Study of the $s \to d\nu\bar{\nu}$ rare hyperon decays in the Standard Model and new physics*

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Abstract: FCNC processes offer important tools to test the Standard Model (SM) and to search for possible new physics. In this work, we investigate the $s \to d\nu\bar{\nu}$ rare hyperon decays in SM and beyond. We find that in SM the branching ratios for these rare hyperon decays range from 10^{-14} to 10^{-11} . When all the errors in the form factors are included, we find that the final branching ratios for most decay modes have an uncertainty of about 5% to 10%. After taking into account the contribution from new physics, the generalized SUSY extension of SM and the minimal 331 model, the decay widths for these channels can be enhanced by a factor of 2 ~ 7.

Keywords: branching ratios, rare hyperon decays, form factors, light-front approach, new physics

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1 Introduction

The flavor changing neutral current (FCNC) transitions provide a critical test of the Cabibbo-Kobayashi-Maskawa (CKM) mechanism in the Standard Model (SM), and allow to search for possible new physics. In SM, the FCNC transition $s \to d\nu\bar{\nu}$ proceeds through the Z-penguin and electroweak box diagrams, and thus the decay probabilities are strongly suppressed. In this case, a precise study allows to perform very stringent tests of SM and ensures large sensitivity to potential new degrees of freedom.

A large number of studies have been performed of the $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ processes, and reviews of these two decay modes can be found in [1–6]. On the theoretical side, using the most recent input parameters, the SM predictions for the two branching ratios are [7]

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm SM} = (8.4 \pm 1.0) \times 10^{-11}, \tag{1}$$

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm SM} = (3.4 \pm 0.6) \times 10^{-11}. \tag{2}$$

The dominant uncertainty comes from the CKM matrix elements and the charm contribution. On the experimental side, the NA62 experiment at the CERN SPS has reported the first search for $K^+ \to \pi^+\nu\bar{\nu}$ using the decay-in-flight technique, and the corresponding observed upper limit is [8] :

$$\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu})_{\rm exp} < 14 \times 10^{-10}, \quad \text{at 95\% CL.} \tag{3}$$

Similarly, the E391a collaboration reported the 90% C.L. upper bound [9]

$$\mathcal{B}(K_L \to \pi^0\nu\bar{\nu})_{\rm exp} \leqslant 2.6 \times 10^{-8}. \tag{4}$$

The KOTO experiment, an upgrade of the E391a experiment, aims at a first observation of the $K_L \to \pi^0\nu\bar{\nu}$ decay at J-PARC around 2020 [3, 10]. Given the goal of a 10% precision by NA62, the authors of Ref. [11] intend to carry out lattice QCD calculations to determine the long-distance contributions to the $K^+ \to \pi^+\nu\bar{\nu}$ amplitude.

Analogous to $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, the rare hyperon decays $B_i \to B_f\nu\bar{\nu}$ also proceed via $s \to d\nu\bar{\nu}$ at the quark level, and thus offer important tools to test SM and to search for possible new physics. Compared to the widely considered $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$, there are few studies devoted to rare hyperon decays $B_i \to B_f\nu\bar{\nu}$. This work aims to perform a preliminary theoretical research of the rare hyperon decays both in and beyond SM.

A study of the hyperon decays at the BESIII experiment is proposed using the hyperon parents of the J/ψ de-

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cay. The electron-positron collider BEPCII provides a clean experimental environment. About 10^6-10^8 hyperons, Λ, Σ, Ξ and Ω, are produced in the J/ψ and $\psi(2S)$ decays with the proposed data samples at the BESIII experiment. Based on these samples, the sensitivity of the measurement of the branching ratios of hyperon decays is in the range of 10^{-5}-10^{-8}. The author of Ref. [12] proposed that rare decays and decays with invisible final states may be probed.

The paper is organized as follows. In Sec. 2, our computing framework is presented. Sec. 3 is devoted to performing the numerical calculations. The branching ratios of several rare hyperon decays are calculated in SM. The new physics contribution, the Minimal Supersymmetric Standard Model (MSSM) and the minimal 331 model, are considered. We also discuss possible uncertainties from the form factors. The last section contains a short summary.

2 Theoretical framework

The next-to-leading order (NLO) effective Hamiltonian for $s\to d\nu\bar{\nu}$ reads [13]:

$$\mathcal{H}_{\rm eff}=\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\theta_W}\sum_{l=e,\mu,\tau}[V_{cs}^*V_{cd}X_{NL}^l + V_{ts}^*V_{td}X(x_t)](\bar{s}d)_{V-A}(\bar{\nu}_l\nu_l)_{V-A}+{\rm h.c.}, \tag{5}$$

where $X(x_t)$ and X_{NL}^l are relevant for the top and the charm contribution, respectively. Their explicit expressions can be found in Ref. [13]. Here, $x_t=m_t^2/m_W^2$. To leading order in α_s, the function $X(x_t)$ relevant for the top contribution reads [14, 15]

$$\begin{aligned}X(x)=&X_0(x)+\frac{\alpha_s}{4\pi}X_1(x),\\ X_0(x)=&\frac{x}{8}\left[-\frac{2+x}{1-x}+\frac{3x-6}{(1-x)^2}\ln x\right],\\ X_1(x)=&-\frac{23x+5x^2-4x^3}{3(1-x)^2}+\frac{x-11x^2+x^3+x^4}{(1-x)^3}\ln x\\ &+\frac{8x+4x^2+x^3-x^4}{2(1-x)^3}\ln^2 x\\ &-\frac{4x-x^3}{(1-x)^2}L_2(1-x)+8x\frac{\partial X_0(x)}{\partial x}\ln x_\mu,\end{aligned} \tag{6}$$

where $x_\mu=\mu^2/M_W^2$ with $\mu=O(m_t)$ and

$$L_2(1-x)=\int_1^x {\rm d}t\frac{\ln t}{1-t}. \tag{7}$$

The function X_{NL}^l corresponds to $X(x_t)$ in the charm sector. It results from the renormalization group (RG) calculation in next-to-leading-order logarithmic approximation (NLLA) and is given as follows:

$$X_{NL}^l=C_{NL}-4B_{NL}^{(1/2)}, \tag{8}$$

where C_{NL} and $B_{NL}^{(1/2)}$ correspond to the Z^0-penguin and the box-type contribution, respectively, given as [16]

$$\begin{aligned}C_{NL}=&\frac{x(m_c)}{32}K_c^{\frac{24}{25}}\left[\left(\frac{48}{7}K_++\frac{24}{11}K_--\frac{696}{77}K_{33}\right)\left(\frac{4\pi}{\alpha_s(\mu)}\right.\right.\\ &\left.+\frac{15212}{1875}(1-K_c^{-1})\right)+\left(1-\ln\frac{\mu^2}{m_c^2}\right)(16K_+-8K_-)\\ &-\frac{1176244}{13125}K_+-\frac{2302}{6875}K_-+\frac{3529184}{48125}K_{33}\\ &\left.+K\left(\frac{56248}{4375}K_+-\frac{81448}{6875}K_-+\frac{4563698}{144375}K_{33}\right)\right],\\ B_{NL}^{(1/2)}=&\frac{x(m_c)}{4}K_c^{\frac{24}{25}}\left[3(1-K_2)\left(\frac{4\pi}{\alpha_s(\mu)}+\frac{15212}{1875}(1-K_c^{-1})\right)\right.\\ &\left.-\ln\frac{\mu^2}{m_c^2}-\frac{r\ln r}{1-r}-\frac{305}{12}+\frac{15212}{625}K_2+\frac{15581}{7500}KK_2\right],\end{aligned} \tag{9}$$

where $r=m_l^2/m_c^2(\mu)$, $\mu=O(m_c)$ and

$$K=\frac{\alpha_s(M_W)}{\alpha_s(\mu)},\quad K_c=\frac{\alpha_s(\mu)}{\alpha_s(m_c)},$$
$$K_+=K^{\frac{6}{25}},\quad K_-=K^{-\frac{12}{25}},\quad K_{33}=K_2=K^{-\frac{1}{25}}. \tag{10}$$

In the following, we consider the transitions between the baryon octet (Ξ, Σ, Σ and N) and the transitions from the baryon decuplet to the octet $\Omega^-\to\Xi^-$.

The transition matrix elements of the vector and axial-vector currents between the baryon octets can be parametrized in terms of six form factors $f_{1,2,3}(q^2)$ and $g_{1,2,3}(q^2)$:

$$\begin{aligned}&\langle B_8'(P',S_z')|\bar{d}\gamma_\mu(1-\gamma_5)s|B_8(P,S_z)\rangle=\\ &\bar{u}(P',S_z')\left[\gamma_\mu f_1(q^2)+i\sigma_{\mu\nu}\frac{q^\nu}{M}f_2(q^2)+\frac{q_\mu}{M}f_3(q^2)\right]u(P,S_z)\\ &-\bar{u}(P',S_z')\left[\gamma_\mu g_1(q^2)+i\sigma_{\mu\nu}\frac{q^\nu}{M}g_2(q^2)+\frac{q_\mu}{M}g_3(q^2)\right]\gamma_5 u(P,S_z),\end{aligned} \tag{11}$$

where $q=P-P'$, and M denotes the mass of the parent baryon octet B_8. The form factors for the $B_8\to B_8'$ transition, $f_i(q^2)$ and $g_i(q^2)$, can be expressed by the following formulas [17]:

$$\begin{aligned}f_m=&aF_m(q^2)+bD_m(q^2),\\ g_m=&aF_{m+3}(q^2)+bD_{m+3}(q^2),\quad (m=1,2,3),\end{aligned} \tag{12}$$

where $F_i(q^2)$ and $D_i(q^2)$, with $i=1,2,\cdots,6$, are different functions of q^2 for each of the six form factors. Some remarks are necessary [17]:

• The constants a and b in Eq. (12) are the $SU(3)$ Clebsch-Gordan coefficients that appear when an octet operator is sandwiched between octet states.

• For $q^2=0$, the form factor $f_1(0)$ is equal to the electric charge of the baryon, therefore $F_1(0)=1$ and $D_1(0)=0$.

• The weak $f_2(0)$ form factor can be computed using the anomalous magnetic moments of proton and neutron (κ_p and κ_n) in the exact $SU(3)$ symmetry. Here,

$F_2(0)=\kappa_p+\frac{1}{2}\kappa_n$ and $D_2(0)=-\frac{3}{2}\kappa_n$.

- $g_1(0)$ is a linear combination of two parameters, F and D.
- Since $g_2^{n\to p}=F_5(q^2)+D_5(q^2)=0$ and $g_2^{\Xi^-\to\Xi^0}=D_5(q^2)-F_5(q^2)=0$, we get $F_5(q^2)=D_5(q^2)=0$. Therefore, all pseudo-tensor form factors g_2 vanish in all decays up to symmetry-breaking effects.
- In the $s\to d\bar{\nu}\nu$ decay, the f_3 and g_3 terms are proportional to the neutrino mass and thus can be neglected for the decays considered in this work.

Since the invariant mass squared of lepton pairs in the hyperon decays is relatively small, it is expected that the q^2 distribution in the form factors has small impact on the decay widths. We list the expressions for f_1, f_2 and g_1 at $q^2=0$ in Table 1.

Hence, Eq. (11) can be rewritten as:

$$\langle B_8'(P',S_z')|\bar{d}\gamma_\mu(1-\gamma_5)s|B_8(P,S_z)\rangle=\\ \bar{u}(P',S_z')\left[\gamma_\mu f_1(q^2)+i\sigma_{\mu\nu}\frac{q^\nu}{M}f_2(q^2)-\gamma_\mu g_1(q^2)\gamma_5\right]u(P,S_z). \tag{13}$$

The helicity amplitudes of the hadronic contribution are defined as

$$H^V_{\lambda',\lambda_V}\equiv\langle B_8'(P',\lambda')|\bar{d}\gamma^\mu s|B_8(P,\lambda)\rangle\epsilon^*_{V\mu}(\lambda_V), \tag{14}$$

$$H^A_{\lambda',\lambda_V}\equiv\langle B_8'(P',\lambda')|\bar{d}\gamma^\mu\gamma_5 s|B_8(P,\lambda)\rangle\epsilon^*_{V\mu}(\lambda_V). \tag{15}$$

Here, $\lambda^{(\prime)}$ denotes the helicity of the parent (daughter) baryon in the initial (final) state, and λ_V is the helicity of the virtual intermediate vector particle. It can be shown that the helicity amplitudes $H^{V,A}_{\lambda',\lambda_V}$ have the following simple forms [19]:

$$\begin{aligned} H^V_{\frac12,0}&=-i\frac{\sqrt{Q_-}}{\sqrt{q^2}}\left[(M+M')f_1-\frac{q^2}{M}f_2\right],\\ H^A_{\frac12,0}&=-i\frac{\sqrt{Q_+}}{\sqrt{q^2}}(M-M')g_1,\\ H^V_{\frac12,1}&=i\sqrt{2Q_-}\left[-f_1+\frac{M+M'}{M}f_2\right],\\ H^A_{\frac12,1}&=-i\sqrt{2Q_+}g_1. \end{aligned} \tag{16}$$

In the above, $Q_\pm=(M\pm M')^2-q^2$, and M (M') is the parent (daughter) baryon mass in the initial (final) state. The amplitudes for the negative helicity are obtained from the relations,

$$H^V_{-\lambda',-\lambda_V}=H^V_{\lambda',\lambda_V},\qquad H^A_{-\lambda',-\lambda_V}=-H^A_{\lambda',\lambda_V}. \tag{17}$$

The complete helicity amplitudes are obtained by

$$H_{\lambda',\lambda_V}=H^V_{\lambda',\lambda_V}-H^A_{\lambda',\lambda_V}. \tag{18}$$

Due to the lack of experimental data for the M_1 and E_2 transitions from the baryon decuplet to the octet, the vector transition matrix element for $\Omega^-\to\Xi^-$ can not be determined. In this work we follow Ref. [18] , and consider only the axial-vector current matrix element [18, 20, 21]:

$$\begin{aligned}\langle\Xi^-(P',S_z')|\bar{d}\gamma_\mu\gamma_5 s|\Omega^-(P,S_z)\rangle=\bar{u}_{\Xi^-}(P',S_z')\Big\{C_5^A(q^2)g_{\mu\nu}\\ +C_6^A(q^2)q_\mu q_\nu+\left[C_3^A(q^2)\gamma^\alpha+C_4^A(q^2)p'\right]\\ \times(q_\alpha g_{\mu\nu}-q_\nu g_{\alpha\mu})\Big\}u^\nu_{\Omega^-}(P,S_z).\end{aligned} \tag{19}$$

Here, $u^\nu_{\Omega^-}(P,S_z)$ represents the Rarita-Schwinger spinor that describes the baryon decuplet Ω^- with spin $\frac{3}{2}$. In Ref. [22] it is shown that $C_3^A(q^2)$ and $C_4^A(q^2)$ are proportional to the mass difference of the initial and final baryons, and thus are suppressed. In the chiral limit, $C_5^A(q^2)$ and $C_6^A(q^2)$ are related by $C_6^A(q^2)=M_N^2C_5^A(q^2)/q^2$ [20]. In our calculations, we use $C_5^A(0)=1.653\pm0.006$ for $\Omega^-\to\Xi^-$, which is the same as $\Omega^-\to\Xi^0$ in the $SU(3)$ limit [18]. The helicity amplitude can then be expressed as:

$$H^A_{\lambda',\lambda_V}=\langle\Xi^-(P',\lambda')|\bar{d}\gamma_\mu\gamma_5 s|\Omega^-(P,\lambda)\rangle\epsilon_V^{\star\mu}(\lambda_V) \tag{20}$$

$$=\bar{u}_{\Xi^-}(P',\lambda')\left[C_5^A(q^2)g_{\mu\nu}+C_6^A(q^2)q_\mu q_\nu\right]u^\nu_{\Omega^-}(P,\lambda)\epsilon_V^{\star\mu}(\lambda_V). \tag{21}$$

Here, $\lambda^{(\prime)}$ and λ_V have the same definition as in Eqs. (14)-(15). It can be shown that the helicity amplitudes H^A_{λ',λ_V} have the following simple forms [19]:

$$\begin{aligned} H^A_{\frac12,0}&=H^A_{-\frac12,0}=i\sqrt{\frac{2Q_+}{3}}\frac{E_V}{\sqrt{q^2}}C_5^A(q^2),\\ H^A_{\frac12,1}&=H^A_{-\frac12,-1}=i\sqrt{\frac{Q_+}{3}}C_5^A(q^2),\\ H^A_{\frac12,-1}&=H^A_{-\frac12,1}=i\sqrt{Q_+}C_5^A(q^2). \end{aligned} \tag{22}$$

Table 1. The form factors for the $B\to B'$ transition, $f_1(0)$, $f_2(0)$ and $g_1(0)$ [17], where the experimental anomalous magnetic moments are $\kappa_p=1.793\pm0.087$ and $\kappa_n=-1.913\pm0.069$ [18], with the two coupling constants $F=0.463\pm0.008$ and $D=0.804\pm0.008$ [18]. Here, g_1/f_1 is positive for the neutron decay, and all other signs are fixed using this sign convention.

$B\to B'$	$\Lambda\to n$	$\Sigma^+\to p$	$\Xi^0\to\Lambda$	$\Xi^0\to\Sigma^0$	$\Xi^-\to\Sigma^-$
$f_1(0)$	$-\sqrt{\frac{3}{2}}$	-1	$\sqrt{\frac{3}{2}}$	$-\frac{1}{\sqrt{2}}$	1
$f_2(0)$	$-\sqrt{\frac{3}{2}}\kappa_p$	$-(\kappa_p+2\kappa_n)$	$\sqrt{\frac{3}{2}}(\kappa_p+\kappa_n)$	$-\frac{1}{\sqrt{2}}(\kappa_p-\kappa_n)$	$\kappa_p-\kappa_n$
$g_1(0)$	$-\sqrt{\frac{3}{2}}(F+D/3)$	$-(F-D)$	$\sqrt{\frac{3}{2}}(F-D/3)$	$-\frac{1}{\sqrt{2}}(F+D)$	$F+D$

The differential decay width for $B \to B'\bar{\nu}\nu$ is given as:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{\mathrm{d}\Gamma_L}{\mathrm{d}q^2} + \frac{\mathrm{d}\Gamma_T}{\mathrm{d}q^2}. \tag{23}$$

Here, $\mathrm{d}\Gamma_L/\mathrm{d}q^2$ and $\mathrm{d}\Gamma_T/\mathrm{d}q^2$ are the longitudinal and transverse parts of the decay width, and their explicit expressions are given by

$$\frac{\mathrm{d}\Gamma_L}{\mathrm{d}q^2} = N\frac{q^2 p'}{12(2\pi)^3 M^2}(|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2), \tag{24}$$

$$\frac{\mathrm{d}\Gamma_T}{\mathrm{d}q^2} = N\frac{q^2 p'}{12(2\pi)^3 M^2}(|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},-1}|^2 + |H_{-\frac{1}{2},1}|^2). \tag{25}$$

In Eqs. (24) and (25), $p' = \sqrt{Q_+Q_-}/2M$ is the magnitude of the momentum of B' in the rest frame of B, and $N = 2N_1(0) + N_1(m_\tau)$ with

$$N_1(m_l) = \left|\frac{G_F}{\sqrt{2}}\frac{\alpha}{2\pi\sin^2\Theta_W}\left(V_{cd}^* V_{cs} X_{NL}^l(m_l) + V_{td}^* V_{ts} X(x_t)\right)\right|^2. \tag{26}$$

Note that we have neglected the electron and muon masses.

One can then obtain the decay width

$$\Gamma = \int_0^{(M-M')^2} \mathrm{d}q^2 \frac{\mathrm{d}\Gamma}{\mathrm{d}q^2}. \tag{27}$$

3 Numerical results and discussion

3.1 Calculations in SM

With the input parameters given in Table 2 and the formulae from the last section, the LO and NLO results for μ_c = 1 GeV, μ_t = 100 GeV , and $\mu_c = 3$ GeV, $\mu_t = 300$ GeV , are listed Table 3.

From the results in Table 3 one can see that:

● The branching ratios of the $s \to d\nu\bar{\nu}$ rare hyperon decays range from 10^{-14} to 10^{-11} .

● For $\mu_c = 1$ GeV,$\mu_t = 100$ GeV , the NLO results are smaller than the LO ones by about 30%, while for $\mu_c = 3$ GeV,$\mu_t = 300$ GeV , the NLO results are larger than the LO ones by about 10%.

● The LO results vary by about 50% from $\mu_c = 1$ GeV, $\mu_t = 100$ GeV to $\mu_c = 3$ GeV,$\mu_t = 300$ GeV, while the NLO ones vary by about 30%. As expected, the NLO results depend less on the mass scales.

● The branching ratio of $\Omega^- \to \Xi^- \nu\bar{\nu}$ is the largest among the 6 channels. It is of the same order as for $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$.

At present, there is a small number of experimental studies, and thus most experimental constraints are less severe. The prospects for rare and forbidden hyperon decays at BESIII were analyzed in a recent publication Ref. [12]. We quote the experimental sensitivity for all decay modes in Table 3 . Unfortunately, one can see that the current BESIII experiment will not be able to probe these hyperon decays. We hope this may be improved at future experimental facilities like the Super Tau-Charm Factory.

3.2 Uncertainties of the form factors

Note that due to the Ademollo-Gatto theorem [25], the form factor $f_1(0)$ does not receive any $SU(3)$ symmetry breaking correction. However, $f_2(0)$ can be computed using the anomalous magnetic moments of proton and neutron (κ_p and κ_n) in the exact $SU(3)$ symmetry. The experimental data for κ_p and κ_n already include the $SU(3)$ symmetry breaking effects [18]:

$$\kappa_p\left[O(m_s^0)\right] = 1.363 \pm 0.069,\ \kappa_p\left[O(m_s^0)\right] = -1.416 \pm 0.049, \tag{28}$$

$$\kappa_p\left[O(m_s^0) + O(m_s^1)\right] = 1.793 \pm 0.087,$$
$$\kappa_p\left[O(m_s^0) + O(m_s^1)\right] = -1.913 \pm 0.069. \tag{29}$$

The uncertainties from κ_p and κ_n in the effect of $SU(3)$ symmetry breaking is approximately 25%. We calculated the effect of κ_p and κ_n on the branching ratio of $\Sigma^+ \to p\nu\bar{\nu}$ in the case of NLO with the energy scale $\mu_c = 1$ GeV and $\mu_t = 100$ GeVsuch that:

Table 2. The input parameters used in this work.

The masses and lifetimes of baryons in the initial and final states [23]				
$m_p = 938.2720813$ MeV	$m_{\Sigma^+} = 1189.37$ MeV	$m_{\Xi^0} = 1314.86$ MeV		
m_n= 939.5654133 MeV	$m_{\Sigma^-} = 1197.45$ MeV	$m_{\Xi^-} = 1321.71$ MeV		
$m_\Lambda = 1115.683$ MeV	$m_{\Sigma^0} = 1192.642$ MeV	$m_{\Omega^-} = 1672.45$ MeV		
$\tau_{\Xi^0} = 2.90 \times 10^{-10}$ s	$\tau_{\Xi^-} = 1.639 \times 10^{-10}$ s	$\tau_{\Omega^-} = 0.821 \times 10^{-10}$ s		
$\tau_\Lambda = 2.632 \times 10^{-10}$ s	$\tau_{\Sigma^+} = 0.8018 \times 10^{-10}$ s			
Physical constants and CKM parameters [23, 24]				
$G_F = 1.16637387 \times 10^{-5}$ GeV^{-2}	$\sin^2\theta_W = 0.23122$	$\alpha_s(m_Z)$= 0.1182	$\alpha \equiv \alpha(m_Z)$= 1/128	
m_τ=1776.86 MeV	$m_c = 1.275$ GeV	$m_t = 173.0$ GeV	m_W=80.379 GeV	m_Z= 91.1876 GeV
$A = 0.836$	$\lambda = 0.22453$	$\bar{\rho} = 0.122$	$\bar{\eta} = 0.355$	

Table 3. The LO, NLO, NLO+SUSY and NLO+M331 results for the branching ratio of rare hyperon decays for $\mu_c = 1$ GeV, $\mu_t = 100$ GeV and $\mu_c = 3$ GeV, $\mu_t = 300$ GeV.

Branching ratio		$\mathcal{B}(\Lambda \to n\nu\bar{\nu})$	$\mathcal{B}(\Sigma^+ \to p\nu\bar{\nu})$	$\mathcal{B}(\Xi^0 \to \Lambda\nu\bar{\nu})$	$\mathcal{B}(\Xi^0 \to \Sigma^0\nu\bar{\nu})$	$\mathcal{B}(\Xi^- \to \Sigma^-\nu\bar{\nu})$	$\mathcal{B}(\Omega^- \to \Xi^-\nu\bar{\nu})$
$\mu_c = 1$ GeV	LO	2.85×10^{-12}	6.88×10^{-13}	1.06×10^{-12}	1.77×10^{-13}	2.17×10^{-13}	1.78×10^{-11}
	NLO	1.98×10^{-12}	5.01×10^{-13}	7.35×10^{-13}	1.24×10^{-13}	1.52×10^{-13}	1.93×10^{-11}
	NLO+SUSY (Set.I)	8.14×10^{-12}	2.06×10^{-12}	3.02×10^{-12}	5.08×10^{-13}	6.23×10^{-13}	7.94×10^{-11}
$\mu_t = 100$ GeV	NLO+SUSY (Set.II)	3.78×10^{-12}	9.55×10^{-13}	1.40×10^{-12}	2.36×10^{-13}	2.89×10^{-13}	3.69×10^{-11}
	NLO+M331	1.24×10^{-11}	3.13×10^{-12}	4.59×10^{-12}	7.71×10^{-13}	9.45×10^{-13}	1.20×10^{-10}
$\mu_c = 3$ GeV	LO	1.10×10^{-12}	2.65×10^{-13}	4.10×10^{-13}	6.83×10^{-14}	8.37×10^{-14}	1.07×10^{-11}
	NLO	1.20×10^{-12}	3.04×10^{-13}	4.46×10^{-13}	7.50×10^{-14}	9.19×10^{-14}	1.17×10^{-11}
	NLO+SUSY (Set.I)	5.85×10^{-12}	1.48×10^{-12}	2.17×10^{-12}	3.65×10^{-13}	4.47×10^{-13}	5.71×10^{-11}
$\mu_t = 300$ GeV	NLO+SUSY (Set.II)	2.35×10^{-12}	5.94×10^{-13}	8.72×10^{-13}	1.47×10^{-13}	1.80×10^{-13}	2.29×10^{-11}
	NLO+M331	1.02×10^{-11}	2.58×10^{-12}	3.80×10^{-12}	6.37×10^{-13}	7.81×10^{-13}	9.95×10^{-11}
BESIII sensitivity [12]		3×10^{-7}	4×10^{-7}	8×10^{-7}	9×10^{-7}	–	2.6×10^{-5}

$$\mathcal{B}(\Sigma^+ \to p\nu\bar{\nu})\left[O(m_s^0)\right] = (4.86\pm0.04)\times10^{-13},$$
$$\mathcal{B}(\Sigma^+ \to p\nu\bar{\nu})\left[O(m_s^0)+O(m_s^1)\right] = (5.01\pm0.08)\times10^{-13}. \quad (30)$$

Next, we consider the uncertainty of the branching ratio of $\Sigma^+ \to p\nu\bar{\nu}$ and $\Lambda \to n\nu\bar{\nu}$ in the case of NLO with the energy scale $\mu_c = 1$ GeV and $\mu_t = 100$ GeV. This uncertainty comes from the parameters $F = 0.463\pm0.008$ and $D = 0.804\pm0.008$ [18] in the form factor $g_1(0)$:

$$\mathcal{B}(\Sigma^+ \to p\nu\bar{\nu}) = (5.01\pm0.12)\times10^{-13},$$
$$\mathcal{B}(\Lambda \to n\nu\bar{\nu}) = (2.03\pm0.05)\times10^{-12}. \quad (31)$$

For the decay $\Omega^- \to \Xi^-\nu\bar{\nu}$, $C_5^A(0) = 1.653\pm0.006$ in the $SU(3)$ symmetry, while $C_5^A(0) = 1.612\pm0.007$ in the $SU(3)$ symmetry breaking [18]. In the case of NLO with the energy scale $\mu_c = 1$ GeV and $\mu_t = 100$ GeV the branching ratio $\mathcal{B}(\Omega^- \to \Xi^-\nu\bar{\nu})$ is then calculated as:

$$\mathcal{B}(\Omega^- \to \Xi^-\nu\bar{\nu})(sy) = (1.84\pm0.01)\times10^{-11},$$
$$\mathcal{B}(\Omega^- \to \Xi^-\nu\bar{\nu})(br) = (1.93\pm0.01)\times10^{-11}. \quad (32)$$

As an illustration of the effects of q^2 distribution in the form factors, we attempt to use the following parametrization for all form factors:

$$F(q^2) = \frac{F(0)}{1-\dfrac{q^2}{m^2}}, \quad (33)$$

with m representing the initial hyperon mass. For example, for the NLO case of $\mu_c = 1$ GeV and $\mu_t = 100$ GeV, we obtain:

$$\mathcal{B}(\Lambda \to n\nu\bar{\nu})(F(0)) = 1.98\times10^{-12},$$
$$\mathcal{B}(\Lambda \to n\nu\bar{\nu})(F(q^2)) = 2.03\times10^{-12},$$
$$\mathcal{B}(\Sigma^+ \to p\nu\bar{\nu})(F(0)) = 5.05\times10^{-13},$$
$$\mathcal{B}(\Sigma^+ \to p\nu\bar{\nu})(F(q^2)) = 5.16\times10^{-13}. \quad (34)$$

We find that the differences between the two cases are small, about a few percent.

When all the above errors in the form factors are included, we find that the final branching ratios for most decay modes have an uncertainty of about 5% to 10% .

3.3 Contribution from MSSM

The effective Hamiltonian for $s \to d\nu\bar{\nu}$ in the generalized supersymmetry (SUSY) extension of SM is given in Eq. (5), with $X(x_t)$ replaced by [26]

$$X_{\text{new}} = X(x_t)+X_H(x_{tH})+C_\chi+C_N. \quad (35)$$

Here, $x_{tH} = m_t^2/m_{H^\pm}^2$, and $X_H(x_{tH})$ corresponds to the charged Higgs contribution. C_χ and C_N denote the chargino and neutralino contributions

$$C_\chi = X_\chi^0 + X_\chi^{LL} R^U_{s_L d_L} + X_\chi^{LR} R^U_{s_L t_R} + X_\chi^{LR*} R^U_{t_R d_L},$$
$$C_N = X_N R^D_{s_L d_L},$$

where X_χ^i and X_N depend on the SUSY masses, and respectively on the chargino and neutralino mixing angles. The explicit expressions for $X_H(x)$, C_χ and C_N can be found in Ref. [26]. The R parameters are defined in terms of mass insertions, and their upper limits are listed in Table 4 [26]. It should be mentioned that the phase ϕ of $R^U_{s_L t_R}$ and $R^U_{t_R d_L}$ is a free parameter which ranges from 0 to 2π. We set $\phi = 0$ as a central result.

The parameters in Table 5 are adopted for detailed

Table 4. Upper limits for the R parameters. Note that the phase of $R^U_{s_L t_R}$ and $R^U_{t_R d_L}$ is unconstrained.

quantity	upper limit
$R^D_{s_L d_L}$	$(-112-55i)\dfrac{m_{\tilde{d}_L}}{500\text{ GeV}}$
$R^U_{s_L d_L}$	$(-112-54i)\dfrac{m_{\tilde{u}_L}}{500\text{ GeV}}$
$R^U_{s_L t_R}$	$\text{Min}\{231\left(\dfrac{m_{\tilde{u}_L}}{500\text{ GeV}}\right)^3, 43\}\times e^{i\phi}, 0<\phi<2\pi$
$R^U_{t_R d_L}$	$37\left(\dfrac{m_{\tilde{u}_L}}{500\text{ GeV}}\right)^2\times e^{i\phi}, 0<\phi<2\pi$

Table 5. Parameters and their ranges used in Ref. [27]. All mass parameters are in GeV.

parameters [27]	the meaning of parameters [27]	the range of parameters [27]	Set.I [27]	Set.II [27]
β	The angle of unitarity triangle	$-180° \leqslant \beta \leqslant 180°$	$\tan\beta = 2$	$\tan\beta = 20$
M_A	CP-odd Higgs boson mass	$150 \leqslant M_A \leqslant 400$	333	260
M_2	$SU(2)$ gaugino mass; we use M_1 GUT-related to M_2	$50 \leqslant M_2 \leqslant 800$	181	750
μ	Supersymmetric Higgs mixing parameter	$-400 \leqslant \mu \leqslant 400$	−375	−344
M_{sl}	Common flavour diagonal slepton mass parameter	$95 \leqslant M_{sl} \leqslant 1000$	105	884
M_{sq}	Common mass parameter for the first two generations of squarks	$240 \leqslant M_{sq} \leqslant 1000$	308	608
$M_{\tilde{t}_L}$	Squark mass parameter for the right stop	$50 \leqslant M_{\tilde{t}_R} \leqslant 1000$	279	338

calculations [27]. The assumption $M_1 \approx 0.5M_2$ was made [28]. With the above parameters, the branching ratios of hyperon decays are listed in Table 3, and are significantly enhanced compared with the SM results. Taking as examples the decays $\Lambda \to n\nu\bar{\nu}$ and $\Omega^- \to \Xi^-\nu\bar{\nu}$ with the energy scale μ_c = 1 GeV and μ_t = 100 GeV , we obtain:

$$\text{NLO}: \mathcal{B}(\Lambda \to n\nu\bar{\nu}) = 1.98\times10^{-12}, \quad \mathcal{B}(\Omega^- \to \Xi^-\nu\bar{\nu}) = 1.93\times10^{-11}, \tag{36}$$

$$\text{Set.I}: \mathcal{B}(\Lambda \to n\nu\bar{\nu}) = 8.14\times10^{-12}, \quad \mathcal{B}(\Omega^- \to \Xi^-\nu\bar{\nu}) = 7.94\times10^{-11}, \tag{37}$$

$$\text{Set.II}: \mathcal{B}(\Lambda \to n\nu\bar{\nu}) = 3.78\times10^{-12}, \quad \mathcal{B}(\Omega^- \to \Xi^-\nu\bar{\nu}) = 3.69\times10^{-11}. \tag{38}$$

Comparing the results of NLO+SUSY (Set. I) and (Set. II) with the ones of NLO, we see that all branching ratios are roughly enhanced by a factor of 4 and 2, respectively. However, none of these results can be probed at the ongoing experimental facilities, like the BESIII experiment [12].

3.4 Contribution from the minimal 331 model

The so-called minimal 331 model is an extension of SM at the TeV scale, where the weak gauge group of SM $SU(2)_L$ is extended to $SU(3)_L$. In this model, a new neutral Z' gauge boson can give very important additional contributions, for it can transmit FCNC at the tree level. In Table 3, we denote this model as M331. More details of this model can be found in Ref. [29]. The minimal 331 model leads to a new term in the effective Hamiltonian [30]:

$$\mathcal{H}^{Z'}_{\rm eff} = \sum_{l=e,\mu,\tau} \frac{G_F}{\sqrt{2}} \frac{\tilde{V}^*_{32}\tilde{V}_{31}}{3}\left(\frac{M_Z}{M_{Z'}}\right)^2 \cos^2\theta_W (\bar{s}d)_{V-A}(\bar{\nu}_l\nu_l)_{V-A} + \text{h.c.}, \tag{39}$$

with $M_{Z'}$ = 1 TeV, $\text{Re}[(\tilde{V}^*_{32}\tilde{V}_{31})^2] = 9.2\times10^{-6}$ and $\text{Im}[(\tilde{V}^*_{32}\tilde{V}_{31})^2] = 4.8\times10^{-8}$ [30]. The other parameters are the same as the SM inputs [23, 24]. The function $X(x_t)$ in Eq. (5) can be redefined as $X(x_t) = X^{\rm SM}(x_t) + \Delta X$ with

$$\Delta X = \frac{\sin^2\theta_W\cos^2\theta_W}{\alpha}\frac{2\pi}{3}\frac{\tilde{V}^*_{32}\tilde{V}_{31}}{V^*_{ts}V_{td}}\left(\frac{M_Z}{M_{Z'}}\right)^2. \tag{40}$$

With the modified function $X(x_t)$ and considering the NLO contribution, the branching ratios of rare hyperon decays in the minimal 331 model can be calculated, as shown in Table 3. The NLO+M331 predictions are much larger than the NLO results in SM, and are two and four times larger than the results of NLO+SUSY (Set. I) and NLO+SUSY (Set. II), respectively.

4 Conclusions

FCNC processes offer important tools to test SM and to search for possible new physics. The two decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ have been widely studied, while the corresponding baryon sector has not been explored. In this work, we studied the $s \to d\nu\bar{\nu}$ rare hyperon decays. We adopted the leading order approximations for the form factors for small q^2 , and derived expressions for the decay width. We applied the decay width formula to both SM and new physics contributions. Different energy scales were considered. The branching ratios in SM range from 10^{-14} to 10^{-11}, and the largest is of the same order as for the decays $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$. When all the errors in the form factors are included, we found that the final branching ratios for most decay modes have an uncertainty of about 5% to 10%. After taking into account the contribution from MSSM, the branching ratios are enhanced by a factor of $2\sim4$. The branching ratios of hyperon decays in the minimal 331 model are seven times larger than the SM results.

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