,

$$J^1_\mu(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} u^{Tj}(x)C\gamma_5 c^k(x) \overleftrightarrow{\partial}_\mu \bar{d}^m(x)\gamma_5 C\bar{c}^{Tn}(x)\,, \tag{4}$$

$$J^2_\mu(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{\sqrt{2}} u^{Tj}(x)C\gamma_\alpha c^k(x) \overleftrightarrow{\partial}_\mu \bar{d}^m(x)\gamma^\alpha C\bar{c}^{Tn}(x)\,, \tag{5}$$

$$\begin{aligned} J^3_\mu(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{2} \Big[& u^{Tj}(x)C\gamma_\mu c^k(x) \overleftrightarrow{\partial}_\alpha \bar{d}^m(x)\gamma^\alpha C\bar{c}^{Tn}(x) \\ & + u^{Tj}(x)C\gamma^\alpha c^k(x) \overleftrightarrow{\partial}_\alpha \bar{d}^m(x)\gamma_\mu C\bar{c}^{Tn}(x)\Big]\,, \end{aligned} \tag{6}$$

$$\begin{aligned} J_{\mu\nu}(x) = \frac{\varepsilon^{ijk}\varepsilon^{imn}}{2\sqrt{2}} \Big[& u^{Tj}(x)C\gamma_5 c^k(x) \overleftrightarrow{\partial}_\mu \bar{d}^m(x)\gamma_\nu C\bar{c}^{Tn}(x) \\ & + u^{Tj}(x)C\gamma_\nu c^k(x) \overleftrightarrow{\partial}_\mu \bar{d}^m(x)\gamma_5 C\bar{c}^{Tn}(x) \\ & - u^{Tj}(x)C\gamma_5 c^k(x) \overleftrightarrow{\partial}_\nu \bar{d}^m(x)\gamma_\mu C\bar{c}^{Tn}(x) \\ & - u^{Tj}(x)C\gamma_\mu c^k(x) \overleftrightarrow{\partial}_\nu \bar{d}^m(x)\gamma_5 C\bar{c}^{Tn}(x)\Big]\,. \end{aligned} \tag{7}$$

Under charge conjugation transform $\widehat{C}$, the currents $J_\mu(x)$ and $J_{\mu\nu}(x)$ have the following properties:

$$\begin{aligned} \widehat{C}J_\mu(x)\widehat{C}^{-1} &= -J_\mu(x)\,, \\ \widehat{C}J_{\mu\nu}(x)\widehat{C}^{-1} &= -J_{\mu\nu}(x)\,, \end{aligned} \tag{8}$$

in other words, they have negative conjugation.

At the hadron side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the interpolating currents $J_\mu(x)$ and $J_{\mu\nu}(x)$ into the correlation functions $\Pi_{\mu\nu}(p)$ and $\Pi_{\mu\nu\alpha\beta}(p)$, respectively, for proper hadronic representation [45–47]. Then, we isolate the ground states and obtain the following expressions:

$$\begin{aligned} \Pi_{\mu\nu}(p) &= \frac{\lambda_Y^2}{M_Y^2-p^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\right)+\cdots\,, \\ &= \Pi_Y(p^2)\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\right)+\cdots\,, \end{aligned} \tag{9}$$

$$\begin{aligned} \Pi_{\mu\nu\alpha\beta}(p) &= \frac{\lambda_Y^2}{M_Y^2\left(M_Y^2-p^2\right)}\left(p^2 g_{\mu\alpha}g_{\nu\beta} - p^2 g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}p_\nu p_\beta - g_{\nu\beta}p_\mu p_\alpha + g_{\mu\beta}p_\nu p_\alpha + g_{\nu\alpha}p_\mu p_\beta\right) \\ &\quad + \frac{\lambda_Z^2}{M_Z^2\left(M_Z^2-p^2\right)}\left(-g_{\mu\alpha}p_\nu p_\beta - g_{\nu\beta}p_\mu p_\alpha + g_{\mu\beta}p_\nu p_\alpha + g_{\nu\alpha}p_\mu p_\beta\right)+\cdots\,, \\ &= \widetilde{\Pi}_Y(p^2)\left(p^2 g_{\mu\alpha}g_{\nu\beta} - p^2 g_{\mu\beta}g_{\nu\alpha} - g_{\mu\alpha}p_\nu p_\beta - g_{\nu\beta}p_\mu p_\alpha + g_{\mu\beta}p_\nu p_\alpha + g_{\nu\alpha}p_\mu p_\beta\right) \\ &\quad + \widetilde{\Pi}_Z(p^2)\left(-g_{\mu\alpha}p_\nu p_\beta - g_{\nu\beta}p_\mu p_\alpha + g_{\mu\beta}p_\nu p_\alpha + g_{\nu\alpha}p_\mu p_\beta\right)\,, \end{aligned} \tag{10}$$

where we apply the definitions of the pole residues λ_Y and λ_Z,

$$\begin{aligned} \langle 0|J_\mu(0)|Y(p)\rangle &= \lambda_Y\,\varepsilon_\mu\,, \\ \langle 0|J_{\mu\nu}(0)|Y(p)\rangle &= \frac{\lambda_Y}{M_Y}\,\varepsilon_{\mu\nu\alpha\beta}\,\varepsilon^\alpha p^\beta\,, \\ \langle 0|J_{\mu\nu}(0)|Z(p)\rangle &= \frac{\lambda_Z}{M_Z}\left(\varepsilon_\mu p_\nu - \varepsilon_\nu p_\mu\right)\,, \end{aligned} \tag{11}$$

where ε_μ denotes the polarization vectors of the tetraquark states Y and Z with quantum numbers $J^{PC}=1^{--}$ and 1^{+-}, respectively. Next, we explicitly project out the components $\Pi_Y(p^2)$ and $\Pi_Z(p^2)$ with the projectors $P_Y^{\mu\nu\alpha\beta}$ and $P_Z^{\mu\nu\alpha\beta}$,

$$\begin{aligned} \Pi_Y(p^2) &= p^2\widetilde{\Pi}_Y(p^2) = P_Y^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p)\,, \\ \Pi_Z(p^2) &= p^2\widetilde{\Pi}_Z(p^2) = P_Z^{\mu\nu\alpha\beta}\Pi_{\mu\nu\alpha\beta}(p)\,, \end{aligned} \tag{12}$$

where

$$P_Y^{\mu\nu\alpha\beta} = \frac{1}{6}\left(g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2}\right),$$
$$P_Z^{\mu\nu\alpha\beta} = \frac{1}{6}\left(g^{\mu\alpha} - \frac{p^\mu p^\alpha}{p^2}\right)\left(g^{\nu\beta} - \frac{p^\nu p^\beta}{p^2}\right) - \frac{1}{6}g^{\mu\alpha}g^{\nu\beta}, \quad (13)$$

and we take the components $\Pi_Y(p^2)$ as we explore the hidden-charm tetraquark states with $J^{PC}=1^{--}$.

At the QCD side, we accomplish the operator product expansion up to the vacuum condensates of dimension 10 and consider the vacuum condensates that are vacuum expectations of the operators of the orders $O(\alpha_s^k)$ with $k\leq 1$ consistently, i.e., we consider $\langle\bar{q}q\rangle$, $\langle\frac{\alpha_s GG}{\pi}\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{q}q\rangle^2$, $\langle\bar{q}q\rangle\langle\frac{\alpha_s GG}{\pi}\rangle$, $\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle^2$, and $\langle\bar{q}q\rangle^2\langle\frac{\alpha_s GG}{\pi}\rangle$. The interested readers can consult Refs. [29, 30] for further details.

Next, we adopt the quark-hadron duality below the continuum thresholds s_0 and s_0' and apply the Borel transform with respect to $P^2=-p^2$ to obtain two QCD sum rules:

$$\lambda_Y^2 \exp\left(-\frac{M_Y^2}{T^2}\right) = \int_{4m_c^2}^{s_0} ds\,\rho_{QCD}(s)\exp\left(-\frac{s}{T^2}\right), \quad (14)$$

$$\lambda_Y^2 \exp\left(-\frac{M_Y^2}{T^2}\right) + \lambda_{Y'}^2 \exp\left(-\frac{M_{Y'}^2}{T^2}\right)$$
$$= \int_{4m_c^2}^{s_0'} ds\,\rho_{QCD}(s)\exp\left(-\frac{s}{T^2}\right), \quad (15)$$

where $\rho_{QCD}(s)$ denotes the QCD spectral densities obtained through dispersion relation, and s_0 and s_0' correspond to the ground states Y and first radial excitations Y', respectively.

We adopt the notations $\tau=\frac{1}{T^2}$, $D^n=\left(-\frac{d}{d\tau}\right)^n$ and use the subscripts 1 and 2 to denote Y and Y' respectively to simplify the expressions. Next, we rewrite the two QCD sum rules in Eqs. (14)−(15) as

$$\lambda_1^2 \exp\left(-\tau M_1^2\right) = \Pi_{QCD}(\tau), \quad (16)$$

$$\lambda_1^2 \exp\left(-\tau M_1^2\right) + \lambda_2^2 \exp\left(-\tau M_2^2\right) = \Pi'_{QCD}(\tau), \quad (17)$$

where $\Pi_{QCD}(\tau)$ and $\Pi'_{QCD}(\tau)$ represent the correlation functions below the continuum thresholds s_0 and s_0', respectively. We derive the QCD sum rules in Eq. (16) with respect to τ to obtain the ground states masses,

$$M_1^2 = \frac{D\Pi_{QCD}(\tau)}{\Pi_{QCD}(\tau)}, \quad (18)$$

then it is straightforward to obtain the ground state masses and pole residues using the two coupled QCD sum rules; see Eqs. (16) and (18) [29, 30].

Next, we derive the QCD sum rules in Eq. (17) with respect to τ to obtain

$$\lambda_1^2 M_1^2 \exp\left(-\tau M_1^2\right) + \lambda_2^2 M_2^2 \exp\left(-\tau M_2^2\right) = D\Pi'_{QCD}(\tau). \quad (19)$$

From Eqs. (17) and (19), we obtain the QCD sum rules,

$$\lambda_i^2 \exp\left(-\tau M_i^2\right) = \frac{\left(D-M_j^2\right)\Pi'_{QCD}(\tau)}{M_i^2-M_j^2}, \quad (20)$$

where $i\neq j$. Then, we derive the QCD sum rules in Eq. (20) with respect to τ to obtain

$$M_i^2 = \frac{\left(D^2-M_j^2 D\right)\Pi'_{QCD}(\tau)}{\left(D-M_j^2\right)\Pi'_{QCD}(\tau)},$$
$$M_i^4 = \frac{\left(D^3-M_j^2 D^2\right)\Pi'_{QCD}(\tau)}{\left(D-M_j^2\right)\Pi'_{QCD}(\tau)}. \quad (21)$$

The squared masses M_i^2 obey the equation

$$M_i^4 - bM_i^2 + c = 0, \quad (22)$$

where

$$b = \frac{D^3\otimes D^0 - D^2\otimes D}{D^2\otimes D^0 - D\otimes D},$$
$$c = \frac{D^3\otimes D - D^2\otimes D^2}{D^2\otimes D^0 - D\otimes D},$$
$$D^j\otimes D^k = D^j\Pi'_{QCD}(\tau)D^k\Pi'_{QCD}(\tau), \quad (23)$$

with subscripts $i=1,2$ and superscripts $j,k=0,1,2,3$. Finally, we solve the simple resulting equation and obtain two solutions, i.e., the masses of the ground states and first radial excitations [35, 48, 49],

$$M_1^2 = \frac{b-\sqrt{b^2-4c}}{2}, \quad (24)$$

$$M_2^2 = \frac{b+\sqrt{b^2-4c}}{2}. \quad (25)$$

We can obtain the ground state masses either from the QCD sum rules in Eq. (18) or in Eq. (24). We prefer the

QCD sum rules in Eq. (18) because there are larger ground state contributions and less uncertainties from the continuum threshold parameters. We obtain the masses and pole residues of the first radial excitations from the two coupled QCD sum rules in Eqs. (20) and (25).

III. NUMERICAL RESULTS AND DISCUSSIONS

We adopted the traditional vacuum condensates $\langle\bar{q}q\rangle = -(0.24\pm0.01\ \text{GeV})^3$, $\langle\bar{q}g_s\sigma Gq\rangle = m_0^2\langle\bar{q}q\rangle$, $m_0^2 = (0.8\pm 0.1)\ \text{GeV}^2$, and $\langle\frac{\alpha_s GG}{\pi}\rangle = 0.012\pm0.003\ \text{GeV}^4$ at the energy scale $\mu = 1$ GeV [45–47, 50]; chose the modified minimum subtracted mass $m_c(m_c) = (1.275\pm0.025)$ GeV from the Particle Data Group [51]; and set $m_u = m_d = 0$. Moreover, we considered the energy scale dependence of the input parameters,

$$
\begin{aligned}
\langle\bar{q}q\rangle(\mu) &= \langle\bar{q}q\rangle(1\ \text{GeV})\left[\frac{\alpha_s(1\ \text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}, \\
\langle\bar{q}g_s\sigma Gq\rangle(\mu) &= \langle\bar{q}g_s\sigma Gq\rangle(1\ \text{GeV})\left[\frac{\alpha_s(1\ \text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}, \\
m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}}, \\
\alpha_s(\mu) &= \frac{1}{b_0 t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2\left(\log^2 t-\log t-1\right)+b_0 b_2}{b_0^4 t^2}\right],
\end{aligned} \tag{26}
$$

where $t = \log\frac{\mu^2}{\Lambda_{\text{QCD}}^2}$, $b_0 = \frac{33-2n_f}{12\pi}$, $b_1 = \frac{153-19n_f}{24\pi^2}$, $b_2 = (2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2)/(128\pi^3)$, and $\Lambda_{\text{QCD}} = 210$ MeV, 292 MeV, and 332 MeV for the flavors $n_f = 5$, 4, and 3, respectively [51, 52]. We chose the flavor $n_f = 4$ because we explored the tetraquark states consisting of the valence quarks u, d, and c. We evolved all the input parameters to the suitable energy scales μ to extract the masses of the hidden-charm tetraquark states, which satisfy the modified energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2-(2\mathbb{M}_c+0.5\ \text{GeV})^2} = \sqrt{M_{X/Y/Z}^2-(4.1\ \text{GeV})^2}$, where $\mathbb{M}$ is the effective charm quark mass [29, 30].

In the scenario of tetraquark states, we can tentatively assign $X(3915)$and $X(4500)$ to be the 1S and 2S states with $J^{PC} = 0^{++}$ [53, 54], respectively; $Z_c(3900)$and $Z_c(4430)$ to be the 1S and 2S states with $J^{PC} = 1^{+-}$, respectively [31, 33, 35]; $Z_c(4020)$ and $Z_c(4600)$ to be the 1S and 2S states with $J^{PC} = 1^{+-}$, respectively [49, 55]; and $X(4140)$ and $X(4685)$ to be the 1S and 2S states with $J^{PC} = 1^{++}$, respectively [56, 57], where the energy gaps between the 1S and 2S states are approximately $0.57-0.59$ GeV.

In Refs. [29, 30], we chose the continuum threshold parameters as $\sqrt{s_0} = M_Y+0.55\sim0.60\pm0.10$ GeV for the hidden-charm tetraquark states with $J^{PC} = 1^{--}$; the ground state contribution can be as large as (49%−81%). Compared with the usually chosen pole contributions (40%−60%) [37–44], the ground state contributions (49%−81%) in the previous analysis were too large in the QCD sum rules for the multiquark states, which may suffer from contaminations from the first radial excitations. In the present calculations, we chose slightly smaller continuum threshold parameters, $\sqrt{s_0} = M_Y+0.50\sim0.55\pm 0.10$ GeV, to reduce the ground state contributions and performed a consistent and detailed analysis. Furthermore, the continuum threshold parameters s_0' were set as $\sqrt{s_0'} = M_{Y'}+0.40\pm0.10$ GeV according to the mass-gap of the ψ' and ψ'' from the Particle Data Group [51].

We searched for the suitable Borel parameters and continuum threshold parameters via trial and error. The pole contributions (PC) and vacuum condensate contributions ($D(n)$) are defined as

$$
\text{PC} = \frac{\int_{4m_c^2}^{s_0/s_0'} ds\rho_{\text{QCD}}(s)\exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{\infty} ds\rho_{\text{QCD}}(s)\exp\left(-\frac{s}{T^2}\right)}, \tag{27}
$$

and

$$
D(n) = \frac{\int_{4m_c^2}^{s_0/s_0'} ds\rho_{\text{QCD,n}}(s)\exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0/s_0'} ds\rho_{\text{QCD}}(s)\exp\left(-\frac{s}{T^2}\right)}, \tag{28}
$$

respectively. Finally, we obtained the Borel windows, continuum threshold parameters, and suitable energy scales and pole contributions, which are listed in Tables 4–5. From the tables, we can see explicitly that the pole contributions of the ground states (ground states plus first radial excited states) are approximately (40%−60%) ((67%−85%)), similar to the values in our previous studies on other tetraquark states [37–44]. The pole dominance criterion is properly satisfied. The contributions from the highest dimensional condensates play a minor role, expressed as $|D(10)| < 3\%$ or $\ll 1\%$ ($< 1\%$ or $\ll 1\%$), for the ground states (the ground states plus first radial excited states). The operator product expansion converges better than that in a previous study of ours [30]. Thus, we can confidently extract reliable tetraquark masses and pole residues.

From Tables 4–7, we can see explicitly that the modified energy scale formula can be properly satisfied, and the relations $\sqrt{s_0} = M_Y+0.50\sim0.55\pm0.10$ GeV and $\sqrt{s_0'} = M_{Y'}+0.40\pm0.10$ GeV are held. Thus, our analysis is consistent.

In Fig. 1, we plot the masses of the ground states and

Table 4. Borel windows T^2, continuum threshold parameters s_0, energy scales of the QCD spectral densities, contributions of the ground states, and values of $D(10)$.

$\|S_{qc}, S_{\bar{q}\bar{c}}; S, L; J\rangle$	μ/GeV	T^2/GeV^2	$\sqrt{s_0}$/GeV	pole(%)	$D(10)$
$\|0,0;0,1;1\rangle$	1.1	$2.6-3.0$	4.75 ± 0.10	$(40-65)$	$<1\%$
$\|1,1;0,1;1\rangle$	1.2	$2.5-2.9$	4.80 ± 0.10	$(39-64)$	$<3\%$
$\frac{1}{\sqrt{2}}(\|1,0;1,1;1\rangle+\|0,1;1,1;1\rangle)$	1.3	$3.0-3.4$	4.85 ± 0.10	$(38-60)$	$\ll1\%$
$\|1,1;2,1;1\rangle$	1.3	$2.7-3.1$	4.85 ± 0.10	$(39-63)$	$<1\%$

Table 5. Borel windows T^2, continuum threshold parameters s_0', energy scales of the QCD spectral densities, contributions of the ground states plus first radial excitations, and values of $D(10)$.

$\|S_{qc}, S_{\bar{q}\bar{c}}; S, L; J\rangle$	μ/GeV	T^2/GeV^2	$\sqrt{s_0'}$/GeV	pole(%)	$D(10)$
$\|0,0;0,1;1\rangle$	2.4	$2.8-3.2$	5.15 ± 0.10	$(67-85)$	$\ll1\%$
$\|1,1;0,1;1\rangle$	2.5	$2.6-3.0$	5.20 ± 0.10	$(67-86)$	$<1\%$
$\frac{1}{\sqrt{2}}(\|1,0;1,1;1\rangle+\|0,1;1,1;1\rangle)$	2.6	$3.0-3.4$	5.25 ± 0.10	$(67-84)$	$\ll1\%$
$\|1,1;2,1;1\rangle$	2.6	$2.7-3.1$	5.25 ± 0.10	$(68-87)$	$\ll1\%$

Table 6. Masses and pole residues of the ground states.

$\|S_{qc}, S_{\bar{q}\bar{c}}; S, L; J\rangle$	M_Y/GeV	$\lambda_Y/(10^{-2}\text{GeV}^6)$
$\|0,0;0,1;1\rangle$	4.24 ± 0.09	2.28 ± 0.42
$\|1,1;0,1;1\rangle$	4.28 ± 0.09	4.80 ± 0.95
$\frac{1}{\sqrt{2}}(\|1,0;1,1;1\rangle+\|0,1;1,1;1\rangle)$	4.31 ± 0.09	2.94 ± 0.50
$\|1,1;2,1;1\rangle$	4.33 ± 0.09	6.55 ± 1.19

Table 7. Masses and pole residues of the first radial excited states.

$\|S_{qc}, S_{\bar{q}\bar{c}}; S, L; J\rangle$	M_Y/GeV	$\lambda_Y/(10^{-2}\text{GeV}^6)$
$\|0,0;0,1;1\rangle$	4.75 ± 0.10	8.19 ± 1.23
$\|1,1;0,1;1\rangle$	4.81 ± 0.10	18.3 ± 3.0
$\frac{1}{\sqrt{2}}(\|1,0;1,1;1\rangle+\|0,1;1,1;1\rangle)$	4.85 ± 0.09	8.63 ± 1.22
$\|1,1;2,1;1\rangle$	4.86 ± 0.10	21.7 ± 3.4

first radial excitations of the hidden-charm tetraquark states with quantum numbers $J^{PC}=1^{--}$. This figure explicitly shows that flat platforms emerge in the Borel windows. Note also that the uncertainties from the Borel parameters are small.

Table 8 presents the possible assignments of the vector tetraquark states based on the QCD sum rules. This table explicitly shows that there is room to accommodate $Y(4750)$, i.e., $Y(4220/4260)$ and $Y(4750)$ can be assigned as the ground state and first radial excited state of the $C\gamma_5\overleftrightarrow{\partial}_\mu\gamma_5 C$ type tetraquark states with $J^{PC}=1^{--}$, respectively. We cannot identify a particle unambiguously with the mass alone. In a future study, we shall investigate the decays of those vector tetraquark candidates with the QCD sum rules to verify the assignments.

Table 8. Masses of the vector tetraquark states and possible assignments; 1P and 2P denote the ground states and first radial excitations, respectively.

$\|S_{qc}, S_{\bar{q}\bar{c}}; S, L; J\rangle$	M_Y/GeV	Assignments
$\|0,0;0,1;1\rangle$ (1P)	4.24 ± 0.09	$Y(4220/4260)$
$\|0,0;0,1;1\rangle$ (2P)	4.75 ± 0.10	$Y(4750)$
$\|1,1;0,1;1\rangle$ (1P)	4.28 ± 0.09	$Y(4220/4320)$
$\|1,1;0,1;1\rangle$ (2P)	4.81 ± 0.10	
$\frac{1}{\sqrt{2}}(\|1,0;1,1;1\rangle+\|0,1;1,1;1\rangle)$ (1P)	4.31 ± 0.09	$Y(4320/4390)$
$\frac{1}{\sqrt{2}}(\|1,0;1,1;1\rangle+\|0,1;1,1;1\rangle)$ (2P)	4.85 ± 0.09	
$\|1,1;2,1;1\rangle$ (1P)	4.33 ± 0.09	$Y(4320/4390)$
$\|1,1;2,1;1\rangle$ (2P)	4.86 ± 0.10	

IV. CONCLUSIONS

In the present study, we chose the diquark-antidiquark type four-quark currents with an explicit P-wave between the diquark and antidiquark pairs to investigate the ground states and first radial excitations of the hidden-charm tetraquark states with quantum numbers $J^{PC}=1^{--}$ via the QCD sum rules. First, we considered the ground states at the hadronic side only and updated the previous analysis by refitting the continuum threshold and Borel parameters. Compared with previous calculations, we obtained better convergent behaviors in the operator product expansion on the QCD side and uniform pole contributions (40%−60%) on the hadronic side. Second, we considered both the ground states and first ra-

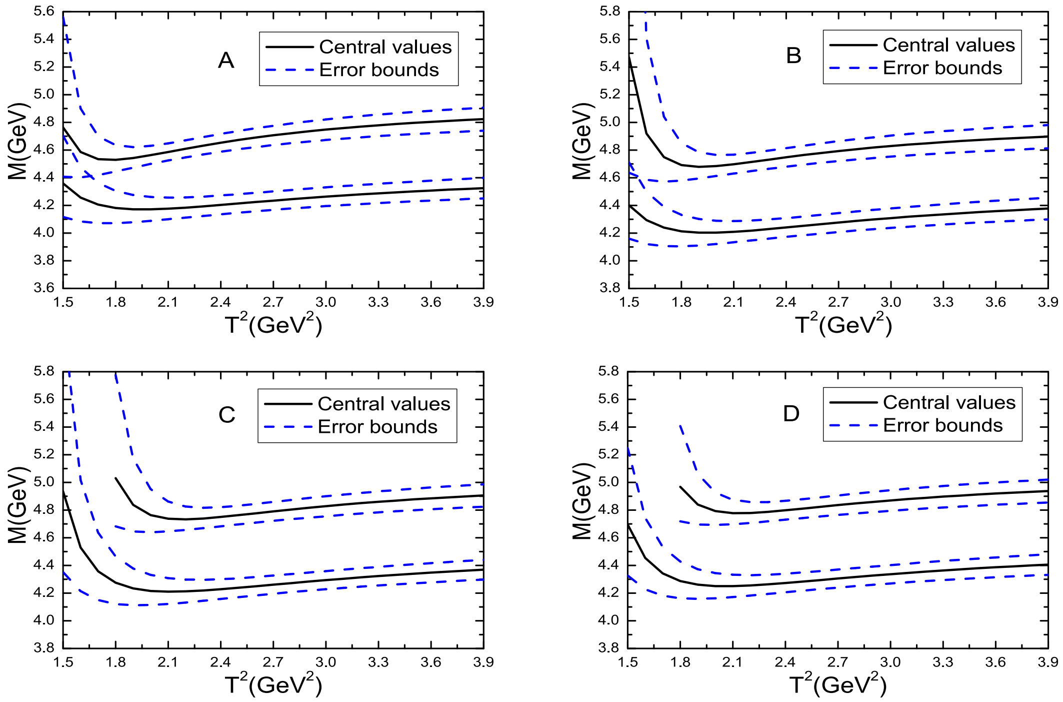

Fig. 1. (color online) Masses of the vector tetraquark states with variations of the Borel parameters T^2, where A, B, C, and D stand for the $|0,0;0,1;1\rangle$, $|1,1;0,1;1\rangle$, $\frac{1}{\sqrt{2}}(|1,0;1,1;1\rangle+|0,1;1,1;1\rangle)$, and $|1,1;2,1;1\rangle$ states, respectively; the lower and upper lines represent the ground states and first radial excitations, respectively.

dial excitations and focused on the first radial excitations, obtaining new predictions. In both present and previous calculations, we used the modified energy scale formula $\mu=\sqrt{M^2_{X/Y/Z}-(4.1\,\text{GeV})^2}$ to select suitable energy scales of the QCD spectral densities to improve the convergent behavior of the operator product expansion and enhance the pole contributions. Overall, we explored and obtained the masses and pole residues of the 1P and 2P vector tetraquark states in a systematic and consistent manner. We obtained the lowest vector tetraquark masses, made possible assignments of the existing Y states, and observed that there indeed exists a hidden-charm tetraquark state with $J^{PC}=1^{--}$ at an energy of approximately 4.75 GeV that can account for the BESIII data.

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