Pure annihilation decay of strange beauty meson into two charm heavy mesons

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Abstract: In this study, the heavy to heavy decay of $B_s^0 \to D^{*+}D^-$ is evaluated through the factorization approach by using the final state interaction as an effective correction. Under the factorization approach, this decay mode occurs only through the annihilation process, so a small amount is produced. Feynman's rules state that six meson pairs can be assumed for the intermediate states before the final meson pairs are produced. By taking into account the effects of twelve final state interaction diagrams in the calculations, a significant correction is obtained. These effects correct the value of the branching ratio obtained by the pure factorization approach from $(2.41 \pm 1.37) \times 10^{-5}$ to $(8.27 \pm 2.23) \times 10^{-5}$. The value obtained for the branching ratio of the $B_s^0 \to D^{*+}D^-$ decay is consistent with the experimental results.

Keywords: standard model, factorization approach, final state interaction

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I. INTRODUCTION

The $B_s^0 \to D^{*+}D^-$ decay has not been observed so far, but an excess of possible $B_s^0 \to D^{*+}D^-$ candidates was seen in a recent measurement of CP violation in $B^0 \to D^{*+}D^-$ decays by the LHCb experiment [1]. LHCb recently measured its branching fraction relative to the $B^0 \to D^{*+}D^-$ decay as [2]

$$\frac{\mathcal{B}(B_s^0 \to D^{*+}D^-)}{\mathcal{B}(B^0 \to D^{*+}D^-)} = 0.137 \pm 0.017 \pm 0.002 \pm 0.006. \quad (1)$$

Using the measured value of the $B^0 \to D^{*+}D^-$ branching fraction from Ref. [3], the $B_s^0 \to D^{*+}D^-$ branching fraction was determined by LHCb collaborations:

$$\mathcal{B}(B_s^0 \to D^{*+}D^-) = (8.41 \pm 1.02 \pm 0.12 \pm 0.39 \pm 0.79) \times 10^{-5}. \quad (2)$$

Assuming prominent contributions from rescattering of, e.g., $D_s^{*+}D_s^-$ states, the branching fraction is predicted to be $(6.1 \pm 3.6) \times 10^{-5}$ [4]. A perturbative QCD approach predicts a much larger branching fraction, i.e., $(3.6 \pm 0.6) \times 10^{-3}$ [5].

This paper presents the calculation of the branching fraction for the $B_s^0 \to D^{*+}D^-$ decay applying the pure factorization approach and then considering the rescattering

of intermediate state effects as significant corrections. The $B_s^0 \to D^{*+}D^-$ decay is forbidden at tree level and its dominant contributions originate from W-exchange and penguin-annihilation diagrams or from rescattering of intermediate states, as shown in Figs. 1 and 2, respectively. The meson pairs $D^0\bar{D}^{*0}$, $\eta_c J/\psi$, $\pi^+\rho^-$, $\pi^0\rho^0$, $D_s^+D_s^{*-}$, and $\bar{K}^0K^{*0}$ are produced from rescattering of intermediate states. Note that in the final state interaction diagrams (see Fig. 3), a vector meson is at the top vertices, and a pseudoscalar meson is at the bottom vertices. To maintain symmetry, the intermediate states should consist of a vector meson at the bottom and a pseudoscalar meson at the top vertices. The contributions of the other modes in which both middle mesons are vectors, or a vector and a pseudoscalar meson, are located at the top and bottom vertices, respectively, and become zero and disappear.

In these processes, the $\pi^+(\rho^+)$, $D^+(D^{*+})$, $D^0(D^{*0})$, $D^+(D^{*+})$, $\bar{K}^0(\bar{K}^{*0})$, and $D_s^+(D_s^{*+})$ mesons are exchanged between the intermediate and final state mesons. The processes of producing these middle and exchanged particles are determined through Feynman diagrams. According to the final state interaction quark model, the Feynman graphs are presented in two channels: t and s. Unlike in D meson decays, in B_s meson decays there is no resonance with energy close to the mass of the B_s meson; consequently, the s channel is suppressed.

The channel t occurs in both uncrossed and crossed ways. In uncrossed channels, two final mesons, D^{*+} and D^-, share one quark and one anti-quark with the same
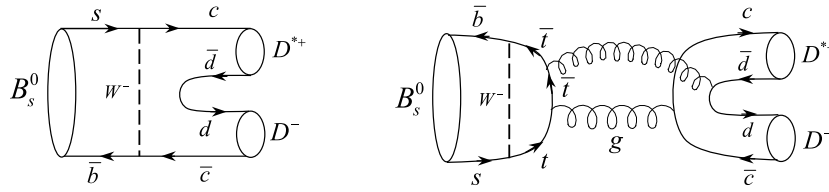
Fig. 1. W-exchange and penguin-annihilation diagrams contributing to $B_s^0 \to D^{*+}D^-$ decay.

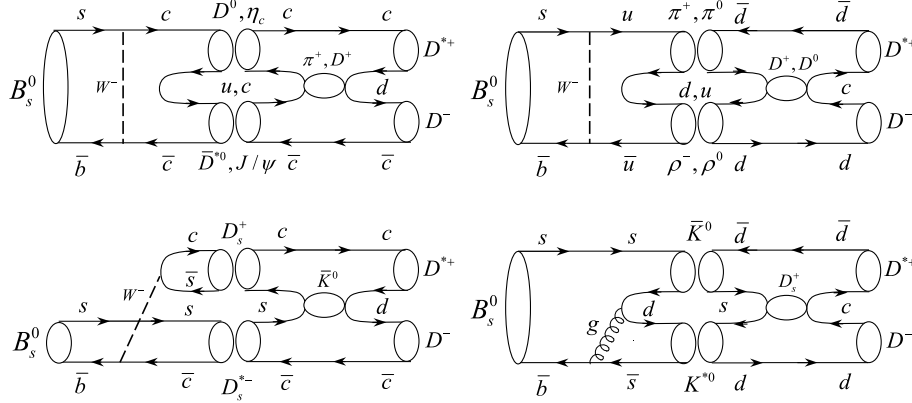**Fig. 2.** Intermediate state rescattering of $B_s^0 \to P_1(p_1)P_2(p_2, \epsilon_2) \to D^{*+}(p_3, \epsilon_3)D^-(p_4)$.
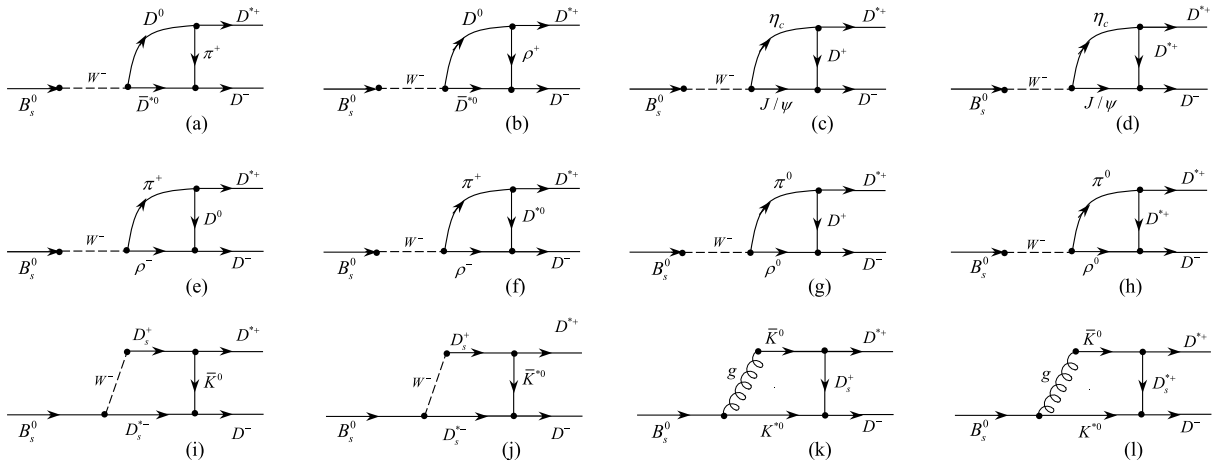

Fig. 3. Hadronic loop level diagrams for long distance uncrossed t channel contribution to $B_s^0 \to D^{*+}D^-$ decay.

flavor (d). In this case, the intermediate mesons are produced by sharing u, c, and s quarks (left graphs in the first and second rows in Fig. 2). If the intermediate mesons exchange a d quark, these mesons are identical to the final mesons, i.e., $B_s^0 \to D^{*+}D^- \to D^{*+}D^-$; therefore, this mode is ignored.

Moreover, in uncrossed channels, the D^{*+} and D^- final mesons can share the c quark and $\bar{c}$ anti-quark, respectively. In this case, the quarks that are shared between the final and middle mesons are d, u, and s. The mode in which the middle mesons exchange quark c is ignored because a duplicate mode of $B_s^0 \to \bar{D}^{*0}\bar{D}^0 \to D^{*+}D^-$ is achieved.

In crossed channels, two final mesons exchange c (or

d) quark and $\bar{c}$ (or $\bar{d}$) anti-quark with intermediate mesons crosswise. Thus, the $B_s^0 \to D^{*+}D^- \to D^{*+}D^-$ process cannot take place. Twelve final state interaction diagrams in an uncrossed t channel are shown in Fig. 3.

In factorization approaches, the perturbative strong phases arising from the penguin graph in $b \to s(d)$ transitions and from the vertex corrections can be in principle sizable. Modeling approaches based on final state interactions are soft rescattering processes. They are calculated through the nonperturbative theory. Therefore, the nonperturbative strong phases induced from power suppressed contributions should be considered. The idea that Hai-Yang Cheng reported about the existence of strong phases in final state interactions is that in charmless B de-

cays, when the intermediate state mesons are charm, the elements of the CKM matrix of these states are dominant, and then, the absorptive part of the final state rescattering amplitude can cause large phases. However, if the final mesons are also charm (which is the case of the current study), the phases cannot have a significant contribution in the rates [6]. An interesting estimate on the basis of the Regge theory is provided by Donoghue *et al.* with this content: final state interactions appear even in the heavy quark limit and soft FSI phases are dominated by inelastic scattering [7]. However, it was later pointed out by Beneke *et al.* within the framework of QCD factorization that the above conclusion holds only for individual rescattering amplitudes. When summing over all possible intermediate states, there exist systematic cancellations in the heavy quark limit such that the strong phases must vanish in the limit $m_b \to \infty$ [8].

II. FACTORIZATION APPROACHES

Under the factorization approaches (Fa), the $B_s^0 \to D^{*+}D^-$ decay occurs only through annihilation processes. These contributions to the matrix elements of the effective weak Hamiltonian can be written in the form [9]

$$\langle D^{*+}D^-|\mathcal{H}_{\text{eff}}|B_s^0\rangle = \mathrm{i}\frac{G_F}{\sqrt{2}}\lambda\langle D^{*+}D^-|\mathcal{T}^{\text{ann}}|B_s^0\rangle, \quad (3)$$

where λ is equal to $V_{cb}^* V_{cs}$ and $V_{tb}^* V_{ts}$ for the current-current tree and penguin level diagrams, respectively. The term $\mathcal{T}^{\text{ann}}$ arises from weak annihilation contributions and the matrix elements of $\langle D^{*+}D^-|\mathcal{T}^{\text{ann}}|B_s^0\rangle$ represent the $b_i(D^*D)$ coefficients multiplied by the f_{B_s}, f_{D^*}, and f_D decay constants. According to Fig. 1, the $B_s^0 \to D^{*+}D^-$ decay includes current-current annihilation ($b_1(D^*D)$), penguin annihilation ($b_4(D^*D)$), and electroweak penguin annihilation ($b_{4,\text{EW}}(D^*D)$) coefficients, so the amplitude of this decay mode is obtained as follows:

$$\mathcal{A}(B_s^0 \to D^{*+}D^-)_{\text{fa}} = \mathrm{i}\frac{G_F}{\sqrt{2}}f_{B_s}f_{D^*}f_D\Big\{b_1 V_{cb}^* V_{cs}$$
$$-\Big(2b_4 + \frac{1}{2}b_{4,\text{EW}}\Big)V_{tb}^* V_{ts}\Big\}, \quad (4)$$

where the quantities b_1, b_4, and $b_{4,\text{EW}}$ depend on the final-state mesons through the light-cone distribution amplitudes entering the expressions for $A_{1,2}^i$ as [9]

$$b_1 = \frac{C_F}{N_c^2}c_1 A_1^i, \quad b_4 = \frac{C_F}{N_c^2}(c_4 A_1^i + c_6 A_2^i),$$
$$b_{4,\text{EW}} = \frac{C_F}{N_c^2}(c_{10}A_1^i + c_8 A_2^i), \quad (5)$$

here c_i denotes the Wilson coefficients, N_c is the color

number, and $C_F = (N_c^2 - 1)/(2N_c)$. By considering a generic b-quark decay in the $B_s \to M_1 M_2$ process, and using the convention that M_2 contains a quark and M_1 contains an antiquark from the weak vertices, it can be found that the type is $B_s \to PV$ (Fig. 1 shows that the D^- meson has an antiquark and the D^{*+} meson has a quark, so they are considered as the pseudoscalar (P) M_1 and vector (V) M_2 mesons, respectively). For such a case, the basic building blocks of $A_{1,2}^i$ are given by [9]

$$A_1^i = 6\pi\alpha_s\int_0^1\mathrm{d}x\mathrm{d}y\Big\{6xy\bar{x}\bar{y}\Big[\frac{1}{y(1-x\bar{y})}+\frac{1}{\bar{x}^2y}\Big]+r_\chi^{D^*}r_\chi^D\frac{x-\bar{x}}{\bar{x}y}\Big\},$$
$$A_2^i = -6\pi\alpha_s\int_0^1\mathrm{d}x\mathrm{d}y\Big\{6xy\bar{x}\bar{y}\Big[\frac{1}{\bar{x}(1-x\bar{y})}+\frac{1}{\bar{x}y^2}\Big]+r_\chi^{D^*}r_\chi^D\frac{x-\bar{x}}{\bar{x}y}\Big\}.$$
$$(6)$$

In the above integrals, there are divergences for each of the final mesons: for the D^- pseudoscalar meson, these divergences are $\int_0^1\mathrm{d}y/y$ and $\int_0^1\ln(y)\mathrm{d}y/y$, which are introduced with parameters X_A^D and $-1/2(X_A^D)^2$, respectively, and similarly for the vector meson of D^{*+} with $y \to \bar{x}$. In general, X_A is allowed for three cases, namely PP, PV, and VP. These values are modeled by using the parameterization [9]

$$X_A = (1 + \rho e^{\mathrm{i}\phi})\ln\frac{m_{B_s}}{\Lambda_h}. \quad (7)$$

The quantity ϕ is an arbitrary strong-interaction phase that may be caused by soft rescattering. The values selected by Beneke and Neubert in Ref. [9] for the three cases, i.e., PP, PV, and VP are $\phi = -55°$ (PP), $\phi = -20°$ (PV), and $\phi = -70°$ (VP), respectively. From the above discussion, it follows that because the final mesons are of PV type; thus, the value of ϕ was set to $-20°$. These authors also evaluated the Wilson coefficients b_i at an intermediate scale $\mu_h \sim (\Lambda_{\text{QCD}}m_b)^{1/2}$ rather than $\mu_h \sim m_b$. Specifically, they used $\mu_h = (\Lambda_h\mu)^{1/2}$ with $\Lambda_h = 0.5$ Gev. The value of the model parameter ρ is limited to $\rho \leqslant 1$, so it was set as $\rho = 0.5$ in this study.

Simple expressions can be obtained for A_1^i and A_2^i [9]:

$$A_1^i \approx -A_2^i = 6\pi\alpha_s\Big[3\Big(X_A - 4 + \frac{\pi^2}{3}\Big) + r_\chi^{D^*}r_\chi^D(X_A^2 - 2X_A)\Big]. \quad (8)$$

The light-cone expansion implies that only leading-twist distribution amplitudes are needed in the heavy-quark limit. There exist however a number of subleading quark-antiquark distribution amplitudes of twist 3 that have large normalization factors for pseudoscalar and vector mesons. For D and D^* mesons, the ratios r_χ^D and

$r_\chi^{D^*}$ are defined as [9]

$$r_\chi^D = \frac{2m_D^2}{(m_b - m_c)(m_c + m_d)}, \quad r_\chi^{D^*} = \frac{2m_D^*}{m_b}\frac{f_{D^*}^\perp}{f_{D^*}}. \quad (9)$$

For the running coupling constant, at two loop order (NLO), the solution of the renormalizaton group equation can always be written in the form

$$\alpha_s = \frac{4\pi}{\beta_0 \ln\frac{\mu^2}{\Lambda_{QCD}^2}}\left[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln\left(\ln\frac{\mu^2}{\Lambda_{QCD}^2}\right)}{\ln\frac{\mu^2}{\Lambda_{QCD}^2}}\right], \quad (10)$$

where $\beta_0 = (11N_c - 2n_f)/3$, $\beta_1 = (34N_c^2 - 10N_c n_f)/3 - 2C_F n_f$. In this study, the running $\alpha_s(\mu)$ is evaluated with $n_f = 5$.

In the $B_s^0 \to D^{*+}D^-$ decay, there is also the possibility of intermediate state rescattering, which greatly contributes to the calculation. Possible decays for the intermedi-

ate states derived from Feynman diagrams are shown in Fig. 4. Therefore, the amplitudes of the middle state decays used in the final state interactions are given by

$$\begin{aligned}
\mathcal{A}(B_s^0 \to D^0\bar{D}^{*0}(\eta_c J/\psi)) =& i\frac{G_F}{\sqrt{2}}f_{B_s}f_D f_{D^*}(f_{\eta_c}f_{J/\psi})[b_1 V_{cb}^* V_{cs} \\
& - (2b_4 + 2b_{4,EW})V_{tb}^* V_{ts}], \\
\mathcal{A}(B_s^0 \to \pi^+\rho^-) =& i\frac{G_F}{\sqrt{2}}f_{B_s}f_\pi f_\rho[b_1 V_{ub}^* V_{us} \\
& - \left(2b_4 + \frac{1}{2}b_{4,EW}\right)V_{tb}^* V_{ts}], \\
\mathcal{A}(B_s^0 \to \pi^0\rho^0) =& i\frac{G_F}{2}f_{B_s}f_\pi f_\rho b_1 V_{ub}^* V_{us}, \\
\mathcal{A}(B_s^0 \to D_s^+ D_s^{*-}) =& i\sqrt{2}G_F m_{D_s^*}(\epsilon_{D_s^*}.p_{B_s})f_{D_s}A_0^{B_s\to D_s^*} \\
& \times (m_{D_s}^2)(a_1 V_{cb}^* V_{cs} - a_4 V_{tb}^* V_{ts}), \\
\mathcal{A}(B_s^0 \to \bar{K}^0 K^{*0}) =& -i\sqrt{2}G_F m_{K^*}(\epsilon_{K^*}.p_{B_s}) \\
& \times f_K A_0^{B_s\to K^*}(m_K^2)a_4 V_{tb}^* V_{ts}.
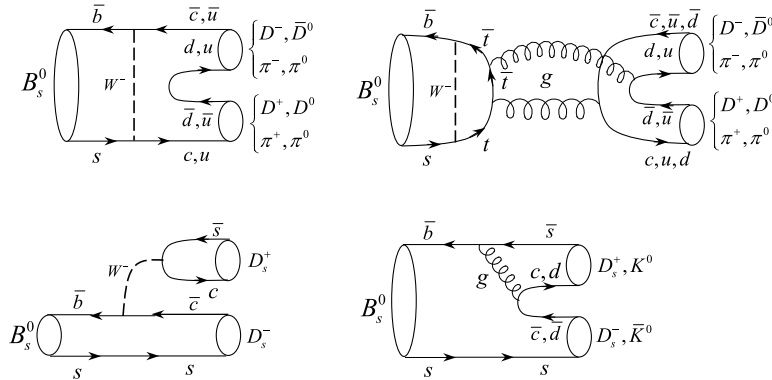\end{aligned}$$

$$(11)$$

Fig. 4. B_s meson decay diagrams to intermediate state mesons in a rescattering process.

III. RESCATTERING OF INTERMEDIATE
STATES

A. Absorptive part of the amplitudes

1. *Mode of $B_s^0 \to D^0(p_1)\bar{D}^{*0}(\epsilon_2, p_2) \to D^{*+}(\epsilon_3, p_3)D^-(p_4)$*
    with π^+ and ρ^+ exchanged mesons

Diagram (3a) in Fig. 3 shows the absorptive part of the amplitude for the $B_s^0 \to D^0(p_1)\bar{D}^{*0}(\epsilon_2, p_2) \to D^{*+}(\epsilon_3, p_3)$ $D^-(p_4)$ mode in the t channel one particle exchange process. In this process, intermediate state decay occurs by

the annihilation topology and the exchanged meson is π^+, so the absorptive part of the amplitude is given by

$$\begin{aligned}
\mathcal{A}bs^{3a} =& \frac{g_{D^*D\pi}^2}{8\pi m_{B_s}}\mathcal{A}(B_s^0 \to D^0\bar{D}^{*0}) \\
& \times \int_{-1}^1 |\vec{p_1}|d(\cos\theta)\frac{F^2(q^2, m_i^2)}{\mathcal{T}^{3a}}\mathcal{H}^{3a}, \quad (12)
\end{aligned}$$

where θ is the angle between $\vec{p_1}$ and $\vec{p_3}$ for which p_1 and p_3 are the four-momentums of the D^0 and D^{*+} mesons, q and m_i are the momentum and mass of the exchange π^+ meson, respectively, and

$$\mathcal{H}^{3a} = (\epsilon_3 . p_1)(\epsilon_2 . p_4)$$
$$= \left(\frac{E_1|\vec{p_3}| - E_3|\vec{p_1}|\cos\theta}{m_{B_s}|\vec{p_3}|}\right)\left(\frac{E_4|\vec{p_2}| - E_2|\vec{p_4}|\cos\theta}{m_{B_s}|\vec{p_2}|}\right),$$
$$\mathcal{T}^{3a} = (p_1 - p_3)^3 - m_\pi^2$$
$$= p_1^2 + p_3^2 - 2p_1^0 p_3^0 + 2p_1.p_2 - m_\pi^2$$
$$= m_D^2 + m_{D^*}^2 - m_\pi^2 - 2E_D E_{D^*} + 2|\vec{p}_D||\vec{p}_{D^*}|\cos\theta. \quad (13)$$

Note that $F(q^2, m_i^2)$ is the form factor defined to account for the off-shell character of the exchange particles, defined as $(\Lambda^2 - m_i^2)/(\Lambda^2 - q^2)$ [10]. Concerning Λ, it is a phenomenological parameter that should not be far from the physical parameters of the exchange particle, m_i and q; it can be written in terms of another phenomenological parameter, for instance, η, as $\Lambda = m_i + \eta\Lambda_{\rm QCD}$. The range of the strong interaction energy scale, $\Lambda_{\rm QCD}$, goes from (90.6 ± 3.4) MeV to (340 ± 8) MeV [11]. We set it as $\Lambda_{\rm QCD} = 225$ MeV. The numerical value of the branching ratio in the final state interaction is very sensitive to the η parameter. According to the exchanged mesons, variable values ranging from 0.5 to 5 can be found. In Ref. [10], the exchanged mesons are D and D^*, so the authors set $\eta = 0.5 - 3.0$. However, the authors of Ref. [12], with the same exchanged mesons, set $\eta \sim 5$. In this regard, in [13], a value of 4 was set for this phenomenological parameter. On the other hand, in Ref. [14], the value of η was selected according to the mass of the meson exchanged: $\eta = 2.2$ for the exchanged particle D^* (or D) and $\eta = 1.1$ for ρ (or π). In the present study, given that the exchanged mesons are $D_{(s)}^{(*)}$, $\pi(\rho)$, and $K^{(*)}$, to select the values of η, Ref. [14] was followed. Similarly, for diagram (3b), in which the decay of the intermediate state also occurs by the annihilation topology, the absorptive part of the amplitude can be written as

$$\mathcal{A}bs^{3b} = \frac{g_{D^*D\rho}^2}{8\pi m_{B_s}}\mathcal{A}(B_s^0 \to D^0\bar{D}^{*0})$$
$$\times \int_{-1}^1 |\vec{p_1}|d(\cos\theta)\frac{F^2(q^2, m_\rho^2)}{\mathcal{T}^{3b}}\mathcal{H}^{3b}, \quad (14)$$

where

$$\mathcal{H}^{3b} = \epsilon_{\mu\nu\alpha\beta}\epsilon_{\rho\sigma\lambda\eta}\epsilon_\rho^\gamma\epsilon_\rho^\sigma\epsilon_3^\mu\epsilon_2^\rho p_1^\alpha p_3^\beta p_2^\lambda p_4^\eta,$$
$$\mathcal{T}^{3b} = m_D^2 + m_{D^*}^2 - m_\rho^2 - 2E_D E_{D^*} + 2|\vec{p}_D||\vec{p}_{D^*}|\cos\theta. \quad (15)$$

2. Mode of $B_s^0 \to \eta_c(p_1)J/\psi(\epsilon_2, p_2) \to D^{*+}(\epsilon_3, p_3)D^-(p_4)$ with D^+ and D^{*+} exchanged mesons

The absorptive part of the amplitude for the $B_s^0 \to \eta_c(p_1) \times J/\psi(\epsilon_2, p_2) \to D^{*+}(\epsilon_3, p_3)D^-(p_4)$ mode with

D^+ and D^{*+} exchanged mesons is obtained as follows (the intermediate state of this process is also based on the annihilation topology):

$$\mathcal{A}bs^{3c(3d)} = \frac{g_{D^*D^{(*)}\eta_c}g_{J/\psi D^{(*)}D}}{8\pi m_{B_s}}\mathcal{A}(B_s^0 \to \eta_c J/\psi)$$
$$\times \int_{-1}^1 |\vec{p_1}|d(\cos\theta)\frac{F^2(q^2, m_{D(D^*)}^2)}{\mathcal{T}^{3c(3d)}}\mathcal{H}^{3a(3d)}, \quad (16)$$

where

$$\mathcal{T}^{3c} = m_{\eta_c}^2 + m_{D^*}^2 - m_D^2 - 2E_{\eta_c}E_{D^*} + 2|\vec{p}_{\eta_c}||\vec{p}_{D^*}|\cos\theta,$$
$$\mathcal{H}^{3d} = \epsilon_{\mu\nu\alpha\beta}\epsilon_{\rho\sigma\lambda\eta}\epsilon_{D^*}^\gamma\epsilon_{D^*}^\sigma\epsilon_3^\mu\epsilon_2^\rho p_1^\alpha p_3^\beta p_2^\lambda p_4^\eta,$$
$$\mathcal{T}^{3d} = m_{\eta_c}^2 - 2E_{\eta_c}E_{D^*} + 2|\vec{p}_{\eta_c}||\vec{p}_{D^*}|\cos\theta. \quad (17)$$

3. Modes of $B_s^0 \to \pi^+(p_1)\rho^-(\epsilon_2, p_2)[\pi^0(p_1)\rho^0(\epsilon_2, p_2)] \to D^{*+}(\epsilon_3, p_3)D^-(p_4)$ with $D^0[D^+]$ and $D^{*0}[D^{*+}]$ exchanged mesons

The intermediate state decays $B_s^0 \to \pi^+(p_1)\times \rho^-(\epsilon_2, p_2)$ and $B_s^0 \to \pi^0(p_1)\rho^0(\epsilon_2, p_2)$ are also transformed into final mesons through processes of annihilation. The absorptive part of the amplitudes for these decay modes are calculated by

$$\mathcal{A}bs^{3e(3f)} = \frac{g_{D^*D^{(*)}\pi}g_{DD^{(*)}\rho}}{8\pi m_{B_s}}\mathcal{A}(B_s^0 \to \pi^+\rho^-)$$
$$\times \int_{-1}^1 |\vec{p_1}|d(\cos\theta)\frac{F^2(q^2, m_{D(D^*)}^2)}{\mathcal{T}^{3e(3f)}}\mathcal{H}^{3a(3d)}, \quad (18)$$

where

$$\mathcal{T}^{3e} = m_\pi^2 + m_{D^*}^2 - m_D^2 - 2E_\pi E_{D^*} + 2|\vec{p}_\pi||\vec{p}_{D^*}|\cos\theta,$$
$$\mathcal{T}^{3f} = m_\pi^2 - 2E_\pi E_{D^*} + 2|\vec{p}_\pi||\vec{p}_{D^*}|\cos\theta. \quad (19)$$

Graphs (3g) and (3h) in Fig. 3 were also calculated for the $B_s^0 \to \pi^0(p_1)\rho^0(\epsilon_2, p_2) \to D^{*+}(\epsilon_3, p_3)D^-(p_4)$ mode using Eqs. (18) and (19), with the difference that the amplitude of $\mathcal{A}(B_s^0 \to \pi^0\rho^0)$ was replaced by $\mathcal{A}(B_s^0 \to \pi^+\rho^-)$.

4. Mode of $B_s^0 \to D_s^+(p_1)D_s^{*-}(\epsilon_2, p_2) \to D^{*+}(\epsilon_3, p_3)D^-(p_4)$ with $\bar{K}^0$ and $\bar{K}^{*0}$ exchanged mesons

The $B_s^0 \to D_s^+(p_1)D_s^{*-}(\epsilon_2, p_2)$ decay, which is considered to be another intermediate state decay, decays through a dominant tree and penguin contributions. The absorptive part of the amplitude for the mode of $B_s^0 \to D_s^+(p_1)D_s^{*-}(\epsilon_2, p_2) \to D^{*+}(\epsilon_3, p_3)D^-(p_4)$ can be ob-

tained by

$$
\begin{aligned}
\mathcal{A}bs^{3i(3j)} =& i\frac{G_F}{\sqrt{2}}\frac{g_{D^*D_sK^{(*)}}g_{D_s^*DK^{(*)}}}{4\pi m_{B_s}} \\
& \times m_{D_s^*}f_{D_s}A_0^{B_s\to D_s^*}(m_{D_s^*}^2)(a_1V_{cb}^*V_{cs}+a_4V_{tb}^*V_{ts}) \\
& \times \int_{-1}^1 |\vec{p_1}|d(\cos\theta)\frac{F^2(q^2,m_{K(K^*)}^2)}{\mathcal{T}^{3i(3j)}}\mathcal{H}^{3i(3j)}, \quad (20)
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{H}^{3i} =& (\epsilon_2.p_1)(\epsilon_2.p_4)(\epsilon_3.p_1) \\
=& \left(-p_1.p_4+\frac{(p_1.p_2)(p_2.p_4)}{m_{D_s^*}^2}\right)\left(\frac{E_1|\vec{p_3}|-E_3|\vec{p_1}|\cos\theta}{m_{B_s}|\vec{p_3}|}\right), \\
\mathcal{H}^{3j} =& m_3^2(p_1.p_2)-(p_1.p_3)(p_2.p_3) \\
& +\left(\frac{E_2|\vec{p_3}|-E_3|\vec{p_2}|\cos\theta}{m_{B_s}|\vec{p_3}|}\right)[(p_{B_s}.p_1)(p_3.p_4) \\
& -(p_{B_s}.p_3)(p_1.p_4)], \\
\mathcal{T}^{3i(3j)} =& m_{D_s}^2+m_{D^*}^2-m_K^2(m_{K^*}^2) \\
& -2E_{D_s}E_{D^*}+2|\vec{p}_{D_s}||\vec{p}_{D^*}|\cos\theta.
\end{aligned}
$$

$$(21)$$

5. *Mode of $B_s^0\to\bar{K}^0(p_1)K^{*0}(\epsilon_2,p_2)\to D^{*+}(\epsilon_3,p_3)D^-(p_4)$*
    with D_s^+ and D_s^{*+} exchanged mesons

The last decay of the intermediate state under consideration, i.e., the decay of $B_s^0\to\bar{K}^0(p_1)K^{*0}(\epsilon_2,p_2)$, occurs through a penguin contribution. Thus, the absorptive part of the amplitude reads

$$
\begin{aligned}
\mathcal{A}bs^{3k(3l)} =& i\frac{G_F}{\sqrt{2}}\frac{g_{D^*D_s^{(*)}K}g_{D_s^{(*)}DK^*}}{4\pi m_{B_s}} \\
& \times m_{K^*}f_KA_0^{B_s\to K^*}(m_K^2)a_4V_{tb}^*V_{ts} \\
& \times \int_{-1}^1 |\vec{p_1}|d(\cos\theta)\frac{F^2(q^2,m_{D_s(D_s)}^2)}{\mathcal{T}^{3k(3l)}}\mathcal{H}^{3k(3j)}, \quad (22)
\end{aligned}
$$

where $\mathcal{H}^{3k}$ is calculated following a procedure similar to that of $\mathcal{H}^{3i}$; concerning $\mathcal{H}^{3k}$, the parameter $m_{K^*}^2$ was employed instead of $m_{D_s^*}^2$, and

$$
\begin{aligned}
\mathcal{T}^{3k(3l)} =& m_K^2+m_{D^*}^2-m_{D_s}^2(m_{D_s}^2)-2E_KE_{D^*} \\
& +2|\vec{p}_K||\vec{p}_{D^*}|\cos\theta.
\end{aligned}
$$

$$(23)$$

B. Dispersive part of the amplitudes

The dispersive part of the intermediate state rescattering amplitudes can be obtained from the absorptive parts

shown in Fig. 3 using the following dispersion relation [14,15]:

$$
\begin{aligned}
\mathrm{Dis}\mathcal{A}(m_{B_s}^2) =& \frac{1}{\pi} \\
& \times \int_s^\infty \frac{\mathcal{A}bs^{3a}(s')+\mathcal{A}bs^{3b}(s')+\mathcal{A}bs^{3c}(s')\ldots+\mathcal{A}bs^{3l}(s')}{s'-m_{B_s}^2}ds'.
\end{aligned}
$$

$$(24)$$

where s' is the square of the momentum carried by the exchanged particle and s is the threshold of intermediate states, in this case $s\sim m_{B_s}^2$.

The total amplitude of the absorptive and dispersive parts of the intermediate state rescattering (Isr) is estimated by

$$
\begin{aligned}
\mathcal{A}(B_s^0\to D^{*+}D^-)_{\mathrm{Isr}} =& i\mathcal{A}bs^{3a}+i\mathcal{A}bs^{3b}+\ldots \\
& +i\mathcal{A}bs^{3l}+\mathrm{Dis}\mathcal{A}(m_{B_s}^2). \quad (25)
\end{aligned}
$$

IV. DECAY RATE AND BRANCHING FRACTION OF DECAY

The decay rate of $B_s^0\to D^{*+}D^-$ in B_s meson rest frame under the factorization approach can be written as [16]

$$
\Gamma(B_s^0\to D^{*+}D^-)_{\mathrm{Fa}} = \frac{1}{8\pi}\frac{|\vec{p}|}{m_{B_s}^2}|\mathcal{A}(B_s^0\to D^{*+}D^-)_{\mathrm{Fa}}|^2, \quad (26)
$$

where $|\vec{p}|$ is the absolute value of the 3-momentum of the D^* or D final mesons, which can be calculated using $\sqrt{(m_{B_s}^2+m_{D^*}^2-m_D^2)^2-4m_{B_s}^2m_{D^*}^2}/(2m_{B_s})$. The branching ratio of the $B_s^0\to D^{*+}D^-$ decay using the factorization method is expressed as

$$
\mathcal{B}(B_s^0\to D^{*+}D^-)_{\mathrm{Fa}} = \frac{\Gamma(B_s^0\to D^{*+}D^-)_{\mathrm{Fa}}}{\Gamma_{\mathrm{tot}}}, \quad (27)
$$

where the value of Γ_{tot} for the B_s meson is $(4.34\pm0.01)\times10^{-13}$ GeV.

Finally, the branching fraction of the $B_s^0\to D^{*+}D^-$ decay applying the factorization model while considering the effects of rescattering of intermediate states is given by

$$
\begin{aligned}
\mathcal{B}(B_s^0\to D^{*+}D^-)_{\mathrm{Fa+Isr}} =& \frac{1}{\Gamma_{\mathrm{tot}}}\frac{|\vec{p}|}{8\pi m_{B_s}^2}|\mathcal{A}(B_s^0\to D^{*+}D^-)_{\mathrm{Fa}} \\
& +\mathcal{A}(B_s^0\to D^{*+}D^-)_{\mathrm{Isr}}|^2. \quad (28)
\end{aligned}
$$

V. NUMERICAL RESULTS AND CONCLUSION

The calculations of the branching fractions are provided next. The input parameters used in this paper are as follows:

masses and decay constants (in units of MeV) [11]

$m_{B_s^0} = 5366.88 \pm 0.14$ $m_{\eta_c} = 2983.9 \pm 0.5$

$m_{D_s^*} = 2112.2 \pm 0.4$ $m_{D^*} = 2006.85 \pm 0.05$

$m_{D_s} = 1968.34 \pm 0.07$ $m_D = 1869.65 \pm 0.05$

$m_{K^*} = 891.66 \pm 0.26$ $m_\rho = 775.26 \pm 0.25$

$m_K = 493.677 \pm 0.016$ $m_\pi = 139.57039 \pm 0.00018$

$m_b = 4180 \pm 0.02$ $m_c = 1270 \pm 20$

$m_d = 4.69 \pm 0.32$ $f_{J/\psi} = 418 \pm 9$ $f_{\eta_c} = 387 \pm 7$

$f_{D^*} = 340 \pm 23$ $f_{D_s} = 294 \pm 7$ $f_{B_s} = 245 \pm 25$

$f_D = 234 \pm 15$ $f_{D^*}^\perp = 216 \pm 14$ $f_\rho = 210 \pm 4$

$f_K = 159.80 \pm 1.84$ $f_\pi = 130.70 \pm 0.46$

CKM matrix elements [11]

$V_{cb} = (41.0 \pm 1.4) \times 10^{-3}$ $V_{ub} = (3.82 \pm 0.24) \times 10^{-3}$

$V_{ts} = (38.8 \pm 1.1) \times 10^{-3}$ $V_{cs} = 0.987 \pm 0.011$

$V_{us} = 0.2245 \pm 0.0008$ $V_{tb} = 1.013 \pm 0.030$

Wilson coefficients ($\mu = m_b$, $\alpha = 1/129$) [17]

$c_1 = 1.081$ $c_4 = -0.036$ $c_6 = -0.042$ $c_8/\alpha = 0.060$

$c_{10}/\alpha = 0.223$

coupling constants [14,18,19,20]

$g_{J/\psi DD} = 7.71$ $g_{J/\psi D^*D} = 8.64$ $g_{D^*D\pi} = 8.84$

$g_{D^*D^*\pi} = 9.08$ $g_{DD\rho} = 2.52$ $g_{D^*D\rho} = 2.82$

$g_{D^*D_sK} = 12.84$ $g_{D^*D_sK^*} = 2.79$ $g_{D_s^*DK^*} = 3.00$

$g_{D_sDK^*} = 2.66$ $g_{D_s^*DK} = 2.89$ $g_{D^*D_sK} = 9.23$

$g_{D^*D\eta_c} = 8.52$

form factors [21,22]

$A_0^{B_s \to K^*}(m_K^2) = 0.306 \pm 0.034$ $A_0^{B_s \to D_s^*}(m_{D_s}^2) = 0.883 \pm 0.012$.

Finally, the branching fractions for the $B_s^0 \to D^{*+}D^-$ decay under the factorization approach and applying the factorization method while considering rescattering of the final state interaction read, respectively, as follows:

$$\mathcal{B}(B_s^0 \to D^{*+}D^-)_{\text{Fa}} = (2.41 \pm 1.37) \times 10^{-5}, \qquad (29)$$

and

$$\mathcal{B}(B_s^0 \to D^{*+}D^-)_{\text{Fa+Isr}} = (8.27 \pm 2.23) \times 10^{-5}. \qquad (30)$$

There are some parameters in the hadronic decays that can be obtained from theoretical estimates, such as transition form factors. In practice, information about form factors often comes from theoretical calculations such as light-cone QCD sum rules or lattice calculations. Thus, they usually present large uncertainties. In this framework, the theoretical uncertainties expressed in Eqs. (29) and (30) arise from the uncertainties in the form factors, CKM matrix elements, decay constant, meson messes, and total decay rate of B_s meson. The corresponding uncertainties in the form factors and CKM matrix elements have more impact on the results.

VI. CONCLUSION

In this study, the contribution of the uncrossed t channel final state interaction, i.e., inelastic re-scattering processes to the branching ratio of $B_s^0 \to D^{*+}D^-$ decay, is calculated. For evaluating these effects, the absorptive and dispersive parts of the hadronic loop level diagrams were considered because the hadrons produced via weak interaction were on their mass shells.

The branching ratio of the $B_s^0 \to D^{*+}D^-$ decay was obtained through the factorization approach and final state interaction. The experimental result of this decay is $\mathcal{B}(B_s^0 \to D^{*+}D^-) = (8.41 \pm 1.02) \times 10^{-5}$. The branching ratios of this decay reported in Refs. [4,5] were $(6.1 \pm 3.6) \times 10^{-5}$ and a much larger value, $(3.6 \pm 0.6) \times 10^{-3}$, respectively. According to the factorization approach and considering the effects of final state interaction as an effective correction, the obtained results are $\mathcal{B}(B_s^0 \to D^{*+}D^-) = (2.41 \pm 1.37) \times 10^{-5}$ and $\mathcal{B}(B_s^0 \to D^{*+}D^-) = (8.27 \pm 2.23) \times 10^{-5}$, respectively.

There are some phenomenological parameters such as η in the calculations on final state interaction effects and in the form factors of the hadronic loop level diagrams. The value of the phenomenological parameter η in the form factor is expected to be of the order of unity and can be determined from the measured rates. For a given exchanged particle, $\eta = 2.2$ was set for the exchanged particles $D_{(s)}^*$ (or $D_{(s)}$) and $\eta = 1.1$ for ρ, K^* (or π, K).

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