

The sixth order cumulant of net-proton number in Binomial distribution at $\sqrt{s_{NN}} = 200$ GeV*

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Abstract: It is proposed that ratios of the sixth order to the second order cumulant (C_6/C_2) of conserved quantities are sensitive to the chiral crossover transition. Recently, the negative C_6/C_2 was obtained in both theoretical Lattice QCD and experiments at $\sqrt{s_{NN}} = 200$ GeV. In this study, we investigate the behavior of net-proton C_6/C_2 in statistical Binomial distribution (BD) at $\sqrt{s_{NN}} = 200$ GeV in Au + Au collisions. With the BD parameters extracted from RHIC/STAR, it is found that C_6/C_2 can be negative. Furthermore, the obtained C_6/C_2 becomes smaller when applying the same magnitude of experimental statistics and calculation method to simulations. In 0-10% centrality, there is a significant difference between the simulated result and theoretical expectation. Based on the extracted parameters and experimentally collected statistics, the baseline of net-proton C_6/C_2 in BD is presented.

Keywords: the sixth order cumulant, net-proton number, Binomial distribution, statistics

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I. INTRODUCTION

One of the main goals of the Beam Energy Scan (BES) Program at the Relativistic Heavy Ion Collider (RHIC) is to explore the phase diagram of Quantum Chromodynamics (QCD) [1]. At vanishing and small values of the chemical potentials for conserved charges, such as net-baryon, net-charge, and net-strangeness, Lattice QCD has predicted that there occurs a smooth crossover from the hadronic phase to the QGP phase [2]. Based on Lattice QCD calculations with physical values of light and strange quark masses, the negative signal of the ratio of the sixth to the second order (C_6/C_2) is observed in the crossover region [3-6]. The QCD-assisted low-energy effective theory and the QCD based models, such as the Polyakov loop extended quark-meson (PQM) and the Nambu-Jona-Lasinio (PNJL) models, also support that the sixth order cumulant is negative near the chiral crossover transition [7-11]. Consequently, C_6/C_2 is a powerful observable to study the QCD phase diagram in experiments.

The STAR Collaboration had reported the preliminary results of net-proton C_6/C_2 [12-14]. In 0-40% centrality, C_6/C_2 is negative at $\sqrt{s_{NN}} = 200$ GeV, which is consistent with the Lattice QCD calculation. However, C_6/C_2 is positive and close to unity at $\sqrt{s_{NN}} = 54.4$ GeV, unlike the QCD predicted negative behavior. In the fu-

ture, with the sufficient statistics accumulated, the analysis of the net-proton C_6/C_2 is possible by the ALICE collaboration at LHC [15]. Before illustrating the physics of measured C_6/C_2 , the contributions of non-phase transition related influences must be subtracted, such as the conservation of the total baryon number, experimental acceptances in terms of kinematic variables, efficiency corrections, and the difference of C_6/C_2 in net-proton and net-baryon. [16-19]. It was shown that C_6/C_2 can be negative for $\sqrt{s_{NN}} \leq 40$ GeV due to the baryon number conservation [20]. In this paper, we will focus on discussions of the BD contribution and influence of the statistics used for data analysis at $\sqrt{s_{NN}} = 200$ GeV.

In nuclear collisions, some basic statistical distributions, such as the Poisson distribution, BD, and Negative Binomial distribution (NBD), are frequently used to describe the shape of the multiplicity distributions [21-24]. Theoretically, in the free hadron gas in equilibrium, the resonance gas in the hadron phase obeys the Poisson distributions. It was also argued that the proton number is given by the superposition of the binomial distribution of the baryon number due to the isospin randomization [25]. Consequently, studies of the cumulants in different statistical distributions can extract information related to the nature of the particle production mechanism.

Recently, cumulants in these statistical distributions

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were widely taken as baselines to help us understand the experimentally measured cumulants, such as the cumulants of the net-proton, net-charge, net-kaon, and net- Λ [26-32]. To date, the studies of cumulants in BD/NBD have been up to the fourth order. It is found that the BD/NBD baselines can quantitatively explain the cumulants of net-charge, net-kaon, and net- Λ in experiment. The BD can also describe the fourth order cumulant of the net-proton number at $\sqrt{s_{NN}} \geq 39$ GeV. Consequently, it is also interesting to study the net-proton C_6/C_2 in BD.

With the data accumulated, STAR has about 850M events for analysis of C_6/C_2 at $\sqrt{s_{NN}} = 200$ GeV [13]. However, the data used in the analysis include three independent parts: (a) around 420M events for 0-80% centrality collected from the year 2010 minimum bias (MB) trigger, (b) around 110M events for 0-10% centrality from the year 2010 central trigger, and (c) around 320M events for 0-80% centrality from the year 2011 MB trigger. In principle, C_6/C_2 in each centrality should be firstly calculated by these three independent parts separately. For each part, C_6/C_2 is obtained by the method of centrality bin width correction (CBWC) [33-35]. It means C_6/C_2 needs to be calculated first in each Refmult3 bin (N_{ch}) to reduce the initial size fluctuation. Then, C_6/C_2 is averaged over all N_{ch} in a given centrality.

A critical issue is that the statistics in each of N_{ch} are still considerably limited even with 320M MB events. Our previous studies showed that we must check whether the statistics in each of N_{ch} are sufficient to satisfy the Central Limit Theorem (CLT) when applying the CBWC method in cumulants analysis [36-38]. It showed that 90M MB events are not sufficient for analysis of net-proton C_6/C_2 at $\sqrt{s_{NN}} = 11.5$ GeV in the UrQMD model. These statistics are of the same order of magnitude as that in each data collection at $\sqrt{s_{NN}} = 200$ GeV at RHIC/STAR. In addition, the required statistics are also related to the detail shape and width of the net-proton multiplicity distributions. Therefore, we need to re-examine if we can obtain reliable C_6/C_2 with 320M MB events.

In this paper, we will start off the discussions from behavior of net-proton C_6/C_2 in BD. At $\sqrt{s_{NN}} = 200$ GeV, it is found that the negative C_6/C_2 can be obtained with the experimentally measured parameters. To study influence of the statistics on C_6/C_2 , a toy simulation about validation of the CLT in C_6/C_2 is demonstrated in Section III. To make a direct comparison with experimental data, more detailed procedures of simulations are described in Section IV. With the experimentally collected statistics and extracted BD parameters, we show that there is a marked drop in C_6/C_2 in 0-10% centrality in Section V. The baseline of C_6/C_2 is also demonstrated. Finally, the summary is presented in Section VI.

II. C_6/C_2 IN BD

In probability theory and statistics, the BD is the discrete probability distribution of the number of successes in a sequence of n independent binary experiments, where the result of each experiment is true with probability p and false with probability $q = 1 - p$ [39]. The probability that the binomial random variable x takes on values in its range can be expressed using the binomial probability function:

$$P(x) = \binom{n}{x} p^x (1-p)^{n-x} = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}, \quad (1)$$

where x corresponds to the number of protons or anti-protons in each event.

Experimentally, if we know the mean μ and variance σ^2 ($\sigma^2 < \mu$ for a BD, while $\sigma^2 > \mu$ for an NBD) of the distribution, then the input parameters of p and n are:

$$p = 1 - \frac{\sigma^2}{\mu} = 1 - \varepsilon, \quad (2)$$

and

$$n = \frac{\mu}{p} = \frac{\mu}{1 - \varepsilon}, \quad (3)$$

where $\varepsilon = \frac{\sigma^2}{\mu}$.

With given μ and ε , the expectations of cumulants from the second to the sixth order can be written as:

$$C_2 = n(p - p^2) = \varepsilon\mu, \quad (4)$$

$$C_3 = n(p - 3p^2 + 2p^3) = \varepsilon\mu(-1 + 2\varepsilon), \quad (5)$$

$$C_4 = n(p - 7p^2 + 12p^3 - 6p^4) = \varepsilon\mu(1 - 6\varepsilon + 6\varepsilon^2), \quad (6)$$

$$C_5 = n(p - 15p^2 + 50p^3 - 60p^4 + 24p^5) = \varepsilon\mu(-1 + 14\varepsilon - 36\varepsilon^2 + 24\varepsilon^3), \quad (7)$$

$$C_6 = n(p - 31p^2 + 180p^3 - 390p^4 + 360p^5 - 120p^6) = \varepsilon\mu(1 - 30\varepsilon + 150\varepsilon^2 - 240\varepsilon^3 + 120\varepsilon^4). \quad (8)$$

If the numbers of protons and anti-protons are independently produced as BD, the net-proton C_6/C_2 can be

expressed as:

$$C_6/C_2 = \frac{C_6^p + C_6^{\bar{p}}}{C_2^p + C_2^{\bar{p}}}. \quad (9)$$

Generally, the expected C_6/C_2 is related to ε_p , $\varepsilon_{\bar{p}}$, μ_p , and $\mu_{\bar{p}}$. Based on these four parameters, one can obtain the expected C_6/C_2 in each centrality. In contrast, the experimental studies had shown that ε_p and $\varepsilon_{\bar{p}}$ are close to each other at $\sqrt{s_{NN}} = 200$ GeV in Au + Au collisions shown in Fig. 1(a) [35]. The error contains the statistical and systematical uncertainties, which is performed by $\sigma_\varepsilon = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2}$. The protons and anti-protons are selected at mid-rapidity ($|y| < 0.5$) within $0.4 < p_T < 2.0$ GeV/c. It shows ε_p and $\varepsilon_{\bar{p}}$ extracted from STAR are consistent with each other. Within $1\sigma_\varepsilon$ of uncertainty, the centrality dependence of ε is weak. To make an appropriate approximation, we assume $\varepsilon = \varepsilon_p = \varepsilon_{\bar{p}}$ in this paper. In this case, the expectation of net-proton C_6/C_2 can be written as:

$$C_6/C_2 = 1 - 30\varepsilon + 150\varepsilon^2 - 240\varepsilon^3 + 120\varepsilon^4. \quad (10)$$

Eq. (10) shows the expectation of C_6/C_2 is only dependent on ε . The effects of μ_p and $\mu_{\bar{p}}$ are canceled. The detailed ε dependence of C_6/C_2 is shown in Fig. 1(b). It shows C_6/C_2 drops drastically with the decrease of ε . It is already negative with $\varepsilon < 0.958$, which is within the experimentally measured range. In addition, C_6/C_2 has a broad range of values, and it can change from positive to negative, within $1\sigma_\varepsilon$ uncertainty of ε . Consequently, it is not suitable to directly give the expectation only based on the unique measured value of ε , without considering its uncertainty. Instead, we must set an interval of ε to study the behavior of C_6/C_2 . Figure 1(a) shows that the upper and lower values of the error bars touch ε at approx-

imately 0.99 and 0.94, respectively. If we assume the range of ε is between 0.94 and 0.99, the expected values of C_6/C_2 are from -0.31 to 0.71 . The negative C_6/C_2 can be obtained in the pure statistical BD. Only the negative signal is not enough to be taken as an indication of a smooth crossover transition.

Here, the obtained C_6/C_2 is the ideal theoretical expectation. In the experiment, the statistics are still a critical issue for the analysis of C_6/C_2 . The satisfaction of the CLT requires to be carefully checked before the data analysis.

III. CENTRAL LIMIT THEOREM IN C_6/C_2

It is known that if the statistics are sufficient for validation of the CLT [40], the experimentally measured results should always be consistent with the true value within 1σ , 2σ , and 3σ of uncertainties with probabilities of 68.3%, 95.5%, and 99.7%, respectively. For most of the measurements, such as the mean of the net-proton number, the required statistics for the CLT are easily achieved in experiments. In this case, within the uncertainties, the mean of the measured observable needs to be a real value and independent of the statistics, i.e.,

$$\langle X \rangle_{n_1} = \langle X \rangle_{n_2} = \langle X \rangle_{n_3}, \quad (11)$$

where X is the measured observable. The subscripts n_1 , n_2 , and n_3 denote different statistics. As an example, Fig. 2(a) shows the simulated $\langle \text{Mean} \rangle$ of the net-proton number as a function of the statistics. The BD is applied in simulations with parameters: $\mu_p = 12.66$, $\mu_{\bar{p}} = 7.5$, $\varepsilon_p = \varepsilon_{\bar{p}} = 0.97$. μ_p and $\mu_{\bar{p}}$ are mean values of proton and anti-proton in 0-10% centrality with $0.4 < p_T < 2.0$ GeV/c and $|y| < 0.5$. For each data point, we randomly and independently generate 50 sub-samples with the fixed statistics to calculate $\langle \text{Mean} \rangle$ as

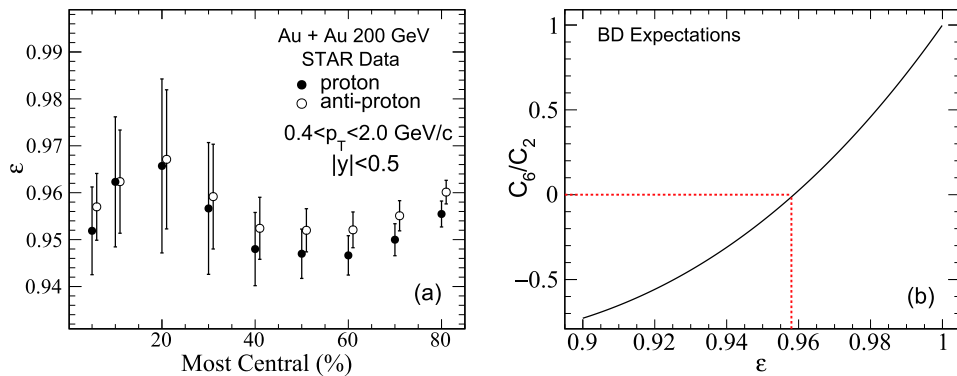


Fig. 1. (color online) The left panel shows ε_p and $\varepsilon_{\bar{p}}$ in different centralities at $\sqrt{s_{NN}} = 200$ GeV in Au + Au collisions measured by RHIC/STAR [35]. The protons and anti-protons are identified at mid-rapidity ($|y| < 0.5$) within $0.4 < p_T < 2.0$ GeV/c. The right panel shows the ε dependence of C_6/C_2 .

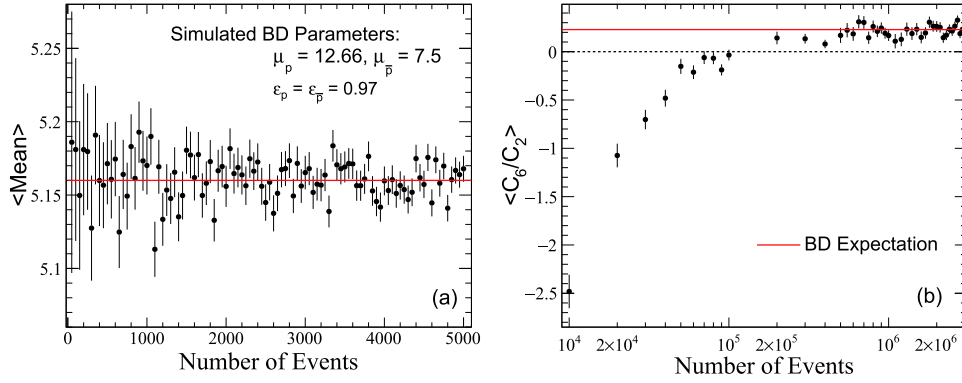


Fig. 2. (color online) Statistics dependence of $\langle \text{Mean} \rangle$ and $\langle C_6/C_2 \rangle$, respectively.

$$\langle \text{Mean} \rangle = \frac{\sum_{i=1}^{i=N} (\text{Mean})_i}{N}, \quad (12)$$

where $(\text{Mean})_i$ is the averaged number of net-proton in i_{th} sub-sample. There are two methods to estimate error ($\langle \text{Mean} \rangle$). One is obtained by the formula of the error propagation:

$$\text{error}(\langle \text{Mean} \rangle) = \frac{\sqrt{\sum_{i=1}^{i=N} \text{error}(\text{Mean})_i^2}}{N}. \quad (13)$$

We can also first calculate the width of $\langle \text{Mean} \rangle$ based on these N results. Then $\text{error}(\langle \text{Mean} \rangle)$ is the width of $\langle \text{Mean} \rangle$ divided by \sqrt{N} . A good agreement is obtained based on these two methods. In this paper, the formula of error propagation is used to measure the statistical uncertainty.

The simulated statistics in Fig. 2(a) are from 50 to 5000 events. The black dashed line is the theoretical expectation. For the mean of the net-proton number analysis, Fig. 2(a) clearly shows that 50 events are sufficient for the validation of the CLT. That is why we do not require to check whether the statistics are sufficient for most observables.

However, it is a challenge for analysis of the high-order cumulants, which are up to the fifth, sixth, or even eighth order [36–38]. When the CBWC method is applied in cumulants calculations, the statistics in each N_{ch} are significantly limited in 0–10% centrality even with a few hundred million MB events. As a crude estimation, supposing there are 1000 N_{ch} bins in 0–10% centrality, the averaged events are only approximately 10000 in each N_{ch} with 100M MB events. If 10000 events are not sufficient for the CLT in C_6/C_2 calculations, the value obtained by the CBWC method is not reliable in 0–10% centrality with 100M MB events.

By using the same simulated parameters as Fig. 2(a), while the simulated sub-sample N is significantly larger

than 50, the statistics dependence of $\langle C_6/C_2 \rangle$ from 10000 to 3M events are demonstrated in Fig. 2(b). Below 0.1M events in each N_{ch} , $\langle C_6/C_2 \rangle$ is systematically smaller than 0 and theoretical BD expectations. Up to 0.5M events in each N_{ch} , $\langle C_6/C_2 \rangle$ is consistent with theoretical expectations within statistical uncertainties.

As we mentioned, the required statistics are also related to the detail shape and width of the net-proton multiplicity distributions [36–38]. The behavior of the statistical dependency in different centralities requires a careful case-by-case study. To directly compare, we study the statistics dependence of C_6/C_2 in BD with the same calculation method as RHIC/STAR.

IV. SIMULATION SETUP

To study the statistics dependence of C_6/C_2 measurement based on the CBWC method, the input parameters for simulations are taken from the UrQMD model at $\sqrt{s_{NN}} = 200$ GeV in Au + Au collisions [41,42].

Figure 3(a) shows the normalized probability distribution of Refmult3, which is used for the centrality selection. To avoid auto-correlation and improve the centrality resolution, Refmult3 is defined by using the number of charged π and K in the final state within pseudo-rapidity $|\eta| < 1.0$ [35]. Figures 3(b) and (c) show the means of proton and anti-proton numbers, $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$, as a function of Refmult3, respectively. As a comparable study to STAR measurements, the protons and anti-protons in the UrQMD model are carried out at mid-rapidity ($|\eta| < 0.5$) in the transverse momentum range $0.4 < p_T < 2.0$ GeV/c. These values of $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ are taken as the input parameters in the following simulations.

For a straightforward comparison between the simulated results and theoretical expectations, we assume that the numbers of protons and anti-protons independently follow BD with $\varepsilon = \varepsilon_p = \varepsilon_{\bar{p}} = 0.97$ in each Refmult3 bin. Finally, the influence of statistics on C_6/C_2 with the whole interval of ε , from 0.94 to 0.99, is studied. The re-

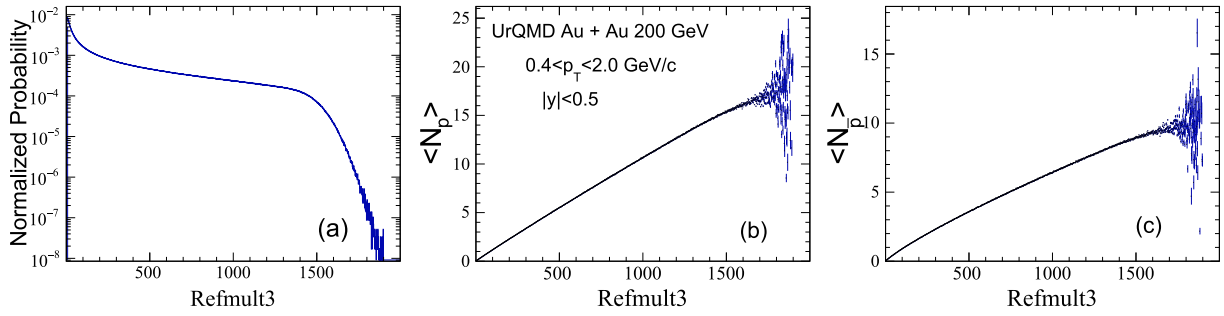


Fig. 3. (color online) The left panel shows normalized probability distribution of Refmult3 in UrQMD model in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The middle and right panels show $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ as a function of Refmult3, respectively.

quired number of MB events is 300 million (M). Under these assumptions, the simulated procedures are implemented as follows:

(a) Multiplying the normalized probability of Refmult3 by 300 M. The corresponding number of events in j_{th} Refmult3 bin is n_j .

(b) According to n_j , the values of $\langle N_p \rangle$ and $\langle N_{\bar{p}} \rangle$ in j_{th} Refmult3 bin, the numbers of protons and anti-protons, are independently and randomly generated event-by-event by BD in each Refmult3 bin.

(c) Calculating $(C_6/C_2)_j$ and its error in each Refmult3 bin. Its error is estimated by the delta theorem method [43]. Then the centrality dependence of C_6/C_2 and error(C_6/C_2) in each centrality are obtained based on the CBWC method.

(d) Repeating the procedures of (b) and (c) N times. Here, N should be large enough to reduce the uncertainty of averaged C_6/C_2 , $\langle C_6/C_2 \rangle$. We set $N = 1000$ in the following.

(e) Calculating the centrality dependence of $\langle C_6/C_2 \rangle$ and error($\langle C_6/C_2 \rangle$). The formula of error propagation is used to estimate the statistical uncertainty.

To demonstrate more details about the simulations, Figs. 4(a) and (b) show 200 results of 1000 samples in 0-10% and 30%-40% centralities, respectively. To reduce the statistical error and obtain a more stable result, the value of $\langle C_6/C_2 \rangle$, shown by the solid black line in each panel, is derived from the total 1000 samples. The black dashed lines are 1σ limits of C_6/C_2 . Both of these two plots clearly demonstrate that C_6/C_2 randomly fluctuates around $\langle C_6/C_2 \rangle$. In Fig. 4(a), the probabilities of C_6/C_2 that lie outside 1σ and 2σ of $\langle C_6/C_2 \rangle$ are about 31.0% and 5.0%, respectively. Meanwhile, 30.0% and 4.0% of the observations lie outside the 1σ and 2σ standard deviations, as shown in Fig. 4(b). Theoretically, for N independent observations (x_1, x_2, \dots, x_N) with the same expectation, they are approximated with a normal distribution. In this case, 31.7% and 4.5% of the observations are outside 1σ and 2σ standard deviations of the expectation, respectively. Consequently, our results are consistent with the theoretical studies. It confirms the validity of our

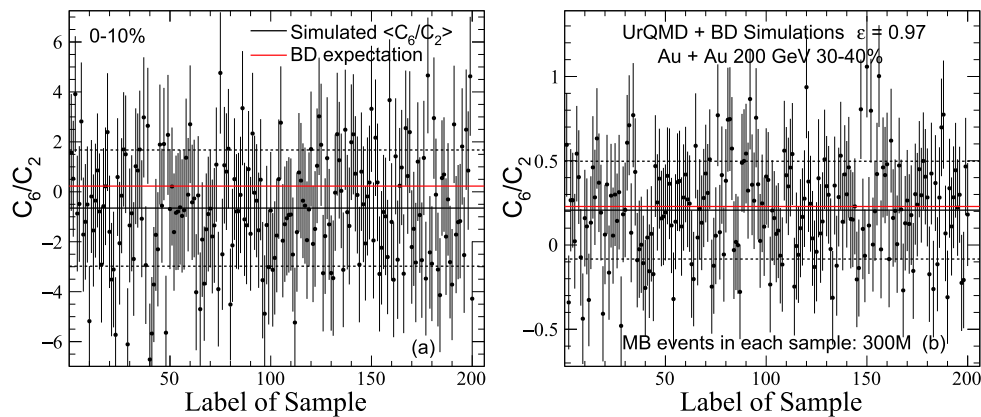


Fig. 4. (color online) C_6/C_2 and their statistical errors of 200 samples in 0-10% and 30%-40% centrality, respectively. Each of C_6/C_2 is calculated by the CBWC method with 300M MB events. The protons and anti-protons are independently and randomly generated from the BD with $\varepsilon = 0.97$. The input parameters for simulations are taken from UrQMD model at $\sqrt{s_{NN}} = 200$ GeV in Au + Au collisions.

simulations.

The solid red line in each panel is the theoretical BD expectation. It is about 0.233 with $\varepsilon = 0.97$. In principle, if the statistics in each Refmult3 bin are sufficient, the final $\langle C_6/C_2 \rangle$ should be consistent with 0.233 in all centralities. However, Figs. 4(a) and (b) show the disagreement of C_6/C_2 between the simulated $\langle C_6/C_2 \rangle$ and theoretical BD expectation. The difference of C_6/C_2 in 0-10% centrality is more significant than that in 30%-40% centrality. Comprehensive studies of the statistics dependence of C_6/C_2 are shown in the following section.

V. STATISTICS DEPENDENCE OF $\langle C_6/C_2 \rangle$ IN BD

As we mentioned, the experimental data collected for C_6/C_2 analysis include three different runs: 300M MB events, 420M MB events, and 110M from the central trigger. Here, 100M events from the central trigger can be a reference to 1000M MB events in our simulations. As a direct comparison, we can study the statistical dependence of $\langle C_6/C_2 \rangle$ with the magnitude of a few hundred million.

By considering $\varepsilon = 0.97$ to the simulations, Figs. 5(a) to (f) show statistical dependence of $\langle C_6/C_2 \rangle$ in different centralities from 100M to 1000M MB events. In 0-10% centrality, Fig. 5(a) shows that all of the results are systematically smaller than the solid black line, which is the theoretical expectation. Below 500M MB events, the values of $\langle C_6/C_2 \rangle$ first increase as the statistics increase, and they are all negative. From 600M to 1000M MB events, $\langle C_6/C_2 \rangle$ is consistent with zero with 2σ of the statistical uncertainty. Consequently, with the current statistics at

RHIC/STAR, C_6/C_2 may be under-estimated in 0-10% centrality at $\sqrt{s_{NN}} = 200$ GeV.

In 10%-20% centrality, Fig. 5(b) shows that the values of $\langle C_6/C_2 \rangle$ are still systematically smaller than that in BD. However, it becomes positive above 200M MB events. It suggests that the effect of the statistics becomes weaker. In 20%-30% and 30%-40% centralities, $\langle C_6/C_2 \rangle$ is only slightly smaller than that in BD, as shown in Figs. 5(c) and (d). Finally, they are in agreement with each other in peripheral collisions shown in Figs. 5(e) and (f).

Figure 5 shows that the influence of statistics on $\langle C_6/C_2 \rangle$ is strongest in central collisions, comparing to the mid-central and peripheral collisions. In 0-10% centrality, $\langle C_6/C_2 \rangle$ is still smaller than the expectation even with 1000M MB events. $\langle C_6/C_2 \rangle$ is still about 0, and we do not observe the convergence even with 1000M MB events. It is due to the wider distribution of net-proton number and the smaller statistics in each N_{ch} in 0-10% centrality. It can be further understood from Fig. 2(b), in which the simulated parameters are also extracted from mean values of proton and anti-proton in 0-10% centrality in the UrQMD model. As the same estimation as in Section II, with 1000M MB events, the averaged statistics in each N_{ch} is around 0.1 M, assuming that there are 1000 N_{ch} bins in 0-10% centrality. Figure 2(b) also shows that $\langle C_6/C_2 \rangle$ is also about 0 with 0.1M events in each N_{ch} , which is consistent with that shown in Fig. 5(a). In contrast, Fig. 2(b) shows that $\langle C_6/C_2 \rangle$ is consistent with the theoretical expectations above 0.5M events, which suggests that 5000M MB events are required if the CBWC method in each N_{ch} is applied. That is why the con-

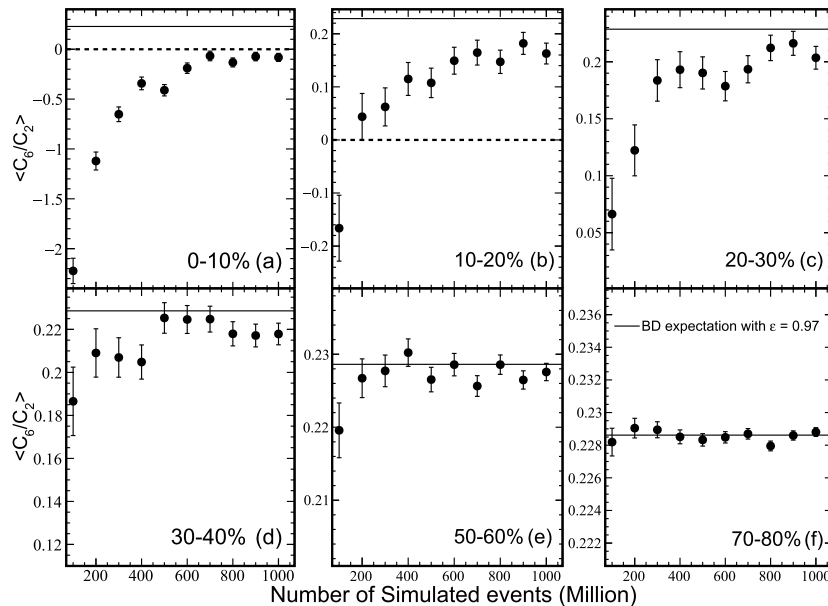


Fig. 5. Statistical dependence of $\langle C_6/C_2 \rangle$ in BD with $\varepsilon = 0.97$. The statistical error is calculated from formula of the error propagation. The black solid lines are the corresponding theoretical BD expectations derived from Eq. (10).

vergence is not observed in Fig. 5(a).

One possible alternative is to reduce the number of bins when applying CBWC method, such as the centrality bin width of each of $\delta 0.5$, $\delta 1.0$, and $\delta 2.5$. It is beyond the scope of this paper to examine through the transport model or the experimental data directly.

Because the influence of the statistics cannot be neglected, it is necessary to study C_6/C_2 in BD with the current statistics at RHIC/STAR. With 300M MB events, the blue filled band of Fig. 6 shows centrality dependence of $\langle C_6/C_2 \rangle$ with ε from 0.94 to 0.99. The solid and dashed lines are theoretically expected C_6/C_2 with $\varepsilon = 0.99$ and 0.94, respectively. In 0-10% and 10-20% centralities, the simulated results are systematically smaller than those with the ideal theoretical expectations. Particularly, there is a marked drop in $\langle C_6/C_2 \rangle$ in 0-10% centrality. With $\varepsilon = 0.99$, the simulated $\langle C_6/C_2 \rangle$ is still negative, while the theoretical expectation is about 0.71. Meanwhile, this phenomenon is also observed with the same simulation method in Skellam distribution (SD), shown by circular solid points in Fig. 6. The simulated $\langle C_6/C_2 \rangle$ is consistent with zero while it must be unity in SD. It further confirms that C_6/C_2 can be under-estimated in 0-10% centrality. With insufficient statistics, C_6/C_2 can vary from a positive value to a negative one.

Together with the same statistics as RHIC/STAR, the negative C_6/C_2 is more significant in BD. The blue filled band shown in Fig. 6 almost covers the current preliminary measured C_6/C_2 within large experimental uncertainties [12]. It can better describe the experimentally measured C_6/C_2 , comparing to the SD baseline. Nonetheless, it cannot completely reproduce the experimentally measured results. First, the experimental data show C_6/C_2 decreases from peripheral to central collisions [12]. In our simulations, the variation range of $\langle C_6/C_2 \rangle$ is significantly large, although ε is just from 0.94 to 0.99. It is difficult to study the feature of the centrality dependence of C_6/C_2 . Second, besides the statistics and BD, C_6/C_2 can still be affected by many other complex contributions. We must correct those contributions well before relating the experimentally measurements to our calculations. Third, we assume that the proton and anti-proton are produced independently. However, they must have correlation, such as from resonance decay, which is not considered in this paper. A direct comparison between the experimentally measured results and this baseline requires further studies. In contrast, our studies also imply that we must exclude factors of non-phase transition related influences well before connecting the measured C_6/C_2 to the theoretical Lattice QCD calculations.

VI. SUMMARY

We studied the behavior of the net-proton C_6/C_2 in BD in Au + Au collisions at $\sqrt{s_{NN}} = 200$ GeV. By a reas-

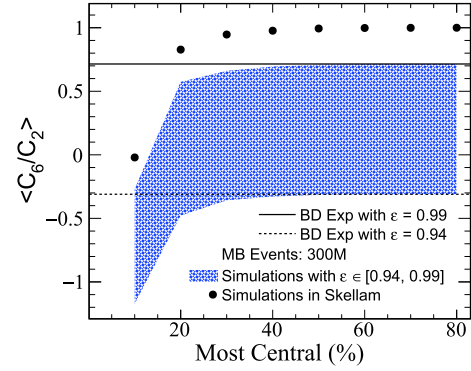


Fig. 6. (color online) Centrality dependence of $\langle C_6/C_2 \rangle$ in BD with ε from 0.94 to 0.99 and in SD, respectively. Each $\langle C_6/C_2 \rangle$ is averaged from 10000 randomly and independently simulated samples, with 300M MB events in each sample.

onable approximation of $\varepsilon = \varepsilon_p = \varepsilon_{\bar{p}}$, the net-proton C_6/C_2 is only dependent on ε in BD. With protons and anti-protons carried out in the mid-rapidity $|y| < 0.5$ and $0.4 < p_T < 2.0$ GeV/c in experiment, ε_p and $\varepsilon_{\bar{p}}$ are close to each other and the values are between 0.94 and 0.99 in all centralities. In this region, ε has a significant effect on C_6/C_2 . The values of C_6/C_2 decrease from 0.71 to -0.31 with ε from 0.99 down to 0.94. The negative C_6/C_2 is observed in the pure statistical BD. It suggests that the negative C_6/C_2 is not sufficient to be considered as an indication of a smooth crossover transition.

We also simulated the statistics dependence of $\langle C_6/C_2 \rangle$ based on the CBWC calculation method. Our method can compare the simulated results and theoretical expectations in a straightforward manner. The significantly dropped signal is observed in 0-10% centrality. With 300M MB events, the simulated C_6/C_2 is negative, while it is about 0.71 with $\varepsilon = 0.99$ in BD. This phenomenon is also observed with the same method of simulation in SD. The simulated $\langle C_6/C_2 \rangle$ is consistent with zero, while the theoretical expectation is unity. Moreover, with 1000M MB events, the simulated $\langle C_6/C_2 \rangle$ is consistent with zero, while the theoretical expectation is about 0.233 with $\varepsilon = 0.97$ in BD. Consequently, with the current statistics at RHIC/STAR, C_6/C_2 may be under-estimated in 0-10% centrality at $\sqrt{s_{NN}} = 200$ GeV.

With the experimental collected statistics, the negative C_6/C_2 is more significant in BD. It shows that the values of C_6/C_2 have a broad range, which can change from positive to negative. Comparing the baselines of C_6/C_2 in SD and BD, the BD could better describe the experimentally measured C_6/C_2 . Within large uncertainties at RHIC/STAR, the obtained baseline almost covers the current preliminary measured C_6/C_2 . Consequently, the exclusions of the non-phase transition related influences are required when using C_6/C_2 to study the chiral phase transition.

References

- [1] M. M. Aggarwal *et al.*, (STAR Collaboration), arXiv: 1007.2613 [nucl-ex]
- [2] Y. Aoki, G. Endrodi, Z. Fodor *et al.*, *Nature* **443**, 675 (2006)
- [3] M. Cheng *et al.*, *Phys. Rev. D* **79**, 074505 (2009)
- [4] C. Schmidt, *prog. Theor. Phys. Suppl.* **186**, 563 (2010)
- [5] A. Bazavov *et al.* (HotQCD Collaboration), *Phys. Rev. D* **95**, 054504 (2017)
- [6] A. Bazavov *et al.* (HotQCD Collaboration), *Phys. Rev. D* **101**, 074502 (2020)
- [7] Kenji Morita *et al.*, *Phys. Rev. C* **88**, 034903 (2013)
- [8] V. Skokov, B. Friman, and K. Redlich, *Phys. Lett. B* **708**, 179 (2012)
- [9] B. Friman *et al.*, *Eur. Phys. J. C* **71**, 1694 (2011)
- [10] Wei-jie Fu, Yu-xin Liu, and Yue-Liang Wu, *Phys. Rev. D* **81**, 014028 (2010)
- [11] Wei-jie Fu *et al.*, arXiv: 2101.06035 [hep-ph]
- [12] Toshihiro Nonaka (for the STAR Collaboration), *Nucl. Phys. A* **1005**, 121882 (2021)
- [13] Roli Esha (for the STAR Collaboration), *Nucl. Phys. A* **967**, 457 (2017)
- [14] M. S. Abdallah *et al.*, (STAR Collaboration), arXiv: 2105.14698 [nucl-ex]
- [15] Z. Citron *et al.* (ALICE Collaboration), CERN Yellow Rep. Monogr. **7**, 1159 (2019), arXiv:1812.06772 [hep-ph]
- [16] Volker Koch and Volodymyr Vovchenko, arXiv: 2102.08506 [nuclth]
- [17] Adam Bzdak, Volker Koch, and Vladimir Skokov, *Phys. Rev. C* **87**, 014901 (2013)
- [18] P. Garg *et al.*, *Phys. Lett. B* **726**, 691 (2013)
- [19] Adam Bzdak, Romain Holzmann, and Volker Koch, *Phys. Rev. C* **94**, 064907 (2016)
- [20] P. Braun-Munzinger *et al.*, *Nucl. Phys. A* **1008**, 122141 (2021)
- [21] P. Braun-Munzinger *et al.*, *Phys. Rev. C* **84**, 064911 (2011)
- [22] P. Braun-Munzinger *et al.*, *Phys. Lett. B* **747**, 292 (2015)
- [23] B. Abelev *et al.*, (ALICE Collaboration), *Eur. Phys. J. C* **75**, 146 (2015)
- [24] Premomoy Ghosh, *Phys. Rev. D* **85**, 054017 (2012)
- [25] M. Kitazawa and M. Asakawa, *Phys. Rev. C* **86**, (2012)
- [26] Terence J. Tarnowsky and Gary D. Westfall, *Phys. Lett. B* **724**, 51 (2013)
- [27] Xiaofeng Luo, Bedangadas Mohanty, and Nu Xu, *Nucl. Phys. A* **931**, 808 (2014)
- [28] L. Adamczyk *et al.* (STAR Collaboration), *Phys. Rev. Lett* **112**, 032302 (2014)
- [29] L. Adamczyk *et al.* (STAR Collaboration), *Phys. Rev. Lett* **113**, 092301 (2014)
- [30] A. Adare *et al.* (PHENIX Collaboration), *Phys. Rev. C* **93**, 011901(R) (2016)
- [31] L. Adamczyk *et al.* (STAR Collaboration), *Phys. Lett. B* **785**, 551 (2018)
- [32] J. Adam *et al.* (STAR Collaboration), *Phys. Rev. C* **102**, 024903 (2020)
- [33] X. Luo (For the STAR Collaboration), *J. Phys. : Conf. Ser.* **316**, 012003 (2011)
- [34] Xiaofeng Luo and Nu Xu, *Nucl. Sci. Tech.* **28**, 112 (2017)
- [35] M. S. Abdallah *et al.*, (STAR Collaboration), arXiv: 2101.12413 [nucl-ex]
- [36] Lizhu Chen, Zhiming Li, and Yuanfang Wu, *J. Phys. G* **41**, 105107 (2014)
- [37] Lizhu Chen *et al.*, *J. Phys. G* **42**, 065103 (2015)
- [38] Lizhu Chen, Zhiming Li, Fenping Cui *et al.*, *Nucl. Phys. A* **957**, 60 (2017)
- [39] Papoulis, A. Probability, Random Variables, and Stochastic Processes, 2nd ed. New York, 1984
- [40] John A. Rice, *Mathematical Statistics and Data Analysis*, third ed., Duxbury Press, ISBN 0-534-39942-8, 2007
- [41] S. A. Bass *et al.*, *Prog. Part. Nucl. Phys.* **41**, 255 (1998)
- [42] M. Bleicher *et al.*, *J. Phys. G* **25**, 1859 (1999)
- [43] X. Luo, *J. Phys. G* **39**, 025008 (2012)