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Magnetic moments of hidden-charm strange pentaquark states*

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Abstract: In this study, the magnetic moments of hidden-charm strange pentaquark states with quantum numbers $J^P=\frac{1}{2}^{\pm}$, $\frac{3}{2}^{\pm}$, $\frac{5}{2}^{\pm}$, and $\frac{7}{2}^{+}$ are calculated in the molecular, diquark-diquark-antiquark, and diquark-triquark models. The numerical results demonstrate that the magnetic moments change for different spin-orbit couplings within the same model and when involving different models with the same angular momentum.

Keywords: magnetic moment, pentaquark, molecular model, diquark, triquark

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I. INTRODUCTION

The quark model is a successful theory that physicists have used to explain the inner structures of mesons and baryons and predict the tetraquark and pentaquark. Over the past decade, significant theoretical and experimental progress has been made in the exploration of multiquark states, with several exotic hadronic states being experimentally observed [1–8].

In 2015, the LHCb Collaboration observed pentaquark states in the $J/\psi p$ invariant mass spectrum of $\Lambda_b^0\to J/\psi K^- p$ decays. The two candidates of the hidden-charm pentaquark are $P_c(4380)$ and $P_c(4450)$, whose J^P has an opposite parity with $(\frac{3}{2}^-,\frac{5}{2}^+)$, [5]. In 2019, the $P_c(4450)$ pentaquark structure was confirmed, and observations revealed that it comprised of two peaks, $P_c(4440)$ and $P_c(4457)$, with a statistical significance of 5.4σ [6]. Meanwhile, the LHCb Collaboration reported a new pentaquark state observation, $P_c(4312)$, with a statistical significance of 7.3σ. In 2021, the LHCb Collaboration found evidence for a new structure, $P_c(4337)$, in $B_s^0\to J/\psi p\bar{p}$ decays, with a final significance of 3.1σ [7]. The mass and width of $P_c(4337)$ are $4337\,^{+7}_{-4}\,^{+2}_{-2}$ MeV and $29\,^{+26}_{-12}\,^{+14}_{-14}$ MeV, respectively, and the parity and angular momentum of $P_c(4337)$ were predicted with $J^P=\frac{1}{2}^+$ [9].

Since discovering P_c states, theorists have shown great interest in explaining the nature of pentaquarks. For instance, in [10], the authors systematically studied the mass spectrum of P_c states using the chromomagnetic model, and in [11], the magnetic moments of P_c states were calculated in different color-flavor structures. In addition, the parity and angular momentum of P_c states were predicted by employing the quark delocalization color screening model [12]. $P_c(4312)$ can be identified as the hidden-charm molecular state $\Sigma_c\bar{D}$ with $J^P=\frac{1}{2}^-$, and $P_c(4440)$ and $P_c(4457)$ can be identified as the hidden-charm molecular states $\Sigma_c\bar{D}^*$ with $J^P=\frac{1}{2}^-$ and $\frac{3}{2}^-$, respectively.

As more exotic hadrons were observed, theorists have attempted to explain their mass spectra using the one-boson-exchange model [13], QCD sum rules [14–19], and effective field theory [20–24]. In general, the inner structure of pentaquarks has been classified as molecular [16, 19, 20, 25–36], diquark-diquark-antiquark [15, 37–40], and diquark-triquark models [41–43].

In 2020, the LHCb Collaboration observed the hidden–charm strange pentaquark $P_{cs}(4459)$ in the $J/\psi\Lambda$ mass spectrum through an amplitude analysis of the $\Xi_b^-\to J/\psi\Lambda K^-$ decay [8]. The mass and width are $4458.8\pm2.9^{+4.7}_{-1.1}$ MeV and $17.3\pm6.5^{+8.0}_{-5.7}$ MeV, respect-

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ively, and after an in-depth study of $P_{cs}(4459)$, the structure was proved to have two resonances, with masses of 4454.9 ± 2.7 MeV and 4467.8 ± 3.7 MeV and widths of 7.5 ± 9.7 MeV and 5.2 ± 5.3 MeV, respectively. However, the parity and angular momentum of $P_{cs}(4459)$ have not been determined experimentally. The predictions of $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ have been given based on QCD sum rules [19], the chiral quark model [25], and the strong decay behaviors of $P_{cs}(4459)$ [35].

The encoding of the pentaquark magnetic moment includes helpful details about the charge and magnetization distributions inside hadrons, which assist in analyzing their geometric configurations. In Ref. [44], the author studied the magnetic moments and transition magnetic moments of hidden-charm pentaquark states with coupled channel effects and D wave contributions; this is important because magnetic moments help us understand the inner structure of pentaquarks. In this study, we calculate the magnetic moments of P_{cs} based on the above three models.

The remainder of this paper is organized as follows. Sec. II discusses the color factor and color configuration, and Sec. III introduces the wave function ofP_{cs}. Sec. IV calculates the magnetic moments of P_{cs} in the molecular, diquark-diquark-antiquark, and diquark-triquark models. Finally, Sec. V summarizes this study.

II. COLOR FACTOR AND COLOR CONFIGURATION

The quark level involves chromomagnetic interactions. Therefore, we use the color factor f to indicate whether the color force is attractive or repulsive.

Regarding the quark-quark color interaction, the color factor f is

$$f(ik \to jl) = \frac{1}{4}\sum_{a=1}^{8}\lambda^a_{ji}\lambda^a_{lk}, \tag{1}$$

where λ^a denotes Gell-Mann matrices, and the quark colors are labeled by i, j, k, and l. The potential is

$$V_{qq}(r) \approx +f\frac{\alpha_s}{r}. \tag{2}$$

Considering the quark-antiquark color interaction, the color factor $\widetilde{f}$ is

$$\widetilde{f}(ik \to jl) = -\frac{1}{4}\sum_{a=1}^{8}\lambda^a_{ji}\lambda^a_{lk}. \tag{3}$$

The potential is

$$V_{q\bar{q}}(r) \approx +\widetilde{f}\frac{\alpha_s}{r}. \tag{4}$$

In Table 1, we list the color factors of the multiplet in the $SU(3)$ color representation.

Table 1. Color factor values for color representation.

$3_c \otimes \bar{3}_c$	$1_c \oplus 8_c$
color factor	$-\frac{4}{3}$ $\frac{1}{6}$
$3_c \otimes 3_c$	$6_c \oplus \bar{3}_c$
color factor	$\frac{1}{3}$ $-\frac{2}{3}$
$3_c \otimes 3_c \otimes 3_c$	$1_c \oplus 8_{1c} \oplus 8_{2c} \oplus 10_c$
color factor	-2 $-\frac{1}{2}$ $-\frac{1}{2}$ 1
$3_c \otimes 3_c \otimes \bar{3}_c$	$3_{1c} \oplus 3_{2c} \oplus \bar{6}_c \oplus 15_c$
color factor	$-\frac{4}{3}$ $-\frac{4}{3}$ $-\frac{1}{3}$ 2

Color confinement implies that physical hadrons are singlets. Under this restriction, we divide the pentaquark states into the following three categories:

1. Molecular model

Each cluster of the molecular model forms a quasi-bound cluster. In other words, clusters of the molecular model tend to be color singlets. We observe that $f_{1_c} < f_{8_c}$ in the color representation of the quark and antiquark; hence, it is easier to form a singlet state than octet states. Similarly, $f_{1_c} < f_{8_{1c}}/f_{8_{2c}} < f_{10_c}$ in the three-quark color representation, and thus it is easier to form a singlet state than other states. Therefore, from the molecular model, we have two configurations, $(c\bar{c})(q_1q_2q_3)$ and $(\bar{c}q_1)(cq_2q_3)$, where q denotes the u, d, s quark.

2. Diquark-Diquark-antiquark model

The diquark prefers to form $\bar{3}_c$ by comparing the color factors of 6_c and $\bar{3}_c$. Similarly, $\bar{3}_c \otimes \bar{3}_c$ prefers to form 3_c. Hence, we have $\bar{3}_c(\mathcal{D}) \otimes \bar{3}_c(\mathcal{D}) \otimes \bar{3}_c(\mathcal{A})$ to form a color singlet, where $\mathcal{D}$ and $\mathcal{A}$ represent the diquark and antiquark, respectively. Thus, the pentaquark configuration is $(cq_1)(q_2q_3)(\bar{c})$, represented by the diquark-diquark-antiquark model.

3. Diquark-triquark model

The triquark involves two quarks and an antiquark, which distinguish it from the molecule model. In this case, $f_{3_{1c}}/f_{3_{2c}} < f_{\bar{6}_c} < f_{15_c}$ is the color representation of the triquark quark, and we have $3_c(\mathcal{T}) \otimes \bar{3}_c(\mathcal{D})$ to form a color singlet, where $\mathcal{T}$ represents a triquark. Thus, the pentaquark configurations represented by the diquark-triquark model are $(c\bar{c}q_1)(q_2q_3)$ and $(cq_1)(\bar{c}q_2q_3)$.

The separation of c and $\bar{c}$ into distinct confinement volumes provides a natural suppression mechanism for the pentaquark widths [6]. Thus, we do not consider $(\bar{c}c)(q_1q_2q_3)$ and $(\bar{c}cq_1)(q_2q_3)$.

III. WAVE FUNCTION OF HIDDEN-CHARM STRANGE PENTAQUARK STATES

In this study, we investigate pentaquark states in the $SU(3)_f$ frame. The overall wavefunction for a bounded multiquark state, while accounting for all degrees of freedom, can be written as

$$\psi_{\text{wavefunction}} = \phi_{\text{flavor}}\chi_{\text{spin}}\varepsilon_{\text{color}}\eta_{\text{space}}.$$

Owing to Fermi statistics, the overall wavefunction above must be antisymmetric.

The molecular model of the pentaquark is composed of mesons and baryons, which must be color singlets because of color confinement. The relationship between spin and flavor is $\phi_{\text{flavor}}\chi_{\text{spin}}$ = symmetric because the color wavefunction is antisymmetric and the spatial wavefunction is symmetric in the ground state. We study the P_{cs} state in a $SU(3)_f$ frame. There are two configurations for q_2q_3, where q_2q_3 forms the $\bar{3}_f$ and 6_f flavor representations with the total spin S = 0 and 1, respectively. When q_2q_3 forms 6_f, it is combined with q_1 to form the flavor representation $6_f \otimes 3_f = 10_f \oplus 8_{1f}$, whereas when q_2q_3 forms $\bar{3}_f$, it is combined with q_1 to form the flavor representation $\bar{3}_f \otimes 3_f = 8_{2f} \oplus 1_f$. After inserting $[c\bar{c}]$ and the Clebsch-Gordan coefficients, we apply the same method to the $(cq_1)(q_2q_3)(\bar{c})$ and $(cq_1)(\bar{c}q_2q_3)$configurations and obtain the flavor wave function ofP_{cs} in 8_{1f} and 8_{2f}. The results are reported in Table 2.

IV. MAGNETIC MOMENTS OF THE HIDDENCHARM STRANGE PENTAQUARK

A. Magnetic moments of the molecular model with the configuration$(\bar{c}q_1)(cq_2q_3)$

Because quarks are fundamental Dirac fermions, the operators of the total magnetic moments and z-component are

$$\hat{\mu} = Q\frac{e}{m}\hat{S}, \qquad \hat{\mu}_z = Q\frac{e}{m}\hat{S}_z. \tag{5}$$

As mentioned above, we do not consider the orbital excitation in the bound state; hence, the orbital excitation lies between the meson and baryon. The total magnetic moment formula can be written as

$$\hat{\mu} = \hat{\mu}_{\mathcal{B}} + \hat{\mu}_{\mathcal{M}} + \hat{\mu}_l, \tag{6}$$

where the subscripts $\mathcal{B}$ and $\mathcal{M}$ represent the baryon and meson, respectively, and l is the orbital excitation between the meson and baryon. The specific forms of the

Table 2. Flavor wave function of hidden–charm strange pentaquark states in different models.

Model	Multiplet	(I,I_3)	Wave function
Molecular model	8_{1f}	(1,0)	$\frac{1}{\sqrt{6}}[(\bar{c}d)(c\{us\})+(\bar{c}u)(c\{ds\})]-\sqrt{\frac{2}{3}}(\bar{c}s)(c\{ud\})$
		(0,0)	$\frac{1}{\sqrt{2}}[(\bar{c}u)(c\{ds\})-(\bar{c}d)(c\{us\})]$
	8_{2f}	(1,0)	$\frac{1}{\sqrt{2}}\{(\bar{c}d)(c[us])+(\bar{c}u)(c[ds])\}$
		(0,0)	$\frac{1}{\sqrt{6}}\{(\bar{c}d)(c[us])-(\bar{c}u)(c[ds])-2(\bar{c}s)(c[ud])\}$
Diquark-diquark-antiquark model	8_{1f}	(1,0)	$\frac{1}{\sqrt{6}}[(cd)\{us\}\bar{c}+(cu)\{ds\}\bar{c}]-\sqrt{\frac{2}{3}}(cs)\{ud\}\bar{c}$
		(0,0)	$\frac{1}{\sqrt{2}}[(cu)\{ds\}\bar{c}-(cd)\{us\}\bar{c}]$
	8_{2f}	(1,0)	$\frac{1}{\sqrt{2}}\{(cd)[us]\bar{c}+(cu)[ds]\bar{c}\}$
		(0,0)	$\frac{1}{\sqrt{6}}\{(cd)[us]\bar{c}-(cu)[ds]\bar{c}-2(cs)[ud]\bar{c}\}$
Diquark-triquark model	8_{1f}	(1,0)	$\frac{1}{\sqrt{6}}[(cd)(\bar{c}\{us\})+(cu)(\bar{c}\{ds\})]-\sqrt{\frac{2}{3}}(cs)(\bar{c}\{ud\})$
		(0,0)	$\frac{1}{\sqrt{2}}[(cu)(\bar{c}\{ds\})-(cd)(\bar{c}\{us\})]$
	8_{2f}	(1,0)	$\frac{1}{\sqrt{2}}\{(cd)(\bar{c}[us])+(cu)(\bar{c}[ds])\}$
		(0,0)	$\frac{1}{\sqrt{6}}\{(cd)(\bar{c}[us])-(cu)(\bar{c}[ds])-2(cs)(\bar{c}[ud])\}$

magnetic moments can be written as

$$\hat{\mu}_{\mathcal{B}} = \sum_{i=1}^{3} \mu_i g_i \hat{S}_i, \tag{7}$$

$$\hat{\mu}_{\mathcal{M}} = \sum_{i=1}^{2} \mu_i g_i \hat{S}_i, \tag{8}$$

$$\hat{\mu}_l = \mu_l \hat{l} = \frac{M_{\mathcal{M}}\mu_{\mathcal{B}} + M_{\mathcal{B}}\mu_{\mathcal{M}}}{M_{\mathcal{M}} + M_{\mathcal{B}}}\hat{l}, \tag{9}$$

where g_i is the Lande factor, and $M_{\mathcal{M}}$ and $M_{\mathcal{B}}$ are the meson and baryon masses, respectively. The $(\bar{c}q_1)(cq_2q_3)$ specific magnetic moment formula of the pentaquark in the molecular model is

$$\begin{aligned}\mu =& \langle \psi \,|\, \hat{\mu}_{\mathcal{B}} + \hat{\mu}_{\mathcal{M}} + \hat{\mu}_l \,|\, \psi \rangle = \sum_{SS_z, ll_z} \langle SS_z, ll_z | JJ_z \rangle^2 \\ &\times \Bigg\{ \mu_l l_z + \sum_{\widetilde{S}_{\mathcal{B}}, \widetilde{S}_{\mathcal{M}}} \langle S_{\mathcal{B}}\widetilde{S}_{\mathcal{B}}, S_{\mathcal{M}}\widetilde{S}_{\mathcal{M}} | SS_z \rangle^2 \Bigg[\widetilde{S}_{\mathcal{M}}\Big(\mu_{\bar{c}} + \mu_{q_1}\Big) \\ &+ \sum_{\widetilde{S}_c} \langle S_c\widetilde{S}_c, S_r\widetilde{S}_{\mathcal{B}} - \widetilde{S}_c | S_{\mathcal{B}}\widetilde{S}_{\mathcal{B}} \rangle^2 \\ &\times \Big(g\mu_c\widetilde{S}_c + (\widetilde{S}_{\mathcal{B}} - \widetilde{S}_c)(\mu_{q_2} + \mu_{q_3}) \Big) \Bigg] \Bigg\},\end{aligned} \tag{10}$$

where ψ represents the wave function in Table 2, $S_{\mathcal{M}}$, $S_{\mathcal{B}}$, and S_r are the meson, baryon, and diquark spin inside the baryon, respectively, and $\widetilde{S}$ is the third spin component.

For example, the recently observed $P_{cs}(4459)$ state is supposed to be the $\bar{D}^*\Xi_c$ molecular state in the 8_{2f} representation with $(I, I_3) = (0,0)$. Its flavor wave functions are

$$|P_{cs}\rangle = \frac{1}{\sqrt{6}}\{(\bar{c}d)(c[us]) - (\bar{c}u)(c[ds]) - 2(\bar{c}s)(c[ud])\}. \tag{11}$$

Take $J^P = \frac{1}{2}^-$ $(\frac{1}{2}^+ \otimes 1^- \otimes 0^+)$ as an example. $J_1^{P_1} \otimes J_2^{P_2} \otimes J_3^{P_3}$ correspond to the angular momentum and parity of the baryon, meson, and orbital, respectively.

$$\begin{aligned}\mu =& \langle P_{cs} \,|\, \hat{\mu}_{\mathcal{B}} + \hat{\mu}_{\mathcal{M}} + \hat{\mu}_l \,|\, P_{cs} \rangle \\ =& \langle \frac{1}{2}\frac{1}{2}, 10 | \frac{1}{2}\frac{1}{2} \rangle^2 \Bigg[\langle \frac{1}{2}\frac{1}{2}, 00 | \frac{1}{2}\frac{1}{2} \rangle^2 \Bigg(\frac{1}{6} * \frac{1}{2} g\mu_c + \frac{1}{6} * \frac{1}{2} g\mu_c + \frac{4}{6} * \frac{1}{2} g\mu_c \Bigg) \Bigg] \\ &+ \langle \frac{1}{2} - \frac{1}{2}, 11 | \frac{1}{2}\frac{1}{2} \rangle^2 \Bigg[\Bigg(\frac{1}{6} * (\frac{1}{2} g\mu_{\bar{c}} + \frac{1}{2} g\mu_d) + \frac{1}{6} * (\frac{1}{2} g\mu_{\bar{c}} + \frac{1}{2} g\mu_u) + \frac{4}{6} * (\frac{1}{2} g\mu_{\bar{c}} + \frac{1}{2} g\mu_s) \Bigg) \\ &+ \langle \frac{1}{2} - \frac{1}{2}, 00 | \frac{1}{2} - \frac{1}{2} \rangle^2 \Bigg(\frac{1}{6} * -\frac{1}{2} g\mu_c + \frac{1}{6} * -\frac{1}{2} g\mu_c + \frac{4}{6} * -\frac{1}{2} g\mu_c \Bigg) \Bigg] \\ =& \frac{1}{9}(\mu_u + \mu_d + 4\mu_s + 6\mu_{\bar{c}} - 3\mu_c).\end{aligned} \tag{12}$$

In this study, we use the following constituent quark masses [45]:

$$m_u = m_d = 0.336 \text{ GeV},$$
$$m_s = 0.540 \text{ GeV},$$
$$m_c = 1.660 \text{ GeV}.$$

The numerical results with isospin $(I, I_3) = (1,0)$ and $(I, I_3) = (0,0)$ are shown in Tables 3 and 4, respectively.

B. Magnetic moments of the diquark-diquark-antiquark model with the $(cq_1)(q_2q_3)\bar{c}$ configuration

In the diquark-diquark-antiquark model, there are two P-wave excitation modes inside the three-body bound state: ρ and λ excitation. ρ mode P-wave orbital excitation lies between the diquark (cq_1) and diquark (q_2q_3), whereas λ mode P-wave orbital excitation lies between $\bar{c}$ and the center of mass system of (cq_1) and (q_2q_3).

The total magnetic moment formula of the diquark-diquark-antiquark model can be written as

$$\hat{\mu} = \hat{\mu}_H + \hat{\mu}_L + \hat{\mu}_{\bar{c}} + \hat{\mu}_l, \tag{13}$$

where the subscripts H and L represent heavy (cq_1) and light diquarks (q_2q_3), respectively, and l is the orbital excitation. In the diquark-diquark-antiquark model, the specific magnetic moment formula of the pentaquark $(cq_1)(q_2q_3)\bar{c}$ is

Table 3. Magnetic moments of pentaquark states in the molecular model with the wavefunction $\frac{1}{\sqrt{6}}[(\bar{c}d)(c\{us\})+(\bar{c}u)(c\{ds\})]-\sqrt{\frac{2}{3}}(\bar{c}s)(c\{ud\})$ in 8_{1f} and $\frac{1}{\sqrt{2}}\{(\bar{c}d)(c[us])+(\bar{c}u)(c[ds])\}$ in 8_{2f} with isospin $(I,I_3)=(1,0)$. They are in the 8_{1f} representation from $6_f\otimes 3_f=10_f\oplus 8_{1f}$ and the 8_{2f} representation from $\bar{3}_f\otimes 3_f=1_f\oplus 8_{2f}$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}$ correspond to the angular momentum and parity of the baryon, meson, and orbital, respectively. The unit is proton magnetic moments.

$8_{1f}:\ \frac{1}{\sqrt{6}}[(\bar{c}d)(c\{us\})+(\bar{c}u)(c\{ds\})]-\sqrt{\frac{2}{3}}(\bar{c}s)(c\{ud\})$

	${}^2S_{\frac{1}{2}}\ (J^P=\frac{1}{2}^-)$			${}^4S_{\frac{3}{2}}\ (J^P=\frac{3}{2}^-)$			${}^6S_{\frac{5}{2}}\ (J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{3}{2}^+\otimes 1^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{3}{2}^+\otimes 0^-\otimes 0^+$	$\frac{3}{2}^+\otimes 1^-\otimes 0^+$	$\frac{3}{2}^+\otimes 1^-\otimes 0^+$
$(0,1,0)$	0.263	−0.493	0.735	−0.345	0.959	0.460	0.352

	${}^2P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$			${}^4P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$		
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$\frac{3}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$
$(0,1,0)$	−0.145	0.125	−0.289	0.564	−0.172	0.278

	${}^2P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$			${}^4P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$			${}^6P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$\frac{3}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{5}{2}}\otimes 1^-$
$(0,1,0)$	0.177	−0.551	0.669	0.666	−0.276	0.311	0.335

	${}^4P_{\frac{5}{2}}\ (J^P=\frac{5}{2}^+)$			${}^6P_{\frac{5}{2}}\ (J^P=\frac{5}{2}^+)$	${}^6P_{\frac{7}{2}}\ (J^P=\frac{7}{2}^+)$
(Y,I,I_3)	$\frac{1}{2}^+\otimes 1^-\otimes 1^-$	$\frac{3}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{5}{2}}\otimes 1^-$	$\frac{3}{2}^+\otimes 1^-\otimes 1^-$
$(0,1,0)$	−0.403	0.865	0.394	0.292	0.285

$8_{2f}:\ \frac{1}{\sqrt{2}}\{(\bar{c}d)(c[us])+(\bar{c}u)(c[ds])\}$

	${}^2S_{\frac{1}{2}}\ (J^P=\frac{1}{2}^-)$		${}^4S_{\frac{3}{2}}\ (J^P=\frac{3}{2}^-)$	${}^2P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$		${}^4P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$
$(0,1,0)$	0.377	−0.067	0.465	−0.167	−0.007	0.273

	${}^2P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$		${}^4P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$	${}^4P_{\frac{5}{2}}\ (J^P=\frac{5}{2}^+)$
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 0^-]_{\frac{3}{2}}\otimes 1^-$	$\frac{1}{2}^+\otimes 1^-\otimes 1^-$	$\frac{1}{2}^+\otimes 1^-\otimes 1^-$
$(0,1,0)$	0.315	−0.110	0.324	0.422

$$\mu=\langle\psi|\hat{\mu}_H+\hat{\mu}_L+\hat{\mu}_{\bar{c}}+\hat{\mu}_l|\psi\rangle=\sum_{S_z,l_z}\langle SS_z,ll_z|JJ_z\rangle^2\Bigg\{\mu_l l_z+\sum_{\widetilde{S}_{\bar{c}}}\langle S_{\bar{c}}\widetilde{S}_{\bar{c}},S_{\mathcal{G}}\widetilde{S}_{\mathcal{G}}|SS_z\rangle^2\Big[g\widetilde{S}_{\bar{c}}\mu_{\bar{c}}$$

$$+\sum_{\widetilde{S}_H,\widetilde{S}_L}\langle S_H\widetilde{S}_H,S_L\widetilde{S}_L|S_{\mathcal{G}}\widetilde{S}_{\mathcal{G}}\rangle^2\Big(\widetilde{S}_H(\mu_c+\mu_{q_1})+\widetilde{S}_L(\mu_{q_2}+\mu_{q_3})\Big)\Big]\Bigg\},\tag{14}$$

where $S_{\mathcal{G}}$ represents the spin of $(cq_1)(q_2q_3)$. The diquark masses are [46]

Table 4. Magnetic moments of pentaquark states in the molecular model with the wavefunction $\frac{1}{\sqrt{2}}[(\bar{c}u)(c\{ds\})-(\bar{c}d)(c\{us\})]$ in 8_{1f} and $\frac{1}{\sqrt{6}}\{(\bar{c}d)(c[us])-(\bar{c}u)(c[ds])-2(\bar{c}s)(c[ud])\}$ in 8_{2f} with isospin $(I,I_3)=(0,0)$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}$ correspond to the angular momentum and parity of the baryon, meson, and orbital, respectively. The unit is proton magnetic moments.

8_{1f}: $\frac{1}{\sqrt{2}}[(\bar{c}u)(c\{ds\})-(\bar{c}d)(c\{us\})]$							
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$			$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$			$^6S_{\frac{5}{2}}(J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{3}{2}^+\otimes 1^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{3}{2}^+\otimes 0^-\otimes 0^+$	$\frac{3}{2}^+\otimes 1^-\otimes 0^+$	$\frac{3}{2}^+\otimes 1^-\otimes 0^+$
(0,0,0)	−0.201	0.126	0.117	−0.113	0.263	0.228	0.352
	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$			$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$			
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$\frac{3}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	
(0,0,0)	0.021	−0.076	−0.076	−0.046	0.171	0.145	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$			$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$			$^6P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$\frac{3}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{5}{2}}\otimes 1^-$
(0,0,0)	−0.270	0.075	0.061	−0.103	0.163	0.145	0.329
	$^4P_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			$^6P_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$	$^6P_{\frac{7}{2}}(J^P=\frac{7}{2}^+)$		
(Y,I,I_3)	$\frac{1}{2}^+\otimes 1^-\otimes 1^-$	$\frac{3}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^+\otimes 1^-]_{\frac{5}{2}}\otimes 1^-$	$\frac{3}{2}^+\otimes 1^-\otimes 1^-$		
(0,0,0)	−0.164	0.189	0.172	0.295	0.296		
8_{2f}: $\frac{1}{\sqrt{6}}\{(\bar{c}d)(c[us])-(\bar{c}u)(c[ds])-2(\bar{c}s)(c[ud])\}$							
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$		$^4S\frac{3}{2}(J^P=\frac{3}{2}^-)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{1}{2}}\otimes 1^-$	$[\frac{1}{2}^+\otimes 1^-]_{\frac{3}{2}}\otimes 1^-$	
(0,0,0)	0.377	−0.531	−0.231	−0.161	0.152	−0.116	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			
(Y,I,I_3)	$\frac{1}{2}^+\otimes 0^-\otimes 1^-$	$[\frac{1}{2}^+\otimes 0^-]_{\frac{3}{2}}\otimes 1^-$	$\frac{1}{2}^+\otimes 1^-\otimes 1^-$	$\frac{1}{2}^+\otimes 1^-\otimes 1^-$			
(0,0,0)	0.324	−0.568	−0.184	−0.268			

$[u,d]=710$ MeV, $\{u,d\}=909$ MeV, $[u,s]=948$ MeV,
$\{u,s\}=1069$ MeV, $[c,q]=1973$ MeV, $\{c,q\}=2036$ MeV,
$[c,s]=2091$ MeV, $\{c,s\}=2158$ MeV.

The numerical results for states with the ρ excitation mode and isospin $(I,I_3)=(1,0)$ and $(I,I_3)=(0,0)$ are presented in Tables 5 and 6, respectively. The numerical results for states with the λ excitation mode and isospin $(I,I_3)=(1,0)$ and $(I,I_3)=(0,0)$ are presented in Tables 7 and 8, respectively.

C. Magnetic moments of the diquark-triquark model with the configuration $(cq_1)(\bar{c}q_2q_3)$

Considering the diquark-triquark model, the total magnetic moment formula is

$$\hat{\mu}=\hat{\mu}_{\mathcal{D}}+\hat{\mu}_{\mathcal{T}}+\hat{\mu}_l. \tag{15}$$

where l is the orbital excitation between the diquark and triquark. The magnetic moment formula of the

Table 5. Magnetic moments of pentaquark states in the diquark-diquark-antiquark model with the wave function $\frac{1}{\sqrt{6}}[(cd)\{us\}\bar{c}+(cu)\{ds\}\bar{c}]-\sqrt{\frac{2}{3}}(cs)\{ud\}\bar{c}$ in 8_{1f} and $\frac{1}{\sqrt{2}}\{(cd)[us]\bar{c}+(cu)[ds]\bar{c}\}$ in 8_{2f} with isospin $(I,I_3)=(1,0)$. They are in the 8_{1f} representation from $6_f\otimes 3_f=10_f\oplus 8_{1f}$ and the 8_{2f} representation from $\bar{3}_f\otimes 3_f=1_f\oplus 8_{2f}$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}\otimes J_4^{P_4}$ correspond to the angular momentum and parity of (cq_1), (q_2q_3), $\bar{c}$, and the orbital, respectively. ρ mode P-wave orbital excitation lies between the diquark (cq_1) and diquark (q_2q_3). The unit is proton magnetic moments.

8_{1f}: $\frac{1}{\sqrt{6}}[(cd)\{us\}\bar{c}+(cu)\{ds\}\bar{c}]-\sqrt{\frac{2}{3}}(cs)\{ud\}\bar{c}$							
	$^2S_{\frac{1}{2}}\,(J^P=\frac{1}{2}^-)$			$^4S_{\frac{3}{2}}\,(J^P=\frac{3}{2}^-)$			$^6S_{\frac{5}{2}}\,(J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$0^+\otimes1^+\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_0\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(0^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_2\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$
$(0,1,0)$	0.514	−0.377	0.368	0.206	−0.013	0.881	0.352
	$^2P_{\frac{1}{2}}\,(J^P=\frac{1}{2}^+)$	$^4P_{\frac{1}{2}}\,(J^P=\frac{1}{2}^+)$	$^2P_{\frac{1}{2}}\,(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}\,(J^P=\frac{1}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
$(0,1,0)$	−0.035	0.046	0.260	0.012	−0.074	0.422	
	$^2P_{\frac{3}{2}}\,(J^P=\frac{3}{2}^+)$	$^4P_{\frac{3}{2}}\,(J^P=\frac{3}{2}^+)$	$^2P_{\frac{3}{2}}\,(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}\,(J^P=\frac{3}{2}^+)$		$^6P_{\frac{3}{2}}\,(J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$
$(0,1,0)$	0.719	0.233	−0.175	0.570	0.005	0.727	0.174
	$^4P^+_{\frac{5}{2}}\,(J^P=\frac{5}{2}^+)$			$^6P_{\frac{5}{2}}\,(J^P=\frac{5}{2}^+)$	$^6P_{\frac{7}{2}}\,(J^P=\frac{7}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$	$1^+\otimes1^+\otimes\frac{1}{2}^-\otimes1^-$		
$(0,1,0)$	0.410	0.190	1.083	0.369	0.554		
8_{2f}: $\frac{1}{\sqrt{2}}\{(cd)[us]\bar{c}+(cu)[ds]\bar{c}\}$							
	$^2S_{\frac{1}{2}}\,(J^P=\frac{1}{2}^-)$		$^4S_{\frac{3}{2}}\,(J^P=\frac{3}{2}^-)$	$^2P_{\frac{1}{2}}\,(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}\,(J^P=\frac{1}{2}^+)$	
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
$(0,1,0)$	−0.377	0.687	0.465	0.137	−0.224	0.256	
	$^2P_{\frac{3}{2}}\,(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}\,(J^P=\frac{3}{2}^+)$	$^4P^+_{\frac{5}{2}}\,(J^P=\frac{5}{2}^+)$			
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$			
$(0,1,0)$	−0.360	0.695	0.344	0.473			

Table 6. Magnetic moments of pentaquark states in the diquark-diquark-antiquark model with the wave function $\frac{1}{\sqrt{2}}[(cu)\{ds\}\bar{c}-(cd)\{us\}\bar{c}]$ in 8_{1f} and $\frac{1}{\sqrt{6}}\{(cd)[us]\bar{c}-(cu)[ds]\bar{c}-2(cs)[ud]\bar{c}\}$ in 8_{2f} with isospin $(I,I_3)=(0,0)$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}\otimes J_4^{P_4}$ correspond to the angular momentum and parity of (cq_1), (q_2q_3), $\bar{c}$, and the orbital, respectively. ρ mode P-wave orbital excitation lies between the diquark (cq_1) and diquark (q_2q_3).

				8_{1f}: $\frac{1}{\sqrt{2}}[(cu)\{ds\}\bar{c}-(cd)\{us\}\bar{c}]$			
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$			$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$			$^6S_{\frac{5}{2}}(J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$0^+\otimes1^+\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_0\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(0^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_2\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$
$(0,0,0)$	0.050	−0.377	0.368	−0.490	−0.013	0.881	0.352
	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
$(0,0,0)$	0.013	−0.287	0.150	−0.098	−0.019	0.478	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^6P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$
$(0,0,0)$	0.094	−0.342	−0.340	0.405	0.197	0.661	0.273
	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			$^6P_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$	$^6P_{\frac{7}{2}}(J^P=\frac{7}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$	$1^+\otimes1^+\otimes\frac{1}{2}^-\otimes1^-$		
$(0,0,0)$	−0.446	0.024	0.918	0.322	0.388		
				8_{2f}: $\frac{1}{\sqrt{6}}\{(cd)[us]\bar{c}-(cu)[ds]\bar{c}-2(cs)[ud]\bar{c}\}$			
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$		$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
$(0,0,0)$	−0.377	0.223	−0.231	0.292	0.091	−0.211	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$			
$(0,0,0)$	−0.126	0.470	−0.070	0.016			

Table 7. Magnetic moments of pentaquark states in the diquark-diquark-antiquark model with the wave function $\frac{1}{\sqrt{6}}[(cd)\{us\}\bar{c}+(cu)\{ds\}\bar{c}]-\sqrt{\frac{2}{3}}(cs)\{ud\}\bar{c}$ in 8_{1f} and $\frac{1}{\sqrt{2}}\{(cd)[us]\bar{c}+(cu)[ds]\bar{c}\}$ in 8_{2f} with isospin $(I,I_3)=(1,0)$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}\otimes J_4^{P_4}$ correspond to the angular momentum and parity of (cq_1), (q_2q_3), $\bar{c}$, and the orbital, respectively. λ mode P-wave orbital excitation lies between $\bar{c}$ and the center of mass system of (cq_1) and (q_2q_3).The unit is proton magnetic moments.

8_{1f}: $\frac{1}{\sqrt{6}}[(cd)\{us\}\bar{c}+(cu)\{ds\}\bar{c}]-\sqrt{\frac{2}{3}}(cs)\{ud\}\bar{c}$							
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$			$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$			$^6S_{\frac{5}{2}}(J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$0^+\otimes1^+\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_0\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(0^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_2\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$
$(0,1,0)$	0.514	−0.377	0.368	0.206	−0.013	0.881	0.352
	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
$(0,1,0)$	0.217	−0.080	0.507	0.259	−0.198	0.299	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^6P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$
$(0,1,0)$	1.096	0.384	0.196	0.941	0.220	0.875	−0.048
	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			$^6P_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$	$^6P_{\frac{7}{2}}(J^P=\frac{7}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$	$1^+\otimes1^+\otimes\frac{1}{2}^-\otimes1^-$		
$(0,1,0)$	0.788	0.560	1.454	0.475	0.924		
8_{2f}: $\frac{1}{\sqrt{2}}\{(cd)[us]\bar{c}+(cu)[ds]\bar{c}\}$							
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$		$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
$(0,1,0)$	−0.377	0.687	0.465	0.525	0.164	0.062	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$			
$(0,1,0)$	0.223	1.277	0.577	1.055			

Table 8. Magnetic moments of pentaquark states in the diquark-diquark-antiquark model with the wave function $\frac{1}{\sqrt{2}}[(cu)\{ds\}\bar{c}-(cd)\{us\}\bar{c}]$ in 8_{1f} and $\frac{1}{\sqrt{6}}\{(cd)[us]\bar{c}-(cu)[ds]\bar{c}-2(cs)[ud]\bar{c}\}$ in 8_{2f} with isospin $(I,I_3)=(0,0)$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}\otimes J_4^{P_4}$ correspond to the angular momentum and parity of (cq_1), (q_2q_3), $\bar{c}$, and the orbital, respectively. λ mode P-wave orbital excitation lies between $\bar{c}$ and the center of mass system of (cq_1) and (q_2q_3).

			8_{1f}: $\frac{1}{\sqrt{2}}[(cu)\{ds\}\bar{c}-(cd)\{us\}\bar{c}]$				
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$			$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$			$^6S_{\frac{5}{2}}(J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$0^+\otimes1^+\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_0\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(0^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_1\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)_2\otimes\frac{1}{2}^-\otimes0^+$	$(1^+\otimes1^+)\otimes\frac{1}{2}^-\otimes0^+$
(0,0,0)	0.050	−0.377	0.368	−0.490	−0.013	0.881	0.352
	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
(0,0,0)	0.334	−0.448	0.469	0.221	−0.179	0.318	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^6P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(0^+\otimes1^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_0\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$
(0,0,0)	0.575	−0.150	0.139	0.884	0.072	0.853	−0.014
	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			$^6P_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$	$^6P_{\frac{7}{2}}(J^P=\frac{7}{2}^+)$		
(Y,I,I_3)	$(0^+\otimes1^+\otimes\frac{1}{2}^-)\otimes1^-$	$((1^+\otimes1^+)_1\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$((1^+\otimes1^+)_2\otimes\frac{1}{2}^-)_{\frac{5}{2}}\otimes1^-$	$1^+\otimes1^+\otimes\frac{1}{2}^-\otimes1^-$		
(0,0,0)	0.035	0.503	1.397	0.459	0.867		
			8_{2f}: $\frac{1}{\sqrt{6}}\{(cd)[us]\bar{c}-(cu)[ds]\bar{c}-2(cs)[ud]\bar{c}\}$				
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$		$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes0^+$	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	
(0,0,0)	−0.377	0.223	−0.231	0.616	0.410	−0.370	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			
(Y,I,I_3)	$0^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{1}{2}}\otimes1^-$	$(1^+\otimes0^+\otimes\frac{1}{2}^-)_{\frac{3}{2}}\otimes1^-$	$1^+\otimes0^+\otimes\frac{1}{2}^-\otimes1^-$			
(0,0,0)	0.359	0.949	0.121	0.495			

Table 9. Magnetic moments of pentaquark states in the diquark-triquark model with the wave function $\frac{1}{\sqrt{6}}[(cd)(\bar{c}\{us\})+(cu)(\bar{c}\{ds\})]-\sqrt{\frac{2}{3}}(cs)(\bar{c}\{ud\})$ in 8_{1f} and $\frac{1}{\sqrt{2}}\{(cd)(\bar{c}[us])+(cu)(\bar{c}[ds])\}$ in 8_{2f} with isospin $(I,I_3)=(1,0)$. They are in the 8_{1f} representation from $6_f\otimes3_f=10_f\oplus8_{1f}$ and the 8_{2f} representation from $\bar{3}_f\otimes3_f=1_f\oplus8_{2f}$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}$ correspond to the angular momentum and parity of the triquark, diquark, and orbital, respectively. The unit is proton magnetic moments.

8_{1f}: $\frac{1}{\sqrt{6}}[(cd)(\bar{c}\{us\})+(cu)(\bar{c}\{ds\})]-\sqrt{\frac{2}{3}}(cs)(\bar{c}\{ud\})$							
	${}^2S_{\frac{1}{2}}\ (J^P=\frac{1}{2}^-)$			${}^4S_{\frac{3}{2}}\ (J^P=\frac{3}{2}^-)$			${}^6S_{\frac{5}{2}}^-\ (J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$\frac{1}{2}^-\otimes0^+\otimes0^+$	$\frac{1}{2}^-\otimes1^+\otimes0^+$	$\frac{3}{2}^-\otimes1^+\otimes0^+$	$\frac{1}{2}^-\otimes1^+\otimes0^+$	$\frac{3}{2}^-\otimes0^+\otimes0^+$	$\frac{3}{2}^-\otimes1^+\otimes0^+$	$\frac{3}{2}^-\otimes1^+\otimes0^+$
$(0,1,0)$	0.522	−0.078	0.051	0.666	0.178	0.188	0.352
	${}^2P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$			${}^4P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$			
(Y,I,I_3)	$\frac{1}{2}^-\otimes0^+\otimes1^-$	$[\frac{1}{2}^-\otimes1^+]_{\frac{1}{2}}\otimes1^-$	$[\frac{3}{2}^-\otimes1^+]_{\frac{1}{2}}\otimes1^-$	$\frac{3}{2}^-\otimes0^+\otimes1^-$	$[\frac{1}{2}^-\otimes1^+]_{\frac{3}{2}}\otimes1^-$	$[\frac{3}{2}^-\otimes1^+]_{\frac{3}{2}}\otimes1^-$	
$(0,1,0)$	−0.137	0.058	0.015	0.080	0.354	0.088	
	${}^2P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$			${}^4P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$			${}^6P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$\frac{1}{2}^-\otimes0^+\otimes1^-$	$[\frac{1}{2}^-\otimes1^+]_{\frac{1}{2}}\otimes1^-$	$[\frac{3}{2}^-\otimes1^+]_{\frac{1}{2}}\otimes1^-$	$\frac{3}{2}^-\otimes0^+\otimes1^-$	$[\frac{1}{2}^-\otimes1^+]_{\frac{3}{2}}\otimes1^-$	$[\frac{3}{2}^-\otimes1^+]_{\frac{3}{2}}\otimes1^-$	$[\frac{3}{2}^-\otimes1^+]_{\frac{5}{2}}\otimes1^-$
$(0,1,0)$	0.577	−0.030	0.098	0.152	0.508	0.157	0.242
	${}^4P_{\frac{5}{2}}\ (J^P=\frac{5}{2}^+)$			${}^6P_{\frac{5}{2}}\ (J^P=\frac{5}{2}^+)$	${}^6P_{\frac{7}{2}}\ (J^P=\frac{7}{2}^+)$		
(Y,I,I_3)	$\frac{1}{2}^-\otimes1^+\otimes1^-$	$\frac{3}{2}^-\otimes0^+\otimes1^-$	$[\frac{3}{2}^-\otimes1^+]_{\frac{3}{2}}\otimes1^-$	$[\frac{3}{2}^-\otimes1^+]_{\frac{5}{2}}\otimes1^-$	$\frac{3}{2}^-\otimes1^+\otimes1^-$		
$(0,1,0)$	0.714	0.233	0.236	0.299	0.370		
8_{2f}: $\frac{1}{\sqrt{2}}\{(cd)(\bar{c}[us])+(cu)(\bar{c}[ds])\}$							
	${}^2S_{\frac{1}{2}}\ (J^P=\frac{1}{2}^-)$		${}^4S_{\frac{3}{2}}\ (J^P=\frac{3}{2}^-)$	${}^2P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$		${}^4P_{\frac{1}{2}}\ (J^P=\frac{1}{2}^+)$	
(Y,I,I_3)	$\frac{1}{2}^-\otimes0^+\otimes0^+$	$\frac{1}{2}^-\otimes1^+\otimes0^+$	$\frac{1}{2}^-\otimes1^+\otimes0^+$	$\frac{1}{2}^-\otimes0^+\otimes1^-$	$[\frac{1}{2}^-\otimes1^+]_{\frac{1}{2}}\otimes1^-$	$[\frac{1}{2}^-\otimes1^+]_{\frac{3}{2}}\otimes1^-$	
$(0,1,0)$	−0.377	0.687	0.465	0.199	−0.184	0.235	
	${}^2P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$		${}^4P_{\frac{3}{2}}\ (J^P=\frac{3}{2}^+)$	${}^4P^+_{\frac{5}{2}}\ (J^P=\frac{5}{2}^+)$			
(Y,I,I_3)	$\frac{1}{2}^-\otimes0^+\otimes1^-$	$[\frac{1}{2}^-\otimes1^+]_{\frac{3}{2}}\otimes1^-$	$\frac{1}{2}^-\otimes1^+\otimes1^-$	$\frac{1}{2}^-\otimes1^+\otimes1^-$			
$(0,1,0)$	−0.307	0.803	0.381	0.558			

Table 10. Magnetic moments of pentaquark states in the diquark-triquark model with the wave function $\frac{1}{\sqrt{2}}[(cu)(\bar{c}\{ds\})-(cd)(\bar{c}\{us\})]$ in 8_{1f} and $\frac{1}{\sqrt{6}}\{(cd)(\bar{c}[us])-(cu)(\bar{c}[ds])-2(cs)(\bar{c}[ud])\}$ in 8_{2f} with isospin $(I,I_3)=(0,0)$. On the third line, $J_1^{P_1}\otimes J_2^{P_2}\otimes J_3^{P_3}$ correspond to the angular momentum and parity of the triquark, diquark, and orbital, respectively. The unit is proton magnetic moments.

8_{1f}: $\frac{1}{\sqrt{2}}[(cu)(\bar{c}\{ds\})-(cd)(\bar{c}\{us\})]$							
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$			$^4S_{\frac{3}{2}}(J^P=\frac{3}{2}^-)$			$^6S^-_{\frac{5}{2}}(J^P=\frac{5}{2}^-)$
(Y,I,I_3)	$\frac{1}{2}^-\otimes 0^+\otimes 0^+$	$\frac{1}{2}^-\otimes 1^+\otimes 0^+$	$\frac{3}{2}^-\otimes 1^+\otimes 0^+$	$\frac{1}{2}^-\otimes 1^+\otimes 0^+$	$\frac{3}{2}^-\otimes 0^+\otimes 0^+$	$\frac{3}{2}^-\otimes 1^+\otimes 0^+$	$\frac{3}{2}^-\otimes 1^+\otimes 0^+$
$(0,0,0)$	0.033	0.574	−0.601	0.910	−0.555	−0.056	0.352
	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$			$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$			
(Y,I,I_3)	$\frac{1}{2}^-\otimes 0^+\otimes 1^-$	$[\frac{1}{2}^-\otimes 1^+]_{\frac{1}{2}}\otimes 1^-$	$[\frac{3}{2}^-\otimes 1^+]_{\frac{1}{2}}\otimes 1^-$	$\frac{3}{2}^-\otimes 0^+\otimes 1^-$	$[\frac{1}{2}^-\otimes 1^+]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^-\otimes 1^+]_{\frac{3}{2}}\otimes 1^-$	
$(0,0,0)$	0.062	−0.126	0.265	0.473	−0.345	−0.064	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$			$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$			$^6P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$
(Y,I,I_3)	$\frac{1}{2}^-\otimes 0^+\otimes 1^-$	$[\frac{1}{2}^-\otimes 1^+]_{\frac{1}{2}}\otimes 1^-$	$[\frac{3}{2}^-\otimes 1^+]_{\frac{1}{2}}\otimes 1^-$	$\frac{3}{2}^-\otimes 0^+\otimes 1^-$	$[\frac{1}{2}^-\otimes 1^+]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^-\otimes 1^+]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^-\otimes 1^+]_{\frac{5}{2}}\otimes 1^-$
$(0,0,0)$	0.143	0.671	−0.503	0.707	−0.363	−0.002	0.212
	$^4P_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$			$^6P_{\frac{5}{2}}(J^P=\frac{5}{2}^+)$	$^6P_{\frac{7}{2}}(J^P=\frac{7}{2}^+)$		
(Y,I,I_3)	$\frac{1}{2}^-\otimes 1^+\otimes 1^-$	$\frac{3}{2}^-\otimes 0^+\otimes 1^-$	$[\frac{3}{2}^-\otimes 1^+]_{\frac{3}{2}}\otimes 1^-$	$[\frac{3}{2}^-\otimes 1^+]_{\frac{5}{2}}\otimes 1^-$	$\frac{3}{2}^-\otimes 1^+\otimes 1^-$		
$(0,0,0)$	1.008	−0.445	0.041	0.313	0.420		
8_{2f}: $\frac{1}{\sqrt{6}}\{(cd)(\bar{c}[us])-(cu)(\bar{c}[ds])-2(cs)(\bar{c}[ud])\}$							
	$^2S_{\frac{1}{2}}(J^P=\frac{1}{2}^-)$		$^4S\frac{3}{2}(J^P=\frac{3}{2}^-)$	$^2P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$		$^4P_{\frac{1}{2}}(J^P=\frac{1}{2}^+)$	
(Y,I,I_3)	$\frac{1}{2}^-\otimes 0^+\otimes 0^+$	$\frac{1}{2}^-\otimes 1^+\otimes 0^+$	$\frac{1}{2}^-\otimes 1^+\otimes 0^+$	$\frac{1}{2}^-\otimes 0^+\otimes 1^-$	$[\frac{1}{2}^-\otimes 1^+]_{\frac{1}{2}}\otimes 1^-$	$[\frac{1}{2}^-\otimes 1^+]_{\frac{3}{2}}\otimes 1^-$	
$(0,0,0)$	−0.377	0.223	−0.231	0.164	−0.035	−0.165	
	$^2P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$		$^4P_{\frac{3}{2}}(J^P=\frac{3}{2}^+)$	$^4P^+_{\frac{5}{2}}(J^P=\frac{5}{2}^+$			
(Y,I,I_3)	$\frac{1}{2}^-\otimes 0^+\otimes 1^-$	$[\frac{1}{2}^-\otimes 1^+]_{\frac{3}{2}}\otimes 1^-$	$\frac{1}{2}^-\otimes 1^+\otimes 1^-$	$\frac{1}{2}^-\otimes 1^+\otimes 1^-$			
$(0,0,0)$	−0.359	0.293	−0.165	−0.196			

Table 11. Magnetic moments of $P_{cs}(4459)$ in the molecular, diquark-diquark-antiquark, and diquark-triquark models in the 8_{2f} representation with isospin $(I,I_3)=(0,0)$.

$P_{cs}(4459)$	Multiplet	Spin-orbit coupling	$I(J^P)$	Magnetic moment	Numerical results
Molecular model	8_{2f}	$\frac{1}{2}^+\otimes 1^-\otimes 0^+$	$0(\frac{1}{2}^-)$	$\frac{1}{9}(6\mu_{\bar{c}}-3\mu_c+\mu_u+\mu_d+4\mu_s)$	−0.531
			$0(\frac{3}{2}^-)$	$\frac{1}{6}(6\mu_c+6\mu_{\bar{c}}+\mu_u+\mu_d+4\mu_s)$	−0.231
Diquark-diquark-antiquark model	8_{2f}	$1^+\otimes 0^+\otimes\frac{1}{2}^-\otimes 0^+$	$0(\frac{1}{2}^-)$	$\frac{1}{9}(6\mu_c-3\mu_{\bar{c}}+\mu_u+\mu_d+4\mu_s)$	0.223
			$0(\frac{3}{2}^-)$	$\frac{1}{6}(6\mu_c+6\mu_{\bar{c}}+\mu_u+\mu_d+4\mu_s)$	−0.231
Diquark-triquark model	8_{2f}	$\frac{1}{2}^-\otimes 1^+\otimes 0^+$	$0(\frac{1}{2}^-)$	$\frac{1}{9}(6\mu_c-3\mu_{\bar{c}}+\mu_u+\mu_d+4\mu_s)$	0.223
			$0(\frac{3}{2}^-)$	$\frac{1}{6}(6\mu_c+6\mu_{\bar{c}}+\mu_u+\mu_d+4\mu_s)$	−0.231

Table 12. Our results and other theoretical results for the magnetic moments of $P_{cs}(4459)$.The unit is proton magnetic moments. A, B, and C correspond to the molecular, diquark-diquark-antiquark, and diquark-triquark models.

Cases	A		B		C	
J^P	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
Our results	−0.531	−0.231	0.223	−0.231	0.223	−0.231
Ref. [44]	−0.062	0.465	−	−	−	−
Ref.	1.75	−	0.34	−	−	−

pentaquark with $(cq_1)(\bar{c}q_2q_3)$ in the diquark-triquark model is

$$\mu=\langle\psi\,|\,\hat{\mu}_{\mathcal{D}}+\hat{\mu}_{\mathcal{T}}+\hat{\mu}_l\,|\,\psi\rangle=\sum_{S_z,l_z}\langle SS_z,ll_z|JJ_z\rangle^2\Bigg\{\mu_l l_z+\sum_{\widetilde{S}_{\mathcal{D}},\widetilde{S}_{\mathcal{T}}}\langle S_{\mathcal{D}}\widetilde{S}_{\mathcal{D}},S_{\mathcal{T}}\widetilde{S}_{\mathcal{T}}|SS_z\rangle^2\Big[\widetilde{S}_{\mathcal{D}}\Big(\mu_c+\mu_{q_1}\Big)$$
$$+\sum_{\widetilde{S}_{\bar{c}}}\langle S_{\bar{c}}\widetilde{S}_{\bar{c}},S_r\widetilde{S}_{\mathcal{T}}-\widetilde{S}_{\bar{c}}|S_{\mathcal{T}}\widetilde{S}_{\mathcal{T}}\rangle^2\Big(g\mu_{\bar{c}}\widetilde{S}_{\bar{c}}+(\widetilde{S}_{\mathcal{T}}-\widetilde{S}_{\bar{c}})(\mu_{q_2}+\mu_{q_3})\Big)\Big]\Bigg\}. \tag{16}$$

where $S_{\mathcal{D}}$, $S_{\mathcal{T}}$, and S_r represent the diquark, triquark, and light diquark spin inside the triquark, respectively. The triquark masses are roughly the sum of the masses of the corresponding diquark and antiquark. The numerical results with isospin $(I,I_3)=(1,0)$ and $(I,I_3)=(0,0)$ are shown in Tables 9 and 10, respectively.

The magnetic moments of $P_{cs}(4459)$ in three configurations are compared, as shown in Table 11. The magnetic moments and numerical results illustrate that the molecular model is distinguishable from the other two models with $0(\frac{1}{2}^-)$ but is indistinguishable with $0(\frac{3}{2}^-)$. The diquark-diquark-antiquark and diquark-triquark models are completely indistinguishable with $0(\frac{1}{2}^-)$ and $0(\frac{3}{2}^-)$. In addition, the magnetic moments of $P_{cs}(4459)$ have been studied in other papers. In Ref. [44], the numerical value in the molecular model was obtained as $\mu_{P_{cs}}=-0.062\mu_N$ with $0(\frac{1}{2}^-)$ and $\mu_{P_{cs}}=0.465\mu_N$ with $0(\frac{3}{2}^-)$. In Ref., the magnetic dipole moments of $P_{cs}(4459)$ in the molecular and diquark-diquark-antiquark models were extracted as $\mu_{P_{cs}}=1.75\mu_N$ and $\mu_{P_{cs}}=0.34\mu_N$, respectively. These numerical results differ from our results of $\mu_{P_{cs}}=-0.531\mu_N$ with $0(\frac{1}{2}^-)$ and $\mu_{P_{cs}}=-0.231\mu_N$ with $0(\frac{3}{2}^-)$ in the molecular model and $\mu_{P_{cs}}=0.223\mu_N$ in the diquark-diquark-antiquark model because of the wave-function and quark mass. We compare the results in Table 12.

V. SUMMARY

Inspired by the recently observed $P_{cs}(4459)$, we systematically calculate the magnetic moments of P_{cs} with $J^P=\frac{1}{2}^\pm,\frac{3}{2}^\pm,\frac{5}{2}^\pm$, and $\frac{7}{2}^+$ in three models: the molecular, diquark-diquark-antiquark, and diquark-triquark models. Comparing the numerical results of the above three models, we observe that the magnetic moments of states with the same quantum numbers are different. Indeed, even within the same model, magnetic moments with different configurations are different. We then compare the magnetic moments of $P_{cs}(4459)$ in three configurations, which have been predicted to involve an S-wave state with $I(J^P)=0(\frac{1}{2}^-)$ and $I(J^P)=0(\frac{3}{2}^-)$. The results show that the molecular model is different from the other two models with $I(J^P)=0(\frac{1}{2}^-)$. These findings highlight that magnetic moments are helpful in determining internal structures when experimental data on P_{cs} keeps accumulating, because magnetic moments encode information about the charge distributions.

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