Monopole effects and high-spin levels in neutron-rich ^{132}Te*

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Abstract: The neutron-rich nuclei near doubly magic ^{132}Sn have attracted considerable interest in both nuclear physics and nuclear astrophysics. For the particle-hole nuclei in this region, the low-lying and high core excitations have been well described by shell model calculations using the extended pairing plus multipole-multipole force model. However, there is a significant difference between experiment and theory in the high-spin level 17^+ of ^{132}Te. We intend to illustrate this difference through monopole interactions. For this purpose, the monopole corrections between $\pi(\nu)0g_{7/2}$, $\nu 1d_{5/2}$ and $\pi(\nu)0h_{11/2}$ are investigated in $^{132-134}$Te, $^{131-133}$Sb, and ^{130}Sn. Some theoretical levels are connected to the (17^+) state of ^{132}Te with the monopole correction (Mc) of $Mc(\nu d_{5/2},\nu h_{11/2})$ and the quadruple-quadruple force between the proton and neutron, i.e., levels $3^-(8^-)$ in ^{130}Sn, level 14^- in ^{132}Te, and level $23/2^-$ in ^{131}Sb. Their observations at lower energies can confirm the datum of level (17^+) in ^{132}Te with an illustration of monopole effects and quadruple-quadruple force.

Keywords: monopole effects, high-spin levels, quadruple-quadruple force

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I. INTRODUCTION

The analysis of neutron-rich isotopes near the doubly magic nucleus ^{132}Sn can provide fascinating findings related to nuclear physics and nuclear astrophysics. The abundance peak appears at $A \sim 130$, which is formed through the rapid neutron capture process [1, 2]. The properties of doubly magic ^{132}Sn have been explored and confirmed in both experiments and theories [3−8]. In this nuclei region, tellurium isotopes have attracted experimental and theoretical research interest [5, 9−13]. For example, the g-factor of ^{134}Te in the 4^+ state was measured, which provides direct insight into the single particle structure [9]. The state (17^+) of ^{132}Te was observed to be 6.166 MeV using the reaction ^{9}Be(^{238}U, f) with a beam energy of 6.2 MeV/u at GANIL [14]. A significant energy difference exists in this state between experiments and shell-model calculations, which should conceal unknown information about the nuclear structure.

In theory, the extended pairing plus multipole-multipole force and monopole correction terms model (EPQQM) provides a suitable method to describe both the low-lying states and core excitations [15−18]. For example, the ordering and energies of the low-lying isomers in ^{129}Cd are predicted and determined by using the implemented phase-imaging ion-cyclotron-resonance method [18, 19]. The 16^+ level in ^{128}Cd is predicted as a spin-trap isomer feeding the known 16^+ of ^{128}In through β^- decay [20]. In addition to monopole interactions, it is necessary to consider the cross-shell excitations to study the properties of these neutron-rich nuclei in this region. For example, identifying the isomer state of level $19/2^+$ at 1942 keV in ^{133}Ba requires the cross-shell orbits lying above the energy gap $N = 82$. As reported in Ref. [21], the interactions without core excitations cannot provide the $B(E1)$ value for the transition from $J^\pi = 19/2^+$ to $J^\pi = 19/2^-$.

This model has an advantage for studying monopole effects by employing monopole correction (Mc) terms. For example, in the southwest quadrant $(Z \leq 50, N \leq 82)$ of ^{132}Sn, the level spectra and the energy gap across $N = 82$ can be modified by monopole correction between neutron orbit $h_{11/2}$ and $f_{7/2}$ [15]. In the northeast quadrant $(Z \geq 50, N \geq 82)$, five monopole terms are used to describe core excitations and high-spin levels, and the states 2^- and 9^- in ^{136,138}Te are predicted as a spin-trap structure coupled by the neutron intruder orbit $i_{13/2}$ [22]. Different effects of tensor forces are also discussed together

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with the monopole-driven shell evolutions [23], as well as other ones in the nucleon-nucleon interaction [24−27]. For the particle-hole nuclei in the northwest region of ^{132}Sn, a suitable interaction has been found, and the spectra of Sb and Te isotopes are well described as single-orbital couplings and cross-shell excitations [18, 28]. The transition probabilities in these nuclei are also calculated and reproduced well through comparisons with the known data.

In this study, we investigate high-spin levels and monopole effects in particle-hole nuclei near ^{132}Sn by employing the interaction in Ref. [18]. The shell-model code NUSHELLX@MSU is used for the calculations [29].

II. HAMILTONIAN AND MONOPOLE EFFECTS

In this work, we use the Hamiltonian in the proton-neutron (pn) representation [18]:

$$\begin{aligned} H =& H_{\rm sp}+H_{P_0}+H_{P_2}+H_{QQ}+H_{OO}+H_{HH}+H_{\rm mc} \\ =& \sum_{\alpha,i}\varepsilon^i_a c^\dagger_{\alpha,i}c_{\alpha,i}-\frac{1}{2}\sum_{J=0,2}\sum_{ii'}g_{J,ii'}\sum_M P^\dagger_{JM,ii'}P_{JM,ii'} \\ &-\frac{1}{2}\sum_{ii'}\frac{\chi_{2,ii'}}{b^4}\sum_M :Q^\dagger_{2M,ii'}Q_{2M,ii'}: \\ &-\frac{1}{2}\sum_{ii'}\frac{\chi_{3,ii'}}{b^6}\sum_M :O^\dagger_{3M,ii'}O_{3M,ii'}: \\ &-\frac{1}{2}\sum_{ii'}\frac{\chi_{4,ii'}}{b^8}\sum_M :H^\dagger_{4M,ii'}H_{4M,ii'}: \\ &+\sum_{a\leq c,ii'}k_{\rm mc}(ia,i'c)\sum_{JM}A^\dagger_{JM}(ia,i'c)A_{JM}(ia,i'c). \end{aligned} \tag{1}$$

Equation (1) includes the single-particle Hamiltonian ($H_{\rm sp}$); the $J=0$ and $J=2$ pairings ($P^\dagger_0P_0$ and $P^\dagger_2P_2$); the quadrupole-quadrupole ($Q^\dagger Q$), octupole-octupole ($O^\dagger O$), and hexadecapole-hexadecapole ($H^\dagger H$) terms; and the monopole corrections ($H_{\rm mc}$). In the pn-representation, $P^\dagger_{JM,ii'}$ and $A^\dagger_{JM}(ia,i'c)$ are the pair operators, while $Q^\dagger_{2M,ii'}$, $O^\dagger_{3M,ii'}$, and $H^\dagger_{4M,ii'}$ are the quadrupole, octupole, and hexadecapole operators, respectively, in which i (i') is an index for protons (neutrons). The parameters $g_{J,ii'}$, $\chi_{2,ii'}$, $\chi_{3,ii'}$, $\chi_{4,ii'}$, and $k_{\rm mc}(ia,i'c)$ are the corresponding force strengths, and b is the harmonic-oscillator range parameter.

The model space includes five orbits ($0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $0h_{11/2}$) for both protons and neutrons. Two extra neutron orbits above the $N=82$ shell, i.e., ($1f_{7/2}$ and $2p_{3/2}$), are added to allow neutron cross-shell excitations. We keep the same parameters of single-particle energies and the two-body force strengths used in Ref. [18]. The monopole interactions were found to be crucial for describing nuclear properties, which are entirely responsible for global saturation properties and single-particle behavior. The neutron-rich nuclei near ^{132}Sn can be divided into four quadrants by the crossing of $Z=50$ and $N=82$. According to nuclei studied previously [15, 22, 30], the monopole corrections are necessary for the hole (particle) nuclei in the southwest (northeast) quadrant of ^{132}Sn.

In southwest quadrant ($Z\leq 50, N\leq 82$), the ground state inversion in ^{129}Cd can be well described by the monopole correction between proton orbit $0g_{9/2}$ and neutron orbit $0h_{11/2}$ with a strength of –0.40 MeV [17], and it was verified using the recently implemented phase-imaging ion-cyclotron-resonance method [19]. Recently, the ground-state inversions from $N=81$ to $N=79$ were explained for the first time by monopole correction between neutron orbits $h_{11/2}$ and $d_{3/2}$. Furthermore, this monopole correction has been found in different isotonic chains of $N=79,80,81$, as all being hole nuclei near ^{132}Sn.

In the northeast quadrant ($Z\geq 50, N\geq 82$), five monopole terms are used to describe core excitations and high-spin levels [22]. For particle-pole nuclei in the northwest quadrant of ^{132}Sn ($Z\geq 50, N\leq 82$), the protons and neutrons occupy the same major shell. The properties of particle-hole nuclei can be well described without additional monopole correlations [28]. Such a situation is confirmed by the electromagnetic transitions of Sb and Te isotopes in the northwest quadrant of ^{132}Sn [28, 31], which is a strict test for shell model calculations. However, in level 17^+ of ^{132}Te, the large difference between experiment and theory motivates us to investigate monopole interactions in the particle-hole nuclei region near ^{132}Sn.

III. HIGH-SPIN LEVELS

In this part, high-spin levels are investigated with the monopole effects and quadruple-quadruple force in $^{132-134}$Te, $^{131-133}$Sb, and ^{130}Sn. The monopole correction alone cannot solve the puzzle of level 17^+; it should be combined with quadrupole correction. As shown in Table 1, there are four different types of level 17^+ under 10 MeV according to the EPQQM model, i.e., Nos. 1 to 4. The possibility of neutron core excitation (config.1 in Table 1) at 8.189 MeV can be excluded to explain the high-spin level 17^+ of ^{132}Te [28]. The given experimental data of ^{134}Te and ^{133}Sb have been reproduced very well as core excitations with a common neutron configuration of $\nu h^{-1}_{11/2}f_{7/2}$ (Fig. 1). If we modify the monopole term of Mc($\nu h^{-1}_{11/2},\nu f_{7/2}$) to explain the level 17^+ of ^{132}Te, the 17 states of neutron core excitations would catastrophically depart from their corresponding data.

After excluding neutron core excitation, we focus on the level at 8.779 MeV coupled by config.2

Table 1. 17^{+} states in ^{132}Te with main configurations. The data are from Ref. [14].

^{132}Te	E_x	(MeV)	Config.	
J^{π}	Th.	Exp.	No.	P(\%)
(17^{+})	8.189	6.166	1. $\pi g^{2}_{7/2}\nu h^{-3}_{11/2}f_{7/2}$	88
	8.779		2. $\pi g_{7/2}h_{11/2}\nu g^{-1}_{7/2}h^{-1}_{11/2}$	76
	9.125		3. $\pi g_{7/2}h_{11/2}\nu d^{-1}_{5/2}h^{-1}_{11/2}$	72
	9.314		4. $\pi h^{2}_{11/2}\nu h^{-2}_{11/2}$	93

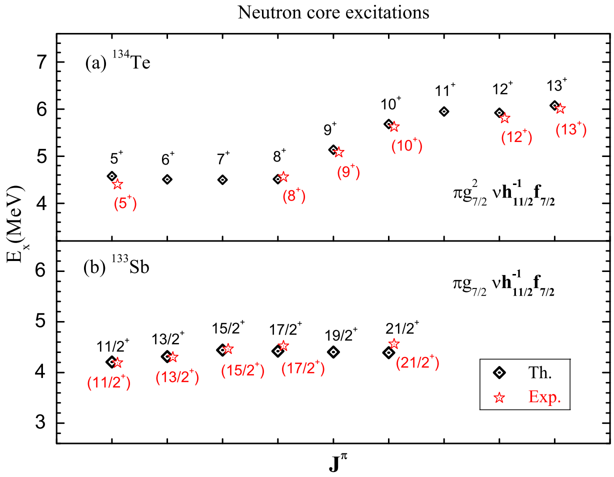

Fig. 1. (color online) Neutron core excitations in ^{134}Te and ^{133}Sb nuclei. Corresponding data are from Ref. [32].

$\pi g_{7/2}h_{11/2}\nu g^{-1}_{7/2}h^{-1}_{11/2}$. This level can be affected by these monopole terms from this configuration, i.e., $Mc(\pi g_{7/2},\pi h_{11/2})$, $Mc(\nu g_{7/2},\nu h_{11/2})$, and $Mc(\pi g_{7/2},\nu h_{11/2})$. The level at 8.779 MeV is abandoned, since the biggest difference is only 0.227 MeV in level 3^{-} of ^{132}Sb (Fig. 2). For the 17^{+} level at 9.314 MeV, it has a new monopole term of Mc$(\pi h_{11/2}\nu h_{11/2})$. This monopole term has almost no effects in levels 17^{+} coupled by config.1, 2, and 3. ($\kappa = 0.1 \sim 0.9$ MeV), while the level 17^{+} at 9.314 MeV drops to 8.3 MeV (Fig. 3). The level at 9.134 MeV is excluded too, because all 17^{+} levels are increased sharply when the strength is $\kappa > 0.9$ MeV. For the last one coupled by config.3, the suitable monopole term $Mc(\nu d_{5/2},\nu h_{11/2})$ is turned up from this configuration. Its monopole effects are investigated in the states of ^{132}Te from 0^{+} to 17^{+} by adding $Mc(\nu d_{5/2},\nu h_{11/2})$ = –2.6 MeV. As shown in Fig. 4, the level 17^{+} is reduced to 6.311 MeV, but the values of the 5^{-} and 7^{-} levels become far from the corresponding data.

As shown above, the monopole interaction alone cannot solve the present puzzle, and we focus on the quadrupole-quadrupole force between the proton and neutron ($QQ_{\pi,\nu}$).

As shown in Fig. 5, the value of the$QQ_{\pi,\nu}$ force equals the quadruple-quadruple force strength divided by $[1/A(^{132}Sn)]^{5/3}$. The level 17^{+} coupled by configuration $\pi g_{7/2}h_{11/2}\nu h^{-1}_{11/2}d^{-1}_{5/2}$ is reduced by 2.121 MeV when the QQ force changes from 300 to –450. The lowest value is

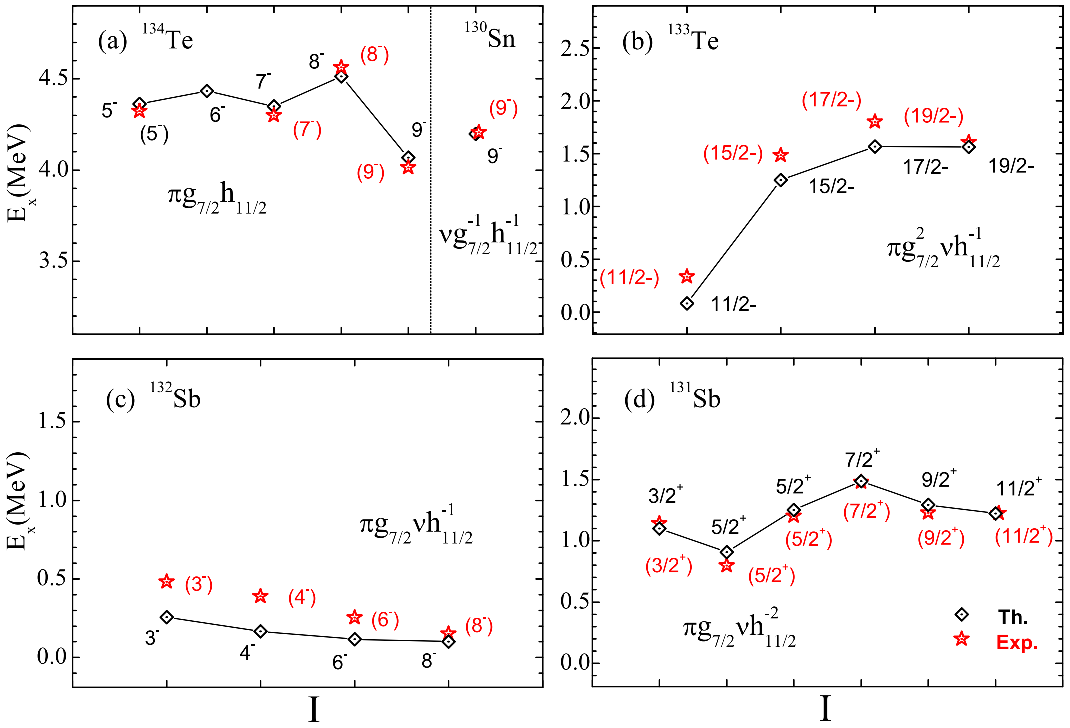

Fig. 2. (color online) Levels produced by particular configurations in ^{134}Te, ^{133}Te, ^{132}Sb, ^{131}Sb, and ^{130}Sn nuclei, in comparison with given data [32].

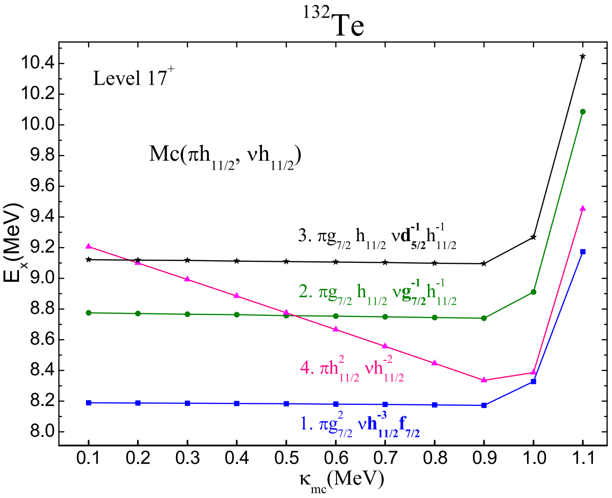

Fig. 3. (color online) Monopole effects of Mc($\pi h_{11/2}\nu h_{11/2}$) in level 17$^+$ of ^{132}Te.

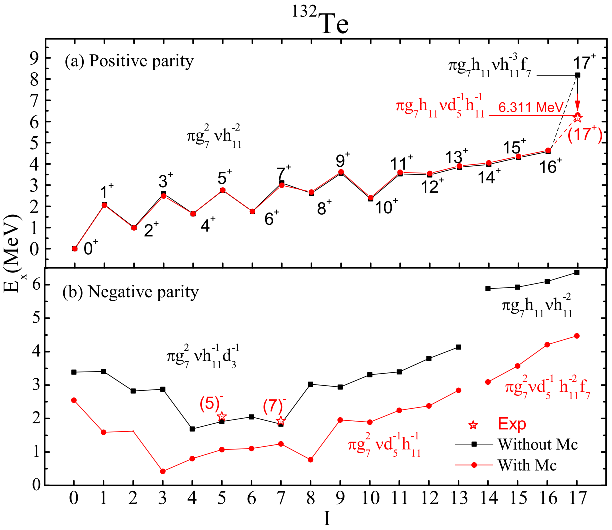

Fig. 4. (color online) Monopole effects of $Mc(\nu d_{5/2},\nu h_{11/2})$ in ^{132}Te. Data marked with stars are from Ref. [32].

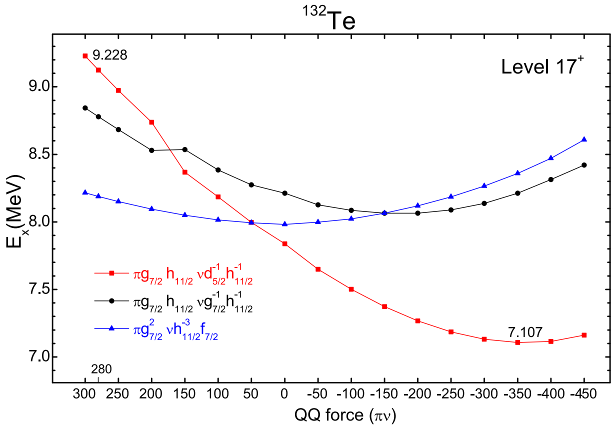

Fig. 5. (color online) Effects of the quadruple-quadruple force between the proton and neutron in level 17$^+$ of ^{132}Te.

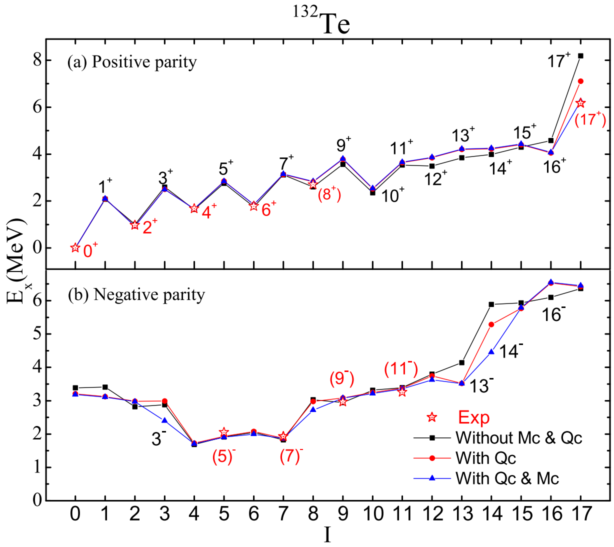

Fig. 6. (color online) $QQ_{\pi,\nu}$ force and monopole effects of $Mc(\nu d_{5/2},\nu h_{11/2})$ in ^{132}Te. Data marked with stars are from Refs. [14, 32].

7.107 MeV, which occurs when $QQ_{\pi\nu} = -350$. It seems the $QQ_{\pi,\nu}$ force provides a new method to explain the large difference in level 17$^+$. As shown in Fig. 6, the datum (17$^+$) is reproduced very well with $QQ_{\pi\nu} = -350$ and $Mc(\nu d_{5/2},\nu h_{11/2}) = -0.8$ MeV. Furthermore, the values of 5$^-$ and 7$^-$ levels are close to data (5)$^-$ and (7)$^-$. The levels 13$^-$ and 14$^-$ drop by approximately 1 MeV with $QQ_{\pi,\nu}$ correction (Q_c). The levels 3$^-$ and 14$^-$ are sensitive to the monopole effects of $Mc(\nu d_{5/2},\nu h_{11/2})$. These levels are connected with datum 17$^+$ by the $QQ_{\pi,\nu}$ force and monopole effects of $Mc(\nu d_{5/2},\nu h_{11/2})$. If they could be observed experimentally, this would be evidence confirming the datum (17$^+$).

For determining the $QQ_{\pi,\nu}$ correction and monopole term $Mc(\nu d_{5/2},\nu h_{11/2})$, an alternative of lack data in ^{132}Te is to study the $QQ_{\pi,\nu}$ correction and monopole effects in other nuclei nearby. The monopole effects of $Mc(\nu d_{5/2},\nu h_{11/2})$ also exist in ^{130}Sn and ^{131}Sb. In ^{130}Sn, the configuration $\nu h^{-1}_{11/2}d^{-1}_{5/2}$ produces levels from 3$^-$ to 8$^-$. With the $Mc(\nu d_{5/2},\nu h_{11/2})$, the configuration percentages have almost no change in levels 3$^-$, 7$^-$, and 8$^-$, while the percentage of level 5$^-$ (6$^-$) drops from 57% (49%) to 52% (39%). Level 3$^-$ is the first state of $J^\pi = 3^-$, and its energy decreases by 1.01 MeV when Mc is added. Here, the $QQ_{\pi,\nu}$ force has no effect on one-shell closed nuclei. If this state could be observed near 2.199 MeV, this would be evidence for considering monopole correction in particle-hole nuclei near ^{132}Sn. The same applies to level 8$^-$ as the first state of $J^\pi = 8^-$.

In ^{131}Sb, levels from 1/2$^-$ to 21/2$^-$ have a main configuration of $\pi g_{7/2}\nu h^{-1}_{11/2}$. As shown in Fig. 7(b), the $QQ_{\pi,\nu}$ correction has obvious effects on levels 1/2$^-$, 3/2$^-$, 5/2$^-$, 21/2$^-$, and 23/2$^-$, while monopole correction Mc has almost no effects except in the case of level 23/2$^-$.

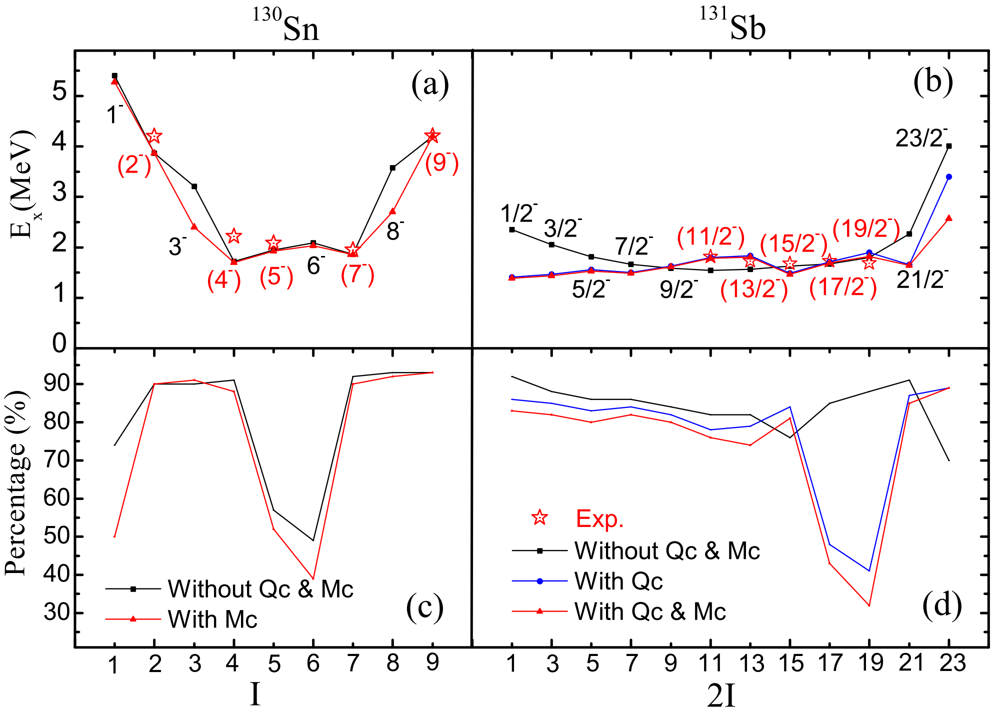

Fig. 7. (color online) $QQ_{\pi,\nu}$ force and monopole effects of $Mc(\nu d_{5/2},\nu h_{11/2})$ in ^{130}Sn and ^{131}Sb. Data marked with stars are from Ref. [32].

The lowest energy level of $J^{\pi}=23/2^-$ drops to 2.570 MeV with Qc and Mc. We are very interested in whether the high-spin state $23/2^-$ can be observed experimentally. This $23/2^-$ level can be used to determine the necessity of $QQ_{\pi,\nu}$ correction in the particle-hole nuclei region and finally explain the large difference in level (17^+) of ^{132}Te.

IV. CONCLUSION

The monopole effects and high-spin levels in $^{132-134}$Te, $^{131-133}$Sb, and ^{130}Sn are investigated. The datum 6.166 MeV of level 17^+ in ^{132}Te is excluded from configurations $\pi g_{7/2}h_{11/2}\nu g_{7/2}^{-1}h_{11/2}^{-1}$ or $\pi h_{11/2}^{2}\nu h_{11/2}^{-2}$. The present work suggests the datum (17^+) coupled by configuration $\pi g_{7/2}h_{11/2}\nu d_{5/2}^{-1}h_{11/2}^{-1}$. Several levels are connected with state (17^+) by monopole effects of $Mc(\nu d_{5/2},\nu h_{11/2})$ and the quadruple-quadruple force between the proton and neutron, i.e., 3^- (8^-) in ^{130}Sn, 14^- in ^{132}Te, and $23/2^-$ in ^{131}Sb. If these states could be observed at lower energies, the lower state (17^+) of ^{132}Te would be explained with quadruple-quadruple correction between the proton and neutron and the increasing strength of the monopole interaction between neutron orbits $d_{5/2}$ and $h_{11/2}$.

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