Precise evaluation of $h \to c\bar{c}$ and axion-like particle production[*]

Shi-Yuan Li(李世渊)[†]   Zhen-Yang Li(栗振洋)[‡]   Peng-Cheng Lu(路鹏程)[§]   Zong-Guo Si(司宗国)[ǂ]

School of Physics, Shandong University, Jinan 250100, China

Abstract: We study the decay of the SM Higgs boson to a massive charm quark pair at the next-to-next-to-leading order QCD and next-to-leading order electroweak. At the second order of QCD coupling, we consider the exact calculation of flavour-singlet contributions where the Higgs boson couples to the internal top and bottom quark. Helpful information on the running mass effects related to Yukawa coupling may be obtained by analyzing this process. High precision production for $h \to c\bar{c}$ within the SM makes it possible to search for new physics that may induce relatively large interactions related to the charm quark. As an example, we evaluate the axion-like particle associate production with a charm quark pair in the Higgs decay and obtain some constraints for the corresponding parameters under some assumptions.

Keywords: Higgs boson, charm quark, QCD, axion-like particle

DOI: 10.1088/1674-1137/ac0c0d

I. INTRODUCTION

Higgs physics plays a significant role in the standard model (SM) testing and search for new physics beyond SM with higher precision. The theoretical and experimental study on the properties of the Higgs boson, including its couplings with SM particles, particularly fermions, has become a primary task of particle physics at the high energy frontier since it was discovered in 2012 at the LHC [1, 2], and considerable efforts have been made. The study of the process of decay of the Higgs boson to heavy quark pairs $b\bar{b}$ and $c\bar{c}$ is important to test the SM and search for high precision new physics. Furthermore, useful information for the running mass effects in the Yukawa coupling can be obtained by analysing these decay processes. The decay width of Higgs boson into massless bottom quarks is known up to the next-to-next-to-next-to-next-to-leading order in QCD [3-5]. The differential decay width for $h \to b\bar{b}$ has been computed to the next-to-next-to-leading order (NNLO) QCD in [6, 7], and next-to-next-to-next-to-leading order in [8] in the limit where the mass of the bottom quark is neglected. The corrections keeping the exact bottom quark mass have been computed to NNLO in [9-12]. The hadronic decays of Higgs boson to the bottom quarks, light quarks, and

gluons at NNLO have been calculated in [13]. For the two loop QCD corrections of $h \to b\bar{b}$, the flavor-singlet contribution from the triangle top and bottom quark loop have to be included. These contributions from the top quark triangle loop are computed approximately using a large quark mass expansions formula [14], and the corresponding exact calculations are also obtained [15]. The calculations at the next-to-leading order (NLO) electroweak (EW) are also completed [16-18]. The measurements of $h \to b\bar{b}$ at LHC are finished [19, 20], and the signal strength $\mu_{h \to b\bar{b}} = 1.01 \pm 0.12 \, (\text{stat.})^{+0.16}_{-0.15} \, (\text{syst.})$ is compatible with SM.

For the evaluation of Higgs boson coupling to the second generation quark, $h \to c\bar{c}$ is a process of great value. Owing to the absence of an observation of Higgs decays to the charm quark, there are some bounds on the charm quark Yukawa coupling. An indirect search for the decay of the Higgs boson to charm quarks via the decay to $J/\psi\gamma$ is presented at the LHC [21-23]. A direct search for the Higgs boson, produced in association with a vector boson (W or Z), and decaying to a charm quark pair is performed by CMS collaboration [24] and ATLAS collaboration [25], respectively. This result is the most stringent limit to date for the inclusive decay of the $h \to c\bar{c}$. Both ATLAS and CMS collaborations present novel

methods for charm-tagging. To test the SM and search for new physics in high precision, the theoretical predictions and experimental measurements on SM Higgs couplings should be completed as precisely as possible. To achieve this, we finish the exact calculation for $h \to c\bar{c}$ up to NNLO QCD including the flavor-singlet contributions from the triangular loops of the bottom and top quarks and those at the NLO EW. However, owing to the discovery of the Higgs boson, the search for additional (pseudo) scalar bosons beyond the SM has attracted increasing interest in collider physics. The axion-like particles (ALPs) are a hypothetical (pseudo) scalar that naturally arises in many extensions of the SM as pseudo Nambu-Goldston bosons. The ALP parameter space has been intensively explored [26-28], covering a wide energy range [29-34]. These experimental searches allow access to several orders of magnitude in the ALP masses and couplings [35], where astrophysics and cosmology impose constraints in the sub-KeV mass range and the most efficient probes in the MeV-GeV range are obtained from experiments acting on the precision frontier [36]. Another aim of this study is to evaluate the ALPs that dominantly couple to the up type quark [37], e.g., charm quark, and obtain some constraints on the corresponding parameters from the ALP associate production with the charm quark pair in the Higgs decay.

This paper is organized as follows. In Sec. II, we study the decay width of $h \to c\bar{c}$ within the SM. First, we use renormalized matrix elements where the QCD coupling is defined in the $\overline{\text{MS}}$ scheme, whereas the charm quark mass and the Yukawa coupling are defined in the on-shell scheme. For the order of α_s^2, we calculate the exact flavor-singlet contributions. To obtain reliable corrections to the decay width of $h \to c\bar{c}$, we express the on-shell Yukawa coupling in terms of the $\overline{\text{MS}}$ Yukawa coupling. At the end of this section, after including the NLO EW corrections, we obtain the decay width of $h \to c\bar{c}$ and compare it with that of $h \to b\bar{b}$. In Sec. III, we evaluate the ALP associate production with the charm quark pair in Higgs decay. Finally, a brief summary is given.

II. PRECISE EVALUATION OF THE $h \to c\bar{c}$ PROCESS

Within the SM, the Higgs interactions with quarks are obtained by considering the Yukawa interactions

$$-\mathcal{L}_Y = \overline{U_R} h_u Q^T (\mathrm{i}\tau_2)\Phi - \overline{D_R} h_d Q^T (\mathrm{i}\tau_2)\tilde{\Phi} + \text{h.c.,} \quad (1)$$

where Φ denotes a complex $SU(2)_L$ doublet Higgs field with hyper charge $Y = 1/2$; τ is the usual 2×2 Pauli matrix; $\tilde{\Phi} = \mathrm{i}\tau_2\Phi^*$. $Q^T = (U_L, D_L)$, where U and D represent three up- and down-type quarks; and $h_{u,d}$ represents Yukawa matrices. By choosing $\Phi = (0, v+h)^T / \sqrt{2}$, we

obtain

$$-\mathcal{L}_{hf\bar{f}} = y_{0,f} h f \bar{f}, \quad f = u, d, s, c, b, t, \quad (2)$$

where $y_{0,f} = m_{0,f}/v$ is the bare Yukawa coupling constant and $m_{0,f}$ is the bare mass of the corresponding quark. The Higgs vacuum expectation value $v = (\sqrt{2}G_F)^{-1/2}$ with G_F the Fermi constant.

In our following analysis, we start by using the renormalized matrix elements where the QCD coupling constant α_s is defined in the $\overline{\text{MS}}$ scheme, whereas the charm quark mass m_c and the related Yukawa coupling constant y_c are defined in the on-shell scheme. Up to NNLO QCD, the decay width of the process $h \to c\bar{c}$ can be written as follows

$$\begin{aligned} \Gamma =& \Gamma_{\text{LO}} + \alpha_s(\mu)\tilde{\Gamma}_1 + \alpha_s^2(\mu)\tilde{\Gamma}_2 \\ &+ \frac{\alpha_s^2(\mu)}{2\pi} \ln\left(\frac{\mu^2}{m_c^2}\right)\left[\frac{11}{6}N_c - \frac{n_f+1}{3}\right]\tilde{\Gamma}_1, \end{aligned} \quad (3)$$

where μ is the renormalization scale. N_c and n_f denote the number of the color and the number of the massless quarks, respectively. The decay width at the leading order (LO) can be expressed as follows

$$\Gamma_{\text{LO}} = N_C \frac{y_c^2}{8\pi m_h^2}(m_h^2 - 4m_c^2)^{3/2}, \quad (4)$$

where m_h and m_c denote the on-shell mass of the Higgs boson and charm quark, respectively. $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ respectively represent the scaled next-to-leading order (NLO) and NNLO decay width of $h \to c\bar{c}$ by the factor out α_s. At the NLO QCD, the scaled decay width $\tilde{\Gamma}_1$ can be expressed as follows [38]

$$\tilde{\Gamma}_1 = \Gamma_{\text{LO}} C_F \delta_1, \quad (5)$$

where $C_F = (N_c^2 - 1)/2/N_c$, and

$$\delta_1 = \frac{1}{\pi}\left\{\frac{A(\beta)}{\beta} - \frac{3 + 34\beta^2 - 13\beta^4}{16\beta^3}\ln(x) - \frac{3(1 - 7\beta^2)}{8\beta^2}\right\}, \quad (6)$$

with $\beta = \sqrt{1 - 4m_c^2/m_h^2}$, $x = (1-\beta)/(1+\beta)$ and

$$\begin{aligned} A(\beta) =& (1+\beta^2)\Big\{4\text{Li}_2(x) + 2\text{Li}_2(-x) + 2\ln(\beta)\ln(x) + \\ & 3\ln\left(\frac{1}{1+\beta}\right)\ln(x)\Big\} - 3\beta\ln\left(\frac{4}{1-\beta^2}\right) - 4\beta\ln(\beta). \end{aligned} \quad (7)$$

At the NLO QCD, the large logarithm related to the ratio of the charm-Higgs mass appears. This effect can be

clearly shown in the limit $m_c \to 0$, i.e.,

$$\delta_1 \simeq \frac{3}{2\pi}\left(\frac{3}{2} - \ln\frac{m_h^2}{m_c^2}\right). \tag{8}$$

The large logarithm $\ln(m_h^2/m_c^2)$ should be absorbed in the running charm quark mass in the Yukawa coupling as discussed below. Γ_{LO}, $\tilde{\Gamma}_1$ and $\tilde{\Gamma}_2$ do not depend on μ. The μ dependence of the last term in Eq. (3) can be obtained from the renormalization group equation.

At the NNLO QCD, the subtraction term of double-real radiation corrections and real-virtual corrections were computed in [39-42]. For the double-virtual corrections, the scaled decay width $\tilde{\Gamma}_2$ receives the flavor-singlet and non-singlet contribution. The flavour-singlet contributions from $h \to c\bar{c}$ (Fig. 1(a)) and $h \to c\bar{c}g$ (Fig. 1(b)) are both UV and IR finite separately. Because we keep the non-zero internal quark mass $m_f(f = c, b, t)$, these three kinds of massive quarks, which couple to the Higgs in the triangle loop, contribute to the decay width of $h \to c\bar{c}$. For the calculation of the case where the Higgs couples to the internal charm quark, i.e., equal masses associated with the inner and outer fermion line, and that of the flavor non-singlet contribution, the strategy and formulas can be found in [9, 43-46]. Here, we focus on discussing the flavor-singlet contribution to $h \to c\bar{c}$ where the Higgs boson couples to the bottom or top quark. For Fig. 1(a) with $Q' = b$ or t, we calculate the exact results using the formula and techniques of [15] where the Mathematica packages PolyLogTools [47], HPL, and GINAC library are necessary. For Fig. 1(b), we finish the calculation independently. In our numerical analysis, we choose $m_h = 125.09$ GeV, $v \simeq 246.2$ GeV, $m_c = 1.68$ GeV, $m_b = 4.78$ GeV, $m_t = 173.34$ GeV, $\alpha_s(m_Z) = 0.118$, and $m_Z = 91.2$ GeV [48]. In Table 1, we list our results $\tilde{\Gamma}_{Q'}^{\text{FS}}$ for the flavor-singlet contribution with $Q' = b$, t where α_s^2 is factored out in the partial decay width. It is found that $\tilde{\Gamma}_b^{\text{FS}}$ is considerably smaller than $\tilde{\Gamma}_t^{\text{FS}}$. For the case where top quark appears in the triangle loop, $m_c \leqslant m_h \leqslant m_t$, and $\tilde{\Gamma}_t^{\text{FS}}$ can be calculated to the leading order in the charm quark mass as an expansion in the inverse powers of the top quark mass [14]. It is found that under this approximation, $\tilde{\Gamma}_t^{\text{FS}} \simeq 0.7417$ MeV which is very close to the exact result. For the triangular loop related to the bottom quark, this approximation is unreliaable. At the NNLO QCD, the other two interesting sub-processes are $h \to c\bar{c}c\bar{c}$ and $h \to b\bar{b}c\bar{c}$. The results for the scaled decay width $\tilde{\Gamma}_{c\bar{c}c\bar{c}}$ and

$\tilde{\Gamma}_{b\bar{b}c\bar{c}}$ where α_s^2 is also factored out are listed in Table 1. Obviously, $\tilde{\Gamma}_{c\bar{c}c\bar{c}}$ is significantly smaller than $\tilde{\Gamma}_{b\bar{b}c\bar{c}}$, but for $\tilde{\Gamma}_{b\bar{b}c\bar{c}}$, the contribution from the interaction between the Higgs boson and bottom quark is dominant. As a result, the contribution from $h \to b\bar{b}c\bar{c}$ should not be included in the decay width of $h \to c\bar{c}$. Up to the NNLO QCD, the ratio of the flavor-singlet contribution with $Q' = b$, t to the total decay width is approximately 4%.

It is known that under the on-shell renormalization scheme for the Yukawa coupling between the SM Higgs and massive quarks, the large logarithm of the fermion-Higgs boson mass ratio can be obtained in the radiative corrections [49]. Therefore, it is not a good idea to study the decay width in terms of the on-shell Yukawa coupling when $m_c/m_h \ll 1$. When using the $\overline{\text{MS}}$ Yukawa coupling

$$\bar{y}_c(\mu) = \bar{m}_c(\mu)/v, \tag{9}$$

where $\bar{m}_c(\mu)$ is the running $\overline{\text{MS}}$ mass and $\mu = m_h$; this kind of large logarithmic effects can be reduced by effectively absorbing all relevant large logarithms into the running mass related to the Yukawa coupling constant. In this study, to obtain reliable corrections to the decay width of $h \to c\bar{c}$, we converted Yukawa coupling constant y_c defined in the on-shell scheme to $\bar{y}_c$ in the $\overline{\text{MS}}$ scheme. The relation between on-shell mass m_c and $\overline{\text{MS}}$ mass $\bar{m}_c(\mu)$ can be written as follows [50]:

$$m_c = \bar{m}_c(\mu)\left[1 + c_1(m_c,\mu)\frac{\alpha_s(\mu)}{\pi} + c_2(m_c,\mu)\left(\frac{\alpha_s(\mu)}{\pi}\right)^2\right] + O(\alpha_s^3), \tag{10}$$

where

$$c_1 = C_{\text{F}}\left(1 + \frac{3}{4}L_c\right),$$

$$c_2 = C_{\text{F}}^2\left[\frac{121}{128} + 3\zeta(2)\left(\frac{5}{8} - \ln 2\right) + \frac{3}{4}\zeta(3) + \frac{27}{32}L_c + \frac{9}{32}L_c^2\right]$$
$$- N_{\text{C}}C_{\text{F}}\left[-\frac{1111}{384} + \frac{\zeta(2)}{2}(1 - 3\ln 2) + \frac{3}{8}\zeta(3) - \frac{185}{96}L_c\right.$$
$$\left.-\frac{11}{32}L_c^2\right] - C_{\text{F}}T_{\text{F}}n_f\left[\frac{71}{96} + \frac{1}{2}\zeta(2) + \frac{13}{24}L_c + \frac{1}{8}L_c^2\right]$$
$$- C_{\text{F}}T_{\text{F}}\left[\frac{143}{96} - \zeta(2) + \frac{13}{24}L_c + \frac{1}{8}L_c^2\right], \tag{11}$$

with $T_{\text{F}} = 1/2$, $L_c = \ln(\mu^2/m_c^2)$, $\zeta(2) = \pi^2/6$ and $\zeta(3) = 1.20205690\cdots$.

To compute the $\overline{\text{MS}}$ Yukawa coupling $\bar{y}_c(\mu)$ for the arbitrary scale μ, we use the solution of the renormaliza-

Table 1. Results for the scaled decay width $\tilde{\Gamma}_b^{FS}$, $\tilde{\Gamma}_t^{FS}$, $\tilde{\Gamma}_{c\bar{c}c\bar{c}}$ and $\tilde{\Gamma}_{b\bar{b}c\bar{c}}$.

| $\tilde{\Gamma}_b^{\text{FS}}/\text{MeV}$	$\tilde{\Gamma}_t^{\text{FS}}/\text{MeV}$	$\tilde{\Gamma}_{c\bar{c}c\bar{c}}/\text{MeV}$	$\tilde{\Gamma}_{b\bar{b}c\bar{c}}/\text{MeV}$
−0.0482	0.7411	0.4262	3.4495
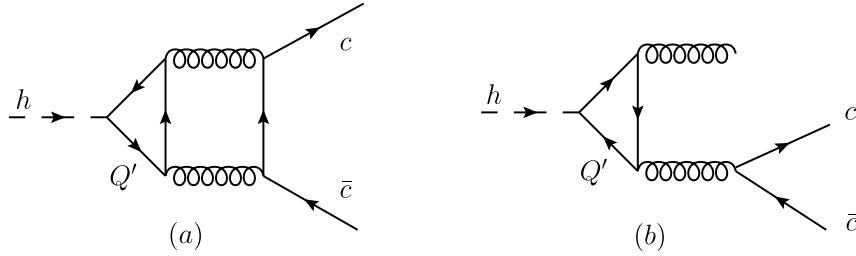
Fig. 1.　The flavor-singlet $O(\alpha_s^2)$ contributions to $h \to c\bar{c}$ (a) and $h \to c\bar{c}g$ (b) with $Q' = t, b, c$.

tion group equation for $\overline{m}_c(\mu)$ at two-loops

$$\overline{m}_c(\mu) = \overline{m}_c(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{1/\beta_0}\left\{1 + \frac{(\gamma_1/\beta_0 - 1/\beta_1)}{\pi\beta_0^2}\right.$$
$$\left. \times [\alpha_s(\mu) - \alpha_s(\mu_0)] + O(\alpha_s^2)\right\}, \qquad (12)$$

where

$$\gamma_1 = \frac{303 - 10n_f}{72}, \quad \beta_0 = \frac{33 - 2n_f}{12}, \quad \beta_1 = \frac{153 - 19n_f}{24}. \quad (13)$$

For the on-shell c-quark mass, we used $\overline{m}_c(\mu = \overline{m}_c) = 1.28$ GeV [48] as the input in Eq. (10), and obtained $m_c = 1.68$ GeV. The results for the dependence of $\overline{m}_c(\mu)$ on μ are displayed in Fig. 2.

Inserting Eq. (10) into Eq. (3), we obtain the decay width for $h \to c\bar{c}$ calculated in terms of the $\overline{\text{MS}}$ Yukawa coupling

$$\overline{\Gamma} = \overline{y}_c^2(\mu)\hat{\Gamma}_0^{c\bar{c}}\left[1 + \frac{\alpha_s(\mu)}{\pi}(\gamma_1^{c\bar{c}} + 2c_1)\right.$$
$$\left. + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2(\gamma_2^{c\bar{c}} + 2c_1\gamma_1^{c\bar{c}} + 2c_2 - 5c_1^2)\right]. \qquad (14)$$

The $\gamma_1^{c\bar{c}}$, $\gamma_2^{c\bar{c}}$, $\hat{\Gamma}_0^{c\bar{c}}$, $\hat{\Gamma}_1^{c\bar{c}}$, and $\hat{\Gamma}_2^{c\bar{c}}$ conventions are similar to those used in [9]. Here, $\gamma_1^{c\bar{c}} = C_F \delta_1 \pi$. In Eq. (14), it can be observed that the large log term $\ln(m_h^2/m_c^2)$ that appears in the α_s order correction is analytically reduced by the log term, $\ln(\mu^2/m_c^2)$, in the $\overline{\text{MS}}$ Yukawa coupling if
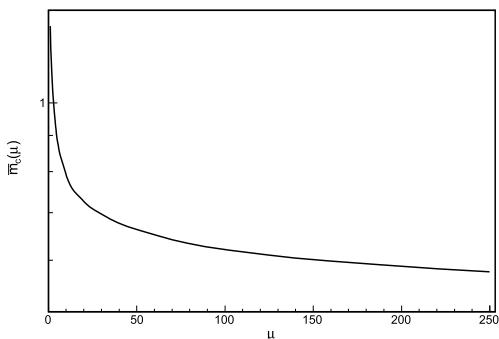
$\mu \sim m_h$. At the NNLO QCD, the corresponding numerical results are listed in Table 2 for three different renormalization scales. In Fig. 3, we show the dependence of $\overline{\Gamma}_I^{c\bar{c}}$ (I = LO, NLO, NNLO) on μ. Here, we use $\overline{\Gamma}_{\text{LO}}^{c\bar{c}}$, $\overline{\Gamma}_{\text{NLO}}^{c\bar{c}}$ and $\overline{\Gamma}_{\text{NNLO}}^{c\bar{c}}$ to respectively denote the LO, NLO, and NNLO QCD corrections in terms of the $\overline{\text{MS}}$ Yukawa coupling. The μ dependence is significantly decreased at the NNLO QCD. We also show the differential distribution of the $c\bar{c}$ invariant mass in Fig. 4, where sub-process $h \to c\bar{c}c\bar{c}$ is not included to avoid confusion. In the small $M_{c\bar{c}}$ region, the NNLO QCD contributions are larger than the NLO QCD corrections because the soft and/or collinear gluon radiation becomes significant.

To obtain the precise result for $h \to c\bar{c}$, the NLO EW corrections should be included along with the NNLO QCD corrections. The corresponding decay width at the NLO EW can be written as follows

$$\Gamma_{\text{EW}}^{c\bar{c}} = \overline{\Gamma}_{\text{LO}}^{c\bar{c}}\left[O_{\text{QED}}^{c\bar{c}} + O_{\text{Weak}}^{c\bar{c}}\right]. \qquad (15)$$

Table 2.　Inclusive decay width of $h \to c\bar{c}$ in terms of $\overline{\text{MS}}$ Yukawa coupling.

	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
$\overline{\Gamma}_{\text{LO}}^{c\bar{c}}$/MeV	0.1033	0.0916	0.0823
$\overline{\Gamma}_{\text{NLO}}^{c\bar{c}}$/MeV	0.1153	0.1103	0.1049
$\overline{\Gamma}_{\text{NNLO}}^{c\bar{c}}$/MeV	0.1162	0.1148	0.1125

Fig. 2.　Dependence of $\overline{m}_c(\mu)$ on μ.
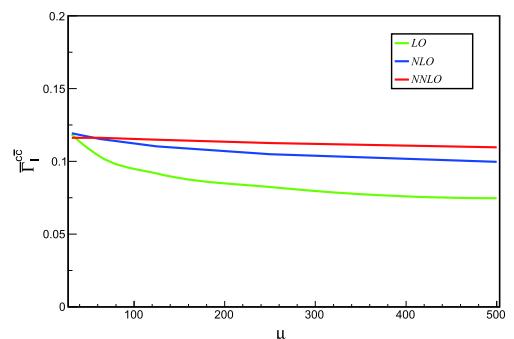
Fig. 3.　(color online) Dependence of Γ_I (I = LO, NL0, NNLO) on μ.
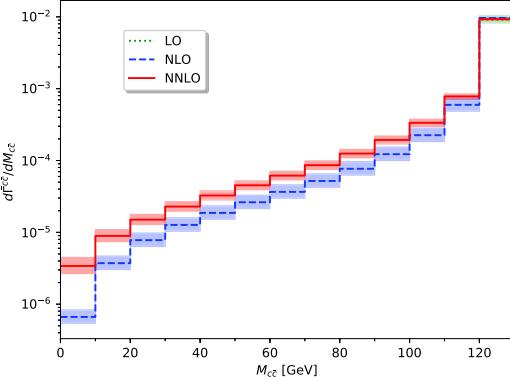
Fig. 4. (color online) Distribution $\mathrm{d}\bar{\Gamma}^{c\bar{c}}/\mathrm{d}M_{c\bar{c}}$ of the $c\bar{c}$ invariant mass at LO (short-dashed), NLO (long-dashed), and NNLO (solid) QCD corrections. The short-dashed, long-dashed, and solid lines correspond to the scale choice $\mu = m_h$, whereas the shaded bands show the effect of varying the renormalization scale between $\mu = m_h/2$ and $\mu = 2m_h$.

The NLO QED correction, $O_{\text{QED}}^{c\bar{c}}$, has a similar form as that of the NLO QCD, i.e.,

$$O_{\text{QED}}^{c\bar{c}} = \alpha Q_c^2 \delta_1 , \tag{16}$$

where α is the QED coupling, and we choose $\alpha(m_Z) = 1/127.934$ [48]. Q_c denotes the electric charge of the charm quark. The coefficient δ_1 at the NLO QED is similar to that at the NLO QCD. The large logarithm $\ln(m_h^2/m_c^2)$ in δ_1 at the NLO QED can be absorbed in the running quark mass as in the QCD corrections. For the NLO weak correction $O_{\text{Weak}}^{c\bar{c}}$, there are several studies on this topic [16, 17]. In this study, we adopt the approximation of [18].

$$O_{\text{Weak}}^{c\bar{c}} = \frac{\alpha}{16m_W^2\pi} \left\{ k_c m_t^2 + m_W^2 \left[-5 + \frac{3}{s_W^2} \ln c_W^2 \right] \right.$$
$$\left. - 8m_Z^2(6v_{Zc\bar{c}}^2 - a_{Zc\bar{c}}^2) \right\} , \tag{17}$$

where $v_{Zc\bar{c}} = I_3^c/2 - Q_c s_W^2$ and $a_{Zc\bar{c}} = I_3^c/2$ with I_3^c denoting the third component of the electroweak isospin of the charm quark, and the coefficient $k_c = 1$. $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$ with the θ_W the weak angle, and we choose $m_W = 80.358$ GeV, and $s_W^2 = 0.2233$ [48] in our numerical calculations. Finally, we obtain the total decay width of the $h \to c\bar{c}$ including the NNLO QCD and NLO EW corrections as follows

$$\Gamma_{\text{total}}^{c\bar{c}} = \bar{\Gamma}_{\text{NNLO}}^{c\bar{c}} + \Gamma_{\text{EW}}^{c\bar{c}} . \tag{18}$$

To combine the precise theoretical predictions and experimental measurements for $h \to f\bar{f}$ with $f = c, b$, we can

abstract some useful information of the running mass effect in the Yukawa couplings. To achieve this, we list the results for the decay width of $h \to c\bar{c}$ at the NNLO QCD and NLO EW, and that of $h \to b\bar{b}$ together with their ratio in Table 3 for three renormalization scales. Ratio $\Gamma_{\text{total}}^{b\bar{b}}/\Gamma_{\text{total}}^{c\bar{c}}$ is approximately twenty. Therefore, it is possible to measure the $h \to c\bar{c}$ events at the LHC because approximately 1.5×10^5 $h \to b\bar{b}$ events have been observed with data samples corresponding to the integrated luminosity of 79.8 fb^{-1} via the Wh and Zh associated production [19, 20]. Therefore, approximately 2.8×10^4 $h \to c\bar{c}$ events will be produced in the similar way but with 300 fb^{-1} integrated luminosity, which make it possible to observe $h \to c\bar{c}$. However, the small decay width of $h \to c\bar{c}$ makes it advantageous to search for new physics that may induce relatively large interactions related to the charm quark, for example, the ALP discussed in the next section.

Table 3. Total decay width $\Gamma_{\text{total}}^{f\bar{f}}$ of $h \to f\bar{f}$ with $f\bar{f} = c\bar{c}, b\bar{b}$ at the NNLO QCD and NLO EW, together with their ratio.

	$\mu = m_h/2$	$\mu = m_h$	$\mu = 2m_h$
$\Gamma_{\text{total}}^{c\bar{c}}$/MeV	0.1165	0.1151	0.1129
$\Gamma_{\text{total}}^{b\bar{b}}$/MeV	2.4248	2.3990	2.3533
$\Gamma_{\text{total}}^{b\bar{b}}/\Gamma_{\text{total}}^{c\bar{c}}$	20.8137	20.8427	20.8441

III. ALP PRODUCTION IN Higgs DECAY

We begin by considering a general ALP with flavor violating couplings to the right-handed up-quarks [37]. To describe such a system, the most general effective field theory is given by the following lagrangian

$$-\mathcal{L} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_a^2}{2}a^2 + \frac{\partial_\mu a}{f_a}\left[(c_{uR})_{ij}\bar{u}_{Ri}\gamma^\mu u_{Rj}\right.$$
$$\left. + c_\Phi \Phi^\dagger i\overleftrightarrow{D}_\mu \Phi \right] - \frac{a}{f_a}\left[c_g \frac{g_3^2}{32\pi^2}G_{\mu\nu}^a \tilde{G}^{\mu\nu a}\right.$$
$$\left. + c_W \frac{g_2^2}{32\pi^2}W_{\mu\nu}^I \tilde{W}^{\mu\nu I} + c_B \frac{g_1^2}{32\pi^2}B_{\mu\nu}\tilde{B}^{\mu\nu} \right], \tag{19}$$

where a and Φ denote the ALP and Higgs field, respectively. g_1, g_2, g_3 are the $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ gauge couplings of the SM, respectively, whereas $B_{\mu\nu}$, $W_{\mu\nu}^I$, $I = 1, 2, 3$, and $G_{\mu\nu}^b$, $b = 1, ...8$, are their corresponding field-strength tensors. $\tilde{B}_{\mu\nu} = 1/2\epsilon_{\mu\nu\alpha\beta}B^{\alpha\beta}$, ..., represent the corresponding dual field. U_{Ri} with $i = 1, 2, 3$ denotes the right-handed SM up-quark of the ith generation. f_a can be treated as a free parameter. c_Φ, c_g, c_W, and c_B are the Wilson coefficients, and c_{uR} is a hermitian matrix. In this model, it is assumed that the ALP is a pseudo Nambu-

Goldstone boson of the spontaneous breaking of a global $U(1)$ symmetry, and that its couplings to the leptons, SM quark doublets, and right-handed down-type quarks vanish. The operator $(\partial^\mu a/f_a)\Phi^\dagger i \overleftrightarrow{D}_\mu \Phi$ can be traded using Higgs field redefinition [31].

We then evaluate the ALP production in association with the charm quark pair in the Higgs boson decay. The differential decay width of the $h \to c\bar{c}a$ process where a denotes the ALP can be obtained as follows

$$\mathrm{d}\Gamma_{h\to c\bar{c}a} = \frac{1}{2m_h}|\mathcal{M}|^2 \mathrm{d}\mathcal{L}ips_3 \,, \tag{20}$$

where $\mathcal{L}ips_3$ is the Lorentz invariant phase space of the final three particles. The matrix element square can be expressed as follows

$$\begin{aligned}
|\mathcal{M}|^2 = &\frac{2y_c^2 \,|(c_{uR})_{22}|^2}{f_a^2}\left\{ m_h^2\left(x_c + x_{\bar{c}} - 1 - \frac{4m_c^2}{m_h^2} + \frac{m_a^2}{m_h^2}\right)\right. \\
&+ \frac{1}{m_h^2}\left[\frac{-m_c^2 m_h^2 (x_c - x_{\bar{c}})^2}{(1-x_c)(1-x_{\bar{c}})}\right. \\
&+ \frac{m_a^2}{m_h^2}\left(\frac{m_c^2(4m_c^2 + m_h^2(1-2x_c))}{(1-x_c)^2}\right. \\
&+ \frac{m_c^2(4m_c^2 + m_h^2(1-2x_{\bar{c}}))}{(1-x_{\bar{c}})^2} \\
&+ \left.\left.\left.\frac{2m_c^2(4m_c^2 - m_h^2(x_c + x_{\bar{c}} - 1))}{(1-x_c)(1-x_{\bar{c}})}\right)\right]\right\},
\end{aligned} \tag{21}$$

where m_a is the ALP mass. $x_c = 2E_c/m_h$ and $x_{\bar{c}} = 2E_{\bar{c}}/m_h$ with $E_c(E_{\bar{c}})$, the $c(\bar{c})$ quark energy in Higgs rest frame. In Eq. (21), the dependence of the ALP mass appears in the form $O(m_a/m_h)^0 + O(m_a/m_h)^2$, where $(m_a/m_h)^2$ is a tiny quantity for $m_a \ll m_h$ so that the height of the distribution is not sensitive to the ALP mass. In the following numerical calculations, we factor out $|(c_{uR})_{22}|^2/f_a^2$ from $\Gamma_{h\to c\bar{c}a}$, i.e.,

$$\Gamma_{h\to c\bar{c}a} = \frac{|(c_{uR})_{22}|^2}{f_a^2}\tilde{\Gamma}_{h\to c\bar{c}a} \,. \tag{22}$$

The experiment searches on the ALP parameter spaces that is m_a and f_a allow access to several orders of magnitude. The joint limit of these two parameters is given experimentally. Direct searches for the ALP and calculations of their effect on the cooling of stars and on the supernova SN1987A impose $f_a \gtrsim 4\times 10^8$ GeV [51]. The thermally produced ALP DM is allowed in sizable parts of the parameter $m_a \gtrsim 154$ eV [52]. In our analysis, f_a is treated as a free parameter that relaxes the restriction of m_a. We choose $m_a = 0.01, 10, 30$ GeV. The results for the differential distribution with respect to $M_{c\bar{c}}$ are shown in Fig. 5. Because there is no exact measurement for $h \to c\bar{c}$,
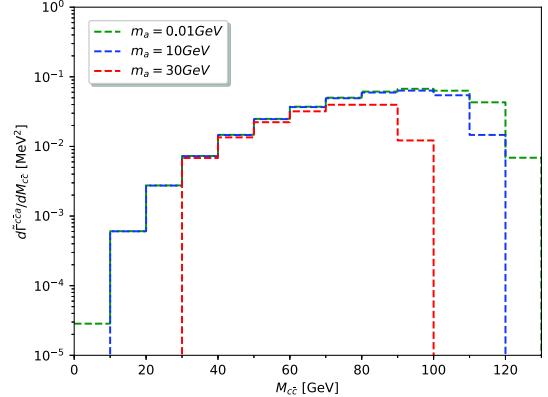
**Fig. 5.** (color online) Distribution $\mathrm{d}\tilde{\Gamma}^{c\bar{c}a}/\mathrm{d}M_{c\bar{c}}$ of the $c\bar{c}$ invariant mass.
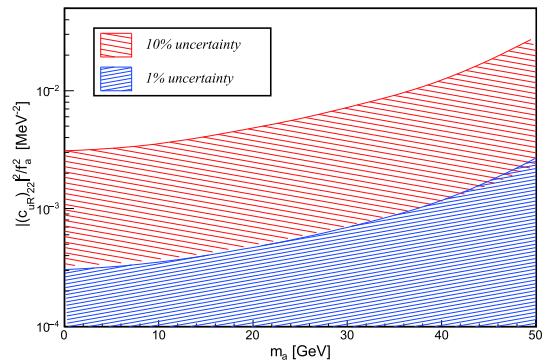

Fig. 6. (color online) Constraints on $|(c_{uR})_{22}|^2/f_a^2$ as a function of m_a, resulting from the conditions $\Gamma_{h\to c\bar{c}a} \leqslant 10\%\Gamma_{\mathrm{total}}^{c\bar{c}}$ and $\Gamma_{h\to c\bar{c}a} \leqslant 1\%\Gamma_{\mathrm{total}}^{c\bar{c}}$, with the allowed regions marked by red and blue, respectively.
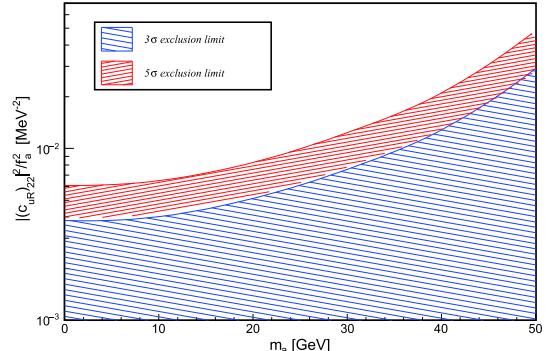

Fig. 7. (color online) 3σ exclusion limit (blue) and 5σ discovery limit (red) of $h \to c\bar{c}a$ at ILC.

to obtain some constrain on the parameters, we set the condition

$$\Gamma_{h\to c\bar{c}a} \leqslant 1\%\Gamma_{\mathrm{total}}^{c\bar{c}} \,, \quad \text{and} \quad 10\%\Gamma_{\mathrm{total}}^{c\bar{c}}. \tag{23}$$

The constraints for $|(c_{uR})_{22}|^2/f_a^2$ with respect to m_a are shown in Fig. 6. A loose constrain for the larger ALP mass can be found. It is also interesting to search for the

ALP via the $h \to c\bar{c}a$ process in the future electron-positron collider, e.g. the Circular Electron-Positron Collider (CEPC) or International Linear Collider (ILC). By comparing the distribution of the $h \to c\bar{c}$ background and $h \to c\bar{c}a$ signal in Fig. 4 and Fig. 5, it can be observed that for the background, the distribution tends to be close to the Higgs mass, whereas the signal process does not. Therefore, for a rough estimation, we require that the invariant mass of c, $\bar{c}$ be less than 110 GeV. Furthermore, we consider the $e^+e^- \to Zh$ process with $Z \to e^+e^-, \mu^+\mu^-$ and $h \to c\bar{c}$ or $c\bar{c}a$ at $\sqrt{s} = 250$ GeV. The 3σ exclusion limit and 5σ discovery limit with the integrated luminosity of 250 fb^{-1} are shown in Fig. 7. When the high precision measurement of $h \to c\bar{c}$ is available, the realistic constrain for the ALP production in $h \to c\bar{c}$ can be obtained.

IV. SUMMARY

Measurements on the $h \to c\bar{c}$ with high statistics at the LHC are still in progress. More theoretical studies on this process are still necessary and significant. Moreover, after the discovery of the SM Higgs, the search for new (pseudo) scalar boson beyond the SM, e.g., ALP, has become a significant topic in particle physics. In this paper, we present the results of the decay widths for $h \to c\bar{c}$ at the NNLO QCD and NLO EW corrections. For the flavor-singlet contributions where the Higgs boson coupled to the bottom and top quark appeared at an order of α_s^2, we provide the exact results, and find that the exact result of the top quark triangle is very close to the approximate result calculated to the large top quark mass expansion. The results for the Yukawa coupling defined in the $\overline{\text{MS}}$ scheme is more reliable and the large logarithmic effect related to the ratio of the charm-Higgs mass is reduced. Finally, we evaluate the ALP associate production with the charm quark pair, and the constrain for the related parameters is estimated by assuming a condition of Eq. (23). At the upgraded LHC, an increasing number of events on the Higgs boson decay will be accumulated, so that precise studies on $h \to c\bar{c}$ and the search for new particles in the Higgs decay become possible.

ACKNOWLEDGEMENT

References

[1] G. Aad *et al.* (ATLAS), Phys. Lett. B **716**, 1-29 (2012), arXiv:1207.7214[hep-ex]

[2] S. Chatrchyan *et al.* (CMS), Phys. Lett. B **716**, 30-61 (2012), arXiv:1207.7235[hep-ex]

[3] P. A. Baikov, K. G. Chetyrkin, and J. H. Kuhn, Phys. Rev. Lett. **96**, 012003 (2006), arXiv:hep-ph/0511063[hep-ph]

[4] J. Davies, M. Steinhauser, and D. Wellmann, Nucl. Phys. B **920**, 20-31 (2017), arXiv:1703.02988[hep-ph]

[5] F. Herzog, B. Ruijl, T. Ueda *et al.*, JHEP **08**, 113 (2017), arXiv:1707.01044[hep-ph]

[6] C. Anastasiou, F. Herzog, and A. Lazopoulos, JHEP **03**, 035 (2012), arXiv:1110.2368[hep-ph]

[7] V. Del Duca, C. Duhr, G. Somogyi *et al.*, JHEP **04**, 036 (2015), arXiv:1501.07226[hep-ph]

[8] R. Mondini, M. Schiavi, and C. Williams, JHEP **06**, 079 (2019), arXiv:1904.08960[hep-ph]

[9] W. Bernreuther, L. Chen, and Z. G. Si, JHEP **07**, 159 (2018), arXiv:1805.06658[hep-ph]

[10] F. Caola, K. Melnikov, and R. Röntsch, Eur. Phys. J. C **79**(12), 1013 (2019), arXiv:1907.05398[hep-ph]

[11] A. Behring and W. Bizoń, JHEP **01**, 189 (2020), arXiv:1911.11524[hep-ph]

[12] G. Somogyi and F. Tramontano, JHEP **11**, 142 (2020), arXiv:2007.15015[hep-ph]

[13] Y. Hu, C. Sun, X. M. Shen *et al.*, arXiv: 2101.08916[hep-ph]

[14] S. A. Larin, T. van Ritbergen, and J. A. M. Vermaseren, Phys. Lett. B **362**, 134-140 (1995), arXiv:hep-ph/9506465 [hep-ph]

[15] A. Primo, G. Sasso, G. Somogyi *et al.*, Phys. Rev. D **99**(5),

054013 (2019), arXiv:1812.07811[hep-ph]

[16] A. Dabelstein and W. Hollik, Z. Phys. C **53**, 507-516 (1992)

[17] J. Fleischer and F. Jegerlehner, Phys. Rev. D **23**, 2001-2026 (1981)

[18] B. A. Kniehl, Nucl. Phys. B **376**, 3-28 (1992)

[19] A. M. Sirunyan *et al.* (CMS), Phys. Rev. Lett. **121**(12), 121801 (2018), arXiv:1808.08242[hep-ex]

[20] M. Aaboud *et al.* (ATLAS), Phys. Lett. B **786**, 59-86 (2018), arXiv:1808.08238[hep-ex]

[21] G. Perez, Y. Soreq, E. Stamou *et al.*, Phys. Rev. D **92**(3), 033016 (2015), arXiv:1503.00290[hep-ph]

[22] G. Aad *et al.* (ATLAS), Phys. Rev. Lett. **114**(12), 121801 (2015), arXiv:1501.03276[hep-ex]

[23] A. M. Sirunyan *et al.* (CMS), Eur. Phys. J. C **79**(2), 94 (2019), arXiv:1810.10056[hep-ex]

[24] A. M. Sirunyan *et al.* (CMS), JHEP **03**, 131 (2020), arXiv:1912.01662[hep-ex]

[25] M. Aaboud *et al.* (ATLAS), Phys. Rev. Lett. **120**(21), 211802 (2018), arXiv:1802.04329[hep-ex]

[26] D. Cadamuro and J. Redondo, JCAP **02**, 032 (2012), arXiv:1110.2895[hep-ph]

[27] M. Millea, L. Knox, and B. Fields, Phys. Rev. D **92**(2), 023010 (2015), arXiv:1501.04097[astro-ph.CO]

[28] L. Di Luzio, F. Mescia, and E. Nardi, Phys. Rev. Lett. **118**(3), 031801 (2017), arXiv:1610.07593[hep-ph]

[29] K. Mimasu and V. Sanz, JHEP **06**, 173 (2015), arXiv:1409.4792[hep-ph]

[30] M. Bauer, M. Neubert, and A. Thamm, JHEP **12**, 044 (2017), arXiv:1708.00443[hep-ph]

[31] I. Brivio, M. B. Gavela, L. Merlo *et al.*, Eur. Phys. J. C **77**(8), 572 (2017), arXiv:1701.05379[hep-ph]

[32]   G. Alonso-Álvarez, M. B. Gavela, and P. Quilez, Eur. Phys. J. C **79**(3), 223 (2019), arXiv:1811.05466[hep-ph]
[33]   L. Harland-Lang, J. Jaeckel, and M. Spannowsky, Phys. Lett. B **793**, 281-289 (2019), arXiv:1902.04878[hep-ph]
[34]   C. Baldenegro, S. Fichet, G. von Gersdorff *et al.*, JHEP **06**, 131 (2018), arXiv:1803.10835[hep-ph]
[35]   I. G. Irastorza and J. Redondo, Prog. Part. Nucl. Phys. **102**, 89-159 (2018), arXiv:1801.08127[hep-ph]
[36]   R. Essig, J. A. Jaros, W. Wester *et al.*, arXiv: 1311.0029[hep-ph]
[37]   A. Carmona, C. Scherb, and P. Schwaller, arXiv: 2101.07803[hep-ph]
[38]   M. Drees and K. i. Hikasa, Phys. Lett. B **240**, 455 (1990), Erratum: Phys. Lett. B **262**, 497 (1991)
[39]   O. Dekkers and W. Bernreuther, Phys. Lett. B **738**, 325-333 (2014), arXiv:1409.3124[hep-ph]
[40]   L. Chen, O. Dekkers, D. Heisler *et al.*, JHEP **12**, 098 (2016), arXiv:1610.07897[hep-ph]
[41]   W. Bernreuther, C. Bogner, and O. Dekkers, JHEP **06**, 032 (2011), arXiv:1105.0530[hep-ph]
[42]   W. Bernreuther, C. Bogner, and O. Dekkers, JHEP **10**, 161 (2013), arXiv:1309.6887[hep-ph]
[43]   W. Bernreuther, R. Bonciani, T. Gehrmann *et al.*, Phys. Rev. D **72**, 096002 (2005), arXiv:hep-ph/0508254[hep-ph]
[44]   W. Bernreuther, R. Bonciani, T. Gehrmann *et al.*, Nucl. Phys. B **723**, 91-116 (2005), arXiv:hep-ph/0504190[hep-ph]
[45]   W. Bernreuther, R. Bonciani, T. Gehrmann *et al.*, Nucl. Phys. B **706**, 245-324 (2005), arXiv:hep-ph/0406046[hep-ph]
[46]   W. Bernreuther, R. Bonciani, T. Gehrmann *et al.*, Nucl. Phys. B **712**, 229-286 (2005), arXiv:hep-ph/0412259[hep-ph]
[47]   C. Duhr and F. Dulat, JHEP **08**, 135 (2019), arXiv:1904.07279[hep-th]
[48]   M. Tanabashi *et al.* (Particle Data Group), Phys. Rev. D **98**(3), 030001 (2018)
[49]   E. Braaten and J. P. Leveille, Phys. Rev. D **22**, 715 (1980)
[50]   N. Gray, D. J. Broadhurst, W. Grafe *et al.*, Z. Phys. C **48**, 673-680 (1990)
[51]   G. G. Raffelt, Lect. Notes Phys. **741**, 51-71 (2008), arXiv:hep-ph/0611350[hep-ph]
[52]   D. Cadamuro, S. Hannestad, G. Raffelt *et al.*, JCAP **02**, 003 (2011), arXiv:1011.3694[hep-ph]