

# Primordial perturbations and non-gaussianities in Hořava-Lifshitz gravity\*

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**Abstract:** We investigate primordial perturbations and non-gaussianities in the Hořava-Lifshitz theory of gravitation. In the UV limit, the scalar perturbation in the Hořava theory is naturally scale-invariant, ignoring the details of the expansion of the Universe. One may thus relax the exponential inflation and the slow-roll conditions for the inflaton field. As a result, it is possible that the primordial non-gaussianities, which are “slow-roll suppressed” in the standard scenarios, become large. We calculate the non-gaussianities from the bispectrum of the perturbation and find that the equilateral-type non-gaussianity is of the order of unity, while the local-type non-gaussianity remains small, as in the usual single-field slow-roll inflation model in general relativity. Our result is a new constraint on Hořava-Lifshitz gravity.

**Keywords:** cosmological perturbation theory, primordial non-gaussianity, Horava gravity, inflation

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## 1 Introduction

A renormalizable theory of gravity was proposed by Hořava [1-3]. This theory reduces to Einstein's general relativity (GR) for large scales, and may be a candidate for the UV completion of general relativity. This theory is renormalizable in the sense that the effective coupling constant is dimensionless in UV. The essential point of this theory is the anisotropic scaling of temporal and spatial coordinates with dynamical critical exponent  $z$ ,

$$t \rightarrow \ell^z t, \quad x^i \rightarrow \ell x^i, \quad (z \geq 1). \quad (1)$$

In  $3+1$  spacetime dimension, the Hořava theory has an ultraviolet fixed point with  $z = 3$ . Since the Hořava theory is analogue to the scalar-field model studied by Lifshitz, in which the full Lorentz symmetry emerges only at the IR fixed point, the Hořava theory is also called the Hořava-Lifshitz theory. Because of this anisotropic scaling, time plays a privileged role in the Hořava theory. In other words, spacetime has a codimension-one foliation structure, which leaves the foliation hypersurfaces of constant time. Thus, contrary to GR, full diffeomorphism invariance is abandoned, and only a subset (of the form of local Galilean invariance) is kept. More precisely, the theory is invariant under the foliation-preserving diffeomorphism defined by

morphism defined by

$$t \rightarrow \tilde{t}(t), \quad x^i \rightarrow \tilde{x}^i(t, x^i). \quad (2)$$

In the infrared (IR), due to a deformation by lower dimensional operators, the theory flows to  $z = 1$ , corresponding to the standard relativistic scale invariance under which the full diffeomorphism, and thus general relativity, is recovered. Non-relativistic scaling allows for many non-trivial scaling theories in dimensions  $D > 2$ . The Hořava-Lifshitz theory allows a theory of gravitation that is scale-invariant in UV, while the standard GR with full diffeomorphism emerges at the IR fixed point.

The Hořava-Lifshitz gravity theory has been intensively investigated [4-18] (see also [19, 20] for reviews and more references therein). In particular, cosmology in the Hořava theory has been studied in [7-13]. Homogeneous vacuum solutions in this theory were obtained in [12], and scalar and tensor perturbations were studied in [8, 10]. In [7, 9], the cosmological evolution in Hořava gravity with scalar-field was extensively studied, and the matter bounce scenario in the Hořava theory was investigated by Brandenberger [11].

As was pointed out in [7], the Hořava theory has at least two important properties. The first is its UV renormalizability, while the second is more interesting for cosmology. The fact that the speed of light is diverging in

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UV implies that exponential inflation is not necessary for solving the horizon problem. Moreover, the short distance structure of perturbations in the Hořava-Lifshitz theory is different from the standard inflation in GR. In particular, in the UV limit, the scalar field perturbation is essentially scale-invariant and is insensitive to the expansion rate of the Universe, as has been addressed in [7-10]. The key point is that the UV renormalizability indicates that the Lagrangian for the non-relativistic scalar field should contain up to six spatial derivatives. Thus, in the UV limit, the dispersion relation is  $\omega^2 \sim k^6$ , which is contrary to  $\omega^2 \sim k^2$  in standard GR. This phenomenon causes different  $k$ -dependence of the two-point correlation function, and thus the scalar perturbation in Hořava gravity is naturally scale-invariant in the UV limit.

In this paper, we extend the previous works on cosmological perturbation theory in Hořava gravity, including non-gaussianities. The 6-parameter  $\Lambda$ CDM model provides an accurate description of the Universe [21]. In particular, the primordial perturbations are assumed to be Gaussian. Deviation from the Gaussian distribution, i.e., primordial non-gaussianity, has not been observed. This puts a strict constraint on any model of the early Universe. The non-Gaussian features of Hořava gravity have not been studied in detail, except in [22] and in [23], which investigated the non-gaussianities of the scalar field and of the gravitational waves in Hořava gravity, respectively. Actually, one of the essential differences of Hořava gravity from Einstein's general relativity is that it contains quadratic curvature terms in the theory. Moreover, the foliation-preserving diffeomorphism does not allow, unfortunately, to choose a spatially-flat gauge as in GR. Thus, in general, the perturbation theory in Hořava gravity is quite involved. On the other hand, the number of dynamical degrees of freedom in the spatial metric is 3 in Hořava gravity (contrary to 2 in GR), with 2 tensor degrees of freedom, as usual, and an additional scalar dynamical degree of freedom (see also [24-26] for the general framework of spatially covariant theories of gravity).

In this work, we focus on the perturbation of the scalar field. Thus, for simplicity, we neglect the spatial metric perturbation. We introduce a scalar field, following the strategy in [7, 9]. We pay special attention to the non-gaussianities in this scalar field model in Hořava gravity. The basic idea is that, as has been addressed before, the divergence of the speed of light and the scale-invariance of the scalar perturbation in Hořava gravity indicate that there is no need to assume an exponential expansion of the Universe. Moreover, the traditional slow-roll conditions are not necessary. While it is well-known that in slow-roll inflationary models non-gaussianity is suppressed by slow-roll parameters [27], and thus too small to be detected (see e.g. [28] for a review of non-gaussian-

ities in cosmological perturbations. Various models have been investigated to generate large non-gaussianities by introducing more complicated kinetic terms [29-32] or more fields [33-38]). However, in Hořava gravity, there are no slow-roll conditions, and thus the “slow-roll suppressed” non-gaussianities can become large. In this work, we focus on the non-gaussianity from the bispectrum, which is defined from the three-point correlation function of the perturbation. We find that the equilateral-type non-gaussianity is roughly of the order of unity, while the local-type non-gaussianity remains very small, as in the usual single-field slow-roll inflation in GR.

The paper is organized as follows. In Section 2, we briefly review the Hořava gravity and set out our conventions. In Section 3, we couple the scalar field to Hořava gravity, and describe the cosmological evolution of the Hořava gravity/scalar matter system. In Section 4, we calculate the scalar field perturbation, including the gravity perturbations. We get the full second-order perturbation action, which reduces to the standard result in the IR limit. In the UV limit, the scalar perturbation is essentially scale-invariant. In Section 5, we calculate the non-gaussianities. Finally, we make a conclusion and discuss several related issues.

## 2 Brief review of Hořava-Lifshitz gravity

In this section we briefly review the Hořava-Lifshitz theory [1]. The dynamical variables in Hořava-Lifshitz gravity are the spatial scalar  $N$ , spatial vector  $N_i$  and spatial metric  $g_{ij}$ . This is similar to the ADM formalism of the metric in standard general relativity, while in Hořava gravity,  $N$ ,  $N_i$  and  $g_{ij}$  are related to the space-time metric as

$$ds^2 = -N^2 c^2 dt^2 + g_{ij} (dx^i + N^i dt) (dx^j + N^j dt), \quad (3)$$

where  $c$  is the speed of light.

The action of Hořava-Lifshitz gravity contains a "kinetic" part and a "potential" part,

$$S = S_K + S_V, \quad (4)$$

with

$$S_K = \frac{2}{\kappa^2} \int dt d^3x \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2), \quad (5)$$

where

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i),$$

is the extrinsic curvature and  $K = g^{ij} K_{ij}$ . The potential terms are given in the “detailed-balance” form

$$S_V = \int dt d^3x \sqrt{g} N \left[ -\frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R_{il} \nabla_j R_k^l - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left( \frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) \right], \quad (6)$$

where  $C_{ij}$  is the Cotton tensor defined by

$$C^{ij} = \frac{\epsilon^{ikl}}{\sqrt{g}} \nabla_k \left( R_l^j - \frac{1}{4} R \delta_l^j \right). \quad (7)$$

Note that in (5)  $\lambda$  is a dimensionless coupling of the theory, and therefore runs.

As mentioned in the introduction, the essential point of the Hořava theory is the anisotropic scaling of temporal and spatial coordinates:  $t \rightarrow \ell^z t$  and  $x^i \rightarrow \ell x^i$ . The classical scaling dimensions of various quantities in the Hořava theory are summarized in Table 1.

Table 1. Summary of the classical scaling dimensions of various quantities in the Hořava-Lifshitz theory.

	$[t]$	$[x^i]$	$[c]$	$[\kappa]$	$[N]$	$[N_i]$	$[g_{ij}]$	$[\Delta]$	$[\partial_t]$	$[\partial_i]$	$[\phi]$
dimension	$-z$	$-1$	$z-1$	$\frac{z-3}{2}$	$0$	$z-1$	$0$	$2$	$z$	$1$	$\frac{3-z}{2}$

### 3 Cosmology with scalar field matter

#### 3.1 Coupling the scalar field to Hořava-Lifshitz gravity

In this work, we couple the scalar-field matter with Hořava-Lifshitz theory following the strategy in [7, 9]. The general structure of the action of the scalar-field matter and Hořava-Lifshitz gravity contains two parts: a quadratic kinetic term with the foliation-preserving diffeomorphisms and a potential term:

$$S^\phi = \int dt d^3x \sqrt{g} N \left[ \frac{1}{2N^2} (\dot{\phi} - N^i \partial_i \phi)^2 + F(\phi, \partial_i \phi, g_{ij}) \right]. \quad (8)$$

The ‘‘potential term’’ is

$$F = -V(\phi) + g_1 \xi_1 + g_{11} \xi_1^2 + g_{111} \xi_1^3 + g_2 \xi_2 + g_{12} \xi_1 \xi_2 + g_3 \xi_3, \quad (9)$$

where  $\xi_i$  and their properties in UV/IR are summarized in Table 2. In Table 2,  $\Delta = g^{ij} \nabla_i \nabla_j$  is the spatial Laplacian, and  $g_1, g_{11}$  etc. can be constant, or in general it can be functions of  $\phi$ . We assume  $g_3 > 0$  in order to guarantee the stability of the perturbation in UV.

In the UV limit ( $z=3$  fixed point),  $\xi_1^3, \xi_1 \xi_2$  and  $\xi_3$  dominate. Thus, in UV the scalar field action takes the form

$$S_{UV}^\phi = \int dt d^3x a^3 N \left[ \frac{1}{2N^2} \dot{\phi}^2 + g_{111} (\partial^i \phi \partial_i \phi)^3 + g_{12} (\partial^i \phi \partial_i \phi) (\Delta \phi)^2 + g_3 (\Delta \phi) (\Delta^2 \phi) \right]. \quad (10)$$

Table 2. Summary of the operators in the non-relativistic scalar field action and their properties under the renormalization group flow from  $z=3$  (UV) to  $z=1$  (IR).

	$O$	scaling dim $[O]$	$z=3$ , UV fixed point	$z=1$ , IR fixed point
	$\dot{\phi}^2$	$z+3$	marginal	marginal
$\xi_1$	$\partial^i \phi \partial_i \phi$	$5-z$	relevant	marginal
$\xi_1^2$	$(\partial^i \phi \partial_i \phi)^2$	$10-2z$	relevant	irrelevant
$\xi_1^3$	$(\partial^i \phi \partial_i \phi)^3$	$15-3z$	marginal	irrelevant
$\xi_2$	$(\Delta \phi)^2$	$7-z$	relevant	irrelevant
$\xi_1 \xi_2$	$(\partial^i \phi \partial_i \phi) (\Delta \phi)^2$	$12-2z$	marginal	irrelevant
$\xi_3$	$(\Delta \phi) (\Delta^2 \phi)$	$9-z$	marginal	irrelevant

#### 3.2 Equations of motion

The full equations of motion for  $N, N_i$  and  $g_{ij}$  have been derived in [7, 9, 12]. For our purpose we focus on the equations of motion for  $N$  and  $N_i$ , which we write here for later convenience :

$$0 = -\frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) - \frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R_{il} \nabla_j R_k^l - \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left( \frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right) - \frac{1}{2N^2} (\dot{\phi} - N^i \partial_i \phi)^2 + F, \quad (11)$$

$$0 = \frac{4}{\kappa^2} \nabla_j (K_i^j - \lambda K \delta_i^j) - \frac{1}{N} (\dot{\phi} - N^i \partial_i \phi) \partial_i \phi.$$

#### 3.3 Cosmological evolution

We now consider the cosmological background evolution in Hořava-Lifshitz gravity. We assume the background to be homogeneous and isotropic, and use the residual invariance under time re-parametrization to set  $N=1$ . We focus on the flat 3-dimensional case. The background values are

$$N=1, \quad N_i=0, \quad g_{ij} = a^2(t) \delta_{ij}, \quad \phi_0 = \phi_0(t). \quad (12)$$

In this background, the action in the gravity sector is significantly simplified. In particular,  $R_{ij} = C_{ij} = 0$  and the spatial covariant derivatives mostly vanish.

The equation of motion for  $N$  to the 0-th order gives

$$3(3\lambda - 1)H^2\alpha + \sigma - \frac{\dot{\phi}_0^2}{2} - V(\phi_0) = 0, \quad (13)$$

where we have denoted

$$\alpha = \frac{2}{\kappa^2}, \quad \sigma = -\frac{3\kappa^2\mu^2\Lambda^3}{8(1-3\lambda)}, \quad (14)$$

and  $H \equiv \dot{a}/a$  is the familiar Hubble parameter. The equation of motion for  $g_{ij}$  gives

$$2(3\lambda - 1)\alpha(\dot{H} + 3H^2/2) + \frac{\dot{\phi}_0^2}{2} - V(\phi_0) = 0. \quad (15)$$

The equation of motion for the scalar field is

$$\ddot{\phi}_0 + 3H\dot{\phi}_0 + \frac{V'_0}{2} = 0. \quad (16)$$

## 4 Cosmological perturbation

We now consider cosmological perturbation in Hořava gravity coupled to scalar-field matter.

As has been addressed before, the action of Hořava gravity is complicated due to the quadratic terms in spatial curvature. We recall that in GR one can choose various gauges to simplify the calculations. However, the case is different in Hořava gravity (see Appendix A for a discussion of gauge transformation and gauge choice in Hořava theory). Since the ‘‘foliation-preserving’’ diffeomorphism is only a subset of the full diffeomorphism in GR, one may expect in general that there are less gauge modes and more physical modes<sup>1)</sup>. Thus, the perturbation theory in Hořava-Lifshitz theory is very involved, but also interesting.

We consider the scalar-field perturbation by neglecting the spatial metric perturbation. We assume that the background scalar field is homogeneous  $\phi_0 = \phi_0(t)$ .

As we focus on the scalar perturbation, we write (in the background  $N = 1$  gauge)<sup>2)</sup>

$$\begin{aligned} N &\equiv 1 + \alpha_1 + \dots, \\ N_i &\equiv \partial_i \beta_1 + \theta_{1i} + \dots. \end{aligned} \quad (17)$$

Here, the subscript ‘‘1’’ denotes the first-order in  $\delta\phi \equiv \delta\phi$ . The constraints in Eq. (11) become

$$\begin{aligned} 0 &= -\frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2) + \sigma - \frac{1}{2N^2}(\dot{\phi} - N^i\partial_i\phi)^2 + F, \\ 0 &= \frac{4}{\kappa^2}\nabla_j(K_i^j - \lambda K\delta_i^j) - \frac{1}{N}(\dot{\phi} - N^i\partial_i\phi)\partial_i\phi. \end{aligned} \quad (18)$$

Solving Eq. (18) up to the first-order of  $\delta\phi$ , we get<sup>3)</sup>

$$\begin{aligned} \alpha_1 &= \frac{(-1 + \lambda)\dot{Q}\dot{\phi}_0 + Q(H(-1 + 3\lambda)\dot{\phi}_0 + (-1 + \lambda)V')}{4H^2\alpha(-1 + 3\lambda) + (-1 + \lambda)\dot{\phi}_0^2}, \\ \partial^2\beta_1 &= a^2 \\ &\times \frac{2H\alpha(1-3\lambda)\dot{Q}\dot{\phi}_0 + Q(6H^2\alpha(1-3\lambda)\dot{\phi}_0 + \dot{\phi}_0^3 + 2H\alpha(1-3\lambda)V')}{2\alpha(4H^2\alpha(-1 + 3\lambda) + (-1 + \lambda)\dot{\phi}_0^2)}, \end{aligned} \quad (19)$$

and  $\theta_{1i} = 0$ , as usual in GR. Note that when  $\lambda \neq 1$ ,  $\alpha_1$  also depends on  $\dot{Q}$ , which is different from GR, where  $\alpha_1 \sim \delta\phi$ . It is useful to note that in the UV limit  $\alpha_1$  and  $\beta_1$  become

$$\begin{aligned} \alpha_1 &\simeq \frac{\sqrt{3}(\lambda - 1)\dot{Q}}{H\sqrt{2\alpha(3\lambda - 1)^{3/2}}}, \\ \partial^2\beta_1 &\simeq -\frac{\sqrt{3}a^2\dot{Q}}{\sqrt{2\alpha(3\lambda - 1)}}, \end{aligned} \quad (20)$$

where we have used the background equations of motion in the UV limit.

### 4.1 Linear perturbation

As we are neglecting the spatial metric perturbation, the gravity sector is greatly simplified

$$S^g = \int dt d^3x a^3 N [\alpha(K_{ij}K^{ij} - \lambda K^2) + \sigma]. \quad (21)$$

After a rather tedious but straightforward calculation, we get the quadratic part of the action for the scalar perturbation  $Q$ :

$$\begin{aligned} S_2[Q] &= \int dt d^3x a^3 \left[ \frac{\gamma}{2}\dot{Q}^2 + \omega\dot{Q}Q + mQ^2 + g_1\partial^i Q\partial_i Q \right. \\ &\quad \left. + g_2(\Delta Q)^2 + g_3(\Delta Q)(\Delta^2 Q) \right], \end{aligned} \quad (22)$$

where

$$\begin{aligned} \gamma &= \frac{H^2\alpha(1-3\lambda)^2 [H^2\alpha(7+3\lambda(3\lambda-4)) - (\lambda-1)(V_0-\sigma)]}{[H^2\alpha(1-3\lambda)^2 - (\lambda-1)(V_0-\sigma)]^2}, \\ \omega &= -\frac{2H^2\alpha(3\lambda-1) - (\lambda-1)(V_0-\sigma)}{2[H^2\alpha(1-3\lambda)^2 - (\lambda-1)(V_0-\sigma)]^2} \\ &\quad \times [6H^3\alpha(1-3\lambda)^2 + H(2-6\lambda)(V_0-\sigma) \\ &\quad + (\lambda-1)\sqrt{6H^2\alpha(-1+3\lambda)-2(V_0-\sigma)V'}], \end{aligned}$$

1) As argued in the original proposal by Hořava [1], the dynamical degrees of freedom in the spatial metric  $g_{ij}$  is 3 (with contrary to 2 in GR) when  $\lambda \neq 1$  and  $1/3$ , which contains 2 usual tensor degrees of freedom and one additional scalar degree of freedom (see also similar arguments based on cosmological context and more general analysis in).

2) In Hořava's original formulation of the theory,  $N$  was assumed to be a function of time only,  $N = N(t)$ . Here in this work, we relax this restriction to assume  $N$  to be function of both temporal and spatial coordinates.

3) At this point, it is useful to compare (19) with the standard results in perturbation theory in GR. It can be seen directly that in the IR fixed point where  $z = 1$ , if we choose  $\lambda = 1$ , (19) reduces to the previous well-known results in GR.

$$\begin{aligned}
 m = & \frac{1}{4\alpha(H^2\alpha(1-3\lambda)^2 - (\lambda-1)(V_0-\sigma))^2} \left\{ 18H^6\alpha^3(1-3\lambda)^4 + (2-2\lambda)(V_0-\sigma)^3 - 2H^2\alpha^2(\lambda-1)(3\lambda-1)(V')^2 \right. \\
 & + \sqrt{6H^2\alpha(-1+3\lambda)-2(V_0-\sigma)}(-4H^3\alpha^2(1-3\lambda)^2V' + 2H\alpha(\lambda-1)(3\lambda-1)(V_0-\sigma)V') \\
 & - 2H^4\alpha^3(1-3\lambda)^4V'' + (V_0-\sigma)(-12H^4\alpha^2(3\lambda-1)^3 + \alpha(\lambda-1)^2(V')^2 + 4H^2\alpha^2(1-3\lambda)^2(\lambda-1)V'') \\
 & \left. + (V_0-\sigma)^2(4H^2\alpha(2+9(-1+\lambda)\lambda) - 2\alpha(\lambda-1)^2V'') \right\}. \quad (23)
 \end{aligned}$$

In (22),  $g_i \equiv g_i(\phi_0)$ . In deriving Eq. (22), we have used the background equations of motion. Moreover, no approximations were made in deriving Eq. (22) and thus it is exact. We can use Eq. (22) to analyze the behavior of perturbations both in the IR or UV limits and in the interpolation era.

#### 4.1.1 IR limit

Taking the IR limit of the full second-order perturbation action Eq. (22) and choosing  $\lambda = 1$ , we get

$$\begin{aligned}
 S_2^{\text{IR}} = & \int dt d^3x a^3 \left[ \frac{1}{2} \dot{Q}^2 + g_1 \partial^i Q \partial_i Q - \frac{H\epsilon}{2\alpha} \dot{Q} Q \right. \\
 & \left. + H^2 \left( \frac{\epsilon^2}{2\alpha^2} - \frac{\sqrt{\epsilon\eta_1}}{2\alpha} - \frac{\eta_2}{2} \right) Q^2 \right], \quad (24)
 \end{aligned}$$

where we have defined the dimensionless parameters

$$\epsilon = \frac{\dot{\phi}^2}{2H^2}, \quad \eta_1 = \frac{1}{2} \left( \frac{V'}{H^2} \right)^2, \quad \eta_2 = \frac{V''}{H^2}. \quad (25)$$

If we further set  $\alpha = \frac{1}{2}$  and choose  $g_1 = -\frac{1}{2}$ , Eq. (24) reduces to the familiar result in the perturbation theory in GR. Especially, the perturbation  $Q$  is scale-invariant when the expansion of the Universe is exponential, and thus with an approximately constant Hubble parameter  $H \approx \text{const.}$

## 4.2 Scale-invariant spectrum in Hořava-Lifshitz era

We now focus on the behavior of the perturbation theory in the UV-limit, where  $\dot{Q}^2$  and  $Q\Delta^3Q$  terms dominate. The perturbation action Eq. (22) becomes rather simple in the UV limit,

$$S_2[Q] = \int dt d^3x a^3 \left( \frac{\gamma}{2} \dot{Q}^2 + g_3 \Delta Q \Delta^2 Q \right), \quad (26)$$

with

$$\gamma = \frac{7+3\lambda(3\lambda-4)}{(3\lambda-1)^2}. \quad (27)$$

Note that  $\gamma$  is now constant. It is convenient to use a new variable  $u$ , defined as  $u \equiv a\sqrt{\gamma}Q$ . After changing into conformal time  $\eta$ , defined by  $dt = a d\eta$ , and going into Fourier space, the second-order perturbation action reads

$$S_2 = \int d\eta \frac{d^3k}{(2\pi)^3} \left[ \frac{1}{2} (u'_k - \mathcal{H}u_k)(u'_{-k} - \mathcal{H}u_{-k}) + \frac{g_3 k^6}{\gamma \alpha^4} u_k u_{-k} \right]. \quad (28)$$

The equation of motion for the perturbation reads

$$u''_k + \left( \frac{g_3 k^6}{\gamma \alpha^4} - \frac{a''}{a} \right) u_k = 0. \quad (29)$$

Here, we assume for simplicity that  $g_3$  is approximately constant. The mode function is

$$u_k(\eta) = \frac{(\gamma/g_3)^{\frac{1}{4}}}{\sqrt{2k^3}} a(\eta) \exp\left(-i \sqrt{\frac{g_3}{\gamma}} k^3 \int^\eta \frac{d\eta'}{a^2(\eta')}\right). \quad (30)$$

The mode function is chosen such that it satisfies the Wronskian normalization condition:

$$u'_k(\eta)u_k^*(\eta) - u_k^*(\eta)u'_k(\eta) = -i. \quad (31)$$

Moreover, the short-time behavior of the mode function Eq. (30) is analogue to that of a positive-frequency oscillator.

The tree-level two-point correlation function of  $Q(\mathbf{k}, \eta)$  is

$$\begin{aligned}
 \langle Q(\mathbf{k}_1, \eta_1) Q(\mathbf{k}_2, \eta_2) \rangle = & (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) \frac{1}{\sqrt{\gamma g_3} 2k_1^3} \\
 & \times \exp\left(-i \sqrt{\frac{g_3}{\gamma}} k_1^3 \int_{\eta_2}^{\eta_1} \frac{d\eta'}{a^2(\eta')}\right), \quad (32)
 \end{aligned}$$

and thus the power spectrum of  $Q$  is given by

$$\langle Q(\mathbf{k}_1, \eta_*) Q(\mathbf{k}_2, \eta_*) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2) P(k_1), \quad (33)$$

with

$$P(k_1) = \frac{1}{\sqrt{\gamma g_3} 2k_1^3}. \quad (34)$$

The so-called dimensionless power spectrum of  $Q$  is

$$\Delta^2(k) \equiv \frac{k^3}{2\pi^2} P(k) = \frac{1}{(2\pi)^2} \frac{1}{\sqrt{\gamma g_3}} \equiv \text{const.} \quad (35)$$

The power spectrum of the scalar perturbation is naturally scale-invariant in the UV limit, ignoring the details of the expansion of the Universe. This feature is contrary to that in GR, where a nearly constant Hubble expansion rate  $H$  is needed to guarantee the scale-invariance of the spectrum. Due to this fact, there is no need to take any ‘‘slow-roll’’-type conditions for the scalar field.

We would like to make some comments here. The crucial picture of the standard inflation is that quantum fluctuations are generated in the subhorizon region ( $k \gg aH$ ), and are then stretched to the cosmological size and become classical ( $k \ll aH$ ). The horizon-exiting point

corresponds to  $k = aH$ . Thus, the ‘‘horizon-exiting’’ process exists only when  $aH$  is an increasing function of time. If we assume a power law inflation  $a \propto t^p$ , it requires  $p > 1$ . This is violated by the curvature, and is only satisfied when the equation of state  $w < -1/3$ . This is why in the standard inflation model we need a slow-rolling scalar field to mimic the cosmological constant and to drive an exponentially expanding background. However, the ‘‘horizon-exiting’’ process occurs generically in the Hořava-Lifshitz era for rather general cosmological backgrounds. From Eq. (29), it is obvious that the perturbation stops oscillating when

$$\frac{k^6}{a^6} \sim H^2,$$

and thus requires that  $a^6 H^2$  is an increasing function of time. Obviously, if we assume a power law expansion  $a \propto t^p$ , this demands  $p > 1/3$ . In terms of the comoving time  $a \propto |\eta|^p$ , we need  $p < 1/2$ . This condition can be satisfied by any matter component with the equation of state  $w < 1$ .

We get the scale-invariant power spectrum in the UV limit from the equation of motion Eq. (29). While one may consider the full equation of motion, the second-order action Eq. (22) can be used. In general, the equation of motion for the perturbation has the following functional form:

$$u_k'' + \left( c_1^2 k^2 + \lambda_1 \frac{\ell^2 k^4}{a^2} + \lambda_2 \frac{\ell^4 k^6}{a^4} - \frac{a''}{a} + m^2 a^2 \right) u_k = 0, \quad (36)$$

where  $c_1$ ,  $\lambda_1$  and  $\lambda_2$  are dimensionless parameters,  $m$  is the effective mass parameter, and  $\ell$  is the length scale of the whole theory. The functional form of the dispersion relation in (36) has been intensively studied in the investigation of trans-Planckian effects [39-42], and also in statistical anisotropy [43]. Although complicated, it is interesting and important to investigate Eq. (36) in order to understand the behavior of the perturbation not only in the UV limit, but also in the interpolation region between UV and IR.

## 5 Non-gaussianities

In this section, we investigate the non-gaussianities, which characterize the interaction of the perturbations.

### 5.1 Bispectrum

We focus on the third-order perturbation action and the three-point correlation function of the perturbation  $Q$ . The third-order action in the gravity sector is

$$S_3^g = \int dt d^3x a^3 \alpha \left[ -\frac{\alpha_1}{a^4} (\partial_i \partial_j \beta_1 \partial_i \partial_j \beta_1 - \lambda (\partial^2 \beta_1)^2) - 2(1-3\lambda) \frac{H}{a^2} \alpha_1^2 \partial^2 \beta_1 - 3(1-3\lambda) H^2 \alpha_1^3 \right], \quad (37)$$

and in the scalar field sector is

$$S_3^\phi = \int dt d^3x a^3 \left\{ \frac{1}{2} [-2\dot{Q} \partial^i \beta_1 \partial_i Q - \alpha_1 (\dot{Q}^2 - 2\dot{\phi} \partial^i \beta_1 \partial_i Q) + 2\dot{\phi} \dot{Q} \alpha_1^2 - \dot{\phi}^2 \alpha_1^3] - \frac{V''''}{6} Q^3 + g'_1 Q \partial^i Q \partial_i Q + g'_2 Q (\Delta Q)^2 + g'_3 Q (\Delta Q) (\Delta^2 Q) + \alpha_1 \left( -\frac{V''}{2} Q^2 + g_1 \partial^i Q \partial_i Q + g_2 (\Delta Q)^2 + g_3 (\Delta Q) (\Delta^2 Q) \right) \right\}, \quad (38)$$

where  $\alpha_1$  and  $\beta_1$  are given in Eq. (19). We are interested in the UV behavior of the perturbation. In the UV limit, after a straightforward calculation, the third-order perturbation action reads,

$$S_3^{\text{UV}}[Q] = \int dt d^3x \frac{a^3}{H} \left[ b_1 \dot{Q}^3 + b_2 \dot{Q} \Delta Q \Delta^2 Q + b_3 \dot{Q} \left( \frac{\partial_i \partial_j}{\partial^2} \dot{Q} \right)^2 \right]. \quad (39)$$

where

$$b_1 = \frac{\sqrt{\frac{3}{2}} (\lambda-1) (8+3\lambda(3\lambda-5))}{2\sqrt{\alpha} (3\lambda-1)^{7/2}},$$

$$b_2 = \frac{\sqrt{\frac{3}{2}} (\lambda-1) g_3}{\sqrt{\alpha} (3\lambda-1)^{3/2}},$$

$$b_3 = -\frac{3\sqrt{3} (\lambda-1)}{2\sqrt{2\alpha} (3\lambda-1)^{5/2}}, \quad (40)$$

which are dimensionless constants (recall that we assume  $g_3$  to be approximately constant). In Eq. (39), the formal operator  $\frac{\partial_i \partial_j}{\partial^2}$  should be understood in momentum space.

After changing into comoving time  $\eta$  and into Fourier space, we have

$$S_3^{\text{UV}} = \int d\eta \prod_{i=1}^3 \frac{d^3 k_i}{(2\pi)^3} (2\pi)^3 \delta^3(\mathbf{k}_{123}) \times \left[ \frac{a}{H} (b_1 + b_3 (\hat{\mathbf{k}}_2 \cdot \hat{\mathbf{k}}_3)^2) Q'(\mathbf{k}_1, \eta) Q'(\mathbf{k}_2, \eta) Q'(\mathbf{k}_3, \eta) - \frac{b_2}{a^3 H} k_2^2 k_3^4 Q'(\mathbf{k}_1, \eta) Q(\mathbf{k}_2, \eta) Q(\mathbf{k}_3, \eta) \right], \quad (41)$$

where we denote  $\mathbf{k}_{123} \equiv \mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3$ .

The three-point correlation function in the cosmological context is evaluated in the so-called ‘‘in-in’’ formalism



$$\langle Q(\mathbf{k}_1, \eta_*) Q(\mathbf{k}_2, \eta_*) Q(\mathbf{k}_3, \eta_*) \rangle = -2 \operatorname{Re} \left[ i \int_{-\infty}^{\eta_*} d\eta' \langle Q(\mathbf{k}_1, \eta_*) Q(\mathbf{k}_2, \eta_*) Q(\mathbf{k}_3, \eta_*) H(\eta') \rangle \right], \quad (42)$$

where  $\eta_*$  is the time when perturbation modes exit the sound horizon, and  $H$  is the Hamiltonian which can be read from Eq. (41) by noting that in the third-order  $H_{(3)} = -L_{(3)}$ . Thus, for three-point interactions described by Eq. (41), we have

$$\langle Q(\mathbf{k}_1, \eta_*) Q(\mathbf{k}_2, \eta_*) Q(\mathbf{k}_3, \eta_*) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \operatorname{Re} \left[ \int_{-\infty}^{\eta_*} d\eta \frac{1}{a^5(\eta) H} e^{-i \sqrt{\frac{25}{\gamma} (k_1^2 + k_2^2 + k_3^2)} \int_{\eta}^{\eta_*} \frac{d\eta'}{a(\eta')}} \right] \mathcal{S}(k_1, k_2, k_3), \quad (43)$$

where we have introduced the ‘‘shape factor’’  $\mathcal{S}(k_1, k_2, k_3)$  defined as

$$\mathcal{S}(k_1, k_2, k_3) \equiv \frac{3b_1}{2\gamma^3} + \frac{b_3}{2\gamma^3} [(\hat{\mathbf{k}}_1 \cdot \hat{\mathbf{k}}_2)^2 + (\hat{\mathbf{k}}_2 \cdot \hat{\mathbf{k}}_3)^2 + (\hat{\mathbf{k}}_3 \cdot \hat{\mathbf{k}}_1)^2] + \frac{b_2}{4\gamma^2 g_3} \left[ \frac{k_1^2(k_2 + k_3) + k_2^2(k_3 + k_1) + k_3^2(k_1 + k_2)}{k_1 k_2 k_3} \right]. \quad (44)$$

For power law expansion  $a(\eta) \propto |\eta|^p$  ( $p \neq 0$ ), the time-integral in Eq. (43) can be evaluated exactly, and the three-point correlation function reads,

$$\langle Q(\mathbf{k}_1, \eta_*) Q(\mathbf{k}_2, \eta_*) Q(\mathbf{k}_3, \eta_*) \rangle = (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times B(k_1, k_2, k_3), \quad (45)$$

with

$$B(k_1, k_2, k_3) = \frac{1-2p}{p} \frac{\gamma \mathcal{S}(k_1, k_2, k_3)}{g_3 (k_1^3 + k_2^3 + k_3^3)^2}, \quad (46)$$

which is the so-called bispectrum.

## 5.2 Non-linear parameter $f_{\text{NL}}$

In practice, it is convenient to introduce non-linear parameters to characterize the non-gaussianities. The dimensionless non-linear parameter  $f_{\text{NL}}$  from the three-point correlation function is defined as

$$B(k_1, k_2, k_3) \equiv \frac{6}{5} f_{\text{NL}}(k_1, k_2, k_3) [P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)], \quad (47)$$

where the power spectrum  $P(k)$  is given in Eq. (33). Note that although dimensionless,  $f_{\text{NL}}$  is in general  $k$  dependent. A straightforward calculation gives

$$f_{\text{NL}}(k_1, k_2, k_3) = \frac{10}{3} \frac{\gamma^2 (1-2p)}{p} \frac{k_1^3 k_2^3 k_3^3}{(k_1^3 + k_2^3 + k_3^3)^3} \mathcal{S}(k_1, k_2, k_3). \quad (48)$$

In the equilateral limit ( $k_1 \approx k_2 \approx k_3$ ),

$$f_{\text{NL}}^{\text{equil}} \approx \frac{5(1-2p)(\lambda-1)(91+3\lambda(-55+36\lambda))}{72p \sqrt{6\alpha}(3\lambda-1)^{3/2}(7+3\lambda(-4+3\lambda))} \sim \mathcal{O}(1), \quad (49)$$

while in the squeezed limit ( $k_1 \ll k_2 \approx k_3$ ),

$$f_{\text{NL}}^{\text{local}} \approx \frac{5(1-2p)(\lambda-1)}{8p \sqrt{6\alpha}(3\lambda-1)^{3/2}} \left( \frac{k_1}{k_2} \right)^2 \ll 1. \quad (50)$$

Thus, in the UV limit, we find that the equilateral-type non-gaussianity is roughly  $\sim \mathcal{O}(1)$ , while the local-type non-gaussianity is very small. This is not surprising, since in Hořava-Lifshitz gravity the scalar-field perturbation in the UV limit is naturally scale-invariant. Thus, no slow-roll condition is needed to guarantee the exponential expansion of the Universe. On the other hand, it is well

known that in standard slow-roll inflationary models in GR non-gaussianities are suppressed by slow-roll parameters [27]. However, in Hořava gravity, no slow-roll parameters are needed. Thus, one may expect the slow-roll suppressed non-gaussianity to be of the order of unity. However, in our simplest scalar-field model with the action given by Eq. (8) and Eq. (9), the temporal kinetic term is canonical, and in general there is no enhancement of the non-gaussianity by non-canonical kinetic terms as in the K-inflation or DBI-inflation models [29-32]. Moreover, the scalar-field action Eq. (8) is mostly ‘‘derivative-coupled’’, thus the local-type non-gaussianity (which characterizes the local couplings of the perturbations in the real space) is small, as expected.

## 6 Conclusion

In this work, we investigated the cosmological perturbation theory in Hořava-Lifshitz gravity and the non-gaussianities from the bispectrum. The most interesting feature of Hořava gravity is that in the UV limit the scalar perturbation is essentially scale-invariant, ignoring the details of the expansion of the Universe. Moreover, together with the fact that the speed of light in the UV limit diverges, there is no need to assume exponential expansion of the early Universe, or the usual scalar-field driven slow-roll inflation. In particular, the slow-roll conditions are not necessary. Thus, one may expect that in the absence of slow-roll conditions, the non-gaussianities can become large. We calculated the three-point correlation function of the scalar perturbation and found that the equilateral-type non-gaussianities are of the order of unity due to the absence of the slow-roll-type conditions, while the local-type non-gaussianities remain small, as in the usual single field inflation in GR.

We focused on the scalar-field perturbation in the Hořava-Lifshitz theory, neglecting the spatial metric perturbations. However, the latter is obviously the most important and interesting part in the Hořava theory. Since in the Hořava theory the dynamical degrees of freedom in the spatial metric part are 3, there is one additional scalar degree of freedom. It is important to investigate the prop-

erty of this additional degree of freedom. Moreover, in this work, we only investigated the behavior of the perturbation in the UV limit at  $z = 3$ , while it is interesting to study the full equations of motion, especially in the interpolating region between UV and IR. We considered the scalar field with canonical temporal kinetic term. However, one could expect enhancement of non-gaussianities if more general kinetic terms are considered. The Hořava-Lifshitz theory, although originating from a

renormalizable quantum gravity in 4 dimensions, may be a potential competitor to the standard inflation theory and deserves further investigation.

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## Appendix A: Gauge transformation, gauge choice and gauge-invariant variables

In this Appendix, we discuss the problem of gauge transformation and gauge choice in the non-relativistic Hořava-Lifshitz theory. The essential point is that in the case of GR we have a larger set of gauge transformations which we can use to choose a gauge. However, in the Hořava-Lifshitz theory, the full diffeomorphism with general coordinate invariance is restricted to a subset, i.e. the so-called “foliation-preserving diffeomorphism”, and thus gives less gauge modes but more physical modes.

In this work, we focus on the scalar-type perturbation. The scalar part of coordinate transformations is:  $\delta\eta = \xi^0$ ,  $\delta x^i = \partial^i \chi$ , with

$$\xi^0 = \xi^0(\eta), \quad \chi = \chi(\eta, x^i) \quad (A1)$$

and the scalar-type “space-time metric” perturbations

$$g_{ij} = a^2 \left[ (1 - 2\psi)\delta_{ij} + \partial_i \partial_j E \right], \quad N = 1 + 2\phi, \quad N_i = \partial_i B. \quad (A2)$$

The essential difference from GR is that in the Hořava theory  $\xi^0$  is a function of time  $\eta$  only.

The gauge transformations is as usual:

$$\Delta\phi = -\frac{1}{a} (a\xi^0)', \quad \Delta\psi = \xi^0 \frac{a'}{a}, \quad \Delta B = \xi^0 - \left(\frac{\chi}{a^2}\right)', \quad \Delta E = -\frac{\chi}{a^2}. \quad (A3)$$

Due to the fact that  $\xi^0$  is a function of  $\eta$  only, the gauge choice is fairly restricted. In particular, (or unfortunately), two familiar gauge-choices -- the “longitudinal gauge” and “spatially-flat gauge” -- are not allowed in the Hořava theory. We can use  $\chi = \chi(\eta, x^i)$  freely to set  $E = 0$ , while in general we cannot use  $\xi^0$  to set  $B = 0$  and get the longitudinal gauge, or to set  $\psi = 0$  and get the spatially-flat gauge.

However, there is still a possible gauge choice in the Hořava theory. First, as we have mentioned, we can choose

$$\chi = a^2 E$$

to get  $\tilde{E} = 0$ . This leaves the question: are we able to choose another gauge condition to set one of  $\phi$ ,  $\psi$  and  $B$  to 0? In Hořava's origin-

al formulation of the theory,  $N = N(t)$  is assumed to be a function of time only. In this case we can choose the proper value of  $\xi^0$  to set the fluctuation of  $N$  to zero, i.e.  $\phi = 0$ . Thus, after gauge transformation with

$$\xi^0 = \frac{1}{a} \left( \int d\eta a\phi(\eta) + c \right),$$

we get  $\tilde{\phi} = 0$ . This does not determine time-slicing unambiguously, but we are left with time reparametrization. If we relax the restriction that  $N$  has to be a function of time only, then there is no gauge condition we can choose. In this case, we are left with 3 non-vanishing variables  $\phi$ ,  $\psi$  and  $B$ .

The two Bardeen potentials

$$\begin{aligned} \Psi &= \psi - \frac{a'}{a} (B - E'), \\ \Phi &= \phi + \frac{1}{a} (a(B - E'))', \end{aligned} \quad (A4)$$

are still gauge-invariant variables in Hořava gravity. This is also because the foliation-preserving diffeomorphism is a subset of the full symmetry in GR. Note that there is an infinite number of gauge-invariant variables, for example, combining  $\Phi$  and  $\Psi$  gives another useful gauge-invariant variable

$$\tilde{\Phi} = \phi + \frac{1}{a} \left( \frac{\psi}{H} \right)'. \quad (A5)$$

For the “space-time” scalar field  $\phi$  (now “scalar” means invariant under “foliation-preserving” diffeomorphism), if we assume that the background value is homogeneous  $\phi_0 = \phi_0(\eta)$ , the gauge transformation for the scalar fluctuation is as usual

$$\Delta(\delta\phi) = -\xi^0 \phi_0'. \quad (A6)$$

The gauge-invariant variable for  $\delta\phi$  is as in GR:

$$Q \equiv \delta\phi + \phi_0' (B - E'). \quad (A7)$$

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