Dynamical interactions of dark energy and dark matter: Yang-Mills condensate and QCD axions *

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Abstract: We analyze a model of cold axion dark matter weakly coupled with a dark gluon condensate, reproducing dark energy. We first review how to recover the dark energy behavior using the functional renormalization group approach, and ground our study in the properties of the effective Lagrangian, to be determined non-perturbatively. Then, within the context of $G_{SM}\times SU(2)_D\times U(1)_{PQ}$, we consider Yang-Mills condensate (YMC) interactions with QCD axions. We predict a transfer of dark energy density into dark matter density, that can be tested in the next generation of experiments dedicated to dark energy measurements. We obtain new bounds on the interactions between the Yang-Mills condensate and axion dark matter from Planck data: the new physics interaction scale related to the axion/gluon condensate mixing is constrained to be higher than the 10^6 GeV energy scale.

Keywords: dark matter, dark energy, axion, peccei-quinn

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1 Introduction

Consolidated observations of Type Ia supernovae (SN Ia) have established that the universe is undergoing a phase of accelerated expansion. The first evidence was provided in 1998 by two independent teams [2, 3]. Since then, analyses exploiting the SN Ia data set [4], combined with cosmic microwave background radiation (CMBR) date [5–8] through the WMAP satellite observations and larger scale structure [9, 10], have strongly corroborated this scenario. Despite the picture being much clearer from an experimental point of view, there is not yet any compelling theoretical explanation of the origin of the current acceleration of the universe (see *e.g.* [11–13]), and the problem has been dubbed "dark energy" (DE) in the literature.

Among many possible descriptions stands the simple hypothesis that DE originates from a Yang-Mills field condensate (YMC). A notable analogy is provided by the Higgs field, but there are important caveats to be considered. The Yang-Mills (YM) field advocated to explain DE does not necessarily match the content of matter of the Standard Model (SM) of particle physics, and might actually represent a different gauge field matter component. A YMC mechanism was first proposed in Ref. [14] to allow a primordial inflationary acceleration of the universe, and was described by the renormalization group improvement (RGI) action on a Friedmann-Lemaître-Robertson-Walker (FLRW) background. The same idea was later adapted to explain DE in Refs. [15, 16], respectively in the perturbative two-loop and three-loop analyses of the effective action of SU(N) YM theory. The strategy deployed in Refs. [15, 16] of accounting for a non-perturbative expansion of the effective action and of retaining only the lower-loop corrections, however, means the results obtained for the YMC are not fully reliable for the infra-red regime needed for DE. Nevertheless, the core of this proposal relies on considering quantum corrections encoded in the effective action, which can be cast in terms of an effective running coupling constant $g=g(\Theta)$, as derived within the RGI framework [18, 19, 19]. The coupling depends on a contraction of the field-strength tensors that plays the role of order-parameter for the YMC, namely $\Theta\equiv-\frac{1}{2}F^a_{\mu\nu}F^{a\mu\nu}$, and enters the density Lagrangian $\mathcal{W}=-\frac{1}{4g^2(\Theta)}F^a_{\mu\nu}F^{a\mu\nu}$. Henceforth, a sum over repeated internal indices a, which run over the dimensions of the Lie-algebra, will be intended. In g a dependence on the square of the renormalization mass-scale κ is also present. The latter only denotes the initial point in the renormalization group

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flow, and must not be confused with a physical scale.

The most serious technical issue plaguing previous analyses [14–16] concerns the stability of the results obtained in the perturbative approach to the computation of the effective action. At higher orders than the three-loop expansion, the appearance of additional terms in the effective action for $SU(\mathrm{N})$ YM theories might spoil the DE behavior, which totally relies on an ultraviolet perturbative expansion. Conversely, summarizing the analysis in Ref. [1], we proceed in Section 2 and Section 3 to show that a fully non-perturbative approach is possible.

First, in Section 2 we prove that under mild and general assumptions, which are basically the existence of a minimum in Θ in the non-perturbative effective Lagrangian, a DE behavior is recovered.

Then in Section 3, by making use of the non-perturbative techniques mutated from the functional renormalization group (FRG) procedure, which is more suitable to be used in the confining infrared limit of the theory, we show that such a minimum indeed exists, at least for the case of $SU(2)$, and we provide an explicit example of the latter. We can state general requirements for the effective density Lagrangian $\mathcal{W}(\Theta)$ that must be fulfilled in order to obtain a YMC model for DE:

PI) $\mathcal{W}(\Theta)$ has a non-trivial minimum at some energy scale $\Theta_0 \approx \Lambda_D^4$;

PII) $\mathcal{W}(\Theta)$ has a perturbative limit, which resembles the one-loop result derived by Savvidy [24];

PIII) $\mathcal{W}(\Theta)$ shows the UV asymptotic behavior ($\Theta \gg \Lambda_D^4$) of being at least linear in Θ, which in turn is linear in the bare Yang-Mills action.

The effective action $\mathcal{W}(\Theta)$ will be in general equipped with a characteristic energy scale Λ_D. A YMC then forms whenever the minimum of $\mathcal{W}(\Theta)$ is reached.

Section 4 and Section 5 are the original parts of this work. In Section 4 we discuss the dark YMC model in the context of a minimal Standard Model (SM) extension $SU(3)_c \times SU(2)_L \times U(1)_Y \times SU(2)_D \times U(1)_{PQ}$, where $U(1)_{PQ}$ is the Peccei-Quinn global axial symmetry, spontaneously broken and associated with a QCD axion.

In Section 5 we emphasize that while DE is described by the dark YMC, the QCD invisible axion provides a good candidate for cold DM. We retain this minimal extension strongly motivated by the strong CP problem. The whole model we present here has an important difference with respect to traditional QCD axion theories: axions can interact with the dark YMC in an EFT framework. In particular, a part of the DE density can be transferred to the DM density during a cosmological time of 1–10 Gyrs or so. We will estimate the rate of this process and its cosmological limits.

In Section 6, we give some conclusions and the outlook for future work.

2 YMC as dark energy

To shed light on the behavior of the YMC, and check whether it can solve the problem of DE, we assume henceforth a flat FLRW universe, the line element of which is cast in terms of comoving coordinates, i.e. $\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j$, with t cosmological time. In the simplest case of a universe filled only with the YMC minimally coupled to gravity, the effective action reads

$$\mathcal{S} = \int \sqrt{-g}\,\left[-\frac{\mathcal{R}}{16\pi G} + \mathcal{W}(\Theta)\right]\,\mathrm{d}^4x, \tag{1}$$

with g the determinant of the metric $g_{\mu\nu}$, and $\mathcal{R}$ the scalar Ricci curvature. By variation of $\mathcal{S}$ with respect to the metric $g^{\mu\nu}$, one obtains the Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, the energy-momentum tensor of the YMC being

$$T^{\mu\nu} = \sum_{a=1}^{3} {}^{(a)}T^{\mu\nu} = \sum_{a=1}^{3} g^{\mu\nu}\mathcal{W}(\Theta) - 2\frac{\partial \mathcal{W}}{\partial \Theta} F_a^{\gamma\mu} F^a{}_\gamma{}^\nu. \tag{2}$$

The YM tensor can be cast in terms of the structure constants f^{abc} of the $SU(\mathrm{N})$ gauge-group under scrutiny, and generally reads $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + f^{abc}A^b_\mu A^c_\nu$. From now on, we focus on the $SU(2)$ gauge-group, for which $f^{abc} = \epsilon^{abc}$. We then pick out a gauge that preserves isotropy and homogeneity of the FLRW background, by assuming gauge fields to be functions only of the cosmological time t, and choosing their components $A_0 = 0$ and $A^a_i = \delta^a_i A(t)$. The YM tensor then assumes a simplified form, and its non-vanishing components read $F_i^{0a} = E/3$. The order parameter Θ can be cast in a simple form, *i.e.* $\Theta = E^2$, and the energy-momentum tensor is found to be isotropic, with energy and pressure densities given by

$$\rho_{\mathrm{YMC}} = -\mathcal{W}(\Theta) + 2\mathcal{W}'(\Theta)\Theta, \tag{3}$$

$$p_{\mathrm{YMC}} = \mathcal{W}(\Theta) - \frac{2}{3}\mathcal{W}'(\Theta)\Theta. \tag{4}$$

Consequently, the equation of state (EOS) of the YMC is immediately recovered to be

$$w_{\mathrm{YMC}} \equiv \frac{p_{\mathrm{YMC}}}{\rho_{\mathrm{YMC}}} = -\frac{\mathcal{W} - \frac{2}{3}\mathcal{W}'\Theta}{\mathcal{W} - 2\mathcal{W}'\Theta} = -\frac{1 - \frac{2}{3}\frac{\mathcal{W}'}{\mathcal{W}}\Theta}{1 - 2\frac{\mathcal{W}'}{\mathcal{W}}\Theta}. \tag{5}$$

If we require the YM theory to condense (property PI in Section 1), then the function $\mathcal{W}(\Theta)$ must have a non-trivial minimum, which implies that $\mathcal{W}'$ vanishes at some Θ_0. At Θ_0, the YMC has an EOS proper for the cosmological constant, since $w_{\mathrm{YMC}} = -1$.

In the high-energy-scale regime $\Theta \gg \Lambda_D^4$, as a consequence of property PIII, we recover for the condensate an EOS of radiation for the YMC, *i.e.* characterized by

$w_{\rm YMC}=1/3$, in analogy with the perturbative analysis [14–16].

We may generalize this analysis, and resort to a description of the universe that takes into account YMC, matter and radiation, treated in terms of their EOS. Since we have assumed *ab initio* the universe to be flat, fractional densities must sum up to the identity, *i.e.* $\Omega_{\rm YMC}+\Omega_m+\Omega_r=1$, with fractional energy densities defined as $\Omega_{\rm YMC}\equiv\rho_{\rm YMC}/\rho_{tot}$, $\Omega_m\equiv\rho_m/\rho_{tot}$, $\Omega_r\equiv\rho_r/\rho_{tot}$, and total energy density $\rho_{tot}\equiv\rho_{\rm YMC}+\rho_m+\rho_r$. The Friedmann equations then read

$$\left(\frac{\dot a}{a}\right)^2=\frac{8\pi G}{3}(\rho_{\rm YMC}+\rho_m+\rho_r), \tag{6}$$

$$\frac{\ddot a}{a}=-\frac{4\pi G}{3}(\rho_{\rm YMC}+3p_{\rm YMC}+\rho_m+\rho_r+3p_r), \tag{7}$$

with *dot* denoting the time-derivative. If we assume no interaction between the three energy components, the dynamical evolution is dictated by energy conservation:

$$\dot\rho_{\rm YMC}+3\frac{\dot a}{a}(\rho_{\rm YMC}+p_{\rm YMC})=0, \tag{8}$$

$$\dot\rho_m+3\frac{\dot a}{a}\rho_m=0, \qquad \dot\rho_r+3\frac{\dot a}{a}(\rho_r+p_r)=0. \tag{9}$$

From Eqs. (9), the standard evolutions of the matter and radiation components easily follow, *i.e.* $\rho_m\propto a^{-3}$ and $\rho_r\propto a^{-4}$, while the evolution of the YMC turns out to be less obvious. Inserting Eq. (3) into Eq. (8) yields

$$\dot\Theta(\mathcal{W}'+2\mathcal{W}''\Theta)+4\frac{\dot a}{a}\mathcal{W}'\Theta=0, \tag{10}$$

a quite compact form that is integrable for any regular enough $\mathcal{W}$. The result is then easily derived:

$$\sqrt{\Theta}\mathcal{W}'(\Theta)=\alpha a^{-2}, \tag{11}$$

where α is a coefficient of proportionality that depends on the initial conditions, to be fine-tuned in order to recover the redshift z at which the universe transits into its dark energy phase. At very high redshift, in the limit $\Theta\gg\Lambda_D^4$, Eq. (11) entails an increase of the order-parameter Θ. Then Eq. (5) encodes the EOS parameter $w_{\rm YMC}\to 1/3$, and the YMC starts behaving as a radiation component, as expected from asymptotic freedom at high energy. At small redshift, the expansion of the universe requires the LHS of Eq. (11) to asymptotically vanish. This occurs for the extremal value of Θ_0, at which the EOS parameter converges towards $w_{\rm YMC}=-1$, which implies a DE behavior.

3 FRG and YMC in the $SU(2)$ extra sector

In the previous section we reviewed the consequences of properties PI-III for the cosmological evolution of the YMC. We shall give hints now that the YMC so far discussed indeed exists, beyond the perturbative approximation [14–16].

The FRG approach, a tool developed to study the non-perturbative flow of a QFT, will provide us with the tools necessary to achieve this purpose. The scale-dependence of the flowing action is recovered by solving the FRG equation (FRGE) [26, 27]:

$$\partial_k\Gamma_k=\frac{1}{2}{\rm STr}\left(\Gamma_k^{(2)}+R_k\right)^{-1}\partial_k R_k, \tag{12}$$

in which k is a (running) cutoff scale that allows us to interpolate smoothly between the microscopic action $\Gamma_{k\to\infty}$ and the full quantum effective action $\Gamma_{k\to 0}$; the Super-Trace STr is over all discrete indexes and fields, and encodes summation over the eigenvalues of the Laplacian; and the quantity R_k denotes a mass-like regulator function that suppresses quantum fluctuations with momenta lower than an IR momentum-cutoff-scale k, implementing the Wilsonian renormalization group flow idea with a momentum-shell wise integration of the path-integral.

The FRGE has been extensively applied to SU(N) YM-theories [29–33], and to the study of YMC — see *e.g.* Refs. [34–36] and Ref. [37] (the most recent work).

It is impossible to solve Eq. (12) exactly, unless we deploy minimal approximations. We might make an *ansatz* on the functional form of the effective action to solve Eq. (12), but this would shift away from our purpose of determining $\mathcal{W}(\Theta)$.

We shall instead proceed to reconstruct $\mathcal{W}(\Theta)$, taking into account the energy-flow generated by the propagator of the bare action, rather than from the propagator of the full (unknown) effective action. In perturbation theory this is equivalent to working at one loop, so we expect to reproduce a result that is similar to one in Ref. [24]. A similar analytical calculation in the full flow equation is beyond currently available techniques. To perform a better analysis, some form of interpolated propagator from the UV (free) regime to the IR (interactive) regime should be deployed [37]. This would amount to using numerical methods and an effective description. Although conclusions on the DE behavior will be quantitatively affected by this approximation, we believe that these arguments strengthen the existence of the minimum in the effective action, and thus the appearance of the DE phase.

The computation is performed in the framework of the background field method. The two key ingredients that enter into Eq. (12) are a wise choice of the YM background field (following Ref. [37] we will use a self-dual background to avoid unnecessary complication with negative eigenvalues of the Laplacian) and the deployment of the simplest possible regulator function (mass-like cutoff $R_k(\mathcal{D})=k^2$ in all the sectors of the trace). The resulting effective Lagrangian (after identifying the characteristic energy scale with Λ_D) is the following:

$$\mathcal{W}(\Theta)=\frac{g^2\Theta}{2\pi^2}\int_0^\infty\frac{\mathrm{d}s}{s}\mathrm{e}^{-s\sqrt{\frac{\Lambda_D^4}{g^2\Theta}}}\left(\frac{1}{4\sinh^2(s)}+1-\frac{1}{4s^2}\right), \quad (13)$$

where the coupling constant g appearing in the equation is the bare coupling constant we used as an initial condition for the integration of the RG flow. For a detailed derivation of Eq. (13) we refer to Ref. [1].

From Fig. 1 it is evident that Eq. (13) has a non-zero global minimum. The exact position of this minimum can be computed numerically, and in terms of dimensionless quantities is found to be $\frac{g^2\Theta_0}{\Lambda_D^4}\approx 0.361$, which is consistent with what we expected from the property PI stated in Section 1.

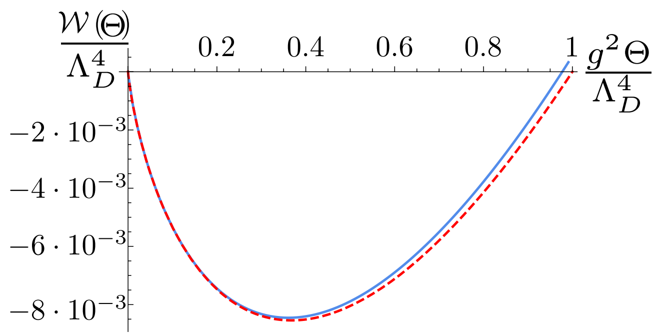

Fig. 1. (color online) Plot of function (13) (continuous blue line) and the one-loop [24] (dashed red line). Notice the presence of a non-zero global minimum for $g^2\Theta_0/\Lambda_D^4\approx 0.361$.

Moreover it is possible, as expected, to reproduce the one-loop result derived by Savvidy [24] by computing the asymptotic expansion of $\mathcal{W}(\Theta)$ for small values of the YM UV coupling constant g. In this limit:

$$\mathcal{W}(\Theta)\approx\frac{11}{48\pi^2}g^2\Theta\mathrm{Log}\left(\frac{\Lambda_D^4}{g^2\Theta}\right). \quad (14)$$

Finally, we verify that the $SU(2)$ YMC so far discussed evolves from a radiation-like component to a DE one. We come back to Eq. (11), and estimate the characteristic scale of the condensate Λ_D by comparing the "predicted" YMC fractional energy density at low redshift with the measured DE fractional energy density $\Omega_\Lambda = 0.735$. We then find that for a wide range of initial conditions — the parameter α in Eq. (11) — $\Lambda_D\approx 3.2h^{1/2}10^{-3}$ eV. As noticed in Refs. [14–16], this is a very low energy scale compared to typical energy scales in particle physics, thus the $SU(2)$ Yang-Mills interaction must be assumed to describe a dark sector.

We can study the evolution of the YMC energy density and its EOS for different values of α, and still find the same asymptotic values. The value of the cosmological constant and the value of z at the transition epoch to dark energy are known from experimental evidence, and are provided with statistical errors. We can then finetune the parameter α to be consistent with experimental data, in the window allowed by current data. The results are summarized in Fig. 2.

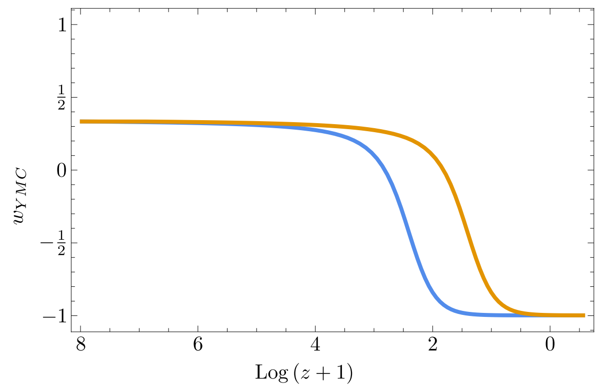

Fig. 2. (color online) The EOS for YMC models of DE with different initial conditions. The increase of the value of the parameter α amounts to a more realistic start of the DE behavior, shifting the curve toward the left. Notice that these numerical plots are realized assuming $\zeta=T_D/T=1$, i.e. the same initial reheating temperature between the ordinary and the dark sector. Assuming instead $\zeta\neq 1$, the redshift scales linearly as $\zeta(1+z)$. For the case $\zeta<<1$ discussed below, this would amount to an additional shift of the curves toward the left of the logarithmic axis.

A stability analysis has been performed for our model in the fully interacting model [1]. The position of the fixed point can be estimated numerically. There exists a unique fixed point for every positive value of the coupling parameter and it is always attractive.

Finally, it is worth commenting on the redshift scale of the gluon condensate formation. This is highly dependent on the initial reheating temperatures of the ordinary sector and the dark gluon sector, i.e on the $\zeta=T_D/T$ ratio. This is a tunable parameter, which depends on the inflaton couplings with the SM particles and the dark gluons. The scenario in which the reheating of SM particles is more efficiently obtained is highly favored. This is simply because the inflaton can have a larger number of decay channels into SM particles than into dark gluons.

In other words, $\zeta<<1$ is the most natural scenario. A similar asymmetric reheating scenario was suggested by Berezhiani *et al* within the framework of asymmetric mirror dark matter [42]. Here, we consider an asymmetric dark energy scenario, since dark energy is provided by the dark gluon condensate with an initial temperature of $T_D<<T$ after inflation reheating. Under these

assumptions, the dark gluon condensate can be formed at a redshift which is much larger than the recombination epoch, since the dark sector effectively expands adiabatically without any efficient transferring of the initial temperatures among the two sectors. Taking into account that the redshift factor is proportional to the temperature, as shown in Fig. 2 we may consider the case in which the condensate forms at a redshift $1+z\sim 10^3$ rather than $1+z\sim 1$. This scenario is phenomenologically preferable in order to correctly fit the CMB peak features, i.e. to have dark energy condensate formation during the recombination epoch. In the following discussions, we will assume $\zeta >> 10^3$ in order to neglect thermal field theory corrections in our considerations, and have dark (gluon) energy already contributing during the recombination epoch.

4 Dark Yang-Mills phase transition

In the previous section, we studied the first order phase transition of the dark Yang-Mills theory in the framework of the FRG approach. From now on we argue how this model can be extended in order to unveil the origin of DM. We start by considering in this section heuristic arguments in favor of a mechanism of evaporation of the gluon condensate of DE into DM. Then in the next section we focus on the instantiation of DM in our model in terms of an axion field coupled to the invisible Yang Mills sector.

In this section we sketch the evaporation of the gluon condensate at finite temperature from the point of view of the glueballs approach. At low temperature gluons are *frozen* (inside the gluon condensate) at a characteristic wavelength Λ_D^{-1}. At temperatures higher that a critical temperature $T_c\simeq\Lambda_D$, gluons have enough kinetic energy to escape from the condensate, and the evaporation process starts. A first order phase transition happens at T_c. For $T<T_c$ the gluon condensate is dominant, while for $T>>T_c$ it can be considered as a gas of free gluons. Formation of a condensate breaks conformal symmetry; dilatons are the only degrees of freedom left, and can be described as being governed by the Lagrangian,

$$\mathcal{L}=\frac{1}{2}(\partial_\mu\rho)^2-V(\rho),$$

in which $V(\rho)$ has a certain minimum at $\rho=\rho_0$. Oscillations around this minimum describe the excitations of a scalar glueball, *i.e.*

$$V(\rho)=\frac{1}{2}M_g^2(\rho-\rho_0)^2+O[(\rho-\rho_0)^3],$$

where M_g is related to the second derivative of V at its minimum.

In the mean field approximation, one can describe the pressure of the dilaton field as

$$P(\rho,T)=-T\mathcal{I}_1,$$

in which

$$\mathcal{I}_1=\int\frac{\mathrm{d}^3p}{(2\pi)^3}\ln[1-\mathrm{e}^{-\omega(\rho)/T}],$$

$$\omega(\rho)=\sqrt{p^2+m^2(\rho)},\qquad m^2(\rho)=\frac{\partial^2V}{\partial\sigma^2},$$

having defined $m^2(\rho_0)=M_g^2$. The thermodynamic potential can be then related to the pressure as

$$\Omega(\rho,T)=V(\rho)-P(\rho,T).$$

For $T>>T_c$, glueballs do not exist and dilaton fluctuations cannot be relevant degrees of freedom of the system. However, gluon momenta experience an infrared cutoff at the scale Λ_D, so that the free-gluon gas pressure reads

$$P(\rho,T)=-(N_c^2-1)T\mathcal{I}_2,$$

in which

$$\mathcal{I}_2=\int\frac{\mathrm{d}^3p}{(2\pi)^3}\ln[1-\mathrm{e}^{-p/T}]\theta[p-\bar{p}(\sigma)].$$

In the latter expression $\bar{p}(\sigma)$ denotes a cutoff that is a function of the dilaton field vev. Furthermore, it is an appropriate function, the asymptotic limits of which are $\bar{p}(\sigma)\to\infty$ for $\rho\to\rho_0$ and $\bar{p}(\sigma)\to$const for $\rho\to 0$ (the cutoff vanishes for $\rho<<\rho_0$).

As emphasized in the literature, these two regimes are expected to be related by a first order phase transition, *i.e.* a discontinuous increase of the dilaton[1)] field profile $\rho(T)$ around T_c [39–41, 43] .

5 Dark gluon condensate and QCD axion condensate

Within the framework of the minimal model considered in the previous sections, we cannot identify any viable DM candidate. In this section, we discuss how to combine our scenario with the QCD invisible axion paradigm. The latter field is associated with the solution of the CP problem, and provides a good candidate for cold and hot DM. Recently a mechanism involving QCD axions for an electroweak scale relaxation was also suggested [46]. We therefore suggest extending the SM by encoding an extra $SU(2)_D\times U(1)_{PQ}$ gauge sector.

It is known that a CP-violating term in QCD $\frac{\theta_{\rm QCD}}{32\pi^2}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}$ can lead to a large neutron electric dipole

1) We are not interested in developing a detailed numerical study of these features here, which nonetheless are still present in the literature for several YM toy-models and/or realistic models. We limit ourself to concluding that an ambiguity in the definition of the S-matrix will be naturally solved if the Universe evolves toward a big crunch rather than an infinite expansion. In fact, during the final contraction, the thermal bath will inevitably reach $T>T_c$, and the condensate will evaporate [44]. However, the introduction of an axion coupled to the dark strong sector can increase the evaporation rate [45], as we will discuss in the next section.

moment. However, measurements of the neutron electric dipole moment actually constrain $\theta_{\rm QCD}<10^{-10}$. Furthermore, the CP-violating term can be shifted away by the ordinary Peccei-Quinn mechanism.

The complete Lagrangian of the $SU(3)_c$ is:

$$\mathcal{L} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}+\frac{\theta_{\rm QCD}}{32\pi^2}G^a_{\mu\nu}\widetilde{G}^{a\mu\nu} +\sum_f \bar{q}_f(i\gamma_\mu D^b_{\mu a}-m_r\delta^b_a)q_{rb},$$

and is invariant under global axial $U(1)_{PQ}$ Peccei-Quinn transformations, when the following shift is implemented:

$$\theta_{\rm QCD}\rightarrow\theta_{\rm QCD}-{\rm arg det}M,$$

M denotes the quark mass matrix with eigenvalues m_r. The shift is equivalent to say that quark mass matrix phases are extra sources of CP violation, *i.e.* that the axial $U(1)_{PQ}$ acts on the vacuum as ${\rm e}^{i\alpha_r\bar{Q}_5}|\theta\rangle = |\theta+{\rm arg det}M\rangle$. The Peccei-Quinn solution of the CP problem consists in promoting $\bar{\theta}_{\rm QCD}=\theta_{\rm QCD}+{\rm arg det}M$ to a dynamical field $\bar{\theta}_{\rm QCD}=a/f_a$, a being the axion field, the Goldstone boson of the spontaneously broken global axial symmetry $U(1)_{PQ}$, and the scale f_a being the spontaneous symmetry breaking scale of $U(1)_{PQ}$. The latter can be minimally realized through a complex scalar $\sigma=\frac{f_a}{\sqrt{2}}{\rm e}^{ia/f_a}$ with a sombrero-like potential. Notice however that the Peccei-Quinn solution is not a symmetry of the quantum theory, and a gets an expectation value induced by the QCD sector, which reads

$$\left\langle\frac{\partial V_{\rm eff}}{\partial a}\right\rangle\Big|_{\langle a\rangle}=\frac{1}{32\pi^2}\frac{1}{f_a}\left\langle G^a_{\mu\nu}\widetilde{G}^{a\mu\nu}\right\rangle\Big|_{\langle a\rangle}=0.$$

The effective potential has a periodicity $\theta-\frac{\langle a\rangle}{f_a}$, so that a is forced to get an expectation value $\langle a\rangle=f'_a\theta'$. For instance, the axion effective potential in the dilute gas approximation is $V_{\rm eff}(a)\simeq K\cos(a/f_a)$, where $K\sim\Lambda^4_{\rm QCD}$. Thus the physical axion field turns out to be $\tilde{a}=a-\langle a\rangle$, and the strong sector generates a mass term for $\tilde{a}$ as

$$m_a^2=\left\langle\frac{\partial^2 V_{\rm eff}}{\partial a^2}\right\rangle\Big|_{\langle a\rangle}=\frac{1}{32\pi^2}\frac{1}{f_a}\frac{\partial}{\partial a}\left\langle G^a_{\mu\nu}\tilde{G}^{a\mu\nu}\right\rangle\Big|_{\langle a\rangle},$$

which can be also expressed as

$$m_a^2=\frac{A^2}{f_a^2}\frac{VK}{V+K{\rm Tr}M^{-1}}.$$

In the latter expression A is the color anomaly of the $U(1)_{PQ}$ current, (A=1 for $N_f=N_c$), K is related to $V_{\rm eff}$ as mentioned above, M is the quark mass matrix, and $V\sim\langle\bar{q}q\rangle\sim\Lambda^3_{\rm QCD}$. By virtue of V, related to the chiral symmetry breaking, the axion gets a small mixing with pseudo-Goldstone bosons π,η. Using $(m_u+m_d)\langle\bar{q}q\rangle=m_\pi^2 f_\pi^2$ (neglecting the s-quark contribution) we can rewrite the axion mass in the useful form

$$m_a=\frac{Az^{1/2}}{1+z}\frac{f_\pi}{f_a}m_\pi\simeq A\left(\frac{10^6\,{\rm GeV}}{f_a}\right)\times 6{\rm eV},$$

with $z=m_u/m_d\simeq0.6$ and m_π, f_π the pion mass and decay constant respectively.

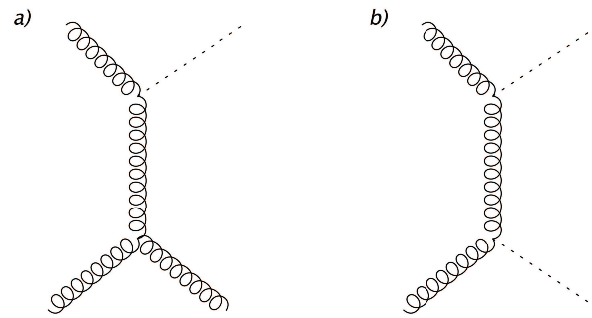

Fig. 3. Feynman diagrams triggered at tree-level by the Chern-Simons term: curly lines represent dark gluons, while dashed lines represent axions. Decay of DE into DM are originated by dgluon-dgluon scattering, with the exchange of intermediate dgluon quanta, and the production of dgluon and axion particles (a) or two axions (b). The two diagrams are shown in the s-channel representation, having in mind that the t and the u channels must also be considered. The former process is first order in the characteristic energy scale of the theory $\mathcal{M}$, while the latter is suppressed by an extra power of $\mathcal{M}$. These perturbative diagrams are dressed by all the non-perturbative corrections arising from the gluonic condensate. These interactions then act as mixing portals among the axion and the dark gluon condensates.

Limits on the masses of QCD invisible axions (KSVZ and DFZX models) are constrained in the range 10^{-1}–10meV, while $f_a\simeq10^9$–7×10^{10}GeV, from axion cold DM production from the misalignment mechanism ADMX, CAST, telescope searches, globular cluster stars, white dwarf cooling, SN1987A [47], and CMB constraints [48].

Within the framework of effective field theory, we can introduce an interaction term of QCD axions with the dark gluon condensate $\mathcal{O}_{aF\tilde{F}}=\frac{1}{\mathcal{M}}aF_{\mu\nu}\widetilde{F}^{\mu\nu}$. Notice that this additional operator does not spoil the analysis in Section 3. In fact its only non-vanishing contribution to the quadratic part is proportional to $\frac{\delta}{\delta A_\mu}F\widetilde{F}$, which is the first variation of a topological term. In principle a dark $SU(2)_D$ sector has a CP violating term. This term induces an extra contribution to the axion mass coming from dark gluon condensate, *i.e.*

$$\Delta m_a^2=\frac{1}{32\pi^2}\frac{1}{\mathcal{M}}\frac{\partial}{\partial a}\left\langle F^a_{\mu\nu}\tilde{F}^{a\mu\nu}\right\rangle\Big|_{\langle a\rangle}.$$

As a consequence it turns out that $m_a\sim\Lambda_D^2/\mathcal{M}+$

$\sqrt{m_q \Lambda_{\rm QCD}^3}/f_a$. Contrary to the scale f_a, $\mathcal{M}$ is not directly constrained by coupling with SM particles. However, the extra contribution will be much smaller than the ordinary QCD one, and thus can be neglected: $\Delta m << \Lambda_D \simeq 10^{-4}$eV.

Nonetheless, the interaction term $\mathcal{O}_{aF\tilde{F}}$ is a gateway from DE to DM. By virtue of this interaction, a dgluon, namely a gluon of the dark sector, can be converted into an axion in the dark condensate. This is a process $gg \to ga$ mediated by an off-shell dgluon, *i.e.* three-dgluon and axion-bigluon vertices.

The YMC and the axion condensate can be effectively described by two fluid equations, coupled by an interaction drag force that reads

$$\boldsymbol{F} = -(1+w)\sigma\gamma^2\rho\mathbf{v}, \tag{15}$$

where $\mathbf{v}$ is the average relative velocity among dark matter and dark energy particles.

The linear scalar perturbation theory equations of the two dark fluids in conformal Newtonian gauge [66–68] can be cast as

$$\begin{aligned}\theta'_{\rm YMC} &= 2\mathcal{H}\theta_{\rm YMC} + k^2\frac{\delta_{\rm YMC}}{1+w_{\rm YMC}} + k^2\Psi \\ &\quad - a n_{\rm DM}\theta(\theta_{\rm YMC} - \theta_{\rm DM}),\end{aligned} \tag{16}$$

$$\begin{aligned}\theta'_{\rm DM} &= -\mathcal{H}\theta_{\rm DM} + k^2\Psi \\ &\quad + \frac{\rho_{\rm YMC}}{\rho_{\rm DM}}(1+w_{\rm YMC}) a n_{\rm DM}\sigma(\theta_{\rm YMC} - \theta_{\rm DM}),\end{aligned} \tag{17}$$

where $n_{\rm DM}$ is the number density of the axions and σ is the scattering cross section between DE and DM.

From CMB limits, we infer the strong bound on the cross section for an axion mass of meV (see Fig. 4 and Fig. 5) as:

$$\sigma \leqslant 10^{-41}\,{\rm cm}^2.$$

In Fig. 6, we show the allowed parameter space of the cross-section as a function of the axion mass.

It is worth emphasizing that the cross-section is connected to the microscopic theory. Dark gluons and axions share the interaction term $aG\tilde{G}/\mathcal{M}$, which induces a mixing conversion of the dark gluon condensate into the axion condensate. The three-level Feynman diagrams shown in Fig. 3 must be dressed by non-perturbative gluon corrections — gluons are indeed in a confined regime. The process in Fig. 3(b) gets an extra $1/\mathcal{M}$ suppression with respect to the diagram in Fig. 3(a), and thus can be safely neglected. A realistic expression for the cross-section, sufficient to infer the order of magnitude of the $\mathcal{M}$ scale, can be written as follows[1]:

$$\sigma = \int {\rm d}\Gamma_{12}\mathcal{F}_{g_1}\mathcal{F}_{g_2}|A(gg \to ga)|^2_{M_g}, \tag{18}$$

where ${\rm d}\Gamma_{12}$ is the usual phase space integral measure, $A(gg \to ga)$ are all the possibile tree-level amplitudes in the s, t and u channels with a virtual gluon portal, and $|A|^2_{M_g}$ reminds us that an effective mass term for the propagating gluon has been inserted. The last assumption is highly motivated by non-perturbative lattice analysis, providing strong evidence that in the confined regime the gluons that are propagating dynamically acquire an effective mass scale of the type $M_g \sim O(1)\Lambda_D$ — see e.g. Refs. [69–71]. Above, we also defined two effective form factors $\mathcal{F}_{g_{1,2}} \equiv \mathcal{F}(p_{1,2})$ for the incoming gluons,

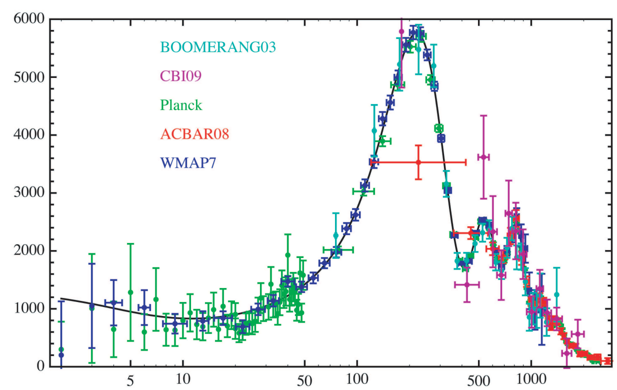

Fig. 4. (color online) The $T_0^2 l(l+1)C_{TT}/(2\pi)10^{-12}{\rm K}^2$ lensed power spectra with interaction cross section $\sigma = 10^{-42}{\rm cm}^2$ and $\sigma = 0$ (null interaction), compared with current data from Planck, WMAP7, BOOMERANG03, CBI09, and ACBAR08. Within the resolution of our integration methods, the two spectra cannot be distinguished from one another. The case of an interaction among dark energy and dark matter condensates of $\sigma = 10^{-42}{\rm cm}^2$ is perfectly compatible with all current data. This plot was produced using a modified version of the CMBQuick Mathematica package [51], with results in agreement with the publicly available codes Monte Python and CLASS [52–54]).

1) It is worth commenting on some technical issues related to these rough estimates. For the dark gluon condensate at finite temperature, one should be aware of the existence of thermal bath corrections to the axion emission processes. These corrections are negligible for $T_D = \zeta T << \Lambda_D$, but will be relevant at earlier cosmological times, when $T_D \sim {\rm few}\,\Lambda_D \simeq 10^{-4}$eV. These corrections can be computed by the general formalism of thermal QFT [49]. Indeed, the thermal production rate of axions can be calculated from the imaginary part of its propagator Π, since $\gamma_a = \frac{{\rm d}\Gamma}{{\rm d}V} = -2\int\frac{{\rm d}^3p}{(2\pi)^3 2E}\Pi^<(p)$, in which $\Pi^< = f_B(E){\rm Im}\Pi$. The thermal axion production rate can be obtained by thermally averaging the scattering rate $gg \to ga$, or equivalently can be evaluated from the two-loop corrections to the axion propagator induced by gluons. Similar calculations for standard QCD axions were performed in Ref. [50]. Complete calculations of these contributions in our scenario are beyond the purpose of this article.

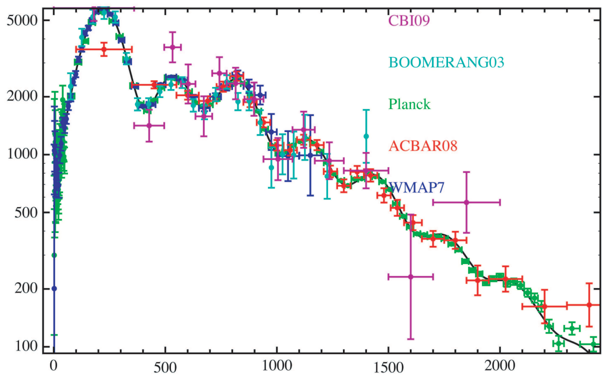

Fig. 5. (color online) The $T_0^2 l(l+1)C_{TT}/(2\pi)10^{-12}\mathrm{K}^2$ lensed power spectra with interaction parameters $\sigma=10^{-42}\mathrm{cm}^2$ and $\sigma=0$ (null interaction), compared with current data from Planck, WMAP7, BOOMERANG03, CBI09, and ACBAR08. Compared with Fig. 1, here the large l-mode part of the spectrum is displayed with higher resolution. Again, a $\sigma=10^{-42}\mathrm{cm}^2$ interaction is compatible with current data. This plot was produced using a modified version of the CMBQuick Mathematica package [51], with results in agreement with the publicly available codes Monte Python and CLASS [52–54].

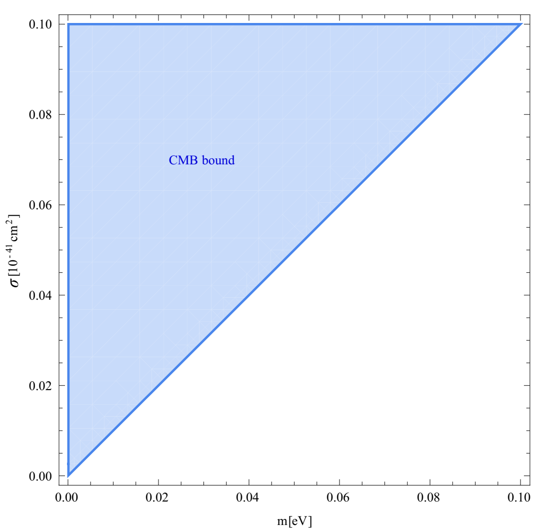

Fig. 6. (color online) The allowed parameter space of the cross-section and axion mass from CMB measurements. The cross section is displayed in $10^{-41}\mathrm{cm}^2$, and the axion mass in units of eV.

dressing perturbative gluons with non-perturbative corrections in the condensate. In other words, they consider the statistical distribution of the gluon momenta inside the condensate. Within our assumptions, a reasonable expression for the form factor, motivated by the evidence of lattice Yang-Mills theory analyses at $T<<\Lambda_D$, is represented by

$$\mathcal{F}(p)=\frac{1}{\mathrm{e}^{\frac{p}{\Lambda_D}}-1}.$$

Statistically, gluons within the condensate are frozen in the same quantum state with a de Broglie wave length of $\lambda_g=O(1)\Lambda_D^{-1}$ — see Section 4. In our model, we obtain a result that can be trusted up to $O(1)$ corrections,

$$\sigma=0.010495\left(\frac{\Lambda_D}{\mathcal{M}}\right)^2\Lambda_D^{-2}, \tag{19}$$

which implies a strong bound on the interaction scale of $\mathcal{M}>3.16{\cdot}10^6\,\mathrm{GeV}$.

6 Conclusions

In this paper we have investigated the possibility that dark energy is the vacuum energy of a YMC formed by a dark sector, and that at the same time a QCD invisible axion coupled to the latter sector provides a good candidate for cold DM. Both the hypotheses are highly motivated by string phenomenology. For instance, cancellation of the vacuum energy contributions from the other gauge sectors can be explained while resorting to $E8\times E8$ heterotic superstring theory with an asymmetric Higgs sector, or alternatively parallel intersecting D-brane worlds in open superstring theories[1)].

Merging the axion model with the dark gluon energy model provides a consistent explanation of the strong CP problem in QCD, cold DM and DE. The strong CP problem solution, which involves the standard axion model, is not spoiled by the introduction of the axion coupling with the dark gluon condensate. The Peccei-Quinn symmetry introduced shifts out the QCD θ-term. Furthermore, the axion can be coupled with the dark strong sector, receiving an extra contribution to its mass term. As commented above, we found that such a contribution is well compatible with any current bound from cosmology, axion condensate production during the QCD phase transition, and experimental constraints.

We have characterized the non-perturbative effective action and shown that, if it has a minimum in Θ, a YMC forms and the model can actually work as DE at small z. If the effective action scales at least like the bare YM action for high energy scales, at large z it entails the EOS of radiation. Internal consistency requires that perturbative one-loop results must be still recovered in the appropriate asymptotic limit. In Ref. [1] these three general requirements have been checked successfully for a $SU(2)$ YM-theory, by deploying non-perturbative FRG

1) New intriguing implications in particle physics and cosmology of exotic stringy instantons have been studied within the framework of intersecting D-brane models. In particular, exotic instantons can generate new effective operators not allowed at perturbative level, as an effective Majorana mass for the neutron, while the proton is not destabilized [55–58].

techniques.

In the original part of this work, we have then sought a relation between YMC models of DE and DM models. In particular, we have shown that within the framework of a model $G_{SM}\times SU(2)_D\times U(1)_{PQ}$, QCD axions can be emitted by the YMC in a cosmological time. This causes a tiny conversion of a part of the DE density into cold DM density. In particular, using the Planck data we have inferred a strong bound on the cross section of interaction between dark energy and dark matter, of about $\sigma\leqslant 10^{-41}\,\text{cm}^2$, corresponding to an effective interaction scale of $3\times10^6\,\text{GeV}$. This distinguishes our model from traditional axion DM models and from ΛCDM. Our proposal can be tested and limited by next generations of experiments dedicated to accurate measurements[1)] of DE. In particular, a new generation of surveys beyond the Dark Energy Surveys [62] can provide a test-bed for such a proposal. The EUCLID collaboration promises to improve the sensitivity of such a direction [63]. As pointed out in Refs. [64, 65], further experiments testing dark energy will be DESI and LSST. A complete analysis of our model accounting for current data is in preparation in a companion paper. We suggest that comparing CMB with data from observations of IA supernovae redshifts will enable measurement of such an interaction among axion condensates and DE. For example, the w-parameter of dark energy, because of the interaction portal to dark matter, has to slowly decrease with cosmological time. Combining future observations of new surveys, mentioned above, with Planck data could provide another test of our model.

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