

Possible Odderon exchange in pp and $\bar{p}p$ elastic collisions at the TOTEM experiment*

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Abstract: Based on the Froissart-Martin theorem, the Regge theory and the possible Odderon exchange, the total cross-section σ_{tot} and the ratio of the real to imaginary parts of the forward scattering amplitude ρ in the pp and $\bar{p}p$ elastic collisions in the TOTEM energy region are studied in the FPO model. We consider the contributions of the Froissart bound and of the Pomeron, Reggeon and Odderon exchange terms in the scattering amplitude of the pp and $\bar{p}p$ elastic collisions. Using the Odderon intercept $\alpha_{\text{O}}(0) = 0.5$, our theoretical predictions are in good agreement with the recent results of the TOTEM experiment. These results show that the Odderon, corresponding to the odd elastic scattering amplitude, is likely to exist.

Keywords: Odderon exchange, unitarity, the FPO model

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1 Introduction

The search for the Odderon has concentrated for a long time on the difference between particle-particle and antiparticle-particle scattering at high energies. The t dependence of pp and $\bar{p}p$ elastic collisions was measured at the CERN ISR and SPS [1–3], and a marked difference was found between the differential cross-sections for pp and $\bar{p}p$ elastic collisions at $\sqrt{s} = 52.8$ GeV. $(\frac{d\sigma}{dt})_{\bar{p}p}$ is smooth for that momentum transfer, but $(\frac{d\sigma}{dt})_{pp}$ has a characteristic dip around $|t| = 1.35$ GeV². The total cross-section σ_{tot}^{pp} and the ratio of the real to imaginary parts of the forward scattering amplitude ρ_{pp} at $\sqrt{s} = 2.76$ TeV, 7 TeV and 8 TeV were measured by the TOTEM collaboration at the LHC [4–9]. Recently, the TOTEM collaboration measured the total cross-section $\sigma_{\text{tot}} = (110.6 \pm 3.4)$ mb [10], and the ratio of the real to imaginary parts of the forward scattering amplitude $\rho = \text{Re}/\text{Im} = (0.098 \pm 0.01)$ [11] for the pp elastic collisions at $\sqrt{s} = 13$ TeV. There is some experimental evidence for the Odderon, related to the scattering amplitude with odd parity $C = -1$ under charge conjugation. In particular, the TOTEM experimental data show a possible contribution of the odd signature amplitude to the total cross-section. Evidence of the Odderon would lead to many interesting aspects of

high energy scattering in QCD.

Let us make a very short review of the Odderon. Regge proposed a new method for calculating the scattering amplitude in quantum mechanics in 1959 [12, 13]. The Regge theory assumes that the scattering amplitude is an asymptotic function of the center-of-mass energy \sqrt{s} and four-momentum transfer t . It relates the orbital angular momentum J and the mass M of particles with the same quantum numbers (charge Q , baryon B , strangeness, isospin I , etc.) as $J = \alpha_{\text{R}}(t = m^2)$, the Regge trajectory [12, 13]. For two-hadron collision process $1 + 2 \rightarrow 3 + 4$, the total cross-section increases slowly with energy $\sigma_{\text{tot}}(s) \sim s^{\alpha_{\text{R}}-1}$ and should vanish for $s \rightarrow \infty$. In fact, the total cross-section does not vanish as the energy increases. The interaction between colliding particles is explained by the exchange between them, the so-called Regge trajectory. In the Pomeron exchange model of the 1960's, the Regge theory framework was widely studied. The Pomeron is different from the intercept ($\alpha_{\text{R}} \leq 1$) of the Regge trajectory for all existing particles in nature, and its intercept is $\alpha_{\text{P}} \simeq 1.08 > 1$. The Pomeron is assumed to be an object composed of two gluons in a colorless state with a C -parity of $C = +1$. Hence the Pomeron may have a close relative with a C -parity of $C = -1$. The relative, the Odderon, was proposed in 1973 by Lukaszuk and

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Nicolescu [14].

QCD is the fundamental theory of the strong interaction. It was discovered in early 1973. In perturbative QCD, the Odderon is considered to be composed of three interacting reggeized gluons in a symmetric color state [15, 16]. In many theories, it is assumed that the Odderon leads to a difference between the total cross-sections of particle-particle and particle-antiparticle collisions, and many models have been proposed [17–24] to explain the experimental data for the pp and $\bar{p}p$ elastic collisions. Does the Odderon exist? If the existence of the Odderon is experimentally confirmed, it would become a very interesting object in high energy scattering in QCD.

We attempt to build a relationship between the Regge theory and the QCD dynamics. We first describe the pp and $\bar{p}p$ elastic collisions at high energy by using the crossing symmetry and unitarity [25–27]. In Sec. 2, we introduce a very important consequence of analyticity - the crossing symmetry - and present the related formalism of the total cross-section σ_{tot} and the ratio of the real to imaginary parts of the forward scattering amplitude ρ in the FPO model. In Sec. 3, we give our theoretical predictions and a comparison with the TOTEM experimental results. A comparison with other calculations of the Odderon intercept is given in Sec. 4. Our summary and concluding remarks are presented in Sec. 5.

2 Crossing symmetry and the Odderon

In quantum field theory, the amplitude of the two-to-two elastic collisions process in the u -channel $1 + \bar{4} \rightarrow 3 + \bar{2}$ (Fig. 1(b)) can be obtained from the amplitude of the two-to-two elastic collisions process in the s -channel $1 + 2 \rightarrow 3 + 4$ (Fig. 1(a)). As a consequence of analyticity - the crossing symmetry relationship - we have

$$F^{1+\bar{4} \rightarrow 3+\bar{2}}(s, t, u) = F^{1+2 \rightarrow 3+4}(u, t, s). \quad (1)$$

The fermion-antifermion elastic collision with an amplitude $F^{1+\bar{2} \rightarrow 1+\bar{2}}(s, t)$ can be obtained from the fermion-fermion elastic collision with an amplitude $F^{1+2 \rightarrow 1+2}(s, t)$ by crossing to the u -channel

$$F^{1+\bar{2} \rightarrow 1+\bar{2}}(s, t) = F^{1+2 \rightarrow 1+2}(s, t). \quad (2)$$

Let us now define two new amplitudes F_{\pm} using $F^{12}(s, t)$ and $F^{1\bar{2}}(s, t)$ such that

$$F_{\pm}(s, t) = \frac{1}{2}(F^{12}(s, t) \pm F^{1\bar{2}}(s, t)), \quad (3)$$

$$F^{12}(s, t) = F_+(s, t) + F_-(s, t), \quad (4)$$

$$F^{1\bar{2}}(s, t) = F_+(s, t) - F_-(s, t). \quad (5)$$

For fermion-fermion and fermion-antifermion elastic collisions, the amplitude F_+ is the same. It has an even (or positive) C -parity, $C = +1$. It is considered that its main

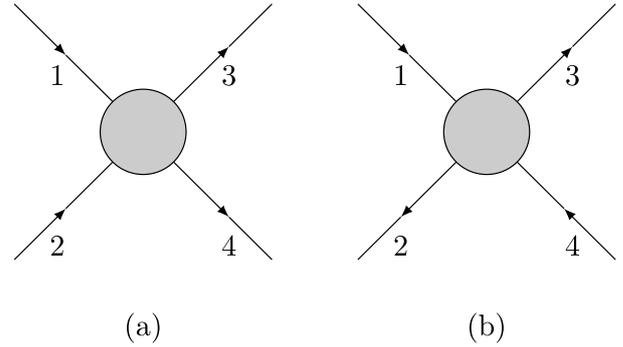


Fig. 1. Feynman diagram of the crossing symmetry in the two-to-two elastic collisions.

contribution is from the Froissart bound, a Reggeon-Pomeron exchange at high energy. The amplitude F_- changes sign from fermion-fermion to fermion-antifermion elastic collisions, and corresponds to an odd (or negative) C -parity, $C = -1$. It is dominated by the Odderon exchange and its contribution does not decrease rapidly at high energy. F_+ and F_- are called even and odd amplitudes under the crossing symmetry.

2.1 Amplitude F_{pp}^{pp}

Proton is a compound fermion. The proton-proton and proton-antiproton elastic collision process satisfies the crossing symmetry in quantum field theory.

Let us now define two amplitudes F_{\pm} for the pp and $\bar{p}p$ elastic collisions

$$F_{\pm}(s, t) = \frac{1}{2}(F^{pp}(s, t) \pm F^{\bar{p}p}(s, t)). \quad (6)$$

From Eq. (6), we get the following expressions for the amplitudes of pp and $\bar{p}p$ elastic collisions composed of the even amplitude F_+ and the odd amplitude F_- under the crossing symmetry as

$$F_{pp}(s, t) = F_+(s, t) + F_-(s, t), \quad (7)$$

$$F_{\bar{p}p}(s, t) = F_+(s, t) - F_-(s, t). \quad (8)$$

2.2 Unitarity and the Odderon

In the 1960's, the process $pp \rightarrow p + X_1 + X_2 + \dots$ (groups of particles) $+ p$, has been recognized [28, 29] as shown in Fig. 2(a, b).

Large rapidity gaps (LRG) separate small groups of particles (X_i). These groups of particles may be Reggeons, Pomerons and Odderons. The gaps between them may cause a problem of unitarity.

A consequence of unitarity is the Froissart-Martin theorem [25, 26]. The total hadronic cross-section grows as a logarithm of energy. Specifically

$$\sigma_{\text{tot}} \leq \frac{\pi}{m_{\pi}^2} \log^2 \left(\frac{s}{s_0} \right), \quad (9)$$

where s_0 is a free parameter, $\tilde{s} = \frac{s}{s_0}$. The usual Froissart

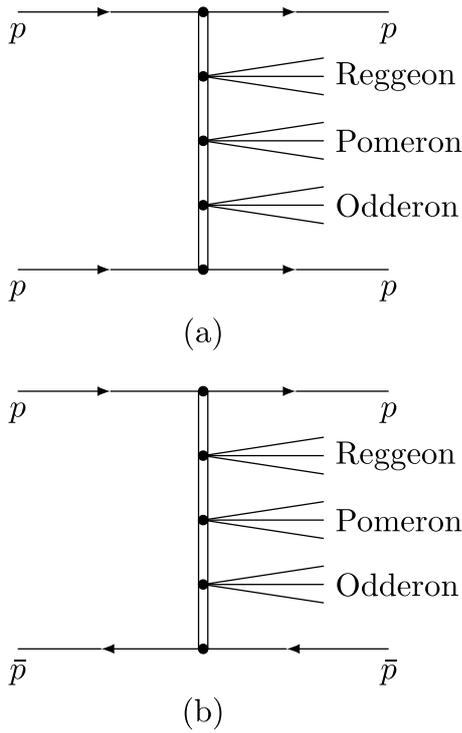


Fig. 2. The elastic collision process of $p + p \rightarrow p + p$ and $\bar{p} + p \rightarrow \bar{p} + p$.

bound coefficient is $m_\pi^2 / m_p^2 = 20$ mb. The total hadronic cross-section rapidly increases and is 10 barns at the LHC energy.

In the Regge theory [12, 13], the scattering amplitude $F(s, t)$ for the process $1 + 2 \rightarrow 3 + 4$ can be expressed by $\alpha(t)$ as

$$F_{1+2 \rightarrow 3+4} = \frac{\eta + e^{-i\pi\alpha(t)}}{2} \beta(t) \cdot s^{\alpha(t)}. \quad (10)$$

$\beta(t)$ in Eq. (10) is a product of two factors which depend only on the coupling of the exchanged object to the scattering particles, as shown in Fig. 3.

$$\beta(t) = \beta_{12}(t)\beta_{34}(t), \quad (11)$$

$$\alpha(t) = \alpha_0 + \alpha' t. \quad (12)$$

$\alpha(t)$ in Eq. (10) is the Regge trajectory of a particle; α_0 is the intercept of the particle trajectory, and α' its slope.

We define our Froissart Pomeron Odderon model (FPO) such that the contributions of the Froissart bound and of the Pomeron, Reggeon and Odderon exchange are taken into account in the forward amplitude $F(s, t)$ of the pp and $\bar{p}p$ elastic collisions.

The even forward amplitudes F_+ of the pp and $\bar{p}p$ elastic collisions are given as the sum of three parts: the first part, F_+^H , saturates at the asymptotic bound of the Froissart-Martin theorem. The second part F_+^P is given by the Pomeron pole with an intercept of about 1. The third part, F_+^R , describes the conventional Regge poles and

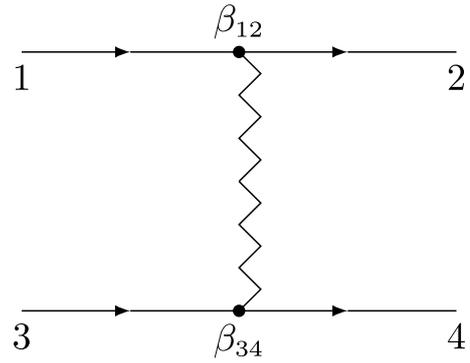


Fig. 3. Illustration of the Reggeon, Pomeron and Odderon exchange.

cuts.

$$F_+ = F_+^H + F_+^P + F_+^R. \quad (13)$$

F_+^H in Eq. (13) denotes the contribution of the Froissart bound to the forward scattering amplitude of the pp and $\bar{p}p$ elastic collisions. The capital H is for "Heisenberg".

$$\begin{aligned} \frac{1}{is} F_+^H(s, t) = & A \ln^2 \bar{s} \frac{2J_1(K_+ \bar{\tau})}{K_+ \bar{\tau}} \exp(b_1^+ t) \\ & + B \ln \bar{s} J_0(K_+ \bar{\tau}) \exp(b_2^+ t) \\ & + H [J_0(K_+ \bar{\tau}) - (K_+ \bar{\tau}) J_1(K_+ \bar{\tau})] \exp(b_3^+ t), \end{aligned} \quad (14)$$

where J_0, J_1 are the Bessel functions, and A, B, H, K_+ and b_k ($k = 1, 2, 3$) are constants.

$$\bar{s} = \frac{s}{s_0} \exp\left(-\frac{1}{2} i\pi\right), \quad \text{with } s_0 = 1 \text{ GeV}^2, \quad (15)$$

$$\bar{\tau} = \sqrt{-\frac{t}{t_0}} \ln \bar{s}, \quad \text{with } t_0 = 1 \text{ GeV}^2. \quad (16)$$

F_+^P and F_+^R in Eq. (13) denote the Pomeron and Reggeon contributions to the forward scattering amplitude of the pp and $\bar{p}p$ elastic collisions, and are expressed as:

$$\frac{1}{s} F_+^P(s, t) = C \exp(\beta_P t) \left[i - \cot\left(\frac{\pi}{2} \alpha_P(t)\right) \right] \left(\frac{s}{s_0}\right)^{(\alpha_P(t)-1)}. \quad (17)$$

$$\alpha_P(t) = \alpha_P(0) + \alpha'_P t, \quad (18)$$

with

$$\alpha_P(0) = 1 \quad \text{and} \quad \alpha'_P = 0.25 \text{ GeV}^{-2}, \quad (19)$$

$$\begin{aligned} \frac{1}{s} F_+^R(s, t) = & D \exp(\beta_R t) \\ & \times \left[i \sin\left(\frac{\pi}{2} \alpha_R(t)\right) - \cos\left(\frac{\pi}{2} \alpha_R(t)\right) \right] \left(\frac{s}{s_0}\right)^{(\alpha_R(t)-1)}. \end{aligned} \quad (20)$$

$$\alpha_R(t) = \alpha_R(0) + \alpha'_R t, \quad (21)$$

with

$$\alpha_R(0) = 0.5, \quad (22)$$

where C, D, β_P and β_R are constants.

The conventional odd forward scattering amplitude of the pp and $\bar{p}p$ elastic collisions in Regge theory is obtained by the Odderon exchange

$$F_- = F_-^O. \quad (23)$$

$$\frac{1}{s} F_-^O(s, t) = E \exp(\beta_O t) \times \left[\sin\left(\frac{\pi}{2} \alpha_O(t)\right) + i \cos\left(\frac{\pi}{2} \alpha_O(t)\right) \right] \left(\frac{s}{s_0}\right)^{(\alpha_O(t)-1)}, \quad (24)$$

where E and β_O are constants, and

$$\alpha_O(t) = \alpha_O(0) + \alpha'_O t, \quad (25)$$

with

$$\alpha_O(0) = 0.5. \quad (26)$$

2.3 The total cross-section and the ratio of the real to imaginary parts of the forward scattering amplitude

Based on the optical theorem for purely elastic collisions, the total cross-section is related to the elastic forward scattering amplitude $F(s, t = 0)$

$$\sigma_{\text{tot}} = \frac{1}{s} \text{Im} F(s, t = 0). \quad (27)$$

We can then obtain the even and odd cross-sections under the crossing symmetry from the even and odd forward scattering amplitudes F_+ and F_- :

$$\sigma_{\text{even}} = \frac{1}{s} \text{Im} F_+(s, t = 0), \quad (28)$$

$$\sigma_{\text{odd}} = \frac{1}{s} \text{Im} F_-(s, t = 0). \quad (29)$$

The total cross-section for the pp and $\bar{p}p$ elastic collisions is composed of even and odd cross-sections under the crossing symmetry:

$$\sigma_{\text{tot}}^{pp} = \sigma_{\text{even}} + \sigma_{\text{odd}}, \quad (30)$$

$$\sigma_{\text{tot}}^{\bar{p}p} = \sigma_{\text{even}} - \sigma_{\text{odd}}. \quad (31)$$

The Froissart bound contribution to the total cross-section for the pp and $\bar{p}p$ elastic collisions can now be written as:

$$\sigma^H = A \ln^2 \tilde{s} + B \ln \tilde{s}. \quad (32)$$

The Pomeron, Reggeon and Odderon exchange terms in the scattering amplitude lead to the cross-section expressed by the Pomeron, Reggeon and Odderon trajectory as

$$\sigma^P \propto (s)^{\alpha_P(0)-1}, \quad \text{with } \alpha_P(0) = 1, \quad (33)$$

$$\sigma^R \propto (s)^{\alpha_R(0)-1}, \quad \text{with } \alpha_R(0) = 0.5. \quad (34)$$

Here, $\alpha_P(0)$ and $\alpha_R(0)$ are the intercepts of the Pomeron trajectory $\alpha_P(t)$ and Reggeon trajectory $\alpha_R(t)$.

The Odderon exchange contribution to the total cross-section of the pp and $\bar{p}p$ elastic collisions contains the odd forward amplitude F_- , and is given by

$$\sigma^O \propto (s)^{\alpha_O(0)-1}, \quad \text{with } \alpha_O(0) = 0.5, \quad (35)$$

$\alpha_O(0)$ is the intercept of the Odderon trajectory $\alpha_O(t)$.

The difference between the total cross-sections of the pp and $\bar{p}p$ elastic collisions is the Odderon term, which is due to the odd forward scattering amplitude. In other words, the odd cross-sections of the pp and $\bar{p}p$ elastic collisions change sign under the crossing symmetry. The even and odd cross-sections can be written as

$$\begin{aligned} \sigma_{\text{even}} &= \frac{1}{s} \text{Im} [F_+^H + F_+^P + F_+^R] \\ &= \text{Im} i \left\{ A \left[\ln \frac{s}{s_0} - i \frac{\pi}{2} \right]^2 + B \left[\ln \frac{s}{s_0} - i \frac{\pi}{2} \right] \right\} \\ &\quad + C \left(\frac{s}{s_0} \right)^{\alpha_P(0)-1} + D \sin\left(\frac{\pi}{2} \alpha_R(0)\right) \left(\frac{s}{s_0} \right)^{\alpha_R(0)-1}, \quad (36) \end{aligned}$$

$$\sigma_{\text{odd}} = \frac{1}{s} \text{Im} F_-^O = E \cos\left(\frac{\pi}{2} \alpha_O(0)\right) \left(\frac{s}{s_0} \right)^{\alpha_O(0)-1}. \quad (37)$$

The corresponding total cross-sections σ_{tot} can be rewritten respectively as

$$\begin{aligned} \sigma_{\text{tot}}^{pp} &= \sigma_{\text{even}} + \sigma_{\text{odd}} \\ &= A \left[\ln^2 \left(\frac{s}{s_0} \right) - \frac{\pi^2}{4} \right] + B \ln \frac{s}{s_0} \\ &\quad + C \left(\frac{s}{s_0} \right)^{\alpha_P(0)-1} + D \sin\left(\frac{\pi}{2} \alpha_R(0)\right) \left(\frac{s}{s_0} \right)^{\alpha_R(0)-1} \\ &\quad + E \cos\left(\frac{\pi}{2} \alpha_O(0)\right) \left(\frac{s}{s_0} \right)^{\alpha_O(0)-1}, \quad (38) \end{aligned}$$

$$\begin{aligned} \sigma_{\text{tot}}^{\bar{p}p} &= \sigma_{\text{even}} - \sigma_{\text{odd}} \\ &= A \left[\ln^2 \left(\frac{s}{s_0} \right) - \frac{\pi^2}{4} \right] + B \ln \frac{s}{s_0} \\ &\quad + C \left(\frac{s}{s_0} \right)^{\alpha_P(0)-1} + D \sin\left(\frac{\pi}{2} \alpha_R(0)\right) \left(\frac{s}{s_0} \right)^{\alpha_R(0)-1} \\ &\quad - E \cos\left(\frac{\pi}{2} \alpha_O(0)\right) \left(\frac{s}{s_0} \right)^{\alpha_O(0)-1}. \quad (39) \end{aligned}$$

The ratio of the real to imaginary parts of the forward scattering amplitude $\rho(s)$ is defined as [30],

$$\rho(s) = \frac{\text{Re} F(s, t = 0)}{\text{Im} F(s, t = 0)}. \quad (40)$$

In the FPO model, it can be written for the pp and $\bar{p}p$ elastic collisions as:

$$\rho_{pp} = \frac{s \left[A\pi \ln \frac{s}{s_0} + B\frac{\pi}{2} - D \cos\left(\frac{\pi}{2}\alpha_R(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_R(0)-1} - E \sin\left(\frac{\pi}{2}\alpha_O(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_O(0)-1} \right]}{s \left\{ A \left[\ln^2\left(\frac{s}{s_0}\right) - \frac{\pi^2}{4} \right] + B \ln \frac{s}{s_0} + C \left(\frac{s}{s_0}\right)^{\alpha_r(0)-1} + D \sin\left(\frac{\pi}{2}\alpha_R(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_R(0)-1} + E \cos\left(\frac{\pi}{2}\alpha_O(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_O(0)-1} \right\}}, \quad (41)$$

$$\rho_{\bar{p}p} = \frac{s \left[A\pi \ln \frac{s}{s_0} + B\frac{\pi}{2} - D \cos\left(\frac{\pi}{2}\alpha_R(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_R(0)-1} + E \sin\left(\frac{\pi}{2}\alpha_O(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_O(0)-1} \right]}{s \left\{ A \left[\ln^2\left(\frac{s}{s_0}\right) - \frac{\pi^2}{4} \right] + B \ln \frac{s}{s_0} + C \left(\frac{s}{s_0}\right)^{\alpha_r(0)-1} + D \sin\left(\frac{\pi}{2}\alpha_R(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_R(0)-1} - E \cos\left(\frac{\pi}{2}\alpha_O(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_O(0)-1} \right\}}. \quad (42)$$

σ_{tot} and ρ are therefore parametrized in terms of five parameters A, B, C, D and E .

3 Numerical results

The total cross-section σ_{tot} and the ratio of the real to imaginary parts of the forward scattering amplitude ρ in the pp and $\bar{p}p$ elastic collisions are calculated using the above formulism. The fitted values of the parameters of our FPO model in Eqs. (38, 39, 41, 42) are given in Table 1. Comparison between our theoretical predictions and the experimental data are shown in Fig. 4 and Fig. 5.

Table 1. The fit values of the parameters used in the FPO model.

parameters/mb	fitted number	error
A	0.25	± 0.05
B	0.1964	± 0.0056
C	38.0	± 1.6
D	48.0	± 10.0
E	40.2910	± 2.0005

As can be seen from Fig. 4, there is a good agreement with all experimental data [1–11], including the new data by TOTEM for σ_{tot}^{pp} at $\sqrt{s} = 2.76, 7, 8$ and 13 TeV [4–11]. However, the ATLAS and cosmic data are incompatible with the TOTEM data and our theoretical predictions. The ATLAS measurement for σ_{tot}^{pp} at $\sqrt{s} = 8$ TeV is (96.07 ± 0.92) mb, but the new TOTEM measurement for σ_{tot}^{pp} at $\sqrt{s} = 8$ TeV is (102.9 ± 2.3) mb and (101.5 ± 2.1) mb. Our theoretical predictions are consistent with the new TOTEM data. In Table 2, we show all TOTEM data compared with the FPO predictions.

We show in Fig. 5 the ratio of the real to imaginary parts of the forward scattering amplitude ρ as a function of the center-of-mass energy \sqrt{s} . The theoretical curve from the FPO model is also given in Fig. 5. One can see a good agreement between the TOTEM measurement of ρ^{pp} (0.12 ± 0.3) at $\sqrt{s} = 8$ TeV and our prediction. However, the TOTEM value of ρ^{pp} (0.098 ± 0.01) at $\sqrt{s} =$

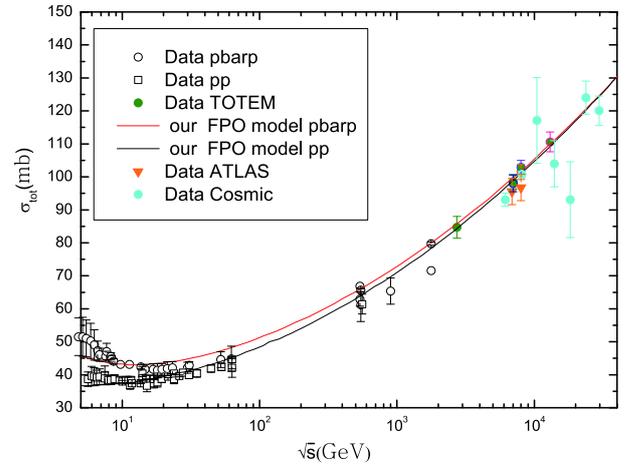


Fig. 4. (color online) The total cross-section of the pp and $\bar{p}p$ elastic collisions (in mb) as a function of the center-of-mass energy \sqrt{s} (in GeV) given by the FPO model. Measurements by ATLAS and TOTEM in the whole energy range explored by the LHC [1–11], and the cosmic ray data are also shown.

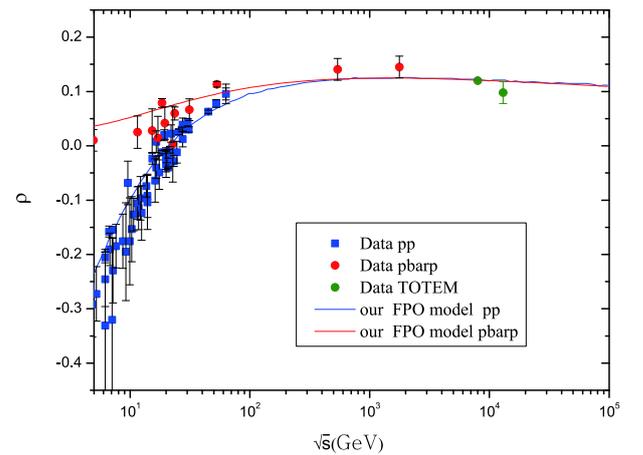


Fig. 5. (color online) The ratio of the real to imaginary parts of the forward scattering amplitude ρ in the pp and $\bar{p}p$ elastic collisions as a function of the center-of-mass energy \sqrt{s} (in GeV) given by the FPO model. Measurements by TOTEM in the whole energy range explored by the LHC [1–11] are also shown.

Table 2. TOTEM measurements for σ_{tot} and ρ and the results of the FPO model.

\sqrt{s}/TeV	$\sigma_{\text{tot}}^{pp}/\text{mb}$	$\sigma_{\text{tot}}^{pp}/\text{mb}$	ρ_{pp}	ρ_{pp}
	TOTEM	FPO	TOTEM	FPO
2.76	84.7±3.3	85.4	–	–
7	98.6±2.2	99.7	–	–
	98.0±2.5			
8	101.5±2.3	101.9	0.12±0.3	0.121
	102.9±2.1			
13	110.6±3.4	109.9	0.098±0.01	0.110

13 TeV is smaller than our theoretical prediction.

4 Comparison with other calculations of the Odderon intercept

In Ref. [31], the Reggeon-Pomeron exchange model is proposed to describe the proton- antiproton elastic scattering

$$F^{pp}(s, t) = F^R(s, t) + F^P(s, t). \quad (43)$$

The total cross-section in the Reggeon-Pomeron model is the sum of two terms:

$$\sigma_{\text{tot}} = Xs^{\alpha_R(0)-1} + Ys^{\alpha_P(0)-1}, \quad (44)$$

where the Pomeron intercept $\alpha_P(0) = 1.08$ and the Reggeon intercept $\alpha_R(0) = 0.5$ fit the experimental data perfectly in the region of center-of-mass energy $5.3 \text{ GeV} \leq \sqrt{s} \leq 2 \text{ TeV}$.

The total cross-section of the pp and $\bar{p}p$ elastic collisions in our FPO model is given by the sum of four parts:

$$\sigma_{\bar{p}p}^{pp}(\text{tot}) = \sigma^H + Cs^{\alpha_r(0)-1} + Ds^{\alpha_R(0)-1} \pm Es^{\alpha_O(0)-1}. \quad (45)$$

We expect a value of the Odderon intercept of about 0.5 in the fit. It is interesting that the Odderon intercept is much smaller than the Pomeron intercept.

Various calculations were performed of the Odderon intercept. The Bartels Kwiciński Praszalowicz (BKP) equation [32, 33] is known to give an excellent description of the Odderon in perturbative QCD. The Odderon singularity is the solution of the BKP equation for three color symmetric gluons. The Odderon intercept α_O is calculated by the variational method as

$$\alpha_O(0) = 1 + \omega_{\text{BKP}}(0) = 1 + \frac{N_c \alpha_s}{\pi} \frac{3}{2} \epsilon_3, \quad (46)$$

where $N_c = 3$, and $\alpha_s = 0.2$ is the QCD coupling constant.

The BKP equation has two explicit solutions. The intercept α_O of the BLC Odderon solution [34] is equal to 1,

$$\alpha_O(0) = 1, \quad (47)$$

while the JW solution [35] gives the Odderon intercept as

$$\alpha_O(0) = 0.96. \quad (48)$$

Let us assume that the Odderon intercept in our FPO model is the solution of the BKP equation. Our result is compatible with $-\frac{3}{2}\epsilon_3 = -2.625$, which is a supercritical Odderon, so that the Odderon intercept is $\alpha_O = 0.5$.

We compare our result to other calculations in Table 3. A number of possible solutions were found in [34–43]. Different schemes of variational calculation were proposed, so that the corresponding Odderon intercepts are also different. As can be seen from Table 3, the Odderon intercept lies in the interval $1 - (N_c \alpha_s / \pi) 2.625 \leq \alpha_O(0) \leq 1 + (N_c \alpha_s / \pi) 4.16$.

Table 3. Comparison of our result with other calculations of the Odderon intercept.

	$-\frac{3}{2}\epsilon_3$	$\alpha_O(0) = 1 + \frac{N_c \alpha_s}{\pi} \frac{3}{2} \epsilon_3$
our result	-2.625	0.5
Ref. [38]	≥ -2.0	≥ 0.6
Refs. [39, 40]	≥ -0.45	≥ 0.91
Ref. [41]	≥ -0.339	≥ 0.94
Ref. [35]	-0.24717	0.96
Ref. [42]	0	1
Ref. [34]	≥ 0.37	≥ 1.07
Refs. [36, 37]	2.41	1.46
Ref. [43]	≤ 4.16	≤ 1.79

5 Summary and concluding remarks

We propose a new approach to investigate the phenomenological Odderon based on the crossing symmetry, the Froissart-Martin theorem and the conventional Regge theory, which we call the Froissart Pomeron Odderon model (FPO).

We have calculated the total cross-section and the ratio of the real to imaginary parts of the forward scattering amplitude in the pp and $\bar{p}p$ elastic collisions in the FPO model. With a smaller number of adjustable parameters than other models, the FPO model provides a fit to all existing experimental data for the high energy pp and $\bar{p}p$ elastic collisions, including the new TOTEM data for σ_{tot}^{pp} at $\sqrt{s} = 2.76, 7, 8$ and 13 TeV , and the new TOTEM data for ρ [4–11]. The results show that the Odderon, which includes the odd amplitude F_- , predicts the difference between the pp and $\bar{p}p$ elastic collisions.

We assume an Odderon intercept $\alpha_O(0) = 0.5$, which corresponds to the odd amplitude F_- in the Regge limit. Many models have studied the Odderon intercept [34–43], with the values of the intercept of exactly one or close to one. We also considered two cases in our model, a non-maximal Odderon with an intercept equal to 0.5 based on the fit results, and an Odderon which couples to

external particles. We will continue to study the case of the maximal Odderon, and the Odderon intercept in general.

The Odderon is identified as a triple-gluon exchange in pp and $\bar{p}p$ scattering. The new TOTEM data for σ_{tot} and ρ could be considered as an experimental proof of the

Odderon. Our next goal is to extend the FPO model to the nuclear slope parameter [44] and the differential cross-section $\frac{d\sigma}{dt}$ [45], and use the Odderon to explain the dip mechanism in the differential cross-section $\frac{d\sigma}{dt}$ at high energies.

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