

# Weak decays of heavy baryons in the light-front approach<sup>\*</sup>

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**Abstract:** In this work, we analyse semi-leptonic and non-leptonic weak decays of the heavy baryons:  $\Lambda_b, \Xi_b, \Omega_b$  and  $\Lambda_c, \Xi_c, \Omega_c$ . For non-leptonic decay modes, we study only the factorizable channels induced by the external  $W$ -emission. The two spectator quarks in the baryonic transitions are treated as a diquark and form factors are calculated in the light-front approach. Using the results for form factors, we also calculate some corresponding semi-leptonic and non-leptonic decay widths. We find that our results are comparable with the available experimental data and other theoretical predictions. Decay branching fractions for many channels are found to reach the level  $10^{-3} \sim 10^{-2}$ , which is promising for discovery in future measurements at BESIII, LHCb and Belle II. The  $SU(3)$  symmetry in semi-leptonic decays is examined and sources of symmetry breaking are discussed.

**Keywords:** heavy baryons, weak decay, light-front approach

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## 1 Introduction

Quite recently, the LHCb collaboration announced the discovery of the doubly charmed baryon  $\Xi_{cc}^{++}$  [1]. Undoubtedly this discovery will open a new door to study strong interactions in the presence of a pair of heavy quarks. Accordingly, it has triggered great theoretical interest in studying doubly heavy baryons from different aspects [2–21].

Inspired by this discovery, we also expect a renaissance in the study of singly bottom or charm baryons. Particularly, there has been rapid progress in the study of  $\Lambda_c$  decays at BESIII [22–27] and some recent studies on  $\Lambda_b$  and  $\Lambda_c$  decays by LHCb can be found in Refs. [28–33]. It is anticipated that many more decay modes will be established in future. Thus, there is high demand for an up-to-date theoretical analysis, and this work aims to provide such an analysis.

The quark model is a very successful tool in classifying mesons and baryons. A heavy baryon is composed of one heavy quark  $c/b$  and two light quarks. Light flavor  $SU(3)$  symmetry arranges the singly heavy baryons into the presentations  $\mathbf{3} \otimes \mathbf{3} = \mathbf{6} \oplus \bar{\mathbf{3}}$ , as can be seen from Fig. 1. For charmed baryons, the irreducible representation  $\bar{\mathbf{3}}$  is composed of  $\Lambda_c^+$  and  $\Xi_c^{+,0}$  while the sextet is composed of  $\Sigma_c^{+,+,0}$ ,  $\Xi_c^{+,',0}$  and  $\Omega_c^0$ . They all have spin

$1/2$ , but only 4 of them predominantly weakly decay:  $\Lambda_c^+$  and  $\Xi_c^{+,0}$  in the representation  $\bar{\mathbf{3}}$  and  $\Omega_c^0$  in the representation  $\mathbf{6}$ . Others can decay into the lowest-lying states via strong or electromagnetic interactions. This is similar for bottom baryons. In this work, we will focus on weak decays of singly heavy baryons and more explicitly we will consider only the following channels:

- charm sector:

$$\begin{aligned}\Lambda_c^+(cud) &\rightarrow n(dud)/\Lambda(sud), \\ \Xi_c^+(cus) &\rightarrow \Sigma^0(dus)/\Lambda(dus)/\Xi^0(sus), \\ \Xi_c^0(cds) &\rightarrow \Sigma^-(dds)/\Xi^-(sds), \\ \Omega_c^0(css) &\rightarrow \Xi^-(dss);\end{aligned}$$

- bottom sector:

$$\begin{aligned}\Lambda_b^0(bud) &\rightarrow p(uud)/\Lambda_c^+(cud), \\ \Xi_b^0(bus) &\rightarrow \Sigma^+(uus)/\Xi_c^+(cus), \\ \Xi_b^-(bds) &\rightarrow \Sigma^0(uds)/\Lambda(uds)/\Xi_c^0(cds), \\ \Omega_b^-(bss) &\rightarrow \Xi^0(uss)/\Omega_c^0(css).\end{aligned}$$

In the above, we have listed the quark contents of the baryons in the brackets, and placed the quarks that participate in the weak decay in the first place.

The light baryons in the final state are composed of 3

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light quarks and belong to the baryon octet. Their wave functions, including the flavor and spin spaces, have the form [34]

$$\mathcal{B}_8 = \sqrt{\frac{1}{2}}(\mathbf{p}_S \chi(M_S) + \mathbf{p}_A \chi(M_A)). \quad (1)$$

Here  $\mathbf{p}_{S(A)}$  stands for the mixed symmetric (antisymmetric)  $\mathbf{8}$  in the  $SU(3)$  representation decomposition  $\mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3} = \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1}$  in flavor space, while  $\chi(M_{S(A)})$  stands for the mixed symmetric (antisymmetric)  $\mathbf{2}$  in the  $SU(2)$  representation decomposition  $\mathbf{2} \otimes \mathbf{2} \otimes \mathbf{2} = \mathbf{4} \oplus \mathbf{2} \oplus \mathbf{2}$  in spin space. Here the ‘‘mixed symmetric (antisymmetric)’’ means the state is symmetric (antisymmetric) under interchange of the first two quarks. The wave functions for baryons in the initial and final states are collected in Appendix A.

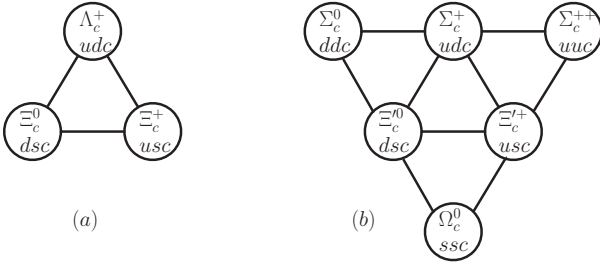


Fig. 1. Anti-triplets (a) and sextets (b) of charmed baryons with one charm quark and two light quarks. The diagrams are similar for the baryons with a bottom quark.

On the theoretical side, the singly heavy baryon decays have been investigated by various theoretical methods, and some of them can be found in Refs. [35–56]. In this work, we will adopt the light-front approach. This method has been widely used to study the properties of mesons [57–74]. Its application to baryons can be found in Refs. [75–78]. In the transition form factors, the two spectator quarks do not change and can be viewed as a diquark. In this diquark scheme, the two quarks are treated as a whole system, and thus its role is similar to that of the antiquark in the meson case, see Fig. 2. In a process like  $\Lambda_b \rightarrow \Lambda_c$ , the light quarks  $u$  and  $d$  are considered to form a scalar diquark, which is denoted by  $[ud]$ , while in a process like  $\Omega_b \rightarrow \Omega_c$ , the light  $s$  quarks are believed to form an axial-vector diquark, which is denoted by  $ss$ .

Some recent works have investigated the singly heavy baryon decays with the help of flavor  $SU(3)$  symmetry [79–82]. Based on the available data, the  $SU(3)$  analysis can predict a great number of decay modes ranging from semi-leptonic decays to multi-body non-leptonic decays. However, in the case of  $c$  quark decays,  $SU(3)$  symmetry breaking effects are sizable and cannot be omitted. A quantitative study of  $SU(3)$  symmetry breaking effects will be conducted within the light-front approach.

The rest of the paper is arranged as follows. In Section 2, we will briefly present the framework of the light-front approach under the diquark picture, and also give the wave function overlap factors. Our results are given in Section 3, including the results for form factors, predictions on semi-leptonic and non-leptonic decay widths, and detailed discussions on the  $SU(3)$  symmetry and sources of symmetry breaking. A brief summary is given in the last section.

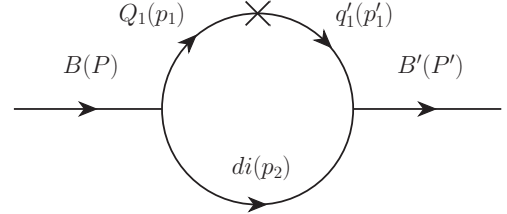


Fig. 2. Feynman diagrams for baryon-baryon transitions in the diquark picture.  $P^{(\prime)}$  is the momentum of the incoming (outgoing) baryon,  $p_1^{(\prime)}$  is the initial (final) quark momentum,  $p_2$  is the diquark momentum, and the cross mark denotes the corresponding vertex of the weak interaction.

## 2 Theoretical framework

In this section, we will briefly overview the theoretical framework for form factors: the light-front approach. More details can be found in Refs. [75] and [4]. It is necessary to point out that the physical form factor should be multiplied by a factor due to the overlap of wave functions in the initial and final states.

### 2.1 Form factors

The transition matrix elements are parameterized as

$$\begin{aligned} \langle B'(P', S'_z) | V_\mu | B(P, S_z) \rangle &= \bar{u}(P', S'_z) \left[ \gamma_\mu f_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M} f_2(q^2) + \frac{q_\mu}{M} f_3(q^2) \right] u(P, S_z), \\ \langle B'(P', S'_z) | A_\mu | B(P, S_z) \rangle &= \bar{u}(P', S'_z) \left[ \gamma_\mu g_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{M} g_2(q^2) + \frac{q_\mu}{M} g_3(q^2) \right] \gamma_5 u(P, S_z), \end{aligned} \quad (2)$$

where  $q = P - P'$ ,  $M$  denotes the mass of the parent baryon  $B$ , and  $f_i, g_i$  are form factors.

In the light-front approach, hadron states are expanded in terms of quark states superposed with a wave func-

tion, where the momentum and other quantum numbers are considered simultaneously. Then the weak transition matrix element can be obtained as

$$\begin{aligned} \langle B'(P', S'_z) | (V-A)_\mu | B(P, S_z) \rangle = & \int \{d^3 p_2\} \frac{\phi'^*(x', k'_\perp) \phi(x, k_\perp)}{2\sqrt{p_1^+ p_1'^+ (p_1 \cdot \bar{P} + m_1 M_0) (p_1' \cdot \bar{P}' + m_1' M_0)}} \\ & \times \bar{u}(\bar{P}', S'_z) \bar{\Gamma}'(\not{p}'_1 + m'_1) \gamma_\mu (1 - \gamma_5) (\not{p}_1 + m_1) \Gamma u(\bar{P}, S_z). \end{aligned} \quad (3)$$

From Eqs. (2) and (3), we can extract the explicit expressions of form factors:

$$\begin{aligned} f_1(q^2) &= \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \frac{\phi'(x', k'_\perp) \phi(x, k_\perp) [k_\perp \cdot k'_\perp + (x_1 M_0 + m_1)(x'_1 M_0 + m'_1)]}{\sqrt{[(m_1 + x_1 M_0)^2 + k_\perp^2] [(m'_1 + x'_1 M_0)^2 + k_\perp'^2]}}, \\ g_1(q^2) &= \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \frac{\phi'(x', k'_\perp) \phi(x, k_\perp) [-k_\perp \cdot k'_\perp + (x_1 M_0 + m_1)(x'_1 M_0 + m'_1)]}{\sqrt{[(m_1 + x_1 M_0)^2 + k_\perp^2] [(m'_1 + x'_1 M_0)^2 + k_\perp'^2]}}, \\ \frac{f_2(q^2)}{M} &= \frac{1}{q_\perp^2} \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \frac{\phi'(x', k'_\perp) \phi(x, k_\perp) [-(m_1 + x_1 M_0) k'_\perp \cdot q_\perp + (m'_1 + x'_1 M_0) k_\perp \cdot q_\perp]}{\sqrt{[(m_1 + x_1 M_0)^2 + k_\perp^2] [(m'_1 + x'_1 M_0)^2 + k_\perp'^2]}}, \\ \frac{g_2(q^2)}{M} &= \frac{1}{q_\perp^2} \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \frac{\phi'(x', k'_\perp) \phi(x, k_\perp) [-(m_1 + x_1 M_0) k'_\perp \cdot q_\perp - (m'_1 + x'_1 M_0) k_\perp \cdot q_\perp]}{\sqrt{[(m_1 + x_1 M_0)^2 + k_\perp^2] [(m'_1 + x'_1 M_0)^2 + k_\perp'^2]}} \end{aligned} \quad (4)$$

for a scalar diquark involved in the initial and final baryons, or

$$\begin{aligned} f_1(q^2) &= \frac{1}{8P^+ P'^+} \int \frac{dx_2 d^2 k_\perp}{2(2\pi)^3} \frac{\varphi'(x', k'_\perp) \varphi(x, k_\perp)}{6\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0) (p_1' \cdot \bar{P}' + m_1' M_0)}} \\ & \times \text{Tr}[(\bar{\not{P}} + M_0) \gamma^+ (\bar{\not{P}}' + M_0') \gamma_5 \gamma_\alpha (\not{p}'_1 + m'_1) \gamma^+ (\not{p}_1 + m_1) \gamma_5 \gamma_\beta] \left( \frac{p_2^\alpha p_2^\beta}{m_2^2} - g^{\alpha\beta} \right), \\ g_1(q^2) &= \frac{1}{8P^+ P'^+} \int \frac{dx_2 d^2 k_\perp}{2(2\pi)^3} \frac{\varphi'(x', k'_\perp) \varphi(x, k_\perp)}{2\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0) (p_1' \cdot \bar{P}' + m_1' M_0)}} \\ & \times \text{Tr}[(\bar{\not{P}} + M_0) \gamma^+ \gamma_5 (\bar{\not{P}}' + M_0') \gamma_5 \gamma_\alpha (\not{p}'_1 + m'_1) \gamma^+ \gamma_5 (\not{p}_1 + m_1) \gamma_5 \gamma_\beta] \left( \frac{p_2^\alpha p_2^\beta}{m_2^2} - g^{\alpha\beta} \right), \\ \frac{f_2(q^2)}{M} &= -\frac{1}{8P^+ P'^+} \frac{iq_\perp^i}{q_\perp^2} \int \frac{dx_2 d^2 k_\perp}{2(2\pi)^3} \frac{\varphi'(x', k'_\perp) \varphi(x, k_\perp)}{2\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0) (p_1' \cdot \bar{P}' + m_1' M_0)}} \\ & \times \text{Tr}[(\bar{\not{P}} + M_0) \sigma^{i+} (\bar{\not{P}}' + M_0') \gamma_5 \gamma_\alpha (\not{p}'_1 + m'_1) \gamma^+ (\not{p}_1 + m_1) \gamma_5 \gamma_\beta] \left( \frac{p_2^\alpha p_2^\beta}{m_2^2} - g^{\alpha\beta} \right), \\ \frac{g_2(q^2)}{M} &= \frac{1}{8P^+ P'^+} \frac{iq_\perp^i}{q_\perp^2} \int \frac{dx_2 d^2 k_\perp}{2(2\pi)^3} \frac{\varphi'(x', k'_\perp) \varphi(x, k_\perp)}{2\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0) (p_1' \cdot \bar{P}' + m_1' M_0)}} \\ & \times \text{Tr}[(\bar{\not{P}} + M_0) \sigma^{i+} \gamma_5 (\bar{\not{P}}' + M_0') \gamma_5 \gamma_\alpha (\not{p}'_1 + m'_1) \gamma^+ \gamma_5 (\not{p}_1 + m_1) \gamma_5 \gamma_\beta] \left( \frac{p_2^\alpha p_2^\beta}{m_2^2} - g^{\alpha\beta} \right) \end{aligned} \quad (5)$$

if an axial-vector diquark is involved.

## 2.2 Spin and flavor wave functions

In the last subsection, we have presented the explicit expressions of form factors. The physical transition form factor should be multiplied by the corresponding overlap factor:

$$f_1^{\text{physical}}(q^2) = \text{the overlap factor} \times f_1^{\text{in Sec. 2.1}}(q^2). \quad (6)$$

From the discussions in Appendix A, we can obtain these factors, and the corresponding results are collected in Table 1.

## 3 Numerical results and discussion

All inputs to calculate the form factors will be collected in the first subsection. What follows is the numerical results for form factors, semi-leptonic and non-leptonic processes. Some remarks will also be given. The last subsection is devoted to an  $SU(3)$  analysis for semi-leptonic processes.

### 3.1 Inputs

The quark masses used in the model are given as

$$\begin{aligned} m_u = m_d = 0.25 \text{ GeV}, \quad m_s = 0.37 \text{ GeV}, \\ m_c = 1.4 \text{ GeV}, \quad m_b = 4.8 \text{ GeV}. \end{aligned}$$

These values are widely adopted in Refs. [66–74]. The diquark masses are chosen as

$$\begin{aligned} m_{[ud]} &= 0.50 \text{ GeV}, & m_{[us]} &= m_{[ds]} = 0.60 \text{ GeV}, \\ m_{\{uu\}} &= m_{\{ud\}} = m_{\{dd\}} = 0.77 \text{ GeV}, \\ m_{\{us\}} &= m_{\{ds\}} = 0.87 \text{ GeV}, & m_{\{ss\}} &= 0.97 \text{ GeV}. \end{aligned}$$

Here the square brackets (curly braces) denote a scalar (an axial-vector) diquark.

Table 1. Overlap factors in the transitions

transition	overlap factors
$\Lambda_c^+(cud) \rightarrow n(dud)/\Lambda(sud)$	$-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$
$\Xi_c^+(cus) \rightarrow \Sigma^0(dus)/\Lambda(dus)/\Xi^0(uss)$	$\frac{1}{2}, \frac{1}{2\sqrt{3}}, -\frac{1}{\sqrt{2}}$
$\Xi_c^0(cds) \rightarrow \Sigma^-(dds)/\Xi^-(sds)$	$\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$
$\Omega_c^0(css) \rightarrow \Xi^-(dss)$	$-\frac{1}{\sqrt{3}}$
$\Lambda_b^0(bud) \rightarrow p(uud)/\Lambda_c^+(cud)$	$\frac{1}{\sqrt{2}}, 1$
$\Xi_b^0(bus) \rightarrow \Sigma^+(uus)/\Xi_c^+(cus)$	$\frac{1}{\sqrt{2}}, 1$
$\Xi_b^-(bds) \rightarrow \Sigma^0(uds)/\Lambda(uds)/\Xi_c^0(cds)$	$\frac{1}{2}, -\frac{1}{2\sqrt{3}}, 1$
$\Omega_b^-(bss) \rightarrow \Xi^0(uss)/\Omega_c^0(css)$	$-\frac{1}{\sqrt{3}}, 1$

The shape parameters are given as

$$\begin{aligned} \beta_{b[ud]} &= 0.66 \text{ GeV}, & \beta_{b[us]} &= \beta_{b[ds]} = 0.68 \text{ GeV}, \\ \beta_{b\{ss\}} &= 0.78 \text{ GeV}, \\ \beta_{c[ud]} &= 0.56 \text{ GeV}, & \beta_{c[us]} &= \beta_{c[ds]} = 0.58 \text{ GeV}, \\ \beta_{c\{ss\}} &= 0.66 \text{ GeV}, \\ \beta_{s[ud]} &= 0.45 \text{ GeV}, & \beta_{s[us]} &= \beta_{s[ds]} = 0.46 \text{ GeV}, \\ \beta_{d[ud]} &= 0.40 \text{ GeV}, & \beta_{d[us]} &= \beta_{d[ds]} = 0.41 \text{ GeV}, \\ \beta_{d\{ss\}} &= 0.44 \text{ GeV}, \\ \beta_{u[ud]} &= 0.40 \text{ GeV}, & \beta_{u[us]} &= \beta_{u[ds]} = 0.41 \text{ GeV}, \\ \beta_{u\{ss\}} &= 0.44 \text{ GeV}. \end{aligned}$$

Some remarks on the above parameters are in order.

1)  $m_{[ud]} = 0.50 \text{ GeV}$  and  $m_{\{ud\}} = 0.77 \text{ GeV}$  are taken from Refs. [75, 77]. Other diquark masses are taken as the above values since the  $s$  quark mass is expected to be 0.1 GeV higher than that of the  $u$  or  $d$  quark.

2) The shape parameters for baryons are constrained by the corresponding ones for mesons [61]. To be specific,  $\beta_{b,di}$  are taken as between  $\beta_{b\bar{s}} = 0.623$  and  $\beta_{b\bar{e}} = 0.886$ ;  $\beta_{c,di}$  are taken as between  $\beta_{c\bar{s}} = 0.535$  and  $\beta_{c\bar{c}} = 0.753$ ;  $\beta_{d,di}$  are taken as between  $\beta_{d\bar{s}} = 0.393$  and  $\beta_{d\bar{c}} = 0.470$ ;  $\beta_{s,di}$  are taken as between  $\beta_{s\bar{s}} = 0.440$  and  $\beta_{s\bar{c}} = 0.535$ .

The masses and lifetimes of the parent baryons are collected in Table 2 and the masses of the daughter baryons are shown in Table 3 [83].

Table 2. Masses and lifetimes of parent baryons.  $\Lambda$  and  $\Xi$  are in the representation  $\mathbf{\bar{3}}$  while  $\Omega$  is in the representation  $\mathbf{6}$ .

baryon	$\Lambda_c^+$	$\Xi_c^+$	$\Xi_c^0$	$\Omega_c^0$	$\Lambda_b^0$	$\Xi_b^0$	$\Xi_b^-$	$\Omega_b^-$
mass/GeV	2.286	2.468	2.471	2.695	5.620	5.792	5.795	6.046
lifetime/fs	200	442	112	69	1466	1464	1560	1570

Table 3. Masses of daughter baryons. They form the baryon octet.

baryon	$p$	$n$	$\Lambda$	$\Sigma^+$	$\Sigma^0$	$\Sigma^-$	$\Xi^0$	$\Xi^-$
mass/GeV	0.938	0.940	1.116	1.189	1.193	1.197	1.315	1.322

Fermi constant and CKM matrix elements are taken from the PDG [83]:

$$\begin{aligned} G_F &= 1.166 \times 10^{-5} \text{ GeV}^{-2}, \\ |V_{ud}| &= 0.974, & |V_{us}| &= 0.225, & |V_{ub}| &= 0.00357, \\ |V_{cd}| &= 0.225, & |V_{cs}| &= 0.974, & |V_{cb}| &= 0.0411. \end{aligned} \quad (7)$$

### 3.2 Form factors

The results for form factors are collected in Table 5 for charmed baryons and Table 6 for bottom baryons. The following expressions have been used to access the  $q^2$  distribution:

$$F(q^2) = \frac{F(0)}{1 \mp \frac{q^2}{m_{\text{fit}}^2} + \delta \left( \frac{q^2}{m_{\text{fit}}^2} \right)^2}, \quad (8)$$

where  $F(0)$  is the form factor at  $q^2 = 0$ .  $m_{\text{fit}}$  and  $\delta$  are two parameters to be fitted from numerical results. For the form factor  $g_2$ , a plus sign is adopted in Eq. (8), otherwise the fitted parameter  $m_{\text{fit}}$  becomes purely imaginary. The minus sign is adopted for all the other situations.

Some comments are in order.

1) Only the scalar diquark contributes to the  $\Lambda_Q$  and  $\Xi_Q$  decays and only the axial-vector diquark contributes to the  $\Omega_Q$  decays, where  $Q = c/b$ .

2) In Tables 5 and 6, the overlap factors are not taken into account. The physical transition form factor should be multiplied by the corresponding overlap factor, see Eq. (6).

3) An advantage of the results in Table 5 is that they can be directly used to explore  $SU(3)$  symmetry and its breaking effects. In fact, if we take the approximations

$$\begin{aligned} m_d &= m_s, \\ m_{[ud]} &= m_{[us]} = m_{[ds]} = m_{\{ss\}}, \\ \beta_{c[ud]} &= \beta_{c[us]} = \beta_{c[ds]} = \beta_{c\{ss\}}, \\ \beta_{d[ud]} &= \beta_{s[ud]} = \beta_{d[us]} = \beta_{s[us]} = \beta_{d[ds]} = \beta_{s[ds]} = \beta_{d\{ss\}} \end{aligned}$$

and

$$m_{\Lambda_c^+} = m_{\Xi_c^+} = m_{\Xi_c^0} = m_{\Omega_c^0},$$

all the form factors will be the same. From the results in Table 5, we can see that the  $SU(3)$  symmetry is not

severely broken.

In Table 4, we compare our results with other theoretical predictions in Refs. [39, 84, 85]. Some comments are given as follows.

1) In Table 4, the physical form factors are shown, see Eq. (6).

2) Our results are comparable to other predictions. However, there still exists an uncertainty about the sign of  $g_2(0)$ . The sign of  $g_2(0)$  in this work is same as that derived by the LCSR method in Ref. [85] but different from those obtained by other quark models. More careful analysis should be performed to fix this problem.

3) The form factors  $f_3$  and  $g_3$  are not obtained in this work because we have taken the  $q^+ = 0$  frame, while another method adopted in Refs. [86, 87] may be applied to extract these form factors.

Table 4. A comparison with other results for the form factors at the maximum recoil  $q^2 = 0$ . The physical form factors are shown in “this work” with the help of Eq. (6).

	$f_1(0)$	$f_2(0)$	$f_3(0)$	$g_1(0)$	$g_2(0)$	$g_3(0)$
$\Lambda_c \rightarrow n$						
this work	0.513	-0.266	-	0.443	-0.034	-
quark model [39]	0.627	-0.259	0.179	0.433	0.118	-0.744
quark model [84]	0.470	-0.246	0.039	0.414	0.073	-0.328
$\Lambda_c \rightarrow \Lambda$						
this work	0.468	-0.222	-	0.407	-0.035	-
quark model [39]	0.700	-0.295	0.222	0.448	0.135	-0.832
quark model [84]	0.511	-0.289	-0.014	0.466	0.025	-0.400
LCSR [85]	0.517	-0.123	-	0.517	-0.123	-

Table 5. Form factors for charmed baryon decays. The plus sign is adopted in the fit formula Eq. (8) for those marked by asterisks, and the minus sign is adopted for all the others.

$F$	$F(0)$	$m_{\text{fit}}$	$\delta$	$F$	$F(0)$	$m_{\text{fit}}$	$\delta$
$f_1^{\Lambda_c^+ \rightarrow n}$	0.726	2.06	0.07	$f_2^{\Lambda_c^+ \rightarrow n}$	-0.376	1.71	0.21
$g_1^{\Lambda_c^+ \rightarrow n}$	0.626	2.40	0.19	$g_2^{\Lambda_c^+ \rightarrow n}$	-0.048	1.42	0.29
$f_1^{\Lambda_c^+ \rightarrow \Lambda}$	0.810	2.19	0.06	$f_2^{\Lambda_c^+ \rightarrow \Lambda}$	-0.384	1.81	0.20
$g_1^{\Lambda_c^+ \rightarrow \Lambda}$	0.705	2.52	0.16	$g_2^{\Lambda_c^+ \rightarrow \Lambda}$	-0.060	1.58	0.25
$f_1^{\Xi_c^+ \rightarrow \Sigma^0}$	0.717	2.00	0.11	$f_2^{\Xi_c^+ \rightarrow \Sigma^0}$	-0.421	1.69	0.22
$g_1^{\Xi_c^+ \rightarrow \Sigma^0}$	0.614	2.34	0.22	$g_2^{\Xi_c^+ \rightarrow \Sigma^0}$	-0.054	1.39	0.32
$f_1^{\Xi_c^+ \rightarrow \Lambda}$	0.717	2.00	0.11	$f_2^{\Xi_c^+ \rightarrow \Lambda}$	-0.421	1.69	0.22
$g_1^{\Xi_c^+ \rightarrow \Lambda}$	0.614	2.34	0.22	$g_2^{\Xi_c^+ \rightarrow \Lambda}$	-0.054	1.39	0.32
$f_1^{\Xi_c^+ \rightarrow \Xi^0}$	0.802	2.13	0.10	$f_2^{\Xi_c^+ \rightarrow \Xi^0}$	-0.431	1.79	0.22
$g_1^{\Xi_c^+ \rightarrow \Xi^0}$	0.694	2.46	0.19	$g_2^{\Xi_c^+ \rightarrow \Xi^0}$	-0.065	1.55	0.28
$f_1^{\Xi_c^0 \rightarrow \Sigma^-}$	0.717	2.00	0.11	$f_2^{\Xi_c^0 \rightarrow \Sigma^-}$	-0.422	1.69	0.22
$g_1^{\Xi_c^0 \rightarrow \Sigma^-}$	0.614	2.34	0.22	$g_2^{\Xi_c^0 \rightarrow \Sigma^-}$	-0.054	1.39	0.32
$f_1^{\Xi_c^0 \rightarrow \Xi^-}$	0.802	2.13	0.10	$f_2^{\Xi_c^0 \rightarrow \Xi^-}$	-0.432	1.79	0.22
$g_1^{\Xi_c^0 \rightarrow \Xi^-}$	0.694	2.46	0.19	$g_2^{\Xi_c^0 \rightarrow \Xi^-}$	-0.065	1.55	0.28
$f_1^{\Omega_c^0 \rightarrow \Xi^-}$	0.653	1.51	0.35	$f_2^{\Omega_c^0 \rightarrow \Xi^-}$	0.620	1.54	0.28
$g_1^{\Omega_c^0 \rightarrow \Xi^-}$	-0.182	1.92	0.08	$g_2^{\Omega_c^0 \rightarrow \Xi^-}$	0.002*	0.40*	0.14*

Table 6. Form factors for bottom baryon decays. The plus sign is adopted in the fit formula Eq. (8) for those marked by asterisks, and the minus sign is adopted for all the others.

$F$	$F(0)$	$m_{\text{fit}}$	$\delta$	$F$	$F(0)$	$m_{\text{fit}}$	$\delta$
$f_1^{\Lambda_b^0 \rightarrow p}$	0.282	4.66	0.30	$f_2^{\Lambda_b^0 \rightarrow p}$	-0.084	3.94	0.37
$g_1^{\Lambda_b^0 \rightarrow p}$	0.273	4.81	0.32	$g_2^{\Lambda_b^0 \rightarrow p}$	-0.012	3.67	0.37
$f_1^{\Lambda_b^0 \rightarrow \Lambda_c^+}$	0.670	5.62	0.23	$f_2^{\Lambda_b^0 \rightarrow \Lambda_c^+}$	-0.132	4.67	0.32
$g_1^{\Lambda_b^0 \rightarrow \Lambda_c^+}$	0.656	5.73	0.24	$g_2^{\Lambda_b^0 \rightarrow \Lambda_c^+}$	-0.012	4.37	0.28
$f_1^{\Xi_b^0 \rightarrow \Sigma^+}$	0.260	4.46	0.34	$f_2^{\Xi_b^0 \rightarrow \Sigma^+}$	-0.086	3.84	0.40
$g_1^{\Xi_b^0 \rightarrow \Sigma^+}$	0.251	4.60	0.36	$g_2^{\Xi_b^0 \rightarrow \Sigma^+}$	-0.012	3.56	0.41
$f_1^{\Xi_b^0 \rightarrow \Xi_c^+}$	0.654	5.42	0.27	$f_2^{\Xi_b^0 \rightarrow \Xi_c^+}$	-0.143	4.59	0.34
$g_1^{\Xi_b^0 \rightarrow \Xi_c^+}$	0.640	5.53	0.28	$g_2^{\Xi_b^0 \rightarrow \Xi_c^+}$	-0.015	4.33	0.30
$f_1^{\Xi_b^- \rightarrow \Sigma^0}$	0.260	4.46	0.34	$f_2^{\Xi_b^- \rightarrow \Sigma^0}$	-0.086	3.84	0.40
$g_1^{\Xi_b^- \rightarrow \Sigma^0}$	0.251	4.60	0.36	$g_2^{\Xi_b^- \rightarrow \Sigma^0}$	-0.012	3.56	0.41
$f_1^{\Xi_b^- \rightarrow \Lambda}$	0.260	4.46	0.34	$f_2^{\Xi_b^- \rightarrow \Lambda}$	-0.086	3.84	0.40
$g_1^{\Xi_b^- \rightarrow \Lambda}$	0.251	4.60	0.36	$g_2^{\Xi_b^- \rightarrow \Lambda}$	-0.012	3.56	0.41
$f_1^{\Xi_b^- \rightarrow \Xi_c^0}$	0.654	5.42	0.27	$f_2^{\Xi_b^- \rightarrow \Xi_c^0}$	-0.143	4.59	0.34
$g_1^{\Xi_b^- \rightarrow \Xi_c^0}$	0.640	5.53	0.28	$g_2^{\Xi_b^- \rightarrow \Xi_c^0}$	-0.015	4.33	0.30
$f_1^{\Omega_b^- \rightarrow \Xi^0}$	0.169	3.30	0.64	$f_2^{\Omega_b^- \rightarrow \Xi^0}$	0.193	3.45	0.49
$g_1^{\Omega_b^- \rightarrow \Xi^0}$	-0.033	4.38	0.20	$g_2^{\Omega_b^- \rightarrow \Xi^0}$	-0.041	4.32	0.65
$f_1^{\Omega_b^- \rightarrow \Omega_c^0}$	0.566	3.92	0.49	$f_2^{\Omega_b^- \rightarrow \Omega_c^0}$	0.531	4.08	0.41
$g_1^{\Omega_b^- \rightarrow \Omega_c^0}$	-0.170	4.80	0.23	$g_2^{\Omega_b^- \rightarrow \Omega_c^0}$	-0.031	9.02	5.05

4) Also note that, in Refs. [39, 84, 85], only a few channels are investigated but this work aims to give a comprehensive investigation of the heavy baryon decays. Only in this way can  $SU(3)$  symmetry and sources of  $SU(3)$  symmetry breaking be seen clearly.

### 3.3 Semi-leptonic results

The differential decay width for the semi-leptonic process reads

$$\frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2} + \frac{d\Gamma_T}{dq^2}, \quad (9)$$

with the polarized decay widths given as

$$\frac{d\Gamma_L}{dq^2} = \frac{G_F^2 |V_{CKM}|^2}{(2\pi)^3} \frac{q^2 p}{24M^2} (|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2), \quad (10)$$

$$\frac{d\Gamma_T}{dq^2} = \frac{G_F^2 |V_{CKM}|^2}{(2\pi)^3} \frac{q^2 p}{24M^2} (|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2). \quad (11)$$

Here the  $q^2$  is the lepton pair invariant mass,  $p = \sqrt{Q_+ Q_-}/2M$ ,  $Q_{\pm} = (M \pm M')^2 - q^2$ , and  $M$  ( $M'$ ) is the mass of the parent (daughter) baryon.

The helicity amplitudes are related to the form factors through the following expressions:

$$H_{\frac{1}{2},0}^V = -i \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left( (M+M')f_1 - \frac{q^2}{M}f_2 \right),$$

$$H_{\frac{1}{2},1}^V = i \sqrt{2Q_-} \left( -f_1 + \frac{M+M'}{M}f_2 \right),$$

$$H_{\frac{1}{2},0}^A = -i \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left( (M-M')g_1 + \frac{q^2}{M}g_2 \right),$$

$$H_{\frac{1}{2},1}^A = i \sqrt{2Q_+} \left( -g_1 - \frac{M-M'}{M}g_2 \right). \quad (12)$$

The negative helicity amplitudes are given as

$$H_{-\lambda',-\lambda_W}^V = H_{\lambda',\lambda_W}^V \quad \text{and} \quad H_{-\lambda',-\lambda_W}^A = -H_{\lambda',\lambda_W}^A. \quad (13)$$

The helicity amplitudes for the left-handed current are obtained as

$$H_{\lambda',\lambda_W} = H_{\lambda',\lambda_W}^V - H_{\lambda',\lambda_W}^A. \quad (14)$$

Numerical results are given in Tables 7 and 9. Comparisons with some recent works [51–53, 80, 81, 84, 88] and the experimental results [83] can be found in Tables 8 and 10.

### 3.4 Non-leptonic results

For non-leptonic decays, we are constrained to consider only the processes of a W boson emitting outward. The naive factorization assumption is employed [89, 90]. The decay width for the  $B \rightarrow B'P$  ( $P$  denotes a pseudoscalar meson) is given as

Table 7. Semi-leptonic decays for charmed baryons.

channels	$\Gamma/\text{GeV}$	$\mathcal{B}$	$\Gamma_L/\Gamma_T$
$\Lambda_c^+ \rightarrow ne^+\nu_e$	$6.62 \times 10^{-15}$	$2.01 \times 10^{-3}$	1.78
$\Lambda_c^+ \rightarrow \Lambda e^+\nu_e$	$5.36 \times 10^{-14}$	$1.63 \times 10^{-2}$	1.96
$\Xi_c^+ \rightarrow \Sigma^0 e^+\nu_e$	$2.79 \times 10^{-15}$	$1.87 \times 10^{-3}$	1.85
$\Xi_c^+ \rightarrow \Lambda e^+\nu_e$	$1.22 \times 10^{-15}$	$8.22 \times 10^{-4}$	1.79
$\Xi_c^+ \rightarrow \Xi^0 e^+\nu_e$	$8.03 \times 10^{-14}$	$5.39 \times 10^{-2}$	1.98
$\Xi_c^0 \rightarrow \Sigma^- e^+\nu_e$	$5.57 \times 10^{-15}$	$9.47 \times 10^{-4}$	1.86
$\Xi_c^0 \rightarrow \Xi^- e^+\nu_e$	$7.91 \times 10^{-14}$	$1.35 \times 10^{-2}$	1.98
$\Omega_c^0 \rightarrow \Xi^- e^+\nu_e$	$2.08 \times 10^{-15}$	$2.18 \times 10^{-4}$	7.94

Table 8. A comparison with some recent works for semi-leptonic charmed decays.

channel	this work	other theoretical predictions	experiment [83]
$\Lambda_c^+ \rightarrow ne^+\nu_e$	$2.01 \times 10^{-3}$	$(2.7 \pm 0.3) \times 10^{-3}$ [80], $2.07 \times 10^{-3}$ [84], $(4.10 \pm 0.26) \times 10^{-3}$ [53]	--
$\Lambda_c^+ \rightarrow \Lambda e^+\nu_e$	$1.63 \times 10^{-2}$	$2.72 \times 10^{-2}$ [88], $(3.80 \pm 0.22) \times 10^{-2}$ [52]	$(3.6 \pm 0.4) \times 10^{-2}$
$\Xi_c^+ \rightarrow \Sigma^0 e^+\nu_e$	$1.87 \times 10^{-3}$	$(0.8 \pm 0.1) \times 10^{-3}$ [80]	--
$\Xi_c^+ \rightarrow \Lambda e^+\nu_e$	$8.22 \times 10^{-4}$	$(2.5 \pm 0.4) \times 10^{-4}$ [80]	--
$\Xi_c^+ \rightarrow \Xi^0 e^+\nu_e$	$5.39 \times 10^{-2}$	$(3.38_{-2.26}^{+2.19}) \times 10^{-2}$ [81], $(3.0 \pm 0.5) \times 10^{-2}$ [80]	--
$\Xi_c^0 \rightarrow \Sigma^- e^+\nu_e$	$9.47 \times 10^{-4}$	$(60 \pm 8) \times 10^{-4}$ [80]	--
$\Xi_c^0 \rightarrow \Xi^- e^+\nu_e$	$1.35 \times 10^{-2}$	$(4.87 \pm 1.74) \times 10^{-2}$ [81], $(11.9 \pm 1.6) \times 10^{-2}$ [80]	--

Table 9. Semi-leptonic decays for bottom baryons.

channels	$\Gamma/\text{GeV}$	$\mathcal{B}$	$\Gamma_L/\Gamma_T$
$\Lambda_b^0 \rightarrow pe^-\bar{\nu}_e$	$1.41 \times 10^{-16}$	$3.14 \times 10^{-4}$	1.25
$\Lambda_b^0 \rightarrow \Lambda_c^+ e^-\bar{\nu}_e$	$3.96 \times 10^{-14}$	$8.83 \times 10^{-2}$	1.71
$\Xi_b^0 \rightarrow \Sigma^+ e^-\bar{\nu}_e$	$1.27 \times 10^{-16}$	$2.83 \times 10^{-4}$	1.27
$\Xi_b^0 \rightarrow \Xi_c^+ e^-\bar{\nu}_e$	$3.97 \times 10^{-14}$	$8.83 \times 10^{-2}$	1.70
$\Xi_b^- \rightarrow \Sigma^0 e^-\bar{\nu}_e$	$6.37 \times 10^{-17}$	$1.51 \times 10^{-4}$	1.27
$\Xi_b^- \rightarrow \Lambda e^-\bar{\nu}_e$	$2.29 \times 10^{-17}$	$5.42 \times 10^{-5}$	1.25
$\Xi_b^- \rightarrow \Xi_c^0 e^-\bar{\nu}_e$	$3.97 \times 10^{-14}$	$9.42 \times 10^{-2}$	1.70
$\Omega_b^- \rightarrow \Xi^0 e^-\bar{\nu}_e$	$1.18 \times 10^{-17}$	$2.82 \times 10^{-5}$	1.72
$\Omega_b^- \rightarrow \Omega_c^0 e^-\bar{\nu}_e$	$1.14 \times 10^{-14}$	$2.72 \times 10^{-2}$	6.26

Table 10. A comparison with some recent works for semi-leptonic bottom decays.

channel	this work	other theoretical predictions	experiment [83]
$\Lambda_b^0 \rightarrow pe^-\bar{\nu}_e$	$3.14 \times 10^{-4}$	$2.9 \times 10^{-4}$ [84], $(4.80 \pm 0.99) \times 10^{-4}$ [51]	$(4.1 \pm 1.0) \times 10^{-4}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ e^-\bar{\nu}_e$	$8.83 \times 10^{-2}$	$6.9 \times 10^{-2}$ [88], $(5.32 \pm 0.35) \times 10^{-2}$ [51]	$(6.2_{-1.3}^{+1.4}) \times 10^{-2}$

$$\Gamma = \frac{p}{8\pi} \left( \frac{(M+M')^2 - m^2}{M^2} |A|^2 + \frac{(M-M')^2 - m^2}{M^2} |B|^2 \right), \quad (15)$$

where  $p$  is the magnitude of the three-momentum of the daughter baryon  $B'$  in the rest frame of the parent baryon  $B$ .  $M$  ( $M'$ ) is the mass of the parent (daughter) baryon. For  $B \rightarrow B'V(A)$  ( $V$  denotes a vector meson while  $A$  denotes an axial-vector meson) decay, the decay width is

$$\Gamma = \frac{p(E'+M')}{4\pi M} \left( 2(|S|^2 + |P_2|^2) + \frac{E^2}{m^2} (|S+D|^2 + |P_1|^2) \right), \quad (16)$$

where  $E$  ( $E'$ ) is the energy of the meson (daughter bary-

on) in the final state, and

$$\begin{aligned} S &= -A_1, \\ P_1 &= -\frac{p}{E} \left( \frac{M+M'}{E'+M'} B_1 + B_2 \right), \\ P_2 &= \frac{p}{E'+M'} B_1, \\ D &= -\frac{p^2}{E(E'+M')} (A_1 - A_2). \end{aligned}$$

$A$ ,  $B$ ,  $A_{1,2}$  and  $B_{1,2}$  are given as

$$\begin{aligned} A &= -\lambda f_P (M-M') f_1(m^2), \\ B &= -\lambda f_P (M+M') g_1(m^2), \end{aligned}$$

$$\begin{aligned}
 A_1 &= -\lambda f_V m \left[ g_1(m^2) + g_2(m^2) \frac{M-M'}{M} \right], \\
 A_2 &= -2\lambda f_V m g_2(m^2), \\
 B_1 &= \lambda f_V m \left[ f_1(m^2) - f_2(m^2) \frac{M+M'}{M} \right], \\
 B_2 &= 2\lambda f_V m f_2(m^2).
 \end{aligned} \tag{17}$$

Here  $\lambda = \frac{G_F}{\sqrt{2}} V_{CKM} V_{q_1 q_2}^* a_1$  with  $a_1 = C_1(\mu_c) + C_2(\mu_c)/3 = 1.07$  [91], the first CKM matrix element corresponds to the process of  $B \rightarrow B'$  and the second comes from the production of the meson.  $M(M')$  is the mass of the parent (daughter) baryon and  $m$  is the mass of the emitted meson. For the decay mode with an axial-vector meson involved,  $f_V$  should be replaced by  $-f_A$  in the expressions of  $A_{1,2}$  and  $B_{1,2}$  in Eqs. (17).

Note that the P-wave meson  $a_1$  emission case is included. The naive factorization can still work for these processes [92].

The masses of the mesons in the final states can be taken from Ref. [83]. The decay constants are adopted as [61, 74, 93]

$$\begin{aligned}
 f_\pi &= 130.4 \text{ MeV}, & f_\rho &= 216 \text{ MeV}, & f_{a_1} &= 238 \text{ MeV}, \\
 f_K &= 160 \text{ MeV}, & f_{K^*} &= 210 \text{ MeV}, \\
 f_D &= 207.4 \text{ MeV}, & f_{D^*} &= 220 \text{ MeV}, \\
 f_{D_s} &= 247.2 \text{ MeV}, & f_{D_s^*} &= 247.2 \text{ MeV}.
 \end{aligned} \tag{18}$$

The numerical results are given in Tables 11, 13 and 14. Comparisons with some recent works [54, 81] and the experimental results [83] can be found in Table 12 and Table 15.

Table 11. Non-leptonic decays for charmed baryons.

channels	$\Gamma/\text{GeV}$	$\mathcal{B}$	channels	$\Gamma/\text{GeV}$	$\mathcal{B}$
$\Lambda_c^+ \rightarrow n\pi^+$	$3.97 \times 10^{-15}$	$1.21 \times 10^{-3}$	$\Lambda_c^+ \rightarrow n\rho^+$	$1.33 \times 10^{-14}$	$4.04 \times 10^{-3}$
$\Lambda_c^+ \rightarrow na_1^+$	$9.88 \times 10^{-15}$	$3.00 \times 10^{-3}$	$\Lambda_c^+ \rightarrow nK^+$	$3.04 \times 10^{-16}$	$9.23 \times 10^{-5}$
$\Lambda_c^+ \rightarrow nK^{*+}$	$6.71 \times 10^{-16}$	$2.04 \times 10^{-4}$	$\Lambda_c^+ \rightarrow \Lambda\rho^+$	$1.39 \times 10^{-13}$	$4.24 \times 10^{-2}$
$\Lambda_c^+ \rightarrow \Lambda\pi^+$	$4.77 \times 10^{-14}$	$1.45 \times 10^{-2}$	$\Lambda_c^+ \rightarrow \Lambda K^+$	$3.47 \times 10^{-15}$	$1.05 \times 10^{-3}$
$\Lambda_c^+ \rightarrow \Lambda K^{*+}$	$6.47 \times 10^{-15}$	$1.97 \times 10^{-3}$	$\Xi_c^+ \rightarrow \Sigma^0 \rho^+$	$6.21 \times 10^{-15}$	$4.17 \times 10^{-3}$
$\Xi_c^+ \rightarrow \Sigma^0 \pi^+$	$1.90 \times 10^{-15}$	$1.28 \times 10^{-3}$	$\Xi_c^+ \rightarrow \Sigma^0 K^+$	$1.45 \times 10^{-16}$	$9.74 \times 10^{-5}$
$\Xi_c^+ \rightarrow \Sigma^0 a_1^+$	$2.96 \times 10^{-15}$	$1.99 \times 10^{-3}$	$\Xi_c^+ \rightarrow \Lambda\rho^+$	$2.41 \times 10^{-15}$	$1.62 \times 10^{-3}$
$\Xi_c^+ \rightarrow \Sigma^0 K^{*+}$	$3.08 \times 10^{-16}$	$2.07 \times 10^{-4}$	$\Xi_c^+ \rightarrow \Lambda K^+$	$5.49 \times 10^{-17}$	$3.69 \times 10^{-5}$
$\Xi_c^+ \rightarrow \Lambda\pi^+$	$7.09 \times 10^{-16}$	$4.76 \times 10^{-4}$	$\Xi_c^+ \rightarrow \Xi^0 \rho^+$	$2.13 \times 10^{-13}$	$1.43 \times 10^{-1}$
$\Xi_c^+ \rightarrow \Lambda a_1^+$	$1.91 \times 10^{-15}$	$1.29 \times 10^{-3}$	$\Xi_c^+ \rightarrow \Xi^0 K^+$	$5.33 \times 10^{-15}$	$3.58 \times 10^{-3}$
$\Xi_c^+ \rightarrow \Lambda K^{*+}$	$1.23 \times 10^{-16}$	$8.24 \times 10^{-5}$	$\Xi_c^+ \rightarrow \Sigma^- \rho^+$	$1.24 \times 10^{-14}$	$2.11 \times 10^{-3}$
$\Xi_c^+ \rightarrow \Xi^0 \pi^+$	$7.30 \times 10^{-14}$	$4.91 \times 10^{-2}$	$\Xi_c^+ \rightarrow \Sigma^- K^+$	$2.90 \times 10^{-16}$	$4.93 \times 10^{-5}$
$\Xi_c^+ \rightarrow \Xi^0 K^{*+}$	$9.85 \times 10^{-15}$	$6.62 \times 10^{-3}$	$\Xi_c^0 \rightarrow \Xi^- \rho^+$	$2.11 \times 10^{-13}$	$3.60 \times 10^{-2}$
$\Xi_c^0 \rightarrow \Sigma^- \pi^+$	$3.80 \times 10^{-15}$	$6.46 \times 10^{-4}$	$\Xi_c^0 \rightarrow \Xi^- K^+$	$5.29 \times 10^{-15}$	$9.01 \times 10^{-4}$
$\Xi_c^0 \rightarrow \Sigma^- a_1^+$	$5.85 \times 10^{-15}$	$9.95 \times 10^{-4}$	$\Omega_c^0 \rightarrow \Xi^- \rho^+$	$4.28 \times 10^{-15}$	$4.49 \times 10^{-4}$
$\Xi_c^0 \rightarrow \Sigma^- K^{*+}$	$6.16 \times 10^{-16}$	$1.05 \times 10^{-4}$	$\Omega_c^0 \rightarrow \Xi^- K^+$	$1.49 \times 10^{-16}$	$1.57 \times 10^{-5}$
$\Xi_c^0 \rightarrow \Xi^- \pi^+$	$7.26 \times 10^{-14}$	$1.24 \times 10^{-2}$			
$\Xi_c^0 \rightarrow \Xi^- K^{*+}$	$9.73 \times 10^{-15}$	$1.66 \times 10^{-3}$			
$\Omega_c^0 \rightarrow \Xi^- \pi^+$	$1.68 \times 10^{-15}$	$1.76 \times 10^{-4}$			
$\Omega_c^0 \rightarrow \Xi^- a_1^+$	$2.51 \times 10^{-15}$	$2.63 \times 10^{-4}$			
$\Omega_c^0 \rightarrow \Xi^- K^{*+}$	$2.03 \times 10^{-16}$	$2.13 \times 10^{-5}$			

Table 12. A comparison with some recent works for non-leptonic charmed decays.

channel	this work	Ref. [81]	experiment [83]
$\Lambda_c^+ \rightarrow \Lambda\pi^+$	$1.45 \times 10^{-2}$	--	$(1.30 \pm 0.07) \times 10^{-2}$
$\Lambda_c^+ \rightarrow \Lambda\rho^+$	$4.24 \times 10^{-2}$	--	$< 6 \times 10^{-2}$
$\Lambda_c^+ \rightarrow \Lambda K^+$	$1.05 \times 10^{-3}$	--	$(0.61 \pm 0.12) \times 10^{-3}$
$\Xi_c^+ \rightarrow \Xi^0 \pi^+$	$4.91 \times 10^{-2}$	$(0.81 \pm 0.40) \times 10^{-2}$	--
$\Xi_c^0 \rightarrow \Xi^- \pi^+$	$1.24 \times 10^{-2}$	$(1.57 \pm 0.07) \times 10^{-2}$	--



Table 13. Non-leptonic decays for  $\Lambda_b$  and  $\Xi_b$ .

channels	$\Gamma/\text{GeV}$	$\mathcal{B}$	channels	$\Gamma/\text{GeV}$	$\mathcal{B}$
$\Lambda_b^0 \rightarrow p\pi^-$	$3.99 \times 10^{-18}$	$8.90 \times 10^{-6}$	$\Lambda_b^0 \rightarrow p\rho^-$	$1.17 \times 10^{-17}$	$2.61 \times 10^{-5}$
$\Lambda_b^0 \rightarrow pa_1^-$	$1.56 \times 10^{-17}$	$3.48 \times 10^{-5}$	$\Lambda_b^0 \rightarrow pK^-$	$3.22 \times 10^{-19}$	$7.18 \times 10^{-7}$
$\Lambda_b^0 \rightarrow pK^{*-}$	$6.02 \times 10^{-19}$	$1.34 \times 10^{-6}$	$\Lambda_b^0 \rightarrow pD^-$	$5.76 \times 10^{-19}$	$1.28 \times 10^{-6}$
$\Lambda_b^0 \rightarrow pD^{*-}$	$8.95 \times 10^{-19}$	$1.99 \times 10^{-6}$	$\Lambda_b^0 \rightarrow pD_s^-$	$1.54 \times 10^{-17}$	$3.44 \times 10^{-5}$
$\Lambda_b^0 \rightarrow pD_s^{*-}$	$2.19 \times 10^{-17}$	$4.88 \times 10^{-5}$			
$\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$	$3.83 \times 10^{-15}$	$8.53 \times 10^{-3}$	$\Lambda_b^0 \rightarrow \Lambda_c^+ \rho^-$	$1.09 \times 10^{-14}$	$2.44 \times 10^{-2}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ a_1^-$	$1.40 \times 10^{-14}$	$3.12 \times 10^{-2}$	$\Lambda_b^0 \rightarrow \Lambda_c^+ K^-$	$3.04 \times 10^{-16}$	$6.78 \times 10^{-4}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ K^{*-}$	$5.59 \times 10^{-16}$	$1.24 \times 10^{-3}$	$\Lambda_b^0 \rightarrow \Lambda_c^+ D^-$	$4.26 \times 10^{-16}$	$9.49 \times 10^{-4}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ D^{*-}$	$6.90 \times 10^{-16}$	$1.54 \times 10^{-3}$	$\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$	$1.10 \times 10^{-14}$	$2.46 \times 10^{-2}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^{*-}$	$1.64 \times 10^{-14}$	$3.65 \times 10^{-2}$			
$\Xi_b^0 \rightarrow \Sigma^+ \pi^-$	$3.55 \times 10^{-18}$	$7.91 \times 10^{-6}$	$\Xi_b^0 \rightarrow \Sigma^+ \rho^-$	$1.05 \times 10^{-17}$	$2.33 \times 10^{-5}$
$\Xi_b^0 \rightarrow \Sigma^+ a_1^-$	$1.41 \times 10^{-17}$	$3.13 \times 10^{-5}$	$\Xi_b^0 \rightarrow \Sigma^+ K^-$	$2.87 \times 10^{-19}$	$6.40 \times 10^{-7}$
$\Xi_b^0 \rightarrow \Sigma^+ K^{*-}$	$5.39 \times 10^{-19}$	$1.20 \times 10^{-6}$	$\Xi_b^0 \rightarrow \Sigma^+ D^-$	$5.30 \times 10^{-19}$	$1.18 \times 10^{-6}$
$\Xi_b^0 \rightarrow \Sigma^+ D^{*-}$	$8.23 \times 10^{-19}$	$1.83 \times 10^{-6}$	$\Xi_b^0 \rightarrow \Sigma^+ D_s^-$	$1.42 \times 10^{-17}$	$3.17 \times 10^{-5}$
$\Xi_b^0 \rightarrow \Sigma^+ D_s^{*-}$	$2.02 \times 10^{-17}$	$4.50 \times 10^{-5}$			
$\Xi_b^0 \rightarrow \Xi_c^+ \pi^-$	$3.76 \times 10^{-15}$	$8.37 \times 10^{-3}$	$\Xi_b^0 \rightarrow \Xi_c^+ \rho^-$	$1.08 \times 10^{-14}$	$2.40 \times 10^{-2}$
$\Xi_b^0 \rightarrow \Xi_c^+ a_1^-$	$1.38 \times 10^{-14}$	$3.08 \times 10^{-2}$	$\Xi_b^0 \rightarrow \Xi_c^+ K^-$	$3.00 \times 10^{-16}$	$6.67 \times 10^{-4}$
$\Xi_b^0 \rightarrow \Xi_c^+ K^{*-}$	$5.51 \times 10^{-16}$	$1.23 \times 10^{-3}$	$\Xi_b^0 \rightarrow \Xi_c^+ D^-$	$4.26 \times 10^{-16}$	$9.49 \times 10^{-4}$
$\Xi_b^0 \rightarrow \Xi_c^+ D^{*-}$	$6.90 \times 10^{-16}$	$1.54 \times 10^{-3}$	$\Xi_b^0 \rightarrow \Xi_c^+ D_s^-$	$1.11 \times 10^{-14}$	$2.46 \times 10^{-2}$
$\Xi_b^0 \rightarrow \Xi_c^+ D_s^{*-}$	$1.64 \times 10^{-14}$	$3.65 \times 10^{-2}$			
$\Xi_b^- \rightarrow \Sigma^0 \pi^-$	$1.78 \times 10^{-18}$	$4.22 \times 10^{-6}$	$\Xi_b^- \rightarrow \Sigma^0 \rho^-$	$5.23 \times 10^{-18}$	$1.24 \times 10^{-5}$
$\Xi_b^- \rightarrow \Sigma^0 a_1^-$	$7.03 \times 10^{-18}$	$1.67 \times 10^{-5}$	$\Xi_b^- \rightarrow \Sigma^0 K^-$	$1.44 \times 10^{-19}$	$3.41 \times 10^{-7}$
$\Xi_b^- \rightarrow \Sigma^0 K^{*-}$	$2.70 \times 10^{-19}$	$6.40 \times 10^{-7}$	$\Xi_b^- \rightarrow \Sigma^0 D^-$	$2.65 \times 10^{-19}$	$6.29 \times 10^{-7}$
$\Xi_b^- \rightarrow \Sigma^0 D^{*-}$	$4.12 \times 10^{-19}$	$9.76 \times 10^{-7}$	$\Xi_b^- \rightarrow \Sigma^0 D_s^-$	$7.13 \times 10^{-18}$	$1.69 \times 10^{-5}$
$\Xi_b^- \rightarrow \Sigma^0 D_s^{*-}$	$1.01 \times 10^{-17}$	$2.40 \times 10^{-5}$			
$\Xi_b^- \rightarrow \Lambda \pi^-$	$6.03 \times 10^{-19}$	$1.43 \times 10^{-6}$	$\Xi_b^- \rightarrow \Lambda \rho^-$	$1.77 \times 10^{-18}$	$4.21 \times 10^{-6}$
$\Xi_b^- \rightarrow \Lambda a_1^-$	$2.39 \times 10^{-18}$	$5.66 \times 10^{-6}$	$\Xi_b^- \rightarrow \Lambda K^-$	$4.88 \times 10^{-20}$	$1.16 \times 10^{-7}$
$\Xi_b^- \rightarrow \Lambda K^{*-}$	$9.15 \times 10^{-20}$	$2.17 \times 10^{-7}$	$\Xi_b^- \rightarrow \Lambda D^-$	$9.02 \times 10^{-20}$	$2.14 \times 10^{-7}$
$\Xi_b^- \rightarrow \Lambda D^{*-}$	$1.40 \times 10^{-19}$	$3.32 \times 10^{-7}$	$\Xi_b^- \rightarrow \Lambda D_s^-$	$2.43 \times 10^{-18}$	$5.75 \times 10^{-6}$
$\Xi_b^- \rightarrow \Lambda D_s^{*-}$	$3.44 \times 10^{-18}$	$8.16 \times 10^{-6}$			
$\Xi_b^- \rightarrow \Xi_c^0 \pi^-$	$3.76 \times 10^{-15}$	$8.93 \times 10^{-3}$	$\Xi_b^- \rightarrow \Xi_c^0 \rho^-$	$1.08 \times 10^{-14}$	$2.56 \times 10^{-2}$
$\Xi_b^- \rightarrow \Xi_c^0 a_1^-$	$1.38 \times 10^{-14}$	$3.28 \times 10^{-2}$	$\Xi_b^- \rightarrow \Xi_c^0 K^-$	$3.00 \times 10^{-16}$	$7.11 \times 10^{-4}$
$\Xi_b^- \rightarrow \Xi_c^0 K^{*-}$	$5.51 \times 10^{-16}$	$1.31 \times 10^{-3}$	$\Xi_b^- \rightarrow \Xi_c^0 D^-$	$4.27 \times 10^{-16}$	$1.01 \times 10^{-3}$
$\Xi_b^- \rightarrow \Xi_c^0 D^{*-}$	$6.91 \times 10^{-16}$	$1.64 \times 10^{-3}$	$\Xi_b^- \rightarrow \Xi_c^0 D_s^-$	$1.11 \times 10^{-14}$	$2.62 \times 10^{-2}$
$\Xi_b^- \rightarrow \Xi_c^0 D_s^{*-}$	$1.64 \times 10^{-14}$	$3.90 \times 10^{-2}$			

Table 14. Non-leptonic decays for  $\Omega_b$ .

channels	$\Gamma/\text{GeV}$	$\mathcal{B}$	channels	$\Gamma/\text{GeV}$	$\mathcal{B}$
$\Omega_b^- \rightarrow \Xi^0 \pi^-$	$6.05 \times 10^{-19}$	$1.44 \times 10^{-6}$	$\Omega_b^- \rightarrow \Xi^0 \rho^-$	$1.73 \times 10^{-18}$	$4.13 \times 10^{-6}$
$\Omega_b^- \rightarrow \Xi^0 a_1^-$	$2.23 \times 10^{-18}$	$5.33 \times 10^{-6}$	$\Omega_b^- \rightarrow \Xi^0 K^-$	$5.00 \times 10^{-20}$	$1.19 \times 10^{-7}$
$\Omega_b^- \rightarrow \Xi^0 K^{*-}$	$8.86 \times 10^{-20}$	$2.11 \times 10^{-7}$	$\Omega_b^- \rightarrow \Xi^0 D^-$	$1.21 \times 10^{-19}$	$2.88 \times 10^{-7}$
$\Omega_b^- \rightarrow \Xi^0 D^{*-}$	$1.13 \times 10^{-19}$	$2.69 \times 10^{-7}$	$\Omega_b^- \rightarrow \Xi^0 D_s^-$	$3.32 \times 10^{-18}$	$7.91 \times 10^{-6}$
$\Omega_b^- \rightarrow \Xi^0 D_s^{*-}$	$2.68 \times 10^{-18}$	$6.40 \times 10^{-6}$			
$\Omega_b^- \rightarrow \Omega_c^0 \pi^-$	$1.68 \times 10^{-15}$	$4.00 \times 10^{-3}$	$\Omega_b^- \rightarrow \Omega_c^0 \rho^-$	$4.54 \times 10^{-15}$	$1.08 \times 10^{-2}$
$\Omega_b^- \rightarrow \Omega_c^0 a_1^-$	$5.37 \times 10^{-15}$	$1.28 \times 10^{-2}$	$\Omega_b^- \rightarrow \Omega_c^0 K^-$	$1.36 \times 10^{-16}$	$3.26 \times 10^{-4}$
$\Omega_b^- \rightarrow \Omega_c^0 K^{*-}$	$2.28 \times 10^{-16}$	$5.44 \times 10^{-4}$	$\Omega_b^- \rightarrow \Omega_c^0 D^-$	$2.66 \times 10^{-16}$	$6.36 \times 10^{-4}$
$\Omega_b^- \rightarrow \Omega_c^0 D^{*-}$	$2.14 \times 10^{-16}$	$5.11 \times 10^{-4}$	$\Omega_b^- \rightarrow \Omega_c^0 D_s^-$	$7.17 \times 10^{-15}$	$1.71 \times 10^{-2}$
$\Omega_b^- \rightarrow \Omega_c^0 D_s^{*-}$	$4.90 \times 10^{-15}$	$1.17 \times 10^{-2}$			

Table 15. A comparison with some recent works for non-leptonic bottom decays.

channel	this work	Ref. [54]	experiment [83]
$\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-$	$8.53 \times 10^{-3}$	$(2.85 \pm 0.54) \times 10^{-3}$	$(4.9 \pm 0.4) \times 10^{-3}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ \rho^-$	$2.44 \times 10^{-2}$	$(0.817 \pm 0.147) \times 10^{-2}$	--
$\Lambda_b^0 \rightarrow \Lambda_c^+ a_1^-$	$3.12 \times 10^{-2}$	$(1.047 \pm 0.178) \times 10^{-2}$	--
$\Lambda_b^0 \rightarrow \Lambda_c^+ K^-$	$6.78 \times 10^{-4}$	$(2.21 \pm 0.40) \times 10^{-4}$	$(3.59 \pm 0.30) \times 10^{-4}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ K^{*-}$	$1.24 \times 10^{-3}$	$(0.422 \pm 0.075) \times 10^{-3}$	--
$\Lambda_b^0 \rightarrow \Lambda_c^+ D^-$	$9.49 \times 10^{-4}$	--	$(4.6 \pm 0.6) \times 10^{-4}$
$\Lambda_b^0 \rightarrow \Lambda_c^+ D_s^-$	$2.46 \times 10^{-2}$	--	$(1.10 \pm 0.10) \times 10^{-2}$
$\Lambda_b^0 \rightarrow p \pi^-$	$8.90 \times 10^{-6}$	--	$(4.2 \pm 0.8) \times 10^{-6}$
$\Lambda_b^0 \rightarrow p K^-$	$7.18 \times 10^{-7}$	--	$(51 \pm 10) \times 10^{-7}$
$\Lambda_b^0 \rightarrow p D_s^-$	$3.44 \times 10^{-5}$	--	$< 48 \times 10^{-5}$

### 3.5 SU(3) analysis for semi-leptonic decays

From the overlap factors above, we would expect the following relations

$$\begin{aligned}
 \frac{2\Gamma(\Lambda_c^+ \rightarrow n e^+ \nu_e)}{|V_{cd}|^2} &= \frac{3\Gamma(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e)}{|V_{cs}|^2} = \frac{4\Gamma(\Xi_c^+ \rightarrow \Sigma^0 e^+ \nu_e)}{|V_{cd}|^2} \\
 &= \frac{12\Gamma(\Xi_c^+ \rightarrow \Lambda e^+ \nu_e)}{|V_{cd}|^2} = \frac{2\Gamma(\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e)}{|V_{cs}|^2} \\
 &= \frac{2\Gamma(\Xi_c^0 \rightarrow \Sigma^- e^+ \nu_e)}{|V_{cd}|^2} = \frac{2\Gamma(\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e)}{|V_{cs}|^2}
 \end{aligned} \tag{19}$$

for the c-baryon sector and

$$\begin{aligned}
 \Gamma(\Lambda_b^0 \rightarrow p e^- \bar{\nu}_e) &= \Gamma(\Xi_b^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e) = 2\Gamma(\Xi_b^- \rightarrow \Sigma^0 e^- \bar{\nu}_e) \\
 &= 6\Gamma(\Xi_b^- \rightarrow \Lambda e^- \bar{\nu}_e), \\
 \Gamma(\Lambda_b^0 \rightarrow \Lambda_c^+ e^- \bar{\nu}_e) &= \Gamma(\Xi_b^0 \rightarrow \Xi_c^+ e^- \bar{\nu}_e) = \Gamma(\Xi_b^- \rightarrow \Xi_c^0 e^- \bar{\nu}_e)
 \end{aligned} \tag{20}$$

for the b-baryon sector, if the flavor SU(3) symmetry is respected. These relations for the charmed baryons are consistent with those in Refs. [79, 80], while the ones for the bottom baryons, as far as we know, are first derived by this work.

In the following, we will investigate the SU(3) symmetry breaking effects. The corresponding results are collected in Tables 16 and 17. Taking  $\Lambda_c^+ \rightarrow n e^+ \nu_e$  and  $\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$  as examples, we can see that:

1) If we have considered the differences of CKM and the overlap factors between these two channels but take all the other parameters as the same, we get the precise SU(3) symmetry prediction  $\Gamma(\Lambda_c^+ \rightarrow n e^+ \nu_e) / (\frac{1}{2}|V_{cd}|^2) = \Gamma(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) / (\frac{1}{3}|V_{cs}|^2)$ . This prediction is also obtained in Refs. [79, 80].

2) If we consider only the difference of daughter baryon mass but take all the other parameters as the same, we get a ratio 0.538. This means that SU(3) symmetry is broken by about 50% between these two modes. The more accurate number is 35% (see Table 16), when all the other relevant impacts are taken into account.

We can see from Tables 16 and 17:

1) The SU(3) symmetry breaking is sizable for c-baryon decays while it is small for the b-baryon decays. This can be understood as due to a much smaller phase space in c-baryon decays, and thus the decay width significantly depends on the mass differences of the baryons in the initial and final states.

2) SU(3) symmetry is broken more severely in the  $c \rightarrow s$  processes than in the  $c \rightarrow d$  processes because of the larger mass of the s quark than the u and d quarks. The typical value of SU(3) symmetry breaking for  $c \rightarrow s$  processes is 35% while that for  $c \rightarrow d$  processes is 15%.

Table 16. Quantitative predictions of SU(3) breaking for semi-leptonic charmed decays.

channels	$\Gamma/\text{GeV(LFQM)}$	$\Gamma/\text{GeV}(SU(3))$	$ \text{LFQM}-SU(3) /SU(3)$
$\Lambda_c^+ \rightarrow n e^+ \nu_e$	$6.62 \times 10^{-15}$	$6.62 \times 10^{-15}$	--
$\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e$	$5.36 \times 10^{-14}$	$8.27 \times 10^{-14}$	35%
$\Xi_c^+ \rightarrow \Sigma^0 e^+ \nu_e$	$2.79 \times 10^{-15}$	$3.31 \times 10^{-15}$	16%
$\Xi_c^+ \rightarrow \Lambda e^+ \nu_e$	$1.22 \times 10^{-15}$	$1.10 \times 10^{-15}$	11%
$\Xi_c^+ \rightarrow \Xi^0 e^+ \nu_e$	$8.03 \times 10^{-14}$	$1.24 \times 10^{-13}$	35%
$\Xi_c^0 \rightarrow \Sigma^- e^+ \nu_e$	$5.57 \times 10^{-15}$	$6.62 \times 10^{-15}$	16%
$\Xi_c^0 \rightarrow \Xi^- e^+ \nu_e$	$7.91 \times 10^{-14}$	$1.24 \times 10^{-13}$	36%

Table 17. Quantitative predictions of SU(3) breaking for semi-leptonic bottom decays.

channels	$\Gamma/\text{GeV(LFQM)}$	$\Gamma/\text{GeV}(SU(3))$	$ \text{LFQM}-SU(3) /SU(3)$
$\Lambda_b^0 \rightarrow p e^- \bar{\nu}_e$	$1.41 \times 10^{-16}$	$1.41 \times 10^{-16}$	--
$\Xi_b^0 \rightarrow \Sigma^+ e^- \bar{\nu}_e$	$1.27 \times 10^{-16}$	$1.41 \times 10^{-16}$	9.9%
$\Xi_b^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$	$6.37 \times 10^{-17}$	$7.05 \times 10^{-17}$	9.6%
$\Xi_b^- \rightarrow \Lambda e^- \bar{\nu}_e$	$2.29 \times 10^{-17}$	$2.35 \times 10^{-17}$	2.6%
$\Lambda_b^0 \rightarrow \Lambda_c^+ e^- \bar{\nu}_e$	$3.96 \times 10^{-14}$	$3.96 \times 10^{-14}$	--
$\Xi_b^0 \rightarrow \Xi_c^+ e^- \bar{\nu}_e$	$3.97 \times 10^{-14}$	$3.96 \times 10^{-14}$	0.25%
$\Xi_b^- \rightarrow \Xi_c^0 e^- \bar{\nu}_e$	$3.97 \times 10^{-14}$	$3.96 \times 10^{-14}$	0.25%

### 3.6 Uncertainties

In this subsection, we will look more carefully at the dependence of our results on the model parameters, taking the  $\Lambda_c^+ \rightarrow \Lambda$  transition as an example. Varying the model parameters  $m_{di} = m_{[ud]}$ ,  $\beta_i = \beta_{c[ud]}$  and  $\beta_f = \beta_{s[ud]}$  by 10% respectively, we arrive at the following error es-

timates for form factors

$$\begin{aligned}
 f_1^{\Lambda_c^+ \rightarrow \Lambda}(0) &= 0.810 \pm 0.007 \pm 0.008 \pm 0.033, \\
 f_2^{\Lambda_c^+ \rightarrow \Lambda}(0) &= -0.384 \pm 0.008 \pm 0.008 \pm 0.015, \\
 g_1^{\Lambda_c^+ \rightarrow \Lambda}(0) &= 0.705 \pm 0.006 \pm 0.015 \pm 0.022, \\
 g_2^{\Lambda_c^+ \rightarrow \Lambda}(0) &= -0.060 \pm 0.003 \pm 0.041 \pm 0.041 \quad (21)
 \end{aligned}$$

and for decay widths

$$\begin{aligned}
 \Gamma(\Lambda_c^+ \rightarrow \Lambda e^+ \nu_e) &= (5.36 \pm 0.03 \pm 0.10 \pm 0.07) \times 10^{-14}, \\
 \Gamma(\Lambda_c^+ \rightarrow \Lambda \pi^+) &= (4.77 \pm 0.08 \pm 0.14 \pm 0.34) \times 10^{-14}, \\
 \Gamma(\Lambda_c^+ \rightarrow \Lambda \rho^+) &= (1.39 \pm 0.01 \pm 0.02 \pm 0.02) \times 10^{-13}, \\
 \Gamma(\Lambda_c^+ \rightarrow \Lambda K^+) &= (3.47 \pm 0.04 \pm 0.11 \pm 0.22) \times 10^{-15}, \\
 \Gamma(\Lambda_c^+ \rightarrow \Lambda K^{*+}) &= (6.47 \pm 0.02 \pm 0.10 \pm 0.19) \times 10^{-15}. \quad (22)
 \end{aligned}$$

Some comments are in order.

1) None of the form factors are very sensitive to the diquark mass  $m_{di}$ .

2)  $g_2$  is one order of magnitude smaller than the other form factors, and it is sensitive to  $\beta_i$  and  $\beta_f$ , while  $f_1$ ,  $f_2$  and  $g_1$  are still not very sensitive to these shape parameters.

3) It can be seen from Eq. (22) that these decay widths are not sensitive to the model parameters. There exists at most about 10% deviation in these decay widths.

## 4 Conclusion

In this work, we have calculated the transition form factors of the singly heavy baryons using the light-front approach under the diquark picture. These form factors are then used to predict semi-leptonic and non-leptonic decays of singly heavy baryons. Most of our results are comparable to the available experimental data and other theoretical results. We have also derived the overlap factors that can be used to reproduce the  $SU(3)$  predictions on semi-leptonic decays. Using the calculated form factors, the  $SU(3)$  symmetry breaking is sizable in the charmed baryon decays while in the bottom baryon case, the  $SU(3)$  symmetry breaking is small. Most of the results in this work can be examined at experimental facilities at BEPCII, LHC or Belle II.

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## Appendices A: Wave functions in initial and final states

### Wave functions in the standard flavor-spin basis

The wave functions in flavor space can also be found in Ref. [94]. The wave functions of the singly heavy baryons in the initial states in the standard flavor-spin basis are given as follows.

For  $\mathcal{B}_{cq}^6$  ( $\Sigma_c^{++0}$  and  $\Omega_c^0$ ), we have

$$|\mathcal{B}_{cq}^6, \uparrow\rangle = (qqc) \left( \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \right), \quad (A1)$$

where  $q=u, d, s$  for  $\Sigma_c^{++0}$  and  $\Omega_c^0$ , respectively.

For  $\mathcal{B}_{cq_1q_2}^6$  ( $\Sigma_c^+$  and  $\Xi_c^{+,0}$ ), we have

$$|\mathcal{B}_{cq_1q_2}^6, \uparrow\rangle = \left( \frac{1}{\sqrt{2}} (q_1q_2 + q_2q_1)c \right) \left( \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \right), \quad (A2)$$

where  $(q_1, q_2) = (u, d), (u, s), (d, s)$  for  $\Sigma_c^+$  and  $\Xi_c^{+,0}$ , respectively.

For  $\mathcal{B}_{cq_1q_2}^3$  ( $\Lambda_c^+$  and  $\Xi_c^{+,0}$ ), we have

$$|\mathcal{B}_{cq_1q_2}^3, \uparrow\rangle = \left( \frac{1}{\sqrt{2}} (q_1q_2 - q_2q_1)c \right) \left( \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \right), \quad (A3)$$

where  $(q_1, q_2) = (u, d), (u, s), (d, s)$  for  $\Lambda_c^+$  and  $\Xi_c^{+,0}$ , respectively.

The wave functions of the baryon octet in the final states in the standard flavor-spin basis are given as follows.

For  $\mathcal{B}_{q_1q_1q_2}$  ( $p, n, \Sigma^{+,-}, \Xi^{0,-}$ ), we have

$$\begin{aligned}
 |\mathcal{B}_{q_1q_1q_2}, \uparrow\rangle &= \frac{1}{\sqrt{2}} \left\{ \left( \frac{1}{\sqrt{6}} (q_1q_2q_1 + q_2q_1q_1 - 2q_1q_1q_2) \right) \right. \\
 &\quad \times \left( \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \right) \\
 &\quad \left. + \left( \frac{1}{\sqrt{2}} (q_1q_2q_1 - q_2q_1q_1) \right) \left( \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \right) \right\}, \quad (A4)
 \end{aligned}$$

where  $(q_1, q_2) = (u, d), (d, u), (u, s), (d, s), (s, u), (s, d)$  for  $p, n, \Sigma^{+,-}, \Xi^{0,-}$ , respectively.

For  $\Sigma^0$  and  $\Lambda$ , we have

$$\begin{aligned}
 |\Sigma^0, \uparrow\rangle &= \frac{1}{\sqrt{2}} \left\{ \left( \frac{1}{\sqrt{12}} (sdu + dsu + sud + usd - 2dus - 2uds) \right) \left( \frac{1}{\sqrt{6}} (\uparrow\downarrow\uparrow + \downarrow\uparrow\uparrow - 2\uparrow\uparrow\downarrow) \right) \right. \\
 &\quad \left. + \left( \frac{1}{2} (-sdu + dsu - sud + usd) \right) \left( \frac{1}{\sqrt{2}} (\uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow) \right) \right\}, \quad (A5)
 \end{aligned}$$

$$|\Lambda, \uparrow\rangle = \frac{1}{\sqrt{2}} \left\{ \left( \frac{1}{2}(sdu+dsu-sud-usd) \right) \left( \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow) \right) + \left( \frac{1}{\sqrt{12}}(sdu-dsu-sud+usd-2dus+2uds) \right) \left( \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow) \right) \right\}. \quad (\text{A6})$$

### Wave functions in the diquark basis

From the coupling of two angular momenta  $j_1=1$  and  $j_2=\frac{1}{2}$ , we know that

$$|J=\frac{1}{2}, M=\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|m_1=1, m_2=-\frac{1}{2}\rangle - \sqrt{\frac{1}{3}}|m_1=0, m_2=\frac{1}{2}\rangle.$$

So, it is natural to define the baryon state with an axial-vector diquark as follows:

$$|q_1(q_2q_3)_A, \uparrow\rangle \equiv \sqrt{\frac{2}{3}}q_1\downarrow(q_2q_3)_{11} - \sqrt{\frac{1}{3}}q_1\uparrow(q_2q_3)_{10}, \quad (\text{A7})$$

where  $(q_2q_3)_{11} = (q_2q_3)(\uparrow\uparrow)$  and  $(q_2q_3)_{10} = (q_2q_3)\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow+\downarrow\uparrow)\right)$ . Meanwhile the baryon state with a scalar diquark can be defined as

$$|q_1(q_2q_3)_S, \uparrow\rangle \equiv q_1\uparrow(q_2q_3)_S, \quad (\text{A8})$$

where  $(q_2q_3)_S = (q_2q_3)_{00} = (q_2q_3)\left(\frac{1}{\sqrt{2}}(\uparrow\downarrow-\downarrow\uparrow)\right)$ .

One can prove the following equations:

$$q_1q_2q_3 \left( \frac{1}{\sqrt{2}}(\uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow) \right) = -\frac{1}{2}|q_1(q_2q_3)_S, \uparrow\rangle - \frac{\sqrt{3}}{2}|q_1(q_2q_3)_A, \uparrow\rangle, \quad (\text{A9})$$

$$q_1q_2q_3 \left( \frac{1}{\sqrt{6}}(\uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow) \right) = -\frac{\sqrt{3}}{2}|q_1(q_2q_3)_S, \uparrow\rangle + \frac{1}{2}|q_1(q_2q_3)_A, \uparrow\rangle. \quad (\text{A10})$$

Equipped with the above expressions, the baryon wave functions in the initial and final states in the diquark basis can be derived as follows.

For  $\mathcal{B}_{cqq}^6$  ( $\Sigma_c^{++}, \Omega_c^0$ ), we have

$$\mathcal{B}_{cqq}^6 = -c(qq)_A, \quad (\text{A11})$$

where  $q=u, d, s$  for  $\Sigma_c^{+,0}$  and  $\Omega_c^0$ , respectively.

For  $\mathcal{B}_{cq_1q_2}^6$  ( $\Sigma_c^+$  and  $\Xi_c^{+,0}$ ), we have

$$\mathcal{B}_{cq_1q_2}^6 = \frac{1}{\sqrt{2}}(-c(q_1q_2)_A - c(q_2q_1)_A), \quad (\text{A12})$$

where  $(q_1, q_2) = (u, d), (u, s), (d, s)$  for  $\Sigma_c^+$  and  $\Xi_c^{+,0}$ , respectively.

For  $\mathcal{B}_{cq_1q_2}^3$  ( $\Lambda_c^+$  and  $\Xi_c^{+,0}$ ), we have

$$\mathcal{B}_{cq_1q_2}^3 = \frac{1}{\sqrt{2}}(c(q_1q_2)_S - c(q_2q_1)_S), \quad (\text{A13})$$

where  $(q_1, q_2) = (u, d), (u, s), (d, s)$  for  $\Lambda_c^+$  and  $\Xi_c^{+,0}$ , respectively.

For  $\mathcal{B}_{q_1q_1q_2}$  ( $p, n, \Sigma^{+,-}$  and  $\Xi^{0,-}$ ), we have

$$\mathcal{B}_{q_1q_1q_2} = \frac{1}{2\sqrt{3}} \left( -q_1(q_1q_2)_A - q_1(q_2q_1)_A + 2q_2(q_1q_1)_A + \sqrt{3}q_1(q_1q_2)_S - \sqrt{3}q_1(q_2q_1)_S \right), \quad (\text{A14})$$

where  $(q_1, q_2) = (u, d), (d, u), (u, s), (d, s), (s, u), (s, d)$  for  $p, n, \Sigma^{+,-}, \Xi^{0,-}$ , respectively.

For  $\Sigma^0$  and  $\Lambda$ , we have

$$\Sigma^0 = \frac{1}{2\sqrt{6}}(2s(ud)_A + 2s(du)_A - d(us)_A - d(su)_A - u(ds)_A - u(sd)_A + \sqrt{3}u(ds)_S - \sqrt{3}u(sd)_S + \sqrt{3}d(us)_S - \sqrt{3}d(su)_S), \quad (\text{A15})$$

$$\Lambda = \frac{1}{2\sqrt{6}}(\sqrt{3}d(us)_A + \sqrt{3}d(su)_A - \sqrt{3}u(ds)_A - \sqrt{3}u(sd)_A + 2s(ud)_S - 2s(du)_S + d(us)_S - d(su)_S - u(ds)_S + u(sd)_S). \quad (\text{A16})$$

### References

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