Testing the variation of the fine structure constant with strongly lensed gravitational waves*

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Abstract: The possible variation of the electromagnetic fine structure constant, α_e, at cosmological scales has aroused great interest in recent years. Strongly lensed gravitational waves (GWs) and their electromagnetic counterparts could be used to test this variation. Under the assumption that the speed of a photon can be modified, whereas the speed of a GW is the same as predicted by general relativity, and they both propagate in a flat Friedman-Robertson-Walker universe, we investigated the difference in time delays of the images and derived the upper bound of the variation of α_e. For a typical lensing system in the standard cosmological models, we obtained $B\cos\theta \leqslant 1.85\times10^{-5}$, where B is the dipolar amplitude and θ is the angle between observation and the preferred direction. Our result is consistent with the most up-to-date observations on α_e. In addition, the observations of strongly lensed GWs and their electromagnetic counterparts could be used to test which types of alternative theories of gravity can account for the variation of α_e.

Keywords: fine structure constant, gravitational wave, gravitational lensing

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1 Introduction

Gravitational waves (GWs), as one of the predictions of general relativity, were detected recently by the advanced LIGO detector [1]. Until now, the LIGO and Virgo Collaborations have directly observed five GW events produced by the merging of compact binary systems [1–5]. The first four events were produced by the merging of binary black hole systems. The last one, GW170817, was produced by the merging of a binary neutron star system, and the corresponding electromagnetic (EM) counterparts were detected by many instruments [6–12]. The observations of GWs can be used to test cosmology and general relativity. One important cosmological quantity, the luminosity distance of the source, can be derived directly from the GW signal. The location of the source can be found from the EM counterparts, and the redshift of the source can be found from the association of the source with its host galaxy.

The information obtained from the GW observations can be used to constrain the cosmological parameters, such as the equation-of-state of dark energy and the Hubble constant [13–17]. In addition, the GW signal has been used to constrain the graviton mass [1]; the relative arrival time between the GW signals of GW170817 and its EM counterparts has been used to constrain the Lorentz invariance violation [18–22]. However, the intrinsic time delay in the emission time of the GW signal and its EM counterpart cannot be measured directly. To test the Lorentz invariance violation more precisely, an approach using strongly lensed GWs has been proposed to cancel the intrinsic time delay [23–25]. This approach requires the GW and its EM counterparts to occur behind a strong gravitational lensing, and the two images are observed.

Such phenomena have not yet been observed by astronomical instruments. The LIGO and Virgo collaborations established a program[5)] for the identification and follow-up of EM counterparts, which activated the campaign to find EM counterparts [26–28]. Ongoing third-generation detectors with higher sensitivity, such as the Einstein Telescope [29], will discover more GW events [30, 31]. The plausibility of such phenomena being observed is discussed in Ref. [25]. It is expected that GWs and their EM counterparts could be observed behind

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strong gravitational lensing in the future.

Testing the constancy of fundamental physical constants is highly important [32]. Analyzing observations of quasar absorption spectra, Webb et al. [33, 34] found that the fine structure constant α_e varies at cosmological scales. However, debates still remain [35, 36]. The GW signals provide us a new window to study the variation of α_e. It is highly interesting to test the variation of α_e by the observations of GW signals and their EM counterparts. Many models have been proposed to explain the variation of α_e. These models can be divided into two types. The first is that the EM field is coupled to another field, such as the quintessence field [37, 38]. The other is that our universe is anisotropic, for example, our universe is a Finsler spacetime instead of a Riemann spacetime [41, 42]. If α_e does vary at cosmological scales, then the observations of the strongly lensed GW signals and their EM counterparts could be used to test which type of model is valid, because the method proposed by Refs. [24, 25] mainly considers the difference between the time delay of two images of the GW signals and their EM counterparts. The first type of model requires that the speeds of a photon and GW are different. Then, the observations of the strongly lensed GW signals and their electromagnetic counterparts will find the difference from the method [24, 25]. The other type of model requires that both photons and GWs propagate in the anisotropic universe with the same anisotropic speed. Then, such observations will not find any difference. In this paper, we discuss these points and show that the observations of the strongly lensed GW signals and their EM counterparts could test the validity of Webb's result.

The arrangement of this paper is as follows: In Section 2, we introduce the basic information of GWs and their EM counterparts, and discuss in detail the method to calculate the difference of time delays between the GW and EM counterparts. Then, in Section 3 we use the method to constrain the variation of α_e and compare it with the observational data. Finally, conclusions and remarks are provided in Section 4.

2 Methodology

Webb et al. [33, 34] showed that the variation of fine structure constant α_e has a dipolar structure in the high red-shift region ($z>1.6$). Recently, Pinho et al. [43] also confirmed that the dipolar variation of α_e is still a good fit to the most up-to-date data. According to their results, the variation of α_e can be expressed as

$$\frac{\triangle\alpha_e}{\alpha_e}=B\cos\theta, \tag{1}$$

where B represents the dipole amplitude and is assumed to be a constant, and θ is the angle between the dipolar direction and the observed direction.

One direct reason for the variation of α_e is variation in the speed of light. Therefore, from (1), the speed of light c is anisotropic with a dipolar structure at the cosmological scale:

$$c_\gamma=c_0/(1+B\cos\theta), \tag{2}$$

where c_0 is the speed of light at the present epoch.

The method proposed by Refs. [24, 25] considers GWs and their EM counterparts propagating through a strong gravitational lensing, and at least two images of the GW event and two images of the EM counterparts are observed. The observations of these phenomena can detect two arrival times of GW events and two arrival times of EM events. The time delay of the two GW events does not depend on the initial emission time of the GW signals. The time delay of the two EM events also does not depend on the initial emission time of the EM signals. Thus, this method does not depend on the intrinsic separation time of the GW and its EM counterparts.

Under general relativity, the two time delays should be the same. If a difference in the two time delays is observed, then it is a sign of new physics. Two reasons will deduce the difference in the two time delays. One reason is that the speeds of gravitons and photons are different. The other is that the geodesics of GWs and photons are different. One type of model, such as that described in Refs. [41, 42], could explain Webb's results by assuming photons propagate in a Finslerian universe. In such a model, both the graviton and photon are massless and their geodesics are the same. Therefore, if Webb's results are confirmed in future astronomical observations and the observations show no difference between the two time delays, then it implies our universe may be Finslerian. Webb's results could be explained by another type of model that assumes the photon speed is modified. In alternative theories of gravity, the speed of both photons and gravitons could be modified. Several approaches could lead to the modifications. For example, the photon speed could be modified if the Lagrange of the EM field possesses a non-minimal coupling form [37, 38]; the speed of gravity could be modified if gravitons couple to background gravitational fields, such as massive gravity [39, 40]. As described in Refs. [37, 38], the graviton or scalar curvature in Lagrange does not couple to background gravitational fields; thus, the graviton speed is unchanged in these models. Because the speeds of light and gravity are different, a difference between the two time delays should occur. In the rest of our paper, we mainly discuss how to test Webb's results with a model-independent method, which is based only on the assumption that the speed of light is different from that of gravity.

The spatial geometry of Friedman-Robertson-Walker

(FRW) spacetime would not greatly affect the difference between the two time delays. In addition, recent data of the Planck satellite suggest a spatially flat universe [44]. For simplicity, we assume that spacetime is depicted by the flat FRW metric

$$\mathrm{d}s^2=c^2\mathrm{d}t^2-a(t)\left[\mathrm{d}r^2+r^2\mathrm{d}\theta^2+r^2\sin^2\theta\mathrm{d}\phi^2\right], \quad (3)$$

where $a(t)$ is the scale factor. Hence, the travel distance of the GW from the emitted moment t_e to observed moment t_0 is

$$r_{\mathrm{GW}}=\int_{t_\mathrm{e}}^{t_0}\frac{c_0}{a(t)}\mathrm{d}t=\frac{c_0}{H_0}\tilde{r}_{\mathrm{GW}}(z), \quad (4)$$

where H_0 is the Hubble constant, and

$$\tilde{r}_{\mathrm{GW}}(z)=\int_0^z\frac{\mathrm{d}z'}{E(z')} \quad (5)$$

is the reduced comoving distance traveled by the graviton, where $E(z)=\sqrt{\Omega_m(1+z)^3+(1-\Omega_m)}$. For the EM counterpart, the photon travels at speed c_γ given in Eq. (2), and the corresponding distance is given as

$$r_\gamma=\int_{t_\mathrm{e}}^{t_0}\frac{c_\gamma}{a(t)}\mathrm{d}t=\frac{c_0}{H_0}\tilde{r}_\gamma, \quad (6)$$

where $\tilde{r}_\gamma=[1-f(B,\theta)]\tilde{r}_{\mathrm{GW}}$ and $f(B,\theta)=B\cos\theta–B^2\cos^2\theta$. Here, we must use the fact that the magnitude B of α_e variation is quite small and expands $f(B,\theta)$ to the second order of B.

In a strong gravitational lensing system, the time delay between two collinear images that are observed on the opposite side of the lens is given as

$$\Delta t=\frac{1+z_l}{2c}\frac{D_lD_s}{D_{ls}}(\theta_A^2-\theta_B^2), \quad (7)$$

where $\theta_A=\theta_\mathrm{E}+\beta$ and $\theta_B=\theta_\mathrm{E}–\beta$ are the radial distances of two images, and β denotes the misalignment angle. Here, D_{ls} denotes the angular diameter distance between the lens and source, and D_l (D_s) denotes the angular diameter distance between the lens (source) and observer. In the singular isothermal sphere lens model, the Einstein ring radius takes the form

$$\theta_\mathrm{E}=4\pi\frac{D_{ls}}{D_s}\frac{\sigma^2}{c^2}, \quad (8)$$

where σ is the one-dimensional velocity dispersion. Combining Eqs. (4), (7), and (8) yields the time delays of two GW signals as

$$\Delta t_{\mathrm{GW}}=\frac{32\pi^2}{H_0}\left(\frac{\sigma}{c_0}\right)^4\frac{\beta\tilde{r}(z_l)\tilde{r}(z_l,z_s)}{\theta_\mathrm{E}\tilde{r}(z_s)}. \quad (9)$$

The EM field with a non-minimal coupling, such as that described in Refs. [37, 38], implies the photon is massive. From the geodesic equation in general relativity, one can find that the deflection of the photon with a small rest mass would be altered with a factor $1+(m_\gamma^2c_0^4/2E_\gamma^2)$, where m_γ and E_γ are the mass and energy of the photon. Thus, the Einstein radius is modified as $\theta_{\mathrm{E},\gamma}=\theta_\mathrm{E}[1+(m_\gamma^2c_0^4/2E_\gamma^2)]$ and the time delay between the two images of EM is given as

$$\Delta t_\gamma=\frac{32\pi^2}{H_0}\left(\frac{\sigma}{c_0}\right)^4\frac{\beta\tilde{r}_\gamma(z_l)\tilde{r}_\gamma(z_l,z_s)}{\theta_\mathrm{E}\tilde{r}_\gamma(z_s)}\left[1+\frac{m_\gamma^2c_0^4}{2E_\gamma^2}\right]. \quad (10)$$

In flat FRW spacetime, the spacetime is Minkowski spacetime locally. Thus, the dispersion relation of massive photons is the same as with other massive particles in Minkowski spacetime. Combining Eq. (2) and the dispersion relation derives the relation between the mass of a photon and its speed as

$$\frac{m_\gamma^2c_0^4}{2E_\gamma^2}=\frac{1}{2}\left(1-\frac{1}{(1+B\cos\theta)^2}\right). \quad (11)$$

By making use of Eqs. (7, 10), and (11), to the second order in B, we obtain the difference in the two time delays as

$$\Delta t_{\mathrm{GW}}-\Delta t_\gamma=\Delta t_{\mathrm{GW}}\frac{3}{2}B^2\cos^2\theta. \quad (12)$$

3 Results

The observational accuracy δT of the difference between the two time delays, i.e., $\Delta t_{\mathrm{GW}}–\Delta t_\gamma$, could yield a constraint on the dipole variation of α_e. From Eq. (12), we find that

$$B\cos\theta\leqslant\left(\frac{2}{3}\frac{\delta T}{\Delta t_{\mathrm{GW}}}\right)^{1/2}. \quad (13)$$

In the strong gravitational lensing systems compiled in Ref. [45], the red-shift ranges are $z_l\in[0.075,1.004]$ for the lens and $z_s\in[0.196,3.596]$ for the source, and the velocity dispersions are in the range $\sigma\in[103,391]$ km/s. Additionally, the source-lens misalignment parameter β/θ_E should not be too large, in order to ensure the formation of multiple images. Piórkowska et al. [46] demonstrated that the maximal value of misalignment parameter β/θ_E is 0.5. For the timing accuracy δT of the time delay, observation has demonstrated that the GW signal can be detected at precision $<10^{-4}$ms [1, 24]. Moreover, the timing precision of promising EM counterparts, such as SGRB and FRB , could be on the order of 10^{-2}–10^3 ms [47, 48]. Thus, the accuracy of the EM time delay determines the ability to test Webb's result. However, because the strongly lensed gravitational waves and their EM counterparts have not been detected, $\delta T=1$ ms could be set as a mediate timing precision of promising EM counterparts to obtain the detection precision for testing the α_e variation.

Considering the ΛCDM cosmology parameters given by the Planck data [44], i.e., $H_0=68$ kms^{-1}Mpc^{-1}, $\Omega_{M0}=0.3$, and using the typical parameters of a strong lensing system ($z_l=1$, $z_s=2$, $\sigma=250$ km/s, and

$\beta/\theta_{\mathrm{E}}=0.1$), and assuming the timing accuracy $\delta T=1$ ms, we obtain the bound of α_{e} variation as

$$B\cos\theta\leqslant 1.85\times 10^{-5}. \tag{14}$$

Webb et al. [33] showed that the magnitude of α_{e} variation is $(0.97^{+0.22}_{-0.20})\times 10^{-5}$. Pinho et al. [43] showed that the magnitude of α_{e} variation is $(0.81\pm 0.17)\times 10^{-5}$. Thus, in the detection precision, the constraint on α_{e} variation, Eq. (14), is consistent with the previous studies on α_{e} variation. This implies that the observations of the difference between the GW time delay and EM time delay are capable of testing whether the α_{e} variation is valid.

The upper limit of the variation of α_{e} measured in the Milky Way is $|\Delta\alpha_{\mathrm{e}}/\alpha_{\mathrm{e}}|<1.1\times 10^{-7}$ [49]. Thus, if Webb's result is correct, there should be a physical mechanism in which α_{e} varies with redshift. In fact, our previous research has shown one such possible physical mechanism [42], where the speed of light depends on the redshift with the form

$$c_\gamma=c_0/(1+B(z)\cos\theta), \tag{15}$$

where

$$B(z)=b_0\int_0^z\frac{1+z'}{\sqrt{\Omega_m(1+z')^3+(1-\Omega_m)}}\mathrm{d}z'=b_0D(z). \tag{16}$$

where $D(z)=\int_0^z\frac{1+z'}{E(z')}dz'$. To the first order of b_0, the difference of time delays measured by the GW and EM windows becomes

$$\Delta t_{\mathrm{GW}}-\Delta t_\gamma=\Delta t_{\mathrm{GW}}b_0\cos\theta F(z_l,z_s), \tag{17}$$

where

$$F=\frac{x(z_s)}{\tilde{r}_{g,ls}}-\frac{x(z_l)}{\tilde{r}_{g,ls}}+\frac{x(z_l)}{\tilde{r}_{g,l}}-\frac{x(z_s)}{\tilde{r}_{g,s}}-D(z_s) \tag{18}$$

where $x(z)=\int_0^z\frac{D(z')}{E(z')}\mathrm{d}z'$. This is different from Eq. (12), where the formula represents that the difference of time delays is proportional to B^2. Eq. (16) shows that, if the variation of α_{e} is independent of the redshift, viz., $D(z)=1$ in Eq. (16), the term that is proportional to B would vanish in Eq. (17) ($F=0$). Then, Eq. (17) reduces to Eq. (12) under the consideration of the second order of B. With the same variable settings as described previously, where $z_s=2$ and $z_l=1$, the upper bound of dipolar variation $b_0\cos\theta\leq 2.08\times 10^{-10}$, which reaches a very high accuracy to test the variation of the fine structure constant.

It should be noted that Webb's result regarding the dipolar variation of α_{e} mainly appears in the high redshift region ($z>1.6$). Furthermore, our method, i.e., testing the variation using the difference between the GW time delay and EM time delay, needs the observation of the EM counterparts of GW signals. However, because of the present sensitivity of the LIGO detector, it is incapable of detecting GW signals with EM counterparts located at redshift $z>1$. Ongoing third-generation detectors such as the Einstein Telescope [29] with higher sensitivity are capable of testing Webb's result.

4 Conclusions and remarks

The associated detection of GWs and their EM counterparts provides a way to test fundamental physics. In this study, we used the method proposed by Refs. [24, 25] to test the possible variation of α_{e}. The method considers the difference between the time delay of two images of the GW event and its EM counterparts. The difference between the speeds of photos and gravitons, and the difference between the geodesics of photons and gravitons, can account for the difference between the two time delays. In an anisotropic universe, the geodesics and speeds of photons and gravitons are modified in the same way. Therefore, if Webb's results are confirmed by future data and the observations show no difference between the two time delays, then it implies our universe may be anisotropic, such as a Finslerian universe.

In this study, we considered that the photon speed is modified, such as electromagnetic field coupling to a quintessence field [37, 38]. In these models, the graviton speed remains the same as that predicted by general relativity, and both gravitons and photons propagate in the same flat FRW universe. It is shown that the dipolar variation of α_{e} has an upper limit, namely, $B\cos\theta\leq 1.85\times 10^{-5}$, which implies that Webb's result can be tested at the current accuracy with this method. In addition, considering that the variation of α_{e} could be a function of the redshift as described by Li and Lin [42], we obtained a bound of α_{e} variation, $b_0\cos\theta\leq 2.08\times 10^{-10}$, which is a higher detection precision to test Webb result.

One should notice, because of the present sensitivity of the LIGO detector, that one cannot find GW signals with EM counterparts located at redshift $z>1$. Webb et al. found that the dipolar variation of α_{e} appears only at the high red-shift ($z>1.6$) region. Thus, it is expected that ongoing third-generation detectors such as the Einstein Telescope could test the validity of Webb's result by observing the difference between the time delay of two images of the GW event and its EM counterparts. In this paper, we only discuss a limit on $B\cos\theta$; the constraint on B and the preferred direction are not considered. If many events of strongly lensed GWs and their EM counterparts are observed in the future, then it is possible to use the data to constrain the dipolar amplitude B and the preferred direction of the universe.

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