Properties of the decay $H \to \gamma\gamma$ using the approximate α_s^4 corrections and the principle of maximum conformality*

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Abstract: The decay channel $H \to \gamma\gamma$ is an important channel for probing the properties of the Higgs boson. In this paper, we analyze its decay width by using the perturbative QCD corrections up to the α_s^4 order with the help of the principle of maximum conformality (PMC). PMC has been suggested in literature for eliminating the conventional renormalization scheme-and-scale ambiguities. After applying PMC, we observe that an accurate renormalization scale independent decay width $\Gamma(H \to \gamma\gamma)$ up to the N^{4}LO level can be achieved. Taking the Higgs mass, $M_{\rm H} = 125.09 \pm 0.21 \pm 0.11$ GeV, given by the ATLAS and CMS collaborations, we obtain $\Gamma(H \to \gamma\gamma)|_{\rm LHC} = 9.364^{+0.076}_{-0.075}$ KeV.

1   Introduction

After the discovery of the Higgs boson at the Large Hadron Collider (LHC) [1–4], the outstanding task is to learn about its properties and confirm them either experimentally or theoretically. Among its various decay modes, the Higgs decay into two photons, $H \to \gamma\gamma$, which can be observed at the LHC or a high luminosity e^+e^- linear collider, provides a clean platform for studying the Higgs properties and for testing the Standard Model.

The Higgs boson couples dominantly to the massive particles and the leading order (LO) term of the total decay width $\Gamma(H \to \gamma\gamma)$ is already at the one-loop level, which conversely makes the high-order pQCD corrections very complicated. The LO, the next-to-leading order (NLO), the N^{2}LO, the approximate N^{3}LO and the

N^{4}LO terms for the total decay width $\Gamma(H \to \gamma\gamma)$ were calculated in Refs. [5–18]. In particular, the fermionic contributions, which form a gauge invariant subset, were given in the N^{3}LO and N^{4}LO terms [18]. As shall be shown below, these state-of-art terms give the opportunity to achieve a more precise prediction of $\Gamma(H \to \gamma\gamma)$. Due to the complexity of high-order pQCD calculations, it is important to use the known fixed-order terms to obtain the perturbative properties as accurately as possible.

Following the standard renormalization group invariance, a pQCD calculable physical observable, corresponding to an infinite order prediction, should be independent of the choice of renormalization scheme and renormalization scale. However, for a fixed-order approximant, one needs to set an optimal scale for comparison with the data. Conventionally, the renormalization scale is chosen as the typical momentum flow of the process, or

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such to eliminate the large log terms. In this simple treatment, the running coupling and its coefficients of the same order cannot be exactly matched, leading to the well-known renormalization scheme-and-scale ambiguities. Due to these ambiguities, the renormalization scale uncertainty is always treated as the key error of a theoretical prediction, which is assumed to decrease when more loop terms are included. As an example, Ref. [18] showed that when going from LO to the approximate N⁴LO, the scale dependence of the total decay width $\Gamma(H \to \gamma\gamma)$ decreases continuously with increasing number of loop terms. However, such a decreasing scale dependence is caused by compensation of scale dependence among different orders, and the exact value for each loop term cannot be obtained by using the "guessing" scale. There are many other problems for such conventional scale-setting treatment [19, 20]. It is thus important to find a proper scale-setting approach to set the renormalization scale so as to achieve a more accurate fixed-order prediction.

The principle of maximum conformality (PMC) [21–25] has been suggested for eliminating the renormalization scheme-and-scale ambiguities. The key idea is to set the correct momentum flow of the process, whose value is independent of the choice of the renormalization scale, based on the renormalization group equation (RGE). Its prediction thus avoids the conventional renormalization scale ambiguities. When one applies PMC, all non-conformal terms that govern the α_s running behavior of the pQCD approximant, should be systematically resummed. The PMC prediction satisfies the renormalization group invariance and all self-consistency conditions of the renormalization group [26]. PMC resums all $\{\beta_i\}$ terms, the divergent renormalon terms which are proportional to $n!\beta_0^n\alpha_s^n$ generally disappear, and a more convergent pQCD series can be naturally achieved. Due to the scheme independent nature of the conformal series and the commensurate scale relations among different observables [27, 28], the PMC predictions are scheme independent. In this paper we adopt PMC to set the renormalization scale for the decay width $\Gamma(H \to \gamma\gamma)$ up to N⁴LO, and show that an accurate scale independent prediction can indeed be achieved. For clarity, we adopt the PMC single-scale approach (PMC-s) [29] for scale-setting.

2   Calculation technology

The total decay width of the Higgs decay into two photons at the one-loop level takes the form

$$\Gamma(H \to \gamma\gamma) = \frac{M_{\mathrm{H}}^3}{64\pi}\left| A_W + \sum_f A_f \right|^2, \tag{1}$$

where M_{H} is the Higgs mass, A_W denotes the contribution

from the purely bosonic diagrams, and A_f stands for the contribution from the amplitudes with $f = (t, b, c, \tau)$, which correspond to top quark, bottom quark, charm quark and τ lepton, respectively.

The higher-order N²LO, N³LO and N⁴LO expressions were given in Refs. [17, 18] for the top quark running mass (m_t). To set the correct momentum flow of the process, only those $\{\beta_i\}$ terms that pertain to RGE should be resummed into α_s. Thus, as was argued in Ref. [30], we transform these terms into the top quark pole mass (M_t) so as to avoid entanglement of the $\{\beta_i\}$ terms from either the top quark anomalous dimension or RGE, and thus avoid the ambiguity in applying PMC. Such a mass transformation can be done by using the relation between m_t and M_t, whose explicit expression up to the α_s^4 order can be found in Ref. [31].

For convenience, we rewrite the total decay width in two parts,

$$\Gamma(H \to \gamma\gamma) = \frac{M_{\mathrm{H}}^3}{64\pi}\left(A_{\mathrm{LO}}^2 + A_{\mathrm{EW}}\frac{\alpha}{\pi} \right) + R(\mu_r), \tag{2}$$

where α is the fine-structure constant.

The LO contribution A_{LO} and the electroweak (EW) correction A_{EW} are [17]

$$A_{\mathrm{LO}} = A_W^{(0)} + A_f^{(0)} + \hat{A}_t A_t^{(0)}, \tag{3}$$

$$A_{\mathrm{EW}} = 2A_{\mathrm{LO}}A_{\mathrm{EW}}^{(1)}, \tag{4}$$

where $A_W^{(0)}$ is the purely bosonic contribution to the amplitude, $A_f^{(0)}$ is the contribution to the amplitude with $f = (b, c, \tau)$, $\hat{A}_t = 2Q_t^2\alpha\sqrt{\sqrt{2}G_{\mathrm{F}}}/\pi$, G_{F} is the Fermi constant, and Q_t is the top quark electric charge. All were calculated in Refs. [5, 6], i.e.

$$A_W^{(0)} = -\frac{\alpha\sqrt{\sqrt{2}G_{\mathrm{F}}}}{2\pi}\left[2 + \frac{3}{\tau_W} + \frac{3}{\tau_W}\left(2 - \frac{1}{\tau_W}\right)f(\tau_W)\right],$$

$$A_f^{(0)} = \sum_{f=c,b,\tau} 3\frac{\alpha\sqrt{\sqrt{2}G_{\mathrm{F}}}}{\pi\tau_f}Q_f^2\left[1 + \left(1 - \frac{1}{\tau_f}\right)f(\tau_f)\right],$$

$$A_t^{(0)} = 1 + \frac{7}{30}\tau_t + \frac{2}{21}\tau_t^2 + \frac{26}{525}\tau_t^3 + \frac{512}{17325}\tau_t^4 + \frac{1216}{63063}\tau_t^5$$
$$+ \frac{128}{9555}\tau_t^6,$$

where

$$f(\tau) = \begin{cases} \mathrm{Arcsin}^2(\sqrt{\tau}) & \text{for} \quad \tau \leqslant 1 \\ -\frac{1}{4}\left[\ln\frac{1 + \sqrt{1 - \tau^{-1}}}{1 - \sqrt{1 - \tau^{-1}}} - i\pi\right]^2 & \text{for} \quad \tau > 1 \end{cases},$$

Q_f denotes the electric charge of $f = (c, b, \tau)$, $\tau_W = M_{\mathrm{H}}^2/(4M_W^2)$, $\tau_t = M_{\mathrm{H}}^2/(4M_t^2)$ and $\tau_f = M_{\mathrm{H}}^2/(4M_f^2)$, and the expression for the NLO electroweak term $A_{\mathrm{EW}}^{(1)}$ can be found in Refs. [31, 32].

The QCD corrections of the decay width $\Gamma(H \to \gamma\gamma)$

are separately represented by $R(\mu_r)$, whose perturbative series up to the $(n+1)$loop level can be written as

$$R_n(\mu_r) = \sum_{i=1}^{n} r_i(\mu) a_s^i(\mu_r), \tag{5}$$

where $a_s = \alpha_s/\pi$, μ_r is the renormalization scale. The perturbative coefficients r_i in the $\overline{\text{MS}}$ scheme up to the α_s^4 order can be derived from Refs. [17, 18]. To apply PMC, the n_f power series (n_f being the active flavor number) of the coefficients r_i should be rewritten into conformal terms and non-conformal β_i terms [24, 25],

$$r_1 = r_{1,0}, \tag{6}$$
$$r_2 = r_{2,0} + r_{2,1}\beta_0, \tag{7}$$
$$r_3 = r_{3,0} + r_{2,1}\beta_1 + 2r_{3,1}\beta_0 + r_{3,2}\beta_0^2, \tag{8}$$

$$r_4 = r_{4,0} + r_{2,1}\beta_2 + 2r_{3,1}\beta_1 + \frac{5}{2}r_{3,2}\beta_0\beta_1$$
$$+ 3r_{4,1}\beta_0 + 3r_{4,2}\beta_0^2 + r_{4,3}\beta_0^3,$$
$$\cdots, \tag{9}$$

where the β pattern at each order is a superposition of RGE, and all coefficients $r_{i,j}$ can be fixed from the n_f power series at the same order by using the degeneracy relations among different orders. $r_{i,0}$ are conformal coefficients which are exactly free of μ_r for the present channel, and $r_{i,j(j\neq 0)}$ are non-conformal coefficients which are functions of μ_r, i.e.,

$$r_{i,j} = \sum_{k=0}^{j} C_j^k \ln^k(\mu_r^2/M_H^2) \hat{r}_{i-k,j-k}, \tag{10}$$

where $\hat{r}_{i,j} = r_{i,j}|_{\mu_r=M_H}$. The needed $\{\beta_i\}$ functions in the $\overline{\text{MS}}$ scheme are given in Refs. [33–41].

Following the standard procedures of the PMC single-scale approach [29], the pQCD corrections of the decay width $\Gamma(H \to \gamma\gamma)$ can be simplified as the following conformal series,

$$R_n(\mu_r)|_{\text{PMC}} = \sum_{i=1}^{n} \hat{r}_{i,0} a_s^i(Q_\star), \tag{11}$$

where $Q_\star$ is the PMC scale. Using the known pQCD corrections up to N^{4}LO, $Q_\star$ can be fixed up to next-to-next-to-leading-log (N^{2}LL) accuracy, i.e.,

$$\ln \frac{Q_\star^2}{M_H^2} = \sum_i T_i a_s^i(M_H), \tag{12}$$

whose first three coefficients with $i = (0,1,2)$ can be determined from the known five-loop QCD corrections of the decay width $\Gamma(H \to \gamma\gamma)$, which are

$$T_0 = -\frac{\hat{r}_{2,1}}{\hat{r}_{1,0}}, \tag{13}$$

$$T_1 = \frac{2(\hat{r}_{2,0}\hat{r}_{2,1} - \hat{r}_{1,0}\hat{r}_{3,1})}{\hat{r}_{1,0}^2} + \frac{(\hat{r}_{2,1}^2 - \hat{r}_{1,0}\hat{r}_{3,2})}{\hat{r}_{1,0}^2}\beta_0, \tag{14}$$

$$T_2 = \frac{4(\hat{r}_{1,0}\hat{r}_{2,0}\hat{r}_{3,1} - \hat{r}_{2,0}^2\hat{r}_{2,1}) + 3(\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,0} - \hat{r}_{1,0}^2\hat{r}_{4,1})}{\hat{r}_{1,0}^3}$$
$$- \frac{\hat{r}_{2,0}\hat{r}_{2,1}^2 + 2(\hat{r}_{2,0}\hat{r}_{2,1}^2 - 2\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,1} - \hat{r}_{1,0}\hat{r}_{2,0}\hat{r}_{3,2})}{\hat{r}_{1,0}^3}\beta_0$$
$$- \frac{3\hat{r}_{1,0}^2\hat{r}_{4,2}}{\hat{r}_{1,0}^3}\beta_0 + \frac{3(\hat{r}_{2,1}^2 - \hat{r}_{1,0}\hat{r}_{3,2})}{2\hat{r}_{1,0}^2}\beta_1$$
$$+ \frac{(\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,2} - \hat{r}_{1,0}^2\hat{r}_{4,3}) + (\hat{r}_{1,0}\hat{r}_{2,1}\hat{r}_{3,2} - \hat{r}_{2,1}^3)}{\hat{r}_{1,0}^3}\beta_0^2. \tag{15}$$

It should be noted that all perturbative coefficients T_i are free of μ_r, and Eq. (12) then indicates that the PMC scale $Q_\star$ is free of μ_r. Together with the fact that the conformal coefficients $\hat{r}_{i,0}$ are also free of μ_r, the PMC approximant $R_n(\mu_r)|_{\text{PMC}}$ is exactly scale independent. Thus the conventional scale ambiguity is eliminated.

As a subtle point, due to the perturbative nature of the PMC scale $Q_\star$, there is a residual scale dependence of the pQCD approximant for the unknown higher-order terms in $Q_\star$ perturbative series. However, this residual scale dependence is different from conventional renormalization scale ambiguity. In fact this residual scale dependence is usually negligible due to both the α_s suppression and exponential suppression. This property has been confirmed in many PMC applications in literature.

3　Numerical results and discussion

For numerical calculations, we take the following central values [42]: the W boson mass $M_W = 80.379$ GeV, the τ lepton mass $M_\tau = 1.77686$ GeV, the b quark pole mass $M_b = 4.78$ GeV, the c quark pole mass $M_c = 1.67$ GeV, the t quark pole mass $M_t = 173.07$ GeV, and the Higgs mass $M_H = 125.9$ GeV. The Fermi constant is $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2} and the fine-structure constant $\alpha = 1/137.035999139$. The magnitude of the EW corrections at the NLO level is about 0.15 KeV [43], which shall be altered to ~0.18 KeV by setting the scale for the fine structure constant to be $M_H/2$, leading to about 20% scale uncertainty for the EW corrections. In the present paper, we shall concentrate on eliminating the scale uncertainty for QCD corrections. The PMC is applicable to QED corrections [19], and if the NNLO and higher order EW corrections are known, one can determine the correct α value by using the PMC via a similar way. To be self-consistent with the N^{4}LO pQCD correction of the total decay width $\Gamma(H \to \gamma\gamma)$, we adopt the four-loop α_s running and $\alpha_s(M_Z = 91.1876\text{GeV}) = 0.1181$ to fix the α_s running behavior.

As a comparison, we present the total decay width $\Gamma(H \to \gamma\gamma)$ up to N^{4}LO with the conventional and PMC

scale-settings in Figs. 1 and 2. In agreement with previous observations, Fig. 1 shows that with the conventional scale-setting the scale dependence becomes smaller and smaller when more loop terms are included. The N^{4}LO total decay width in the conventional scale-setting gives

$$\Gamma(H \to \gamma\gamma)|_{\text{Conv.}} = 9.626^{+0.002}_{+0.002} \text{ KeV}, \tag{16}$$

where the central value is for $\mu_r = M_{\text{H}}$, and the renormalization scale error is for $\mu_r \in [M_{\text{H}}/2, 2M_{\text{H}}]$.

It should be pointed out that the above approximate scale independence of the N^{4}LO total decay width $\Gamma(H \to \gamma\gamma)$ with the conventional scale-setting is caused by large cancellations of the scale dependence among different orders. This can be explicitly seen in Table 1, in which the individual decay widths for LO+EW, NLO,
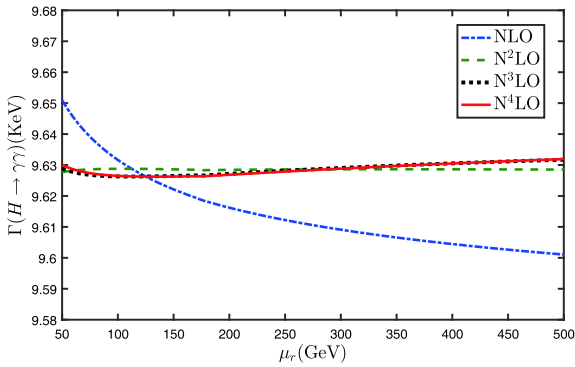

Fig. 1.   (color online) Total decay width $\Gamma(H \to \gamma\gamma)$ versus the initial scale μ_r up to N^{4}LO with the conventional scale-setting.
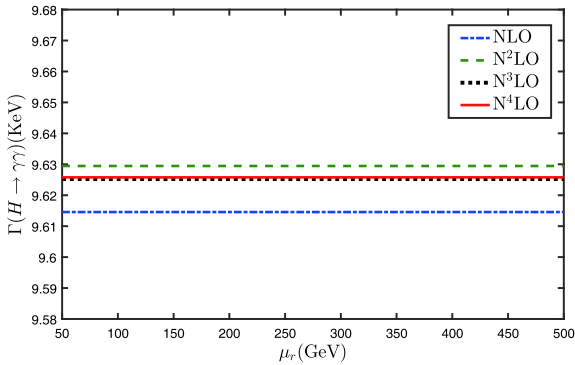

Fig. 2.   (color online) Total decay width $\Gamma(H \to \gamma\gamma)$ versus the initial scale μ_r up to N^{4}LO with the PMC scale-setting.

N^{2}LO, N^{3}LO and N^{4}LO are presented separately. More explicitly, we define a parameter κ_i to measure the scale dependence of the separate decay widths at different orders, i.e.

$$\kappa_i = \frac{\Gamma_i|_{\mu_r=M_{\text{H}}/2} - \Gamma_i|_{\mu_r=2M_{\text{H}}}}{\Gamma_i|_{\mu_r=M_{\text{H}}}}, \tag{17}$$

where the subscript i stands for the individual NLO, N^{2}LO, N^{3}LO and N^{4}LO decay widths. In the conventional scale-setting, we have

$$\kappa_{\text{NLO}} = +20\%, \tag{18}$$

$$\kappa_{\text{N}^2\text{LO}} = -1.2 \times 10^3\%, \tag{19}$$

$$\kappa_{\text{N}^3\text{LO}} = +19\%, \tag{20}$$

$$\kappa_{\text{N}^4\text{LO}} = -1.6 \times 10^3\%. \tag{21}$$

Large magnitude of κ_i indicates that in the conventional scale-setting, there are large scale errors for each order. Due to the cancellation among different orders, the net scale error for the N^{4}LO total decay width is small and is about 0.2%.

On the other hand, as shown in Fig. 2, the PMC prediction is almost scale independent for each order, and the PMC prediction of $\Gamma(H \to \gamma\gamma)$ quickly approaches the scale independent "physical" value due to a faster convergence than the conventional pQCD series. As the magnitudes of the newly added N^{3}LO and N^{4}LO terms are only about 28% and 4% of the N^{2}LO term, whose magnitude is small, our previous N^{2}LO PMC prediction agrees with the present one [44] to a high precision. Table 1 shows that after applying PMC, both the separate decay widths and the total decay width are unchanged for $\mu_r \in [M_{\text{H}}/2, 2M_{\text{H}}]$. The N^{4}LO total decay width with the PMC scale-setting is

$$\Gamma(H \to \gamma\gamma)|_{\text{PMC}} \equiv 9.626 \text{ KeV}. \tag{22}$$

The four-loop and five-loop fermionic contributions are helpful to set an accurate PMC scale. The effective scale $Q_\star$ can be fixed up to the N^{2}LL accuracy by using the known five-loop pQCD corrections, i.e.

$$\ln \frac{Q_\star^2}{M_{\text{H}}^2} = 1.321 - 4.271\alpha_s(M_{\text{H}}) + 21.029\alpha_s^2(M_{\text{H}}). \tag{23}$$

Figure 3 shows the perturbative nature of $Q_\star$, e.g.

Table 1.   Total decay width $\Gamma(H \to \gamma\gamma)$ with the conventional (Conv.) and PMC scale-settings. $\Gamma_{\text{LO+EW}}$, Γ_{NLO}, $\Gamma_{\text{N}^2\text{LO}}$, $\Gamma_{\text{N}^3\text{LO}}$ and $\Gamma_{\text{N}^4\text{LO}}$ are individual decay widths for LO+EW, NLO, N^{2}LO, N^{3}LO and N^{4}LO, respectively. The final row is the total decay width up to N^{4}LO. Three typical scales $\mu_r = M_{\text{H}}/2$, M_{H}, $2M_{\text{H}}$ are adopted.

		$i = $ LO + EW	$i = $ NLO	$i = $ N^{2}LO	N^{3}LO	$i = $ N^{4}LO	total
	$\mu_r = M_{\text{H}}/2$	9.46477	0.17927	−0.01573	−0.00085	0.00083	9.62830
$\Gamma_i(\text{KeV})|_{\text{Conv.}}$	$\mu_r = M_{\text{H}}$	9.46477	0.16133	0.00263	−0.00242	−0.00007	9.62624
	$\mu_r = 2M_{\text{H}}$	9.46477	0.14731	0.01649	−0.00038	−0.00028	9.62791
$\Gamma_i(\text{KeV})|_{\text{PMC}}$	$\mu_r \in [M_{\text{H}}/2, 2M_{\text{H}}]$	9.46477	0.14979	0.01489	−0.00423	0.00056	9.62578

$|Q_\star^{(3)} - Q_\star^{(2)}| < |Q_\star^{(2)} - Q_\star^{(1)}|$, in which $Q_\star^{(1)}$ is at the LL accuracy, $Q_\star^{(2)}$ at the NLL accuracy and $Q_\star^{(3)}$ at the N²LL accuracy. To be self-consistent and to ensure the scheme independence of the PMC prediction, we show in Fig. 3, the N²LO prediction for $Q_\star^{(1)}$, the N³LO for $Q_\star^{(2)}$, and the N⁴LO for $Q_\star^{(3)}$. The approximate scale independence of each order in the PMC scale-setting is caused by the nearly conformal nature of the pQCD series.
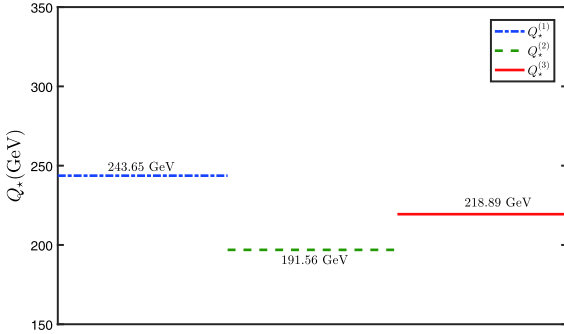

Fig. 3.   (color online) The determined effective scale $Q_\star$. $Q_\star^{(1)}$ is at the LL accuracy, $Q_\star^{(2)}$ at the NLL accuracy and $Q_\star^{(3)}$ at the N²LL accuracy.

It is helpful to be able to estimate the "unknown" higher-order pQCD corrections. The conventional error estimate obtained by varying the scale over a certain range is not reliable, since it only partly estimates the non-conformal contribution but not the conformal one. The Padé approximation approach (PAA) provides a practical way of promoting a finite series of an analytic function [45–47], which was recently suggested to give a reliable prediction of uncalculated high-order terms by using the PMC conformal series [48].

As an attempt, following the approach described in detail in Ref. [48], we give a PAA+PMC prediction for $R_n(M_H)$ by using the preferable $[0/(n-1)]$-type Padé series. The results are presented in Fig. 4, where "PAA" is the predicted $R_n(M_H)$ by using the known $R_{n-1}(M_H)$ series, and "EC" is the prediction by directly using the known PMC $R_n(M_H)$ series. Figure 4 shows that the difference between "EC" and the predicted $R_n(M_H)$ tends to decrease as more higher-order loops are included. The difference between $R_{3,4}(M_H)|_{PAA}$ and $R_{3,4}(M_H)|_{EC}$ is already less than 1%, thus the "exact" value of $R(M_H)|_{EC}$ could be directly taken as $R_5(M_H)|_{PAA}$, i.e.

$$R_5(M_H)|_{PAA} \cong 1.614 \times 10^{-1} \text{ KeV}. \qquad (24)$$

The total decay width is then

$$\Gamma_5(H \to \gamma\gamma)|_{PMC} = \left[9.626 \pm 5.354 \times 10^{-5}\right] \text{ KeV}, \qquad (25)$$

where the error is the PAA+PMC prediction of uncalculated high-order pQCD contributions, which is negligible.

The total decay width $\Gamma(H \to \gamma\gamma)$ versus the Higgs mass M_H is presented in Fig. 5. If we take the Higgs mass
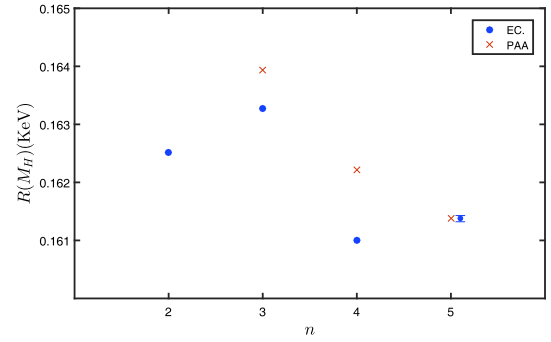

Fig. 4.   (color online) Comparison of the exact ("EC") and the predicted $[0/(n-1)]$-type "PAA" pQCD approximant $R_n(M_H)$ with the PMC scale-setting, showing how the PAA prediction changes when more loop terms are included.
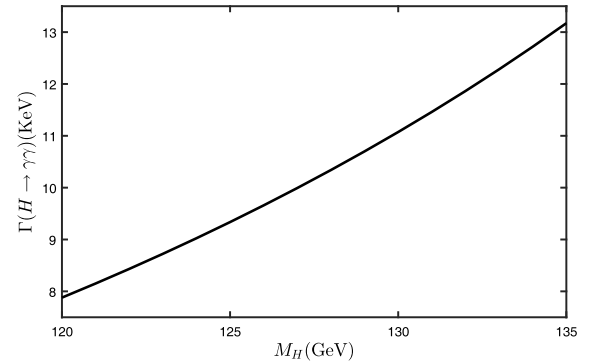

Fig. 5.   The PMC prediction of the decay width $\Gamma(H \to \gamma\gamma)$ versus the Higgs mass M_H.

given by the ATLAS and CMS collaborations [49, 50], i.e. $M_H = 125.09 \pm 0.21 \pm 0.11$ GeV, we obtain

$$\Gamma(H \to \gamma\gamma)|_{LHC} = 9.364^{+0.076}_{-0.075} \text{ KeV}, \qquad (26)$$

As an application, we predict the "fiducial cross-section" of the process $pp \to H \to \gamma\gamma$, which was predicted by the LHC-XS group with the conventional scale-setting [51] and was measured by the ATLAS and CMS collaborations with increasing integrated luminosity [52–54]. A PMC prediction was previously given in Ref. [55] by using $\Gamma(H \to \gamma\gamma)$ up to N²LO. Taking the same parameters as in Refs. [51, 55, 56], e.g. $M_H = 125$ GeV and $M_t = 173.3$ GeV, and by using the present $\Gamma(H \to \gamma\gamma)$ up to N⁴LO, we obtain $\sigma_{fid}(pp \to H \to \gamma\gamma) = 30.1^{+2.3}_{-2.2}$ fb, $38.3^{+2.9}_{-2.8}$ fb, and $85.8^{+5.7}_{-5.3}$ fb for the proton-proton center-of-mass collision energy $\sqrt{S} = 7$, 8 and 13 TeV, respectively. Here, the errors are dominated by the error of the Higgs inclusive cross-section. A comparison with the recent experimental data is shown in Fig. 6. A better agreement with the data at $\sqrt{S} = 7$ and 8 TeV can be achieved by applying PMC. The ATLAS and CMS measurements at $\sqrt{S} = 13$ TeV still have large errors and are in disagreement, and the PMC prediction is closer to the CMS result.
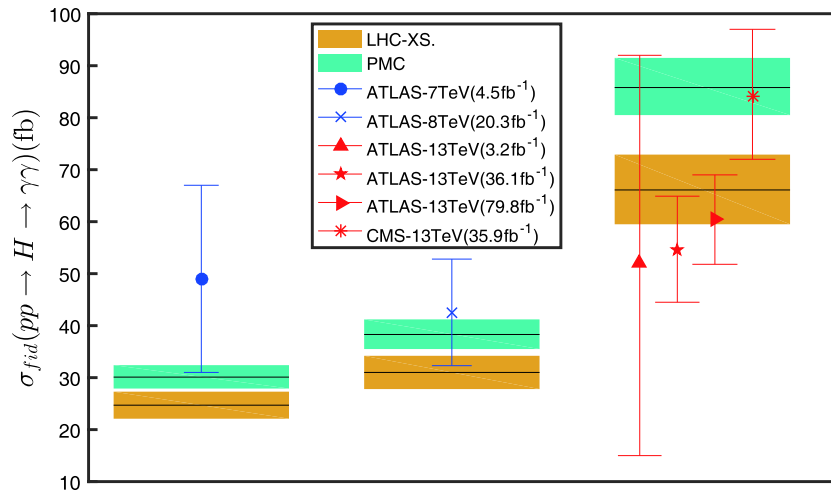
Fig. 6.   (color online) The fiducial cross-section $\sigma_{\rm fid}(pp\to H\to\gamma\gamma)$ using $\Gamma(H\to\gamma\gamma)$ up to N^{4}LO. The LHC-XS prediction [51], the AT-LAS measurements [52, 53] and the CMS measurements [54] are presented for comparison.

4   Summary

In summary, PMC uses the basic RGE to set the correct α_s running behavior. The resultant conformal series is independent of the initial choice of renormalization scale and renormalization scheme, and thus eliminates conventional scheme-and-scale ambiguities.

Using the pQCD corrections up to the N^{4}LO level, we can fix the effective PMC scale up to the N^{2}LL level, and an accurate scheme-and-scale independent prediction for the decay width $\Gamma(H\to\gamma\gamma)$ can be achieved. Due to the

elimination of divergent renormalon terms, the pQCD convergence can be naturally improved by applying PMC. This improvement of pQCD convergence has been found in most PMC applications. However, as shown in the case of the $\gamma\gamma^*\to\eta_c$ form factor [57], where the magnitude of the NNLO term is still larger than the NLO term even after applying PMC, there may remain large logarithmic terms in the resultant PMC conformal series, diluting the pQCD convergence. In these special cases, the conventional resummation approach [58] may help to further improve the pQCD convergence. A detailed discussion on this point is in progress.

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