Analysis of hidden-charm pentaquark molecular states with and without strangeness via the QCD sum rules*

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Abstract: In this study, we investigate the $\bar{D}\Sigma_c$, $\bar{D}\Xi_c'$, $\bar{D}\Sigma_c^*$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Xi_c'$, $\bar{D}^*\Sigma_c^*$, and $\bar{D}^*\Xi_c^*$ pentaquark molecular states with and without strangeness via the QCD sum rules in detail, focusing on the light flavor, $SU(3)$, breaking effects, and make predictions for new pentaquark molecular states besides assigning $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$ self-consistently. In the future, we can search for these pentaquark molecular states in the decay of Λ_b^0, Ξ_b^0 , and Ξ_b^- . Furthermore, we discuss high-dimensional vacuum condensates in detail.

Keywords: pentaquark molecular states, QCD sum rules

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I. INTRODUCTION

In 2015, the LHCb collaboration explored the $\Lambda_b^0 \to J/\psi K^- p$ decay and observed two pentaquark candidates $P_c(4380)$ and $P_c(4450)$ in the $J/\psi p$ mass spectrum with preferred quantum numbers $J^P=3/2^-$ and $5/2^+$, respectively [1]. The Breit-Wigner masses and widths are $M_{P_c(4380)}=4380\pm8\pm29\,\text{MeV}$, $M_{P_c(4450)}=4449.8\pm1.7\pm2.5$ MeV, $\Gamma_{P_c(4380)}=205\pm18\pm86$ MeV, and $\Gamma_{P_c(4450)}=39\pm5\pm19$ MeV, respectively. In 2019, the LHCb collaboration re-investigated a data sample of the $\Lambda_b^0 \to J/\psi K^- p$ decay, which was an order of magnitude larger than that previously analyzed, and observed a narrow pentaquark candidate $P_c(4312)$ in the $J/\psi p$ mass spectrum, and confirmed structure $P_c(4450)$, which consisted of two narrow overlapping peaks $P_c(4440)$ and $P_c(4457)$ [2]. The measured Breit-Wigner masses and widths are

$$\begin{aligned}
P_c(4312):\, & M=4311.9\pm0.7^{+6.8}_{-0.6}\,\text{MeV},\ \Gamma=9.8\pm2.7^{+3.7}_{-4.5}\,\text{MeV},\\
P_c(4440):\, & M=4440.3\pm1.3^{+4.1}_{-4.7}\,\text{MeV},\ \Gamma=20.6\pm4.9^{+8.7}_{-10.1}\,\text{MeV},\\
P_c(4457):\, & M=4457.3\pm0.6^{+4.1}_{-1.7}\,\text{MeV},\ \Gamma=6.4\pm2.0^{+5.7}_{-1.9}\,\text{MeV}.
\end{aligned} \tag{1}$$

Very recently, the LHCb collaboration reported an evidence of a hidden-charm pentaquark candidate $P_{cs}(4459)$ with strangeness $S=-1$ in the $J/\psi\Lambda$ invariant mass spectrum with a statistical significance of 3.1σ in the $\Xi_b^- \to J/\psi K^-\Lambda$ decay using the pp collision data corresponding to a total integrated luminosity of 9fb^{-1} collected in LHCb experiments at centre-of-mass energies of 7, 8, and 13 TeV [3]. Its Breit-Wigner mass and width are

$$P_{cs}(4459):M=4458.8\pm2.9^{+4.7}_{-1.1}\,\text{MeV},\ \Gamma=17.3\pm6.5^{+8.0}_{-5.7}\,\text{MeV}, \tag{2}$$

but the spin and the parity have not been determined yet. $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$ lie slightly below the thresholds of meson-baryon pairs $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Xi_c$, respectively. The nearby meson-baryon thresholds are listed clearly in Table 1, because the D and D^* mesons, $1/2^+$ flavor antitriplet ($\Lambda_c^+(2286)$, $\Xi_c^+(2468)$, and $\Xi_c^0(2471)$), and $1/2^+$ and $3/2^+$ flavor sextets ($\Omega_c(2695)$, $\Xi_c'(2579)$, $\Sigma_c(2453)$) and ($\Omega_c^*(2766)$, $\Xi_c^*(2646)$, $\Sigma_c^*(2518)$) have been well established [4].

As expected, $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$ have been tentatively assigned to the $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Sigma_c^*$, and $\bar{D}^*\Xi_c$ pentaquark molecular states, respectively, based on the contact-range effective field theory [5, 6], one-boson exchange potential model [7-9], (quasipotential) Bethe-Salpeter equation [10-13], effective Lagrangian approach [14, 15], effective-range expansion and resonance compositeness relation [16], Lippmann-Schwinger equation [17, 18], and QCD sum rules [19-23].

At first glance, $M_{\bar{D}^{*0}}+M_{\Xi_c^0}-M_{P_{cs}(4459)}=19\,\text{MeV}$ and

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Table 1. The thresholds of the open-charm meson and baryon pairs, where the unit is MeV.

$(\bar{D}^0\Xi_c^0, \bar{D}^-\Xi_c^+)$	(4336, 4337)
$(\bar{D}^{*0}\Xi_c^0, \bar{D}^{*-}\Xi_c^+)$	(4478, 4478)
$(\bar{D}^0\Sigma_c^+, \bar{D}^-\Sigma_c^{++})$	(4318, 4323)
$(\bar{D}^0\Sigma_c^{*+}, \bar{D}^-\Sigma_c^{*++})$	(4382, 4388)
$(\bar{D}^{*0}\Sigma_c^+, \bar{D}^{*-}\Sigma_c^{++})$	(4460, 4464)
$(\bar{D}^{*0}\Sigma_c^{*+}, \bar{D}^{*-}\Sigma_c^{*++})$	(4524, 4529)
$(\bar{D}^0\Xi_c^{\prime 0}, \bar{D}^-\Xi_c^{\prime +})$	(4444, 4448)
$(\bar{D}^0\Xi_c^{*0}, \bar{D}^-\Xi_c^{*+})$	(4511, 4515)
$(\bar{D}^{*0}\Xi_c^{\prime 0}, \bar{D}^{*-}\Xi_c^{\prime +})$	(4586, 4589)
$(\bar{D}^{*0}\Xi_c^{*0}, \bar{D}^{*-}\Xi_c^{*+})$	(4653, 4656)

$M_{\bar{D}^0}+M_{\Sigma_c^+}-M_{P_c(4312)}=6\,\text{MeV}$, and it is unusual that the $\bar{D}^{*0}\Xi_c^0$ molecular state, which involves exchanges of strange mesons, is more tightly bound than the $\bar{D}^0\Sigma_c^+$ molecular state, which involves exchanges of non-strange mesons. Therefore, we have to introduce coupled-channel effects [7-13, 16-18].

In the QCD sum rules, we usually choose local currents to interpolate tetraquark or pentaquark molecular states having two color-neutral clusters [19-23], which are not necessary to be physical mesons and baryons, and the tetraquark or pentaquark molecular states are not necessary to be loosely bound; they can be compact objects and can lie below or above the corresponding meson-meson or meson-baryon pairs [24].

In contrast, $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$ can also be tentatively assigned to diquark-diquark-antiquark-type (or diquark-triquark-type) pentaquark states in the diquark-model by exploring their masses [25, 26] ([27]) and decay modes [28-30] ([31]) using an effective Hamiltonian or by investigating their masses [32-35, 37], decay [38], and electromagnetic properties [39] using the QCD sum rules.

In Ref. [40], we suggested the hadronic dressing mechanism to compromise pentaquark and pentaquark molecular interpretations based on the calculations of the QCD sum rules and that pentaquark states may have a diquark-diquark-antiquark-type pentaquark core with the typical size of qqq-type baryon states. Moreover, the strong couplings to meson-baryon pairs lead to some pentaquark molecule Fock components; therefore, pentaquark states may spend a rather long time as molecular states. We can choose either diquark-diquark-antiquark-type currents or color-singlet-color-singlet-type five-quark currents to interpolate pentaquark states.

In the scenario of pentaquark molecular states, $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$ are assigned to the $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ pentaquark molecular states, which involve charmed baryon states in flavor sextets $\mathbf{6}_f$ [5, 7, 10, 14, 16-22], whereas $P_{cs}(4459)$ is assigned to the $\bar{D}^*\Xi_c$ pentaquark molecular state, which involves a charmed baryon state in flavor antitriplet $\bar{\mathbf{3}}_f$ [6, 8, 9, 11-13, 15, 23]. Thus the $P_c(4312/4380/4440/4457)$ and $P_{cs}(4459)$ belong to different flavor multiplets, and in the present study, we focus on the flavor sextets, $\mathbf{6}_f$.

In this study, we extend our previous research [22] to investigate the masses and pole residues of the $\bar{D}\Sigma_c$, $\bar{D}\Xi_c'$, $\bar{D}\Sigma_c^*$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Xi_c'$, $\bar{D}^*\Sigma_c^*$, and $\bar{D}^*\Xi_c^*$ pentaquark molecular states with the QCD sum rules by accomplishing operator product expansion up to vacuum condensates of dimension 13 in a consistent manner, take into account vacuum condensates $\langle\bar{q}q\rangle\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}GG\rangle$, and $\langle\bar{q}q\rangle^3\langle\frac{\alpha_s}{\pi}GG\rangle$ neglected in Ref. [22], and revisit the assignments of $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$. Furthermore, we pay much attention to the light flavor, $SU(3)$, breaking effects.

The remainder of this article is arranged as follows: we describe the QCD sum rules for the masses and pole residues of pentaquark molecular states in Section II. In Section III, we present the numerical results and discussions, and Section IV draws our conclusion.

II. QCD SUM RULES FOR THE PENTAQUARK MOLECULAR STATES

We express the two-point correlation functions, $\Pi(p)$, $\Pi_{\mu\nu}(p)$, and $\Pi_{\mu\nu\alpha\beta}(p)$, in the QCD sum rules,

$$\begin{aligned}\Pi(p) &= i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J(x)\bar{J}(0)\right\}|0\rangle\,,\\ \Pi_{\mu\nu}(p) &= i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J_\mu(x)\bar{J}_\nu(0)\right\}|0\rangle\,,\\ \Pi_{\mu\nu\alpha\beta}(p) &= i\int d^4x e^{ip\cdot x}\langle 0|T\left\{J_{\mu\nu}(x)\bar{J}_{\alpha\beta}(0)\right\}|0\rangle\,,\end{aligned}\qquad(3)$$

where currents $J(x)=J^{\bar{D}\Sigma_c}(x)$, $J^{\bar{D}\Xi_c'}(x)$, $J_\mu(x)=J_\mu^{\bar{D}\Sigma_c^*}(x)$, $J_\mu^{\bar{D}\Xi_c^*}(x)$, $J_\mu^{\bar{D}^*\Sigma_c}(x)$, $J_\mu^{\bar{D}^*\Xi_c'}(x)$, $J_{\mu\nu}(x)=J_{\mu\nu}^{\bar{D}^*\Sigma_c^*}(x)$, $J_{\mu\nu}^{\bar{D}^*\Xi_c^*}(x)$,

$$\begin{aligned}J^{\bar{D}\Sigma_c}(x) &= \bar{c}(x)i\gamma_5 u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\alpha d_j(x)\gamma^\alpha\gamma_5 c_k(x)\,,\\ J^{\bar{D}\Xi_c'}(x) &= \bar{c}(x)i\gamma_5 u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\alpha s_j(x)\gamma^\alpha\gamma_5 c_k(x)\,,\end{aligned}\qquad(4)$$

$$\begin{aligned}J_\mu^{\bar{D}\Sigma_c^*}(x) &= \bar{c}(x)i\gamma_5 u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\mu d_j(x)\,c_k(x)\,,\\ J_\mu^{\bar{D}\Xi_c^*}(x) &= \bar{c}(x)i\gamma_5 u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\mu s_j(x)\,c_k(x)\,,\end{aligned}\qquad(5)$$

$$\begin{aligned}J_\mu^{\bar{D}^*\Sigma_c}(x) &= \bar{c}(x)\gamma_\mu u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\alpha d_j(x)\gamma^\alpha\gamma_5 c_k(x)\,,\\ J_\mu^{\bar{D}^*\Xi_c'}(x) &= \bar{c}(x)\gamma_\mu u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\alpha s_j(x)\gamma^\alpha\gamma_5 c_k(x)\,,\end{aligned}\qquad(6)$$

$$J^{\bar{D}^*\Sigma_c^*}_{\mu\nu}(x) = \bar{c}(x)\gamma_\mu u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\nu d_j(x)\,c_k(x) + (\mu\leftrightarrow\nu),$$
$$J^{\bar{D}^*\Xi_c^*}_{\mu\nu}(x) = \bar{c}(x)\gamma_\mu u(x)\,\varepsilon^{ijk}u_i^T(x)C\gamma_\nu s_j(x)\,c_k(x) + (\mu\leftrightarrow\nu), \tag{7}$$

where i, j, and k are color indices. In the present study, we choose color-singlet-color-singlet-type currents $J(x)$, $J_\mu(x)$, and $J_{\mu\nu}(x)$ to interpolate the pentaquark molecular states with spin parities $J^P = 1/2^-$, $3/2^-$, and $5/2^-$, respectively. The currents couple potentially to the pentaquark molecular states having two color-neutral clusters: one has the same quantum numbers as the charmed mesons, and the other has the same quantum numbers as the charmed baryons. They are not physical mesons and baryons, as we choose local five-quark currents, whereas the mesons and the baryons are spatial extended objects and have mean spatial sizes $\sqrt{\langle r^2\rangle}\neq 0$. For example, $\sqrt{\langle r^2\rangle_{E,\Sigma_c^{++}}} = 0.48\,\mathrm{fm}$, $\sqrt{\langle r^2\rangle_{M,\Sigma_c^{++}}} = 0.83\,\mathrm{fm}$, and $\sqrt{\langle r^2\rangle_{M,\Sigma_c^{0}}} = 0.81\,\mathrm{fm}$ from the lattice QCD, where subscripts E and M stand for the electric and magnetic radii, respectively[41], $\sqrt{\langle r^2\rangle_{M,\Sigma_c^{*++}}} = 0.77\,\mathrm{fm}$, $\sqrt{\langle r^2\rangle_{M,\Sigma_c^{0}}} = 0.52\,\mathrm{fm}$, $\sqrt{\langle r^2\rangle_{M,\Sigma_c^{+}}} = 0.81\,\mathrm{fm}$, $\sqrt{\langle r^2\rangle_{M,\Xi_c^{\prime +}}} = 0.55\,\mathrm{fm}$, $\sqrt{\langle r^2\rangle_{M,\Xi_c^{0}}} = 0.79\,\mathrm{fm}$ from the self-consistent $SU(3)$ chiral quark-soliton model [42], $\sqrt{\langle r^2\rangle_{D^+}} = 0.43\,\mathrm{fm}$, and $\sqrt{\langle r^2\rangle_{D^0}} = 0.55\,\mathrm{fm}$ from the light-front quark model [43]. In the present study, although we refer the color-singlet-color-singlet-type pentaquark states as the pentaquark molecular states, they have average spatial sizes as those of the typical heavy mesons and baryons, and are compact objects. For example, a loosely bound $\bar{D}^0\Sigma_c^+$ molecular state with physical meson $\bar{D}^0$ and baryon Σ_c^+ should have average spatial size $\sqrt{\langle r^2\rangle}\geqslant 1.36\,\mathrm{fm}$, which is extremely large to be interpolated by local currents.

Currents $J(0)$, $J_\mu(0)$ and $J_{\mu\nu}(0)$ couple potentially to $1/2^\mp$, $1/2^\pm$, $3/2^\mp$ and $1/2^\mp$, $3/2^\pm$, $5/2^\mp$ hidden-charm pentaquark molecular states $P_{\frac{1}{2}}^\mp$, $P_{\frac{1}{2}}^\pm$, $P_{\frac{3}{2}}^\mp$ and $P_{\frac{1}{2}}^\mp$, $P_{\frac{3}{2}}^\pm$, $P_{\frac{5}{2}}^\mp$, respectively,

$$\langle 0|J(0)|P_{\frac{1}{2}}^-(p)\rangle = \lambda^-_{\frac{1}{2}}U^-(p,s),$$
$$\langle 0|J(0)|P_{\frac{1}{2}}^+(p)\rangle = \lambda^+_{\frac{1}{2}}i\gamma_5U^+(p,s), \tag{8}$$

$$\langle 0|J_\mu(0)|P_{\frac{1}{2}}^+(p)\rangle = f^+_{\frac{1}{2}}p_\mu U^+(p,s),$$
$$\langle 0|J_\mu(0)|P_{\frac{1}{2}}^-(p)\rangle = f^-_{\frac{1}{2}}p_\mu i\gamma_5U^-(p,s),$$
$$\langle 0|J_\mu(0)|P_{\frac{3}{2}}^-(p)\rangle = \lambda^-_{\frac{3}{2}}U^-_\mu(p,s)$$
$$\langle 0|J_\mu(0)|P_{\frac{3}{2}}^+(p)\rangle = \lambda^+_{\frac{3}{2}}i\gamma_5U^+_\mu(p,s), \tag{9}$$

$$\langle 0|J_{\mu\nu}(0)|P_{\frac{1}{2}}^-(p)\rangle = g^-_{\frac{1}{2}}p_\mu p_\nu U^-(p,s),$$
$$\langle 0|J_{\mu\nu}(0)|P_{\frac{1}{2}}^+(p)\rangle = g^+_{\frac{1}{2}}p_\mu p_\nu i\gamma_5U^+(p,s),$$
$$\langle 0|J_{\mu\nu}(0)|P_{\frac{3}{2}}^+(p)\rangle = f^+_{\frac{3}{2}}\left[p_\mu U^+_\nu(p,s)+p_\nu U^+_\mu(p,s)\right],$$
$$\langle 0|J_{\mu\nu}(0)|P_{\frac{3}{2}}^-(p)\rangle = f^-_{\frac{3}{2}}i\gamma_5\left[p_\mu U^-_\nu(p,s)+p_\nu U^-_\mu(p,s)\right],$$
$$\langle 0|J_{\mu\nu}(0)|P_{\frac{5}{2}}^-(p)\rangle = \sqrt{2}\lambda^-_{\frac{5}{2}}U^-_{\mu\nu}(p,s),$$
$$\langle 0|J_{\mu\nu}(0)|P_{\frac{5}{2}}^+(p)\rangle = \sqrt{2}\lambda^+_{\frac{5}{2}}i\gamma_5U^+_{\mu\nu}(p,s), \tag{10}$$

where the $U^\pm(p,s)$, $U^\pm_\mu(p,s)$, and $U^\pm_{\mu\nu}(p,s)$ are the Dirac and Rarita-Schwinger spinors [22, 32-36].

At the hadron side of correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$, and $\Pi_{\mu\nu\alpha\beta}(p)$, we isolate the ground state contributions from the hidden-charm pentaquark molecular states with spin parities $J^P = 1/2^\pm$, $3/2^\pm$, and $5/2^\pm$, respectively, without contamination based on the current-hadron couplings expressed in Eqs. (8)-(10) and obtain the hadronic representation [22, 32-36],

$$\Pi(p) = \lambda^{-2}_{\frac{1}{2}}\frac{\not{p}+M_-}{M_-^2-p^2} + \lambda^{+2}_{\frac{1}{2}}\frac{\not{p}-M_+}{M_+^2-p^2}+\cdots,$$
$$= \Pi^1_{\frac{1}{2}}(p^2)\,\not{p} + \Pi^0_{\frac{1}{2}}(p^2), \tag{11}$$

$$\Pi_{\mu\nu}(p) = \lambda^{-2}_{\frac{3}{2}}\frac{\not{p}+M_-}{M_-^2-p^2}\left(-g_{\mu\nu}\right) + \lambda^{+2}_{\frac{3}{2}}\frac{\not{p}-M_+}{M_+^2-p^2}\left(-g_{\mu\nu}\right)+\cdots,$$
$$= -\Pi^1_{\frac{3}{2}}(p^2)\,\not{p}\,g_{\mu\nu} - \Pi^0_{\frac{3}{2}}(p^2)\,g_{\mu\nu}+\cdots, \tag{12}$$

$$\Pi_{\mu\nu\alpha\beta}(p) = \lambda^{-2}_{\frac{5}{2}}\frac{\not{p}+M_-}{M_-^2-p^2}\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}\right)$$
$$+\lambda^{+2}_{\frac{5}{2}}\frac{\not{p}-M_+}{M_+^2-p^2}\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}\right)+\cdots,$$
$$= \Pi^1_{\frac{5}{2}}(p^2)\,\not{p}\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}\right)$$
$$+\Pi^0_{\frac{5}{2}}(p^2)\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}\right)+\cdots. \tag{13}$$

There are other spinor structures, which are not shown explicitly, and we choose the components corresponding to spinor structures $\not{p}$, 1, $\not{p}g_{\mu\nu}$, $g_{\mu\nu}$, and $\not{p}\left(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}\right)$, $g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}$ in correlation functions $\Pi(p)$, $\Pi_{\mu\nu}(p)$, and $\Pi_{\mu\nu\alpha\beta}(p)$, respectively, to investigate the $J^P = 1/2^\mp$, $3/2^\mp$, and $5/2^\mp$ pentaquark molecular states.

Now, we briefly digress to discuss the isospins of the interpolating currents. From Eqs. (4)-(7), we can see clearly that currents $J^{\bar{D}\Sigma_c}(x)$, $J^{\bar{D}\Sigma_c^*}_\mu(x)$, $J^{\bar{D}^*\Sigma_c}_\mu(x)$, and

$J^{\bar{D}^*\Sigma_c^*}_{\mu\nu}(x)$ without strangeness have the same isospin structures, and currents $J^{\bar{D}\Xi_c^{\prime}}(x)$, $J^{\bar{D}\Xi_c^*}_{\mu}(x)$, $J^{\bar{D}^*\Xi_c^{\prime}}_{\mu}(x)$, and $J^{\bar{D}^*\Xi_c^*}_{\mu\nu}(x)$ with strangeness also have the same isospin structures. Moreover, they can be transformed into each other with the simple replacement, $d \leftrightarrow s$. It is a good objective to explore the light flavor, $SU(3)$, breaking effects. We can rewrite currents $J^{\bar{D}^0\Sigma_c^+}(x)$ and $J^{\bar{D}^0\Xi_c^{\prime +}}(x)$ in terms of the isospin eigenstates as

$$\begin{aligned} J^{\bar{D}^0\Sigma_c^+}(x) &= J_{\bar{D}^0}(x)J_{\Sigma_c^+}(x) = \frac{1}{\sqrt{3}}J^{\frac{1}{2}}_{\bar{D}\Sigma_c}(x) + \sqrt{\frac{2}{3}}J^{\frac{3}{2}}_{\bar{D}\Sigma_c}(x), \\ J^{\bar{D}^0\Xi_c^{\prime +}}(x) &= J_{\bar{D}^0}(x)J_{\Xi_c^{\prime +}}(x) = J^{1}_{\bar{D}\Xi_c^{\prime}}(x), \end{aligned} \tag{14}$$

where

$$\begin{aligned} J^{\frac{1}{2}}_{\bar{D}\Sigma_c}(x) &= \frac{1}{\sqrt{3}}J_{\bar{D}^0}(x)J_{\Sigma_c^+}(x) - \sqrt{\frac{2}{3}}J_{\bar{D}^-}(x)J_{\Sigma_c^{++}}(x), \\ J^{\frac{3}{2}}_{\bar{D}\Sigma_c}(x) &= \sqrt{\frac{2}{3}}J_{\bar{D}^0}(x)J_{\Sigma_c^+}(x) + \frac{1}{\sqrt{3}}J_{\bar{D}^-}(x)J_{\Sigma_c^{++}}(x), \end{aligned} \tag{15}$$

the $J_{\bar{D}^0}(x)$, $J_{\bar{D}^-}(x)$, $J_{\Sigma_c^+}(x)$, $J_{\Sigma_c^{++}}(x)$, and $J_{\Xi_c^{\prime +}}(x)$ are the standard currents for the mesons and baryons, respectively, and superscripts 1/2, 1, and 3/2 stand for the isospins of the interpolating currents. Now let us estimate the isospin breaking effects,

$$\begin{aligned} \langle 0|T\left\{J_{\bar{D}^0\Sigma_c^+}(x)\bar{J}_{\bar{D}^0\Sigma_c^+}(0)\right\}|0\rangle =& \frac{1}{3}\langle 0|T\left\{J^{\frac{1}{2}}_{\bar{D}\Sigma_c}(x)\bar{J}^{\frac{1}{2}}_{\bar{D}\Sigma_c}(0)\right\}|0\rangle \\ &+ \frac{2}{3}\langle 0|T\left\{J^{\frac{3}{2}}_{\bar{D}\Sigma_c}(x)\bar{J}^{\frac{3}{2}}_{\bar{D}\Sigma_c}(0)\right\}|0\rangle, \\ \langle 0|T\left\{J_{\bar{D}^-\Sigma_c^{++}}(x)\bar{J}_{\bar{D}^-\Sigma_c^{++}}(0)\right\}|0\rangle =& \frac{2}{3}\langle 0|T\left\{J^{\frac{1}{2}}_{\bar{D}\Sigma_c}(x)\bar{J}^{\frac{1}{2}}_{\bar{D}\Sigma_c}(0)\right\}|0\rangle \\ &+ \frac{1}{3}\langle 0|T\left\{J^{\frac{3}{2}}_{\bar{D}\Sigma_c}(x)\bar{J}^{\frac{3}{2}}_{\bar{D}\Sigma_c}(0)\right\}|0\rangle, \end{aligned} \tag{16}$$

where current $J_{\bar{D}^-\Sigma_c^{++}}(x) = J_{\bar{D}^-}(x)J_{\Sigma_c^{++}}(x)$. The isospin breaking effects between the vacuum matrix elements, $\langle 0|T\left\{J_{\bar{D}^0\Sigma_c^+}(x)\bar{J}_{\bar{D}^0\Sigma_c^+}(0)\right\}|0\rangle$ and $\langle 0|T\left\{J_{\bar{D}^-\Sigma_c^{++}}(x)\bar{J}_{\bar{D}^-\Sigma_c^{++}}(0)\right\}|0\rangle$, at the quark-gluon level are suppressed by a factor $\frac{1}{4N_c}$, which is of minor importance and can be neglected safely in the large N_c limit. Then, we can estimate (or obtain the conclusion tentatively) that the currents $J^{\frac{1}{2}}_{\bar{D}\Sigma_c}(x)$ and $J^{\frac{3}{2}}_{\bar{D}\Sigma_c}(x)$ couple potentially to the spin-1/2 $\bar{D}\Sigma_c$ pentaquark molecular states with almost degenerated masses but different pole residues; therefore, we will not distinguish isospins $I = 1/2$ and $3/2$ as we are only interested in the molecular masses, just like in previous studies [19-22]. Furthermore, current $J^{1}_{\bar{D}\Xi_c^{\prime}}(x)$ has the quantum numbers $I = 1$ and $I_3 = 1$, and we can add superscript I_3 to distinguish the components in the isospin triplets and singlets,

$$\begin{aligned} J^{1,1}_{\bar{D}\Xi_c^{\prime}}(x) =& J_{\bar{D}^0}(x)J_{\Xi_c^{\prime +}}(x), \\ J^{1,0}_{\bar{D}\Xi_c^{\prime}}(x) =& \frac{1}{\sqrt{2}}J_{\bar{D}^0}(x)J_{\Xi_c^{\prime 0}}(x) + \frac{1}{\sqrt{2}}J_{\bar{D}^-}(x)J_{\Xi_c^{\prime +}}(x), \\ J^{1,-1}_{\bar{D}\Xi_c^{\prime}}(x) =& J_{\bar{D}^-}(x)J_{\Xi_c^{\prime 0}}(x), \\ J^{0,0}_{\bar{D}\Xi_c^{\prime}}(x) =& \frac{1}{\sqrt{2}}J_{\bar{D}^0}(x)J_{\Xi_c^{\prime 0}}(x) - \frac{1}{\sqrt{2}}J_{\bar{D}^-}(x)J_{\Xi_c^{\prime +}}(x), \end{aligned} \tag{17}$$

and they couple potentially to the pentaquark molecular states with almost degenerated masses. Again the isospin breaking effects are suppressed by the factor, $\frac{1}{4N_c}$; therefore, we will not distinguish isospins $I = 1$ and 0.

Now, let us go back to the correlation functions. It is straightforward to obtain the spectral densities at the hadron side through the dispersion relation,

$$\frac{{\rm Im}\Pi^1_j(s)}{\pi} = {\lambda^{-}_j}^2\delta\left(s-M_-^2\right) + {\lambda^{+}_j}^2\delta\left(s-M_+^2\right) = \rho^1_{j,H}(s), \tag{18}$$

$$\begin{aligned} \frac{{\rm Im}\Pi^0_j(s)}{\pi} =& M_-{\lambda^{-}_j}^2\delta\left(s-M_-^2\right) - M_+{\lambda^{+}_j}^2\delta\left(s-M_+^2\right) \\ =& \rho^0_{j,H}(s), \end{aligned} \tag{19}$$

where $j = 1/2$, 3/2, and 5/2, and we add subscript H to represent the hadron side; then, we introduce weight functions $\sqrt{s}\exp\left(-\frac{s}{T^2}\right)$ and $\exp\left(-\frac{s}{T^2}\right)$ to obtain the QCD sum rules at the hadron side,

$$\int_{4m_c^2}^{s_0} {\rm d}s\left[\sqrt{s}\rho^1_{j,H}(s)+\rho^0_{j,H}(s)\right]\exp\left(-\frac{s}{T^2}\right) = 2M_-{\lambda^{-}_j}^2\exp\left(-\frac{M_-^2}{T^2}\right), \tag{20}$$

where s_0 is the continuum threshold parameter and T^2 is the Borel parameter.

It is also straightforward to accomplish the operator product expansion in the deep Euclidian space-time. For the technical details of performing the operator product expansion for the correlation functions in exploring the multiquark states with hidden charm, one can consult Refs. [32-35, 44-49]. If we contract the quark fields in the correlation functions in Eq. (3) with Wick's theorem, we can observe clearly that there are two heavy quark propagators and three light quark propagators. If each heavy quark line emits a gluon and each light quark line contributes a quark-antiquark pair, we obtain a quark-gluon mixed operator $g_sG_{\mu\nu}g_sG_{\alpha\beta}\bar{q}q\bar{q}q\bar{q}q$ with $q=u$, d or s, which is of dimension 13, and leads to vacuum condensates $\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle^2$ and $\langle\bar{q}q\rangle^3\langle\frac{\alpha_s}{\pi}GG\rangle$. It is better to take into account the vacuum condensates up to dimension $n=13$ at least. In contrast, the quark-gluon operators can be counted by fine structure constant $\alpha_s=\frac{g_s^2}{4\pi}$ with orders $O(\alpha_s^k)$, where $k=0$, $1/2$, 1, $3/2$, $\cdots$. In the present study, we take truncations $n\leqslant 13$ and $k\leqslant 1$ consistently, and the quark-gluon operators of orders $O(\alpha_s^k)$ with $k\leqslant 1$ are given full considerations, whereas in the previous study [22], we neglected vacuum condensates $\langle\bar{q}q\rangle\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}GG\rangle$, and $\langle\bar{q}q\rangle^3\langle\frac{\alpha_s}{\pi}GG\rangle$ owing to their small contributions. Furthermore, we take into account the light flavor, $SU(3)$, mass-breaking effects by including the contributions of order $O(m_s)$ consistently.

Now, we briefly digress to discuss the higher dimensional vacuum condensates. In the QED, we deal with a perturbative vacuum, and the vacuum expectation values of normal-ordered electron-photon operators can be set as zero, for example, $\langle 0|:\bar{e}e:|0\rangle=0$, $\langle 0|:\bar{e}\sigma\cdot Fe:|0\rangle=0$, and $\langle 0|:\bar{e}e\bar{e}e:|0\rangle=0$.

In the QCD, we deal with a non-perturbative vacuum and have to resort to the non-zero vacuum expectation values of normal-ordered quark-gluon operators to describe the hadron properties satisfactorily, for example, $\langle 0|:\bar{q}^i_\alpha q^j_\beta:|0\rangle\neq 0$, $\langle 0|:\bar{q}^i_\alpha q^j_\beta g_sG^a_{\mu\nu}:|0\rangle\neq 0$, and $\langle 0|:\bar{q}^i_\alpha q^j_\beta\bar{q}^m_\lambda q^n_\tau:|0\rangle\neq 0$, where i, j, m , and n are color indices and α, β, λ , and τ are the Dirac spinor indices. We usually parameterize vacuum matrix elements in terms of

$$
\begin{aligned}
\langle 0|:\bar{q}^i_\alpha q^j_\beta:|0\rangle &= \frac{1}{12}\langle 0|:\bar{q}q:|0\rangle\delta_{ij}\delta_{\alpha\beta},\\
\langle 0|:\bar{q}^i_\alpha q^j_\beta G^a_{\mu\nu}:|0\rangle &= \frac{1}{192}\langle 0|:\bar{q}g_s\sigma\cdot Gq:|0\rangle\left(\sigma_{\mu\nu}\right)_{\beta\alpha}\frac{\lambda^a_{ji}}{2},\\
\langle 0|:\bar{q}^i_\alpha q^j_\beta\bar{q}^m_\lambda q^n_\tau:|0\rangle &= \frac{\varrho}{144}\langle 0|:\bar{q}q:|0\rangle^2\left(\delta_{ij}\delta_{mn}\delta_{\alpha\beta}\delta_{\lambda\tau}\right.\\
&\quad\left.-\delta_{in}\delta_{jm}\delta_{\alpha\tau}\delta_{\beta\lambda}\right),
\end{aligned}
$$

or $\frac{1}{144}\langle 0|:\bar{q}q\bar{q}q:|0\rangle\left(\delta_{ij}\delta_{mn}\delta_{\alpha\beta}\delta_{\lambda\tau}-\delta_{in}\delta_{jm}\delta_{\alpha\tau}\delta_{\beta\lambda}\right)$, etc., where λ^a are the Gell-mann matrices. Except for the quark condensates, which indicate spontaneous breaking of the Chiral symmetry through the Gell-Mann-Oakes-Renner relation, $f_\pi^2m_\pi^2=-2(m_u+m_d)\langle 0|:\bar{q}q:|0\rangle$ [50], vacuum condensates such as $\langle 0|:\bar{q}g_s\sigma\cdot Gq:|0\rangle$, $\langle 0|:\bar{q}q\bar{q}q:|0\rangle$, $\varrho\langle 0|:\bar{q}q:|0\rangle^2$, $\cdots$ are just parameters introduced by hand to describe the non-pertubative vacuum.

We can parameterize the non-perturbative properties in some manner, and then compare them to the experimental data on multiquark states to obtain the optimal values. In the QCD sum rules for multiquark states, $\varrho\langle\bar{q}q\rangle^2$ plays an important role and influences the convergent behaviors of the operator product expansion and pole contributions remarkably; therefore, remarkably influencing the predictions, and large values of ϱ may destroy the platforms [51]. For example, in the present case, if we take value $\varrho=2(3)$ in the QCD sum rules for the $\bar{D}\Sigma_c$ pentaquark molecular state, we obtain uncertainty $\delta M_P=-0.10(-0.16)\,\text{GeV}$, which is of the same order of the total uncertainty from other parameters, and is a very bad platform (in other words, no platform at all). In the calculations, we observe that the optimal value is $\varrho=1$, and vacuum saturation (factorizaiton) is effective in the QCD sum rules for multiquark states [32-37, 44-49, 51].

In the QCD sum rules for the $q\bar{q}$, $q\bar{Q}$, and $Q\bar{Q}$ mesons, $\varrho\langle\bar{q}q\rangle^2$ is typically accompanied with the fine-structure constant, α_s, and plays a small role, and the deviation from vacuum saturation (factorization) $\varrho=1$, for example, $\varrho=2\sim 3$, cannot make much difference in the numerical predictions, although in some cases the values $\varrho>1$ can lead to better QCD sum rules [52, 53].

Once the corresponding analytical spectral densities, $\rho^1_{j,\mathrm{QCD}}(s)$ and $\rho^0_{j,\mathrm{QCD}}(s)$, at the quark-gluon level are obtained, we can take the quark-hadron duality below the continuum threshold, s_0, and introduce weight functions $\sqrt{s}\exp\left(-\frac{s}{T^2}\right)$ and $\exp\left(-\frac{s}{T^2}\right)$ to obtain the QCD sum rules as follows:

$$
\begin{aligned}
2M_-\lambda_j^{-2}\exp\left(-\frac{M_-^2}{T^2}\right) &= \int_{4m_c^2}^{s_0}\mathrm{d}s\left[\sqrt{s}\rho^1_{j,\mathrm{QCD}}(s)\right.\\
&\quad\left.+\rho^0_{j,\mathrm{QCD}}(s)\right]\exp\left(-\frac{s}{T^2}\right),
\end{aligned}
\tag{21}
$$

where the explicit expressions of spectral densities $\rho^1_{j,\mathrm{QCD}}(s)$ and $\rho^0_{j,\mathrm{QCD}}(s)$ at the quark level are neglected for simplicity.

We differentiate Eq. (21) with respect to $\tau=\frac{1}{T^2}$ and then eliminate pole residues λ_j^- with $j=1/2$, $3/2$, and $5/2$ to obtain the QCD sum rules for the masses of the pentaquark molecular states,

$$
M_-^2=\frac{-\frac{\mathrm{d}}{\mathrm{d}\tau}\int_{4m_c^2}^{s_0}\mathrm{d}s\left[\sqrt{s}\rho^1_{\mathrm{QCD}}(s)+\rho^0_{\mathrm{QCD}}(s)\right]\exp(-\tau s)}{\int_{4m_c^2}^{s_0}\mathrm{d}s\left[\sqrt{s}\rho^1_{\mathrm{QCD}}(s)+\rho^0_{\mathrm{QCD}}(s)\right]\exp(-\tau s)},
\tag{22}
$$

where spectral densities $\rho^1_{\mathrm{QCD}}(s)=\rho^1_{j,\mathrm{QCD}}(s)$ and $\rho^0_{\mathrm{QCD}}(s)=\rho^0_{j,\mathrm{QCD}}(s)$.

III. NUMERICAL RESULTS AND DISCUSSIONS

We take the standard values of the vacuum condensates, $\langle\bar{q}q\rangle=-(0.24\pm 0.01\,\text{GeV})^3$, $\langle\bar{s}s\rangle=(0.8\pm 0.1)\langle\bar{q}q\rangle$, $\langle\bar{q}g_s\sigma Gq\rangle=m_0^2\langle\bar{q}q\rangle$, $\langle\bar{s}g_s\sigma Gs\rangle=m_0^2\langle\bar{s}s\rangle$, $m_0^2=(0.8\pm 0.1)$

GeV^2, and $\langle\frac{\alpha_s GG}{\pi}\rangle = 0.012\pm0.004\,\text{GeV}^4$, at the energy scale of $\mu = 1\,\text{GeV}$ [54-56], and take $\overline{MS}$ masses $m_c(m_c) = (1.275\pm0.025)\,\text{GeV}$ and $m_s(\mu = 2\,\text{GeV}) = (0.095\pm0.005)$ GeV from the Particle Data Group [4]. Furthermore, we take into account the energy-scale dependence of the quark condensates, mixed quark condensates, and $\overline{MS}$ masses according to the renormalization group equation [57],

$$\begin{aligned}
\langle\bar{q}q\rangle(\mu) =&\langle\bar{q}q\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}, \\
\langle\bar{s}s\rangle(\mu) =&\langle\bar{s}s\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{12}{33-2n_f}}, \\
\langle\bar{q}g_s\sigma Gq\rangle(\mu) =&\langle\bar{q}g_s\sigma Gq\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}, \\
\langle\bar{s}g_s\sigma Gs\rangle(\mu) =&\langle\bar{s}g_s\sigma Gs\rangle(1\text{GeV})\left[\frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)}\right]^{\frac{2}{33-2n_f}}, \\
m_c(\mu) =&m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{33-2n_f}}, \\
m_s(\mu) =&m_s(2\text{GeV})\left[\frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})}\right]^{\frac{12}{33-2n_f}}, \\
\alpha_s(\mu) =&\frac{1}{b_0 t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2(\log^2 t-\log t-1)+b_0 b_2}{b_0^4 t^2}\right]
\end{aligned} \tag{23}$$

where

$$t = \log\frac{\mu^2}{\Lambda^2},\ \ b_0 = \frac{33-2n_f}{12\pi},\ \ b_1 = \frac{153-19n_f}{24\pi^2}$$
$$b_2 = \frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3}$$

$\Lambda = 213\,\text{MeV}$, $296\,\text{MeV}$, and $339\,\text{MeV}$ for the flavors, $n_f = 5$, 4, and 3, respectively [4, 57].

In the present study, we investigate the hidden-charm pentaquark molecular states with and without strangeness, and it is good to choose flavor number $n_f = 4$. We evolve all input parameters to typical or special energy scales μ, which satisfy the energy scale formula or a modified energy scale formula [32-35, 44-49] with the updated value of the effective (or constituent) charmed quark mass, $\mathbb{M}_c = 1.85$ GeV [58]. By comparing with the constituent quark masses based on analysis of the J/ψ and Υ mass spectrum with the well-known Cornell potential [59], we introduce an uncertainty $\mathbb{M}_c = 1.85\pm 0.01\,\text{GeV}$. Furthermore, we take into account the light flavor, $SU(3)$, mass-breaking effects, and prefer the modified energy scale formula, $\mu = \sqrt{M^2_{X/Y/Z/P}-(2\mathbb{M}_c)^2}-k\mathbb{M}_s$, with s-quark numbers $k = 0$, 1, 2, 3 and effective s-quark mass $\mathbb{M}_s = 0.2\,\text{GeV}$, which was proved to be effective [60]. Compared to the constituent quark mass, the effective s-quark mass, $\mathbb{M}_s = 0.2\,\text{GeV}$, seems extremely small, as the effective u/d-quark masses $\mathbb{M}_{u/d}$ serve as milestones and have been absorbed into energy scale μ, where value $\mathbb{M}_s = 0.2\,\text{GeV}$ embodies the net $SU(3)$ mass-breaking effects.

We can rewrite the energy scale formula in the form,

$$M^2_{X/Y/Z/P} = (\mu+k\delta)^2+\text{Constants}, \tag{24}$$

where the light flavor $SU(3)$ mass-breaking effects δ have the value of $\mathbb{M}_s$ and the Constants have value $4\mathbb{M}_c^2$, and they are all fitted by the QCD sum rules. μ and $\mathbb{M}_s$ embody the light degrees of freedom, whereas $4\mathbb{M}_c^2$ embodies the heavy degree of freedom. The hidden-charm tetraquark and pentaquark (molecular) states can be divided into heavy and light degrees of freedom [32-35, 45-49]. The predicted tetraquark and pentaquark (molecular) masses and the pertinent energy scales of the QCD spectral densities have Regge-trajectory-like relations [51].

In Ref. [22], we explored the $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$ pentaquark molecular states with the QCD sum rules in detail, and concluded that the energy scale formula, $\mu = \sqrt{M^2_{X/Y/Z/P}-(2\mathbb{M}_c)^2}$, can enhance the pole contributions at the hadron side remarkably and improve the convergent behaviors of the operator product expansion notably. In fact, we take the energy scale formula as a constraint on the predicted molecular masses, which should be obeyed in the QCD sum rules. In the present study, we consider the light flavor, $SU(3)$, mass-breaking effects and utilize the modified energy scale formula, $\mu = \sqrt{M^2_{X/Y/Z/P}-(2\mathbb{M}_c)^2}-k\mathbb{M}_s$, to choose the best energy scales of the spectral densities at the quark-gluon level. We search for the best Borel parameters and continuum threshold parameters to satisfy the two fundamental criteria of the QCD sum rules: pole dominance at the hadron side and convergence of the operator product expansion at the QCD side, via trial and error.

Then, we obtain the Borel parameters, continuum threshold parameter s_0, optimal energy scales of the spectral densities at the quark-gluon level, and pole contributions of the ground-state pentaquark molecular states, which are listed in Table 2. From the table, we can clearly see that the contributions from the ground states are about or larger than 40–60%, and the pole dominance criterion is satisfied very well. For the conventional hadrons, QCD spectral densities $\rho(s)\sim s^n$ with $n\leqslant 1$ and 2 for the mesons and baryons, respectively, and it is easy to satis-

Table 2. Optimal energy scale μ, Borel parameter T^2, continuum threshold parameter s_0, and pole contribution (pole) for hidden-charm pentaquark molecular states.

	J^P	μ/GeV	T^2/GeV^2	$\sqrt{s_0}$/GeV	pole(%)
$\bar{D}\Sigma_c$	$1/2^-$	2.2	$3.1-3.5$	5.02 ± 0.10	$42-64$
$\bar{D}\Xi'_c$	$1/2^-$	2.2	$3.2-3.6$	5.14 ± 0.10	$42-63$
$\bar{D}\Sigma^*_c$	$3/2^-$	2.4	$3.2-3.6$	5.08 ± 0.10	$43-64$
$\bar{D}\Xi^*_c$	$3/2^-$	2.4	$3.3-3.7$	5.21 ± 0.10	$43-64$
$\bar{D}^*\Sigma_c$	$3/2^-$	2.5	$3.3-3.7$	5.16 ± 0.10	$41-62$
$\bar{D}^*\Xi'_c$	$3/2^-$	2.5	$3.4-3.8$	5.29 ± 0.10	$41-61$
$\bar{D}^*\Sigma^*_c$	$5/2^-$	2.6	$3.4-3.8$	5.22 ± 0.10	$40-60$
$\bar{D}^*\Xi^*_c$	$5/2^-$	2.6	$3.5-3.9$	5.35 ± 0.10	$40-60$

fy the pole dominance criterion, because the integral,

$$\int_{\Delta^2}^{s_0} \mathrm{d}s s^n \exp\left(-\frac{s}{T^2}\right), \tag{25}$$

converges quickly even if we choose a large Borel parameter T^2, where Δ^2 is the threshold, and the uncertainty originating from the continuum threshold parameter, s_0, is small. For the multiquark states, QCD spectral densities $\rho(s)\sim s^n$ with $n\leqslant 4$ and 5 for the tetraquark and pentaquark (molecular) states, respectively, and it is very difficult to satisfy the pole dominance criterion, because the integral,

$$\int_{\Delta^2}^{s_0} \mathrm{d}s s^n \exp\left(-\frac{s}{T^2}\right), \tag{26}$$

converges very slowly even if we choose a rather small Borel parameter T^2. In general, we expect to choose $T^2=O(M^2)$, and the integral (or continuum state) is suppressed by a factor $\exp\left(-\frac{s}{T^2}\right)\sim\exp\left(-\frac{M^2}{T^2}\right)\sim e^{-1}$. Thus, for the multiquark states, we have to resort to a much stringent suppression of the continuum states, $T^2\ll M^2$. One may think that such a small Borel parameter might lead to a bad convergent behavior in the operator product expansion.

In Fig. 1, we plot the absolute values of $D(n)$ for the central values of the input parameters listed in Table 2, where the $D(n)$ is defined as

$$D(n)=\frac{\int_{4m_c^2}^{s_0}\mathrm{d}s\rho_n(s)\exp\left(-\frac{s}{T^2}\right)}{\int_{4m_c^2}^{s_0}\mathrm{d}s\rho(s)\exp\left(-\frac{s}{T^2}\right)}, \tag{27}$$

the $\rho_n(s)$ denotes the QCD spectral densities for the vacuum condensates of dimension n, and the total spectral densities, $\rho(s)=\sqrt{s}\rho^1_{\rm QCD}(s)+\rho^0_{\rm QCD}(s)$. From the figure, we can see clearly that although the largest contributions do not come from term $D(0)$ in some cases, the vacuum condensates $\langle\bar{q}q\rangle\langle\bar{q}q\rangle$ and $\langle\bar{q}q\rangle\langle\bar{s}s\rangle$ with dimension six serve as a milestone, and the absolute values of the contributions, $|D(n)|$, with $n\geqslant 6$ decrease monotonically and quickly with the increase in the dimensions, n. Moreover, the value $|D(13)|\approx 0$, and the operator product expansion converges very well. The two basic criteria of the QCD sum rules are satisfied.

In the calculations, we observe that the predicted molecular masses increase monotonically and slowly with the increase in the continuum threshold parameter, s_0, if we fix the Borel parameter, T^2; in contrast, a larger continuum threshold parameter implies a larger pole contribution. We truncate the continuum threshold parameter, s_0, requiring the same pole contributions, approximately 40–60%, in all QCD sum rules so as to reduce the uncertainties originating from the continuum threshold parameter, s_0.

In a previous study [22], we neglected vacuum condensates $\langle\bar{q}q\rangle\langle\frac{\alpha_s}{\pi}GG\rangle$, $\langle\bar{q}q\rangle^2\langle\frac{\alpha_s}{\pi}GG\rangle$, and $\langle\bar{q}q\rangle^3\langle\frac{\alpha_s}{\pi}GG\rangle$, which are of dimensions 7, 10, and 13, respectively, owing to their small contributions. From Fig. 1, we can see clearly that the vacuum condensates of 7, 10, and 13 days play a small role in the Borel windows indeed. We prefer to take into account these contributions because they lead to slightly larger pole contributions, and therefore, more

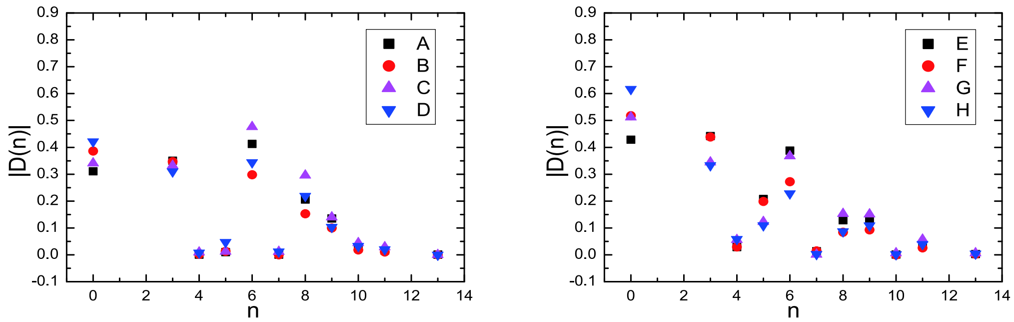

Fig. 1. (color online) The absolute values of the contributions of the vacuum condensates of dimension n, where A, B, C, D, E, F, G, and H denote the pentaquark molecular states, $\bar{D}\Sigma_c$, $\bar{D}\Xi'_c$, $\bar{D}\Sigma^*_c$, $\bar{D}\Xi^*_c$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Xi'_c$, $\bar{D}^*\Sigma^*_c$, and $\bar{D}^*\Xi^*_c$, respectively.

reliable QCD sum rules. In the present study, we intend to explore the $SU(3)$-breaking effects, and it is better to take into account these contributions consistently.

Finally, we take into account all uncertainties of the input parameters, and obtain the masses and pole residues of the $J^P=1/2^-$, $3/2^-$, and $5/2^-$ hidden-charm pentaquark molecular states without and with strangeness, which are summarized explicitly in Table 3 and Fig. 2. In Fig. 2, we plot the predicted masses of the hidden-charm pentaquark molecules without and with strangeness according to variations in the Borel parameter, where the regions between two short vertical lines are the Borel windows. From the figure, we can see clearly that there appear rather flat platforms in the Borel windows and the uncertainties coming from the Borel parameters seem rather small. The uncetainities are compatible with the fact that the Borel parameters are just supplementary parameters, and not physical quantities. Furthermore, in the figure, we also present the experimental values of the masses of $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$ from the LHCb collaboration [1-3].

The pentaquark (molecule) candidates $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, and $P_c(4457)$ are observed in the $J/\psi p$ mass spectrum, and their isospins are $I=1/2$, whereas pentaquark (molecule) candidate $P_{cs}(4459)$ is observed in the $J/\psi\Lambda$ mass spectrum, and its isospin is $I=0$. The present calculations support assigning $P_c(4312)$ to the $\bar{D}\Sigma_c$ pentaquark molecular state with quantum numbers $J^P=1/2^-$ and $I=1/2$, assigning $P_c(4380)$ as the $\bar{D}\Sigma_c^*$ pentaquark molecular state with quantum numbers $J^P=3/2^-$ and $I=1/2$, assigning $P_c(4440/4457)$ as the $\bar{D}^*\Sigma_c$ pentaquark molecular state with quantum numbers $J^P=3/2^-$ and $I=1/2$, assigning the $P_{cs}(4459)$ as the $\bar{D}\Xi_c'$ pentaquark molecular state with quantum numbers $J^P=1/2^-$ and $I=0$. However, we cannot exclude the possibilities of assigning $P_c(4457)$ as the $\bar{D}^*\Sigma_c^*$ pentaquark molecular state with quantum numbers $J^P=5/2^-$ and $I=1/2$ and assigning the $P_{cs}(4459)$ as the $\bar{D}\Xi_c^*$ pentaquark molecular state with quantum numbers $J^P=3/2^-$ and $I=0$ owing to the uncertainties, and refer to Table 3 and Fig. 2. For example, it is marginal to assign the $P_c(4457)$ as the $\bar{D}^*\Sigma_c^*$ pentaquark molecular state with the quantum numbers $J^P=5/2^-$ and $I=1/2$ and as $P_c(4457)$ lies at the bottom of the predicted mass of the $\bar{D}^*\Sigma_c^*$ pentaquark molecular state, see Fig. 2-G.

From Tables 2-3, we can see that the modified energy scale formula, $\mu=\sqrt{M^2_{X/Y/Z/P}-(2\mathbb{M}_c)^2}-k\mathbb{M}_s$ with s-quark numbers $k=0$, 1, 2, 3 and the effective s-quark mass $\mathbb{M}_s=0.2\,\text{GeV}$ is satisfied very well [60]. In contrast, the predicted masses for the pentaquark molecular states without and with strangeness have the relation, $M_{P_{cs}}-M_{P_c}\approx m_s-m_q\approx 0.13\sim0.15\,\text{GeV}$, which is consistent with the light-flavor, $SU(3)$, breaking effects for the heavy baryons in the flavor sextet, $\mathbf{6}_f$, $M_{\Xi_c'}-M_{\Sigma_c}\approx M_{\Xi_c^*}-M_{\Sigma_c^*}\approx m_s-m_q\approx 0.13\,\text{GeV}$ from the Particle Data Group [4].

The present calculations indicate that there may exist $\bar{D}\Sigma_c$ ($\bar{D}\Xi_c'$), $\bar{D}\Sigma_c^*$ ($\bar{D}\Xi_c^*$), $\bar{D}^*\Sigma_c$ ($\bar{D}^*\Xi_c'$), and $\bar{D}^*\Sigma_c^*$ ($\bar{D}^*\Xi_c^*$) pentaquark molecular states with the $J^P=1/2^-$, $3/2^-$, $3/2^-$ and $5/2^-$, respectively, which lie near the corresponding thresholds: $\bar{D}\Sigma_c$ ($\bar{D}\Xi_c'$), $\bar{D}\Sigma_c^*$ ($\bar{D}\Xi_c^*$), $\bar{D}^*\Sigma_c$ ($\bar{D}^*\Xi_c'$) , and $\bar{D}^*\Sigma_c^*$ ($\bar{D}^*\Xi_c^*$), respectively (see Table 3. The two-body strong decay to the corresponding open-charm meson-baryon pairs, such as $\bar{D}\Sigma_c$ ($\bar{D}\Xi_c'$), $\bar{D}\Sigma_c^*$ ($\bar{D}\Xi_c^*$), $\bar{D}^*\Sigma_c$ ($\bar{D}^*\Xi_c'$), and $\bar{D}^*\Sigma_c^*$ ($\bar{D}^*\Xi_c^*$), with the fall-apart mechanism directly, can only take place through the higher tails of the mass distributions and are kinematically suppressed in the phase space, and the widths of these pentaquark molecular states should be narrow. A large width $\Gamma_{P_c(4380)}=205\pm18\pm86\,\text{MeV}$ may indicate that $P_c(4390)$ may correspond to two or more unresolved structures. More experimental data and theoretical studies are still needed to identify $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$, unambiguously.

In the present study, we predict the masses of new pentaquark molecular states, besides reproducing the masses of the existing pentaquark candidates, $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$. We can search for the non-strange $\bar{D}\Sigma_c$, $\bar{D}\Sigma_c^*$, $\bar{D}^*\Sigma_c$, and $\bar{D}^*\Sigma_c^*$

Table 3. Predicted masses and pole residues of the hidden-charm pentaquark molecular states with the possible assignments, where double-? denotes that such assignment is not excluded owing to the uncertainties.

	J^P	M/GeV	$\lambda/(10^{-3}\text{GeV}^6)$	Thresholds/MeV	Assignments
$\bar{D}\Sigma_c$	$1/2^-$	4.32 ± 0.12	2.00 ± 0.36	4318	? $P_c(4312)$
$\bar{D}\Xi_c'$	$1/2^-$	4.45 ± 0.12	2.32 ± 0.42	4443	? $P_{cs}(4459)$
$\bar{D}\Sigma_c^*$	$3/2^-$	4.38 ± 0.12	1.25 ± 0.21	4382	? $P_c(4380)$
$\bar{D}\Xi_c^*$	$3/2^-$	4.51 ± 0.11	1.45 ± 0.25	4510	?? $P_{cs}(4459)$
$\bar{D}^*\Sigma_c$	$3/2^-$	4.46 ± 0.12	2.37 ± 0.40	4460	? $P_c(4440/4457)$
$\bar{D}^*\Xi_c'$	$3/2^-$	4.60 ± 0.11	2.80 ± 0.48	4585	
$\bar{D}^*\Sigma_c^*$	$5/2^-$	4.52 ± 0.12	1.82 ± 0.31	4524	?? $P_c(4457)$
$\bar{D}^*\Xi_c^*$	$5/2^-$	4.67 ± 0.11	2.16 ± 0.37	4652	

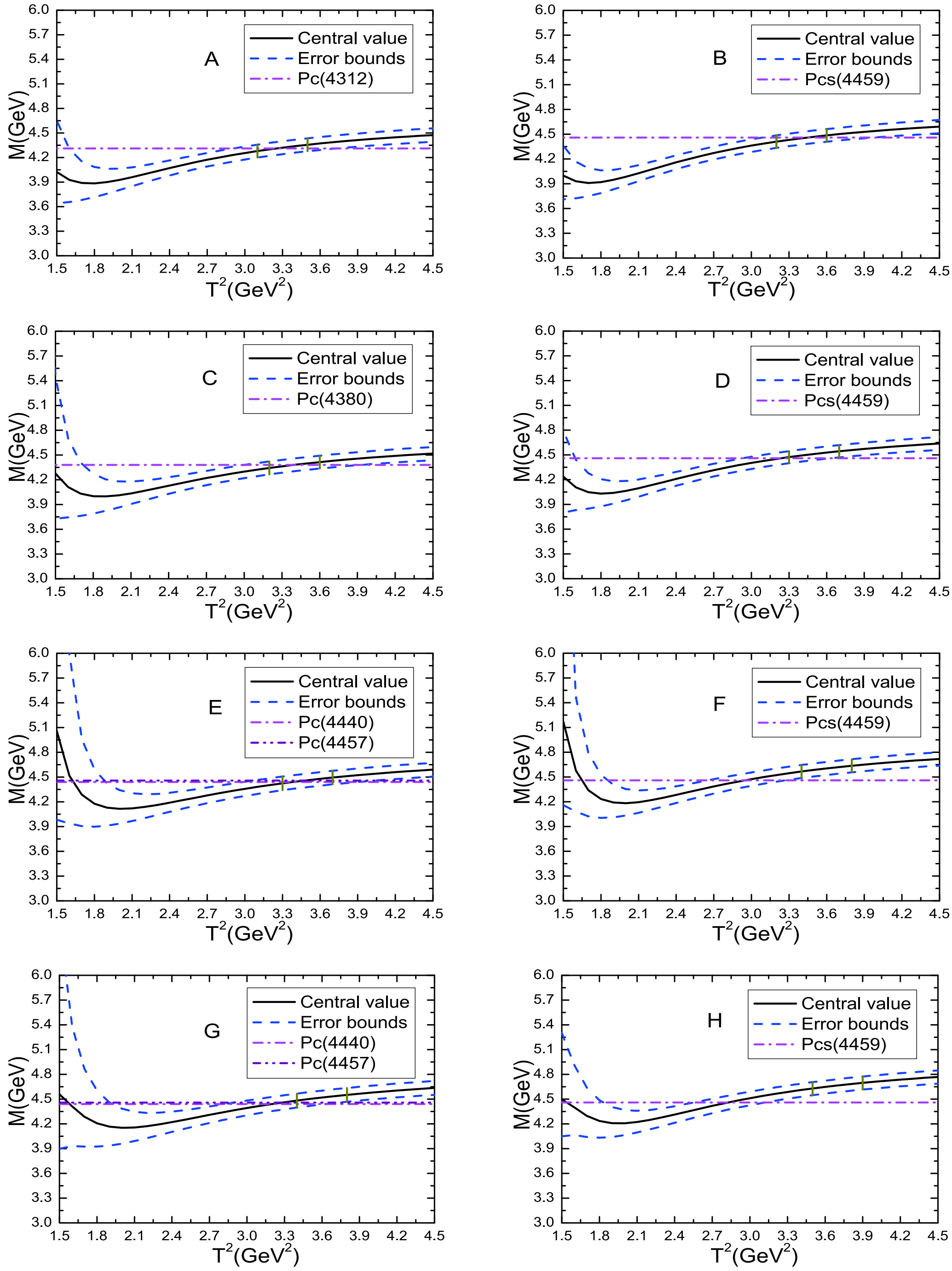

Fig. 2. (color online) Masses of the pentaquark molecular states with variations in the Borel parameter, T^2, where A, B, C, D, E, F, G, and H denote the pentaquark molecular states of $\bar{D}\Sigma_c$, $\bar{D}\Xi_c'$, $\bar{D}\Sigma_c^*$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Xi_c'$, $\bar{D}^*\Sigma_c^*$, and $\bar{D}^*\Xi_c^*$, respectively.

pentaquark molecular states with isospin $I=1/2$ (or $I=3/2$) and with spin parity $J^P=1/2^-$, $3/2^-$, $3/2^-$ and $5/2^-$, respectively in the Λ_b^0 decay,

$$\begin{aligned}\Lambda_b^0 \to\, & pJ/\psi K^-,\, nJ/\psi\bar{K}^0,\, nJ/\psi\bar{K}^0,\, p\eta_c K^-,\, n\eta_c\bar{K}^0,\\ & n\eta_c\bar{K}^0,\Delta^+J/\psi K^-,\,\Delta^0 J/\psi\bar{K}^0,\,\Delta^0 J/\psi\bar{K}^0,\\ & \Delta^+\eta_c K^-,\,\Delta^0\eta_c\bar{K}^0,\,\Delta^0\eta_c\bar{K}^0,\end{aligned} \tag{28}$$

and search for the strange $\bar{D}\Xi_c'$, $\bar{D}\Xi_c^*$, $\bar{D}^*\Xi_c'$, and $\bar{D}^*\Xi_c^*$ pentaquark molecular states with isospin $I=0$ (or $I=1$) and spin parity $J^P=1/2^-$, $3/2^-$, $3/2^-$, and $5/2^-$, respectively, in the Ξ_b^0 and Ξ_b^- decays,

$$\Xi_b^0 \to \Sigma^+ J/\psi K^-, \Sigma^0 J/\psi \bar{K}^0, \Lambda^0 J/\psi \bar{K}^0, \Sigma^+ \eta_c K^-, \Sigma^0 \eta_c \bar{K}^0, \Lambda^0 \eta_c \bar{K}^0, \Sigma^{*+} J/\psi K^-, \Sigma^{*0} J/\psi \bar{K}^0, \Sigma^{*+} \eta_c K^-, \Sigma^{*0} \eta_c \bar{K}^0, \quad (29)$$

$$\Xi_b^- \to \Lambda^0 J/\psi K^-, \Sigma^0 J/\psi K^-, \Sigma^- J/\psi \bar{K}^0, \Lambda^0 \eta_c K^-, \Sigma^0 \eta_c K^-, \Sigma^- \eta_c \bar{K}^0, \Sigma^{*0} J/\psi K^-, \Sigma^{*-} J/\psi \bar{K}^0, \Sigma^{*0} \eta_c K^-, \Sigma^{*-} \eta_c \bar{K}^0. \quad (30)$$

IV. CONCLUSION

In this study, we investigated the $\bar{D}\Sigma_c$, $\bar{D}\Xi'_c$, $\bar{D}\Sigma^*_c$, $\bar{D}\Xi^*_c$, $\bar{D}^*\Sigma_c$, $\bar{D}^*\Xi'_c$, $\bar{D}^*\Sigma^*_c$, and $\bar{D}^*\Xi^*_c$ pentaquark molecular states with and without strangeness using the QCD sum rules in detail by consistently performing operator product expansion up to vacuum condensates of dimension 13. The modified energy scale formula was used to choose the best energy scales of the spectral densities at the quark-gluon levels and make predictions for the masses of the new pentaquark molecular states, besides reproducing the masses of existing pentaquark candidates $P_c(4312)$, $P_c(4380)$, $P_c(4440)$, $P_c(4457)$, and $P_{cs}(4459)$. The present calculations support assigning $P_c(4312)$ as the $\bar{D}\Sigma_c$ pentaquark molecular state with the quantum numbers $J^P = 1/2^-$ and $I = 1/2$, assigning the $P_c(4380)$ is a $\bar{D}\Sigma^*_c$ pentaquark molecular state with quantum numbers $J^P = 3/2^-$ and $I = 1/2$,$P_c(4440/4457)$ as the $\bar{D}^*\Sigma_c$ pentaquark molecular state with quantum numbers $J^P = 3/2^-$ and $I = 1/2$, and $P_{cs}(4459)$ as the $\bar{D}\Xi'_c$ pentaquark molecular state with quantum numbers $J^P = 1/2^-$ and $I = 0$. However, we cannot exclude the possibilities of assigning $P_c(4457)$ as the $\bar{D}^*\Sigma^*_c$ pentaquark molecular state with the quantum numbers $J^P = 5/2^-$ and $I = 1/2$ and assigning $P_{cs}(4459)$ as the $\bar{D}\Xi^*_c$ pentaquark molecular state with quantum numbers $J^P = 3/2^-$ and $I = 0$ due to the uncertainties. In the calculations, we observe that the predicted masses of the pentaquark molecular states without strangeness and with strangeness have a mass gap of approximately $0.13 \sim 0.15\,\text{GeV}$, which is consistent with the light-flavor, $SU(3)$, breaking effects of the heavy baryons in flavor sextet $\mathbf{6}_f$. We can search for both old and new pentaquark molecular states in the decay of Λ_b^0, Ξ_b^0, and Ξ_b^- in the future to conduct more robust investigations and shed light on the nature of P_c and P_{cs} states.

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