HTL resummation in the light cone gauge*

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Abstract: The light cone gauge with light cone variables is often used in pQCD calculations in relativistic heavy-ion collision physics. The Hard Thermal Loops (HTL) resummation is an indispensable technique for hot QCD calculation. It was developed in covariant gauges with conventional Minkowski varaiables; we shall extend this method to the light cone gauge. In the real time formalism, using the Mandelstam-Leibbrant prescription of $(n{\cdot}K)^{-1}$, we calculate the transverse and longitudinal components of the gluon HTL self energy, and prove that there are no infrared divergences. With this HTL self energy, we derive the HTL resummed gluon propagator in the light cone gauge. We also calculate the quark HTL self energy and the resummed quark propagator in the light cone gauge and find it is gauge independent. As application examples, we analytically calculate the damping rates of hard quarks and gluons with the HTL resummed gluon propagator in the light cone gauge and showed that they are gauge independent. The final physical results are identical to those computed in covariant gauge, as they should be.

Keywords: hard thermal loop, light cone gauge, damping rate

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1 Introduction

Bare perturbative QCD theory breaks down at high temperature, and there are some serious problems in gauge theories at finite temperature, such as infrared (IR) singularities and gauge dependent results, when the bare propagators (vertices) are used. The HTL (Hard Thermal Loop) resummed propagators have been developed by Braaten and Pisarski [1]. Some gauge independent physical quantities are given with the HTL resummed propagators in the calculation, other than the bare propagators at finite temperature. If the momentum of the propagators is soft at finite temperature, we should use the HTL resummed propagators. High order loop HTL diagrams can give low order contributions in the coupling constant at finite temperature, which should be resummed. Because of the HTL resummation, medium effects are taken into account, such as the Debye screening caused by the color charges of the QGP. The HTL resummation technique is a great step forward compared to the bare perturbation theory at finite temperature. However, this technique was developed in conventional covariant gauges with Minkowski variables.

In the 1980s, some positive features emerged from studies of perturbative QCD in the light cone gauge [2–5]. The light cone gauge is a non-covariant and physical gauge [6, 7], and is also ghost-free. When multiple gluon emission is calculated in the light cone gauge, because the interference terms among different tree diagrams do not contribute to the leading order in the process of calculating the diagram amplitude, the differential cross section with n-gluon emission in the leading pole approximation has a simple ladder structure at zero temperature [8]. For instance, in deep inelastic processes, only planar diagrams are needed to evaluate the dominant contribution in the leading logarithmic approximation [9]. These nice properties simplify the calculation. Based on the factorization theorem [10], in the light cone gauge a systematic mechanism has been developed to deal with collision processes with perturbative QCD [11–17]. However, the light cone gauge has its disadvantages, such as the spurious singularity of $(n{\cdot}K)^{-1}$, the renormalization and so on.

In heavy ion collision experiments, high transverse momentum partons suffer radiation energy loss and collision energy loss through the hot and dense medium, and then in vacuum fragment into hadrons, which are observed in the mid-rapidity region. These partons lose

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energy and lead to the suppression of the high p_T hadron spectrum, which is called jet quenching. For high transverse momentum jets, the shower partons in the jet cone have large momentum along the axis of the jet, but low momentum perpendicular to the axis, and it is more suitable to calculate some physical quantities in the light cone gauge with light cone variables $K^\mu=(k^+,k^-,\overrightarrow{k_\perp})$, than to use the Coulomb gauge in Minkowski space $K^\mu=(k^0,\overrightarrow{k})$. Many good theoretical studies have been done in the light cone gauge which are consistent with the experimental data [18–25].

To date, we can only calculate the evolution equations of parton distribution functions (PDFs) [26, 27] and parton fragmentation functions (FFs) [28–30], which are both non-perturbative physical quantities, by pertubative QCD. The evolution equations govern the running of PDFs and FFs with a scale Q. Correspondingly, we can extract PDFs and FFs, which are taken to have some fixed value for the relevant hard scale Q, from the experimental data.

For some physical quantities such as the FFs [31], we can extend the case of high energy in vacuum to the case at high temperature. For hard processes, we can use the bare propagators in the light cone gauge at finite temperature. With the HTL resummed gluon propagator in the light cone gauge, we can consider multiple soft gluon scattering among hard partons and hot medium in heavy ion collisions[1)], which contains soft processes as well as hard processes. Soft processes can give a significant correction to the hard processes. The purpose of this paper is to extend the conventional HTL resummation to the light-cone gauge. We shall work out the HTL resummed gluon and quark propagators by calculating the transverse and longitudinal parts of the gluon HTL self energy and quark self energy in the light cone gauge. As application examples, we will compute the damping rates of hard quarks and gluons at high temperature with the HTL resummed propagators and demonstrate their gauge invariance. Thus, this study can serve as a basis for future research with high temperature QCD in the light-cone gauge.

The remainder of the paper is organized as follows. In Section 2 we review the HTL resummed gluon propagator in the Coulomb gauge. In Section 3 the HTL resummed gluon propagator in the light cone gauge is worked out, and then we calculate and analyze the transverse and longitudinal parts of the gluon HTL self energy in the light cone gauge. Via the HTL resummed gluon propagator, we show the transverse and longitudinal spectral functions and the equations of the dispersion relation. In Section 4 we calculate the quark self energy in the HTL approximation, and show that the HTL resummed quark propagator is gauge independent. In Section 5 we analytically calculate the damping rates of hard quarks and gluons with this HTL resummed gluon propagator in the light cone gauge in a particular limit, and demonstrate that in the general case we can have the same result in the light cone gauge and the Coulomb gauge. Our conclusion is given in Section 6. We define the notation $P^\mu=(p^0,\overrightarrow{p})$, etc. In the Appendix, we demonstrate that the light cone term in the gluon HTL self energy has no divergence.

1) Qi Chen, Defu Hou, Xin-nian Wang et al, in preparation

2 HTL resummed gluon propagator in the Coulomb gauge

At zero temperature, covariant gauges have a definite advantage over non-covariant gauges such as the Coulomb gauge or axial gauges. Calculations are simplified considerably due to Lorentz invariance, and the renormalization program can be implemented in practice only in covariant gauge. At finite temperature, Lorentz invariance is broken because the heat bath defines a privileged frame, and renormalization is of secondary importance, so that non-covariant gauges may present useful alternatives to covariant gauge [32].

The HTL resummed gluon propagator has been derived in the Coulomb gauge [33, 34]. The HTL gluon self energy $\Pi^{\mu\nu}(P)$ is expressed as the transverse part and the longitudinal part. The gluon self energy is given by

$$\Pi^{\mu\nu}(P)=-\Pi_{\mathrm{T}}(P)T_P^{\mu\nu}-\frac{1}{n_P^2}\Pi_{\mathrm{L}}(P)L_P^{\mu\nu}, \tag{1}$$

where the transverse projection tensor $T_P^{\mu\nu}$, the longitudinal projection tensor $L_P^{\mu\nu}$, and the four-vector n_P^μ are defined as

$$\begin{aligned} T_P^{\mu\nu} &= g^{\mu\nu}-\frac{P^\mu P^\nu}{P^2}-\frac{n_P^\mu n_P^\nu}{n_P^2}, \\ L_P^{\mu\nu} &= \frac{n_P^\mu n_P^\nu}{n_P^2}, \\ n_P^\mu &= n^\mu-\frac{n\cdot P}{P^2}P^\mu. \end{aligned} \tag{2}$$

The axial vector is

$$n_c^\mu=(n^0,n^1,n^2,n^3)=(1,0,0,0), \tag{3}$$

which specifies the thermal rest frame.

The inverse propagator for general ξ in the Coulomb gauge is

$$\Delta_\xi^{-1}(P)^{\mu\nu}=\Delta^{-1}(P)^{\mu\nu}-\frac{1}{\xi}(P^\mu-P\cdot nn^\mu)(P^\nu-P\cdot nn^\nu), \tag{4}$$

where ξ is an arbitrary gauge parameter.

The inverse propagator reduces in the limit $\xi\to\infty$ to

$$\Delta_\infty^{-1}(P)^{\mu\nu}=-P^2g^{\mu\nu}+P^\mu P^\nu-\Pi^{\mu\nu}(P). \tag{5}$$

$\Delta_\infty^{-1}(P)^{\mu\nu}$ can also be written as

$$\Delta_\infty^{-1}(P)^{\mu\nu}=-\frac{1}{\Delta_{\rm T}(P)}T_P^{\mu\nu}+\frac{1}{n_P^2\Delta_{\rm L}(P)}L_P^{\mu\nu}, \qquad (6)$$

where $\Delta_{\rm T}(P)$ and$\Delta_{\rm L}(P)$ are the transverse and longitudinal propagators:

$$\begin{aligned}\Delta_{\rm T}(P) &= \frac{1}{P^2-\Pi_{\rm T}(P)},\\ \Delta_{\rm L}(P) &= \frac{1}{-n_P^2P^2+\Pi_{\rm L}(P)}.\end{aligned} \qquad (7)$$

The HTL resummed gluon propagator in the Coulomb gauge [34] is

$$\Delta_\xi^{\mu\nu}(P)=-\Delta_{\rm T}(P)T_P^{\mu\nu}+\Delta_{\rm L}(P)n^\mu n^\nu-\xi\frac{P^\mu P^\nu}{(n_P^2P^2)^2}. \qquad (8)$$

By calculating, we can find

$$T_P^{00}=0,T_P^{0i}=T_P^{i0}=0. \qquad (9)$$

So the HTL resummed gluon propagator in the Coulomb gauge can be simplified to

$$\begin{aligned}G_{\rm Ret}^{00}(P) &= \frac{1}{p^2+\Pi_{\rm L}(P)},\\ G_{\rm Ret}^{ij}(P) &= \frac{\delta^{ij}-\hat{p}^i\hat{p}^j}{P^2-\Pi_{\rm T}(P)},\end{aligned} \qquad (10)$$

where $\hat{\overrightarrow{p}}$ is a unit vector in the direction of $\overrightarrow{p}$, $\hat{\overrightarrow{p}}=\frac{\overrightarrow{p}}{|\overrightarrow{p}|}$ and $\hat{\overrightarrow{p}}=(\hat{p}^1,\hat{p}^2,\hat{p}^3)$.

The longitudinal and transverse gluon HTL self energy [35] are

$$\begin{aligned}\Pi_{\rm L}(P) &= m_{\rm D}^2\left[1-\frac{p_0}{2p}\ln|\frac{p_0+p}{p_0-p}|+i\pi\frac{p_0}{2p}\theta(p^2-p_0^2)\right],\\ \Pi_{\rm T}(P) &= \frac{m_{\rm D}^2}{2}\frac{p_0^2}{p^2}\left[1-(1-\frac{p^2}{p_0^2})\frac{p_0}{2p}\left[\ln|\frac{p_0+p}{p_0-p}|-i\pi\theta(p^2-p_0^2)\right]\right],\end{aligned} \qquad (11)$$

where the gluon screening mass $m_{\rm D}^2=\frac{1}{3}(C_{\rm A}+\frac{1}{2}N_{\rm f})g^2T^2$ and $\theta(p^2-p_0^2)$ is the step function.

The imaginary parts in the above equations correspond to the Landau damping, which means that one particle is emitted from the thermal medium and absorbed by the medium.

In the static limit, $p_0\rightarrow 0$, the longitudinal HTL self energy

$$\Pi_{\rm R}^L(p_0\rightarrow 0,p)=m_{\rm D}^2, \qquad (12)$$

which means the Debye screening of the gluon in the plasma.

However, in the static limit, the transverse HTL self energy

$$\Pi_{\rm R}^T(p_0\rightarrow 0,p)=0, \qquad (13)$$

which shows no static magnetic screening.

The self energy tensor $\Pi^{\mu\nu}$ is symmetric in μ and ν and satisfies

$$\begin{aligned}&P_\mu\Pi^{\mu\nu}(P)=0,\\ &g_{\mu\nu}\Pi^{\mu\nu}(P)=-2\Pi_{\rm T}(P)-\frac{1}{n_P^2}\Pi_{\rm L}(P)=-m_{\rm D}^2.\end{aligned} \qquad (14)$$

3 HTL resummed gluon propagator in the light cone gauge

In this section, we derive the HTL resummed gluon propagator in the light cone gauge, and compute the transverse and longitudinal parts of gluon HTL self energy in the real time formalism. In the static limit, we discuss $\Pi^{00}(P)$ in the light cone gauge and the Coulomb gauge. We obtain the pole terms and the cut terms of the transverse and longitudinal spectral functions.

The light cone gauge is an axial and non-covariant gauge [6, 7],

$$n_{\rm l}^2=0, \quad n_{\rm l}\cdot A=0. \qquad (15)$$

The axial vector in the light cone gauge is

$$n_{\rm l}^\mu=(n^0,n^1,n^2,n^3)=(\frac{\sqrt{2}}{2},0,0,-\frac{\sqrt{2}}{2}). \qquad (16)$$

The bare gluon propagator in the light cone gauge is

$$\frac{i\left(-g^{\mu\nu}+\dfrac{n^\mu K^\nu+n^\nu K^\mu}{n\cdot K}\right)}{K^2+i\epsilon}. \qquad (17)$$

Here we use the Mandelstam-Leibbrandt (ML) prescription of $(n\cdot K)^{-1}$ instead of the usual principal-value prescription,

$$\begin{aligned}\frac{1}{n\cdot K} &= \frac{n^*\cdot K}{n\cdot Kn^*\cdot K+i\epsilon}=\frac{1}{n\cdot K+isgn(n^*\cdot K)\epsilon}\\ &= \frac{n_0k_0+\overrightarrow{n}\cdot\overrightarrow{k}}{(n_0k_0)^2-(\overrightarrow{n}\cdot\overrightarrow{k})^2+i\epsilon}.\end{aligned} \qquad (18)$$

where $n_{\rm l}^{*\mu}=(n^0,n^1,n^2,n^3)=(\frac{\sqrt{2}}{2},0,0,\frac{\sqrt{2}}{2})$.

The usual principal-value prescription of $(n\cdot K)^{-1}$ leads to some serious problems, such as violating power counting and other basic criteria, when we calculate the integral of the loop diagram [6].

In the real time mechanism, the time of the field goes from $t=0$ to $t=-i\beta$. The contour can be deformed in order to include the real time axis by going first from $t=0$ to $t=\infty$ above the real time axis and then back to $t=-i\beta$ below the real time axis. So we have double degrees of freedom, one above the real time axis and the other below the real time axis. We get the propagator in the real time formalism [32, 36], which is a 2×2 matrix,

$$\varDelta(K) = \begin{pmatrix}\varDelta_{11} & \varDelta_{12}\\ \varDelta_{21} & \varDelta_{22}\end{pmatrix} = \begin{pmatrix}\dfrac{1}{K^2-m^2+i\epsilon} & 0\\ 0 & \dfrac{-1}{K^2-m^2-i\epsilon}\end{pmatrix} -2\pi i\delta(K^2-m^2) \times\begin{pmatrix} n_{\rm B}(k_0) & \theta(-k_0)+n_{\rm B}(k_0)\\ \theta(-k_0)+n_{\rm B}(k_0) & n_{\rm B}(k_0)\end{pmatrix}. \quad (19)$$

The Bose-Einstein distribution function $n_{\rm B}(k_0)$ is

$$n_{\rm B}(k_0)=\frac{1}{{\rm e}^{|\frac{k_0}{T}|}-1}. \quad (20)$$

For fermions, we have

$$F(K) = (\not{K}+m)\tilde{\varDelta}(K)=(\not{K}+m)\begin{pmatrix}\tilde{\varDelta}_{11} & \tilde{\varDelta}_{12}\\ \tilde{\varDelta}_{21} & \tilde{\varDelta}_{22}\end{pmatrix} = (\not{K}+m)\left[\begin{pmatrix}\dfrac{1}{K^2-m^2+i\epsilon} & 0\\ 0 & \dfrac{-1}{K^2-m^2-i\epsilon}\end{pmatrix} -2\pi i\delta(K^2-m^2) \times\begin{pmatrix} -f(k_0) & \theta(-k_0)-f(k_0)\\ \theta(-k_0)-f(k_0) & -f(k_0)\end{pmatrix}\right], \quad (21)$$

The Fermi-Dirac distribution function $f(k_0)$ is

$$f(k_0)=\frac{1}{{\rm e}^{|\frac{k_0}{T}|}+1}. \quad (22)$$

We use the Keldysh representation in the real time formalism [36, 37]. The retarded propagator, advanced propagator and symmetric propagator for bosons are

$$\begin{aligned}\varDelta_{\rm R}(K)&=\varDelta_{11}-\varDelta_{12}=\frac{1}{K^2-m^2+isgn(k_0)\epsilon},\\ \varDelta_{\rm A}(K)&=\varDelta_{11}-\varDelta_{21}=\frac{1}{K^2-m^2-isgn(k_0)\epsilon},\\ \varDelta_{\rm S}(K)&=\varDelta_{11}+\varDelta_{22}=-2\pi i\delta(K^2-m^2)[1+2n_{\rm B}(k_0)].\end{aligned} \quad (23)$$

The inverse relation for bosons is

$$\begin{aligned}\varDelta_{11}&=\frac{1}{2}[\varDelta_{\rm S}(K)+\varDelta_{\rm A}(K)+\varDelta_{\rm R}(K)],\\ \varDelta_{12}&=\frac{1}{2}[\varDelta_{\rm S}(K)+\varDelta_{\rm A}(K)-\varDelta_{\rm R}(K)],\\ \varDelta_{21}&=\frac{1}{2}[\varDelta_{\rm S}(K)-\varDelta_{\rm A}(K)+\varDelta_{\rm R}(K)],\\ \varDelta_{22}&=\frac{1}{2}[\varDelta_{\rm S}(K)-\varDelta_{\rm A}(K)-\varDelta_{\rm R}(K)].\end{aligned} \quad (24)$$

For fermions, in the Keldysh representation, we only replace the Bose-Einstein distribution function $n_{\rm B}(k_0)$ by the Fermion-Dirac distribution function $-f(k_0)$ in the symmetric propagator, and the retarded propagator and advanced propagator are the same as that of bosons,

$$\tilde{\varDelta}_{\rm S}(K)=\tilde{\varDelta}_{11}+\tilde{\varDelta}_{22}=-2\pi i\delta(K^2-m^2)[1-2f(k_0)]. \quad (25)$$

The inverse relation in Eq. (24) is also applicable for fermions.

3.1 Gluon HTL self energy in the light cone gauge

Explicit computation in different gauges (covariant, Coulomb, temporal) whose axial vectors are all the same, $n_c^\mu=(1,0,0,0)$, has been used to check that the gluon HTL self energy does not depend on the choice of gauge. However, the axial vector in the light cone gauge is different, which brings about some changes.

A massive boson gives rise to the longitudinal polarization state, so that the boson self energy is separated into the longitudinal and transverse parts [8]. The massive gluon self energy in the light cone gauge is made up of the transverse and longitudinal parts,

$$\Pi^{\mu\nu}(P)=-\left[\tilde{T}_P^{\mu\nu}\Pi_{\rm T}(P)+\frac{\tilde{L}_P^{\mu\nu}}{n_P^2}\Pi_{\rm L}(P)\right]. \quad (26)$$

The transverse projection tensor is

$$\tilde{T}_P^{\mu\nu}=g^{\mu\nu}-\frac{n^\mu P^\nu+n^\nu P^\mu}{n\cdot P}+\frac{n^\mu n^\nu P^2}{(n\cdot P)^2}. \quad (27)$$

The longitudinal projection tensor is

$$\tilde{L}_P^{\mu\nu}=-\left[\frac{n^\mu n^\nu P^2}{(n\cdot P)^2}-\frac{n^\mu P^\nu+n^\nu P^\mu}{n\cdot P}+\frac{P^\mu P^\nu}{P^2}\right]. \quad (28)$$

The four-vector n_P^μ is

$$n_P^\mu=\left(g^{\mu\nu}-\frac{P^\mu P^\nu}{P^2}\right)n_\nu=n^\mu-\frac{n\cdot P}{P^2}P^\mu. \quad (29)$$

$\tilde{L}_p^{\mu\nu}$ and $\tilde{T}_p^{\mu\nu}$ satisfy the following relations,

$$\begin{aligned}\tilde{L}_P^{\mu\nu}\tilde{L}_{P\nu}^{\rho} &= \tilde{L}_P^{\mu\rho},\\ \tilde{T}_P^{\mu\nu}\tilde{T}_{P\nu}^{\rho} &= \tilde{T}_P^{\mu\rho},\\ \tilde{T}_P^{\mu\rho}\tilde{L}_{P\mu\sigma} &= 0.\end{aligned} \quad (30)$$

These equations are also suitable for $L_P^{\mu\nu}$ and $T_P^{\mu\nu}$ in Eq. (2) in the Coulomb gauge.

The axial vector n^μ in the light cone gauge is defined in Eq. (16),

$$n_\mu\tilde{T}_P^{\mu\nu}=0,\quad n_\mu\tilde{L}_P^{\mu\nu}\neq 0. \quad (31)$$

$\tilde{L}_P^{\mu\nu}$ and $\tilde{T}_P^{\mu\nu}$ are the longitudinal and transverse projection tensors with respect to the axial vector n^μ in the light cone gauge in Eq. (16). Because the axial vector in the light cone gauge in Eq. (16) is different from the axial vector in the Coulomb gauge in Eq. (3), the longitudinal and transverse projection tensors $\tilde{L}_P^{\mu\nu}$ and $\tilde{T}_P^{\mu\nu}$ are different from those in the Coulomb gauge. Finally, these differences change the expression of the HTL resummed gluon propagator in the light cone gauge.

The inverse propagator in the light cone gauge in the limit $\xi\to\infty$ is

$$\Delta_{\infty}^{-1}(P)^{\mu\nu} = -P^2g^{\mu\nu}+P^{\mu}P^{\nu}-\Pi^{\mu\nu}(P), \tag{32}$$

$$= \left[-P^2+\Pi_{\rm T}(P)\right]\tilde{T}_P^{\mu\nu}+\left[-P^2+\frac{1}{n_P^2}\Pi_{\rm L}(P)\right]\tilde{L}_P^{\mu\nu}. \tag{33}$$

Applying Eqs. (30), (31), we can get the HTL resummed gluon propagator in the light cone gauge

$$\Delta^{\mu\nu}(P)=\frac{\tilde{T}_P^{\mu\nu}}{-P^2+\Pi_{\rm T}(P)}+\frac{-\dfrac{n^{\mu}n^{\nu}P^2}{(n\cdot P)^2}}{-P^2+\dfrac{1}{n_P^2}\Pi_{\rm L}(P)}, \tag{34}$$

where we do not consider the terms containing the gauge parameter ξ.

When $\Pi_{\rm T}(P)=0$ and $\Pi_{\rm L}(P)=0$, the HTL resummed gluon propagator returns back to the bare gluon propagator in Eq. (17).

Due to the relations in Eqs. (26), (27), (28), (30), the transverse and longitudinal gluon HTL self-energies are given by

$$\Pi_{\rm T}(P) = -\frac{1}{2}\tilde{T}_{P\mu\nu}\Pi^{\mu\nu}(P) = -\frac{1}{2}\left[g_{\mu\nu}-\frac{n_{\mu}P_{\nu}+n_{\nu}P_{\mu}}{n\cdot P}+\frac{n_{\mu}n_{\nu}P^2}{(n\cdot P)^2}\right]\Pi^{\mu\nu}(P), \tag{35}$$

$$\frac{1}{n_P^2}\Pi_{\rm L}(P) = -\tilde{L}_{P\mu\nu}\Pi^{\mu\nu}(P) = \left[\frac{P_{\mu}P_{\nu}}{P^2}-\frac{n_{\mu}P_{\nu}+n_{\nu}P_{\mu}}{n\cdot P}+\frac{n_{\mu}n_{\nu}P^2}{(n\cdot P)^2}\right]\Pi^{\mu\nu}(P). \tag{36}$$

$\Pi^{\mu\nu}(P)$ is the sum of the quark loop, the gluon loop and the gluon tadpole in the light cone gauge. Multiplying $\Pi^{\mu\nu}(P)$ by the projection tensors $-\frac{1}{2}\tilde{T}_{P\mu\nu}$ and $-\tilde{L}_{P\mu\nu}$, we can calculate $\Pi_{\rm T}(P)$ and $\frac{1}{n_P^2}\Pi_{\rm L}(P)$ in the HTL approximation.

3.2 Quark loop of gluon HTL self energy in the light cone gauge

The quark loop in Fig. 1 can be expressed as

$$\Pi_{ab}^{\mu\alpha}(P) = -i\frac{1}{2}N_{\rm f}g^2\delta_{ab}\int\frac{d^4K}{(2\pi)^4}{\rm Tr}[\gamma^{\mu}F(K)\gamma^{\alpha}F(K-P)], \tag{37}$$

where $N_{\rm f}$ is the number of active quark flavors and $F(K)$ is the bare quark propagator.

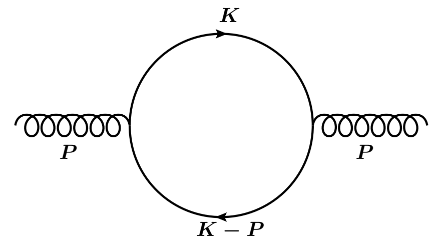

Fig. 1. The quark loop in the light cone gauge.

The retarded self energy in the real time formalism [37, 38] is expressed as

$$\begin{aligned}\Pi_{\rm R}^{\mu\alpha}(P) &= \Pi_{11}^{\mu\alpha}(P)+\Pi_{12}^{\mu\alpha}(P)\\ &= -\frac{i}{2}g^2N_{\rm f}\int\frac{{\rm d}^4K}{(2\pi)^4}{\rm Tr}[\gamma^{\mu}\not{K}\gamma^{\alpha}(\not{K}-\not{P})]\\ &\quad\times[\tilde{\Delta}_{11}(K)\tilde{\Delta}_{11}(K-P)-\tilde{\Delta}_{12}(K)\tilde{\Delta}_{21}(K-P)],\end{aligned} \tag{38}$$

where the RTF Green function $\tilde{\Delta}_{ij}(K)$ refers to the component of the propagator in the real time formalism in Eq. (21).

Multiplying $\Pi^{\mu\alpha}(P)$ by the transverse projection tensor $-\frac{1}{2}\tilde{T}_{P\mu\alpha}$, we can get the transverse self energy $\Pi_{\rm T}(P)$,

$$\begin{aligned}\Pi_{\rm T}(P) &= \frac{i}{4}N_{\rm f}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}8\left[K\cdot P-2\frac{n\cdot K}{n\cdot P}K\cdot P+\frac{(n\cdot K)^2}{(n\cdot P)^2}P^2\right]\\ &\quad\times\left[\tilde{\Delta}_{11}(K)\tilde{\Delta}_{11}(K-P)-\tilde{\Delta}_{12}(K)\tilde{\Delta}_{21}(K-P)\right].\end{aligned} \tag{39}$$

Using the relation in Eq. (24), we can obtain

$$\begin{aligned}&\tilde{\Delta}_{11}(K)\tilde{\Delta}_{11}(K-P)-\tilde{\Delta}_{12}(K)\tilde{\Delta}_{21}(K-P)\\ &= \frac{1}{2}\left[\tilde{\Delta}_{\rm S}(K-P)\tilde{\Delta}_{\rm R}(K)+\tilde{\Delta}_{\rm A}(K-P)\tilde{\Delta}_{\rm S}(K)\right.\\ &\quad\left.+\tilde{\Delta}_{\rm A}(K-P)\tilde{\Delta}_{\rm A}(K)+\tilde{\Delta}_{\rm R}(K-P)\tilde{\Delta}_{\rm R}(K)\right]\\ &= \frac{1}{2}\left[\tilde{\Delta}_{\rm S}(K-P)\tilde{\Delta}_{\rm R}(K)+\tilde{\Delta}_{\rm A}(K-P)\tilde{\Delta}_{\rm S}(K)\right],\end{aligned} \tag{40}$$

where the minus sign in front of the term $\tilde{\Delta}_{12}(K)\tilde{\Delta}_{21}(K-P)$ comes from the vertex of the type 2 fields [39]. The k_0 integrals of $\tilde{\Delta}_{\rm A}(K-P)\tilde{\Delta}_{\rm A}(K)$ and $\tilde{\Delta}_{\rm R}(K-P)\tilde{\Delta}_{\rm R}(K)$ reduce to zero.

Replacing K by $P-K$ in the first term and using $\tilde{\Delta}_{\rm R}(P-K)=\tilde{\Delta}_{\rm A}(K-P)$, this expression can be simplified further,

$$\begin{aligned}\Pi_{\rm T}(P) &= \frac{i}{4}N_{\rm f}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}8\left[K\cdot P-2\frac{n\cdot K}{n\cdot P}K\cdot P+\frac{(n\cdot K)^2}{(n\cdot P)^2}P^2\right]\\ &\quad\times\tilde{\Delta}_{\rm S}(K)\tilde{\Delta}_{\rm A}(K-P)\\ &= \frac{i}{4}N_{\rm f}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}8\left[K\cdot P-2\frac{n\cdot K}{n\cdot P}K\cdot P+\frac{(n\cdot K)^2}{(n\cdot P)^2}P^2\right]\\ &\quad\times(-2\pi i)\delta(K^2)[1-2f(k_0)]\\ &\quad\times\frac{1}{(K-P)^2-isgn(k_0-p_0)\epsilon}.\end{aligned} \tag{41}$$

In the calculation, we use the HTL approximation, i.e. the high temperature limit. The internal momentum K is hard, and the external momentum P is soft. The transverse part of the quark loop in the light cone gauge in the HTL approximation from Eq. (35) is obtained by

$$\begin{aligned}\Pi_{\rm T}(P) &= \frac{i}{4}N_{\rm f}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}8\left[K{\cdot}P-2\frac{n{\cdot}K}{n{\cdot}P}K{\cdot}P+\frac{(n{\cdot}K)^2}{(n{\cdot}P)^2}P^2\right]\\&\quad\times\tilde{\Delta}_{\rm S}(K)\tilde{\Delta}_{\rm A}(K-P)\\&= \frac{1}{12}N_{\rm f}g^2T^2\left[\frac{(p_0)^2}{p^2}-\frac{p_0P^2}{2p^3}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\right].\end{aligned}\tag{42}$$

Similarly, the longitudinal part of the quark loop in the light cone gauge in the HTL approximation is

$$\begin{aligned}\frac{1}{n_P^2}\Pi_{\rm L}(P) &= \frac{i}{2}g^2N_{\rm f}\int\frac{{\rm d}^4K}{(2\pi)^4}\left[8\frac{(n{\cdot}K)^2}{(n{\cdot}P)^2}P^2-16\frac{n{\cdot}K}{n{\cdot}P}K{\cdot}P\right.\\&\quad\left.+8\frac{(K{\cdot}P)^2}{P^2}-4K{\cdot}P+4K^2\right]\tilde{\Delta}_{\rm S}(K)\tilde{\Delta}_{\rm A}(K-P)\\&= -\frac{1}{6}N_{\rm f}g^2T^2\left[\frac{P^2}{p^2}-\frac{p_0P^2}{2p^3}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\right].\end{aligned}\tag{43}$$

3.3 Gluon loop and gluon tadpole of gluon HTL self energy in the light cone gauge

The gluon loop and the gluon tadpole in Fig. 2 are expressed as

$$\begin{aligned}\Pi_{ab}^{\mu\alpha}(P) &= \frac{i}{2}\int\frac{{\rm d}^4K}{(2\pi)^4}V^{\mu\nu\rho}(P,-K,K-P)id_{\nu\beta}(K)V^{\beta\gamma\alpha}\\&\quad\times(K,P-K,-P)id_{\rho\gamma}(K-P)G(K)G(K-P)\\&\quad+\frac{i}{2}\int\frac{{\rm d}^4K}{(2\pi)^4}id_{\rho\sigma}(K)G(K)\delta^{cd}V_{abcd}^{\mu\alpha\rho\sigma}.\end{aligned}\tag{44}$$

where $d_{\nu\beta}(K)G(K)$, $d^{\rho\gamma}(K–P)G(K–P)$ and $d_{\rho\sigma}(K)G(K)$ are all the bare gluon propagators.

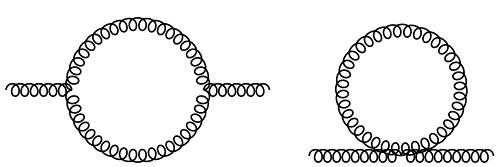

Fig. 2. The gluon loop and the gluon tadpole in the light cone gauge.

The tensor of the bare gluon propagator in the light cone gauge is

$$d_{\nu\beta}(K)=-g_{\nu\beta}+\frac{n_\nu K_\beta+n_\beta K_\nu}{n{\cdot}K}.\tag{45}$$

The three-gluon vertexes are

$$\begin{aligned}&V^{\mu\nu\rho}(P,-K,K-P)\\&= gf^{acd}\left[g^{\mu\nu}(P+K)^\rho+g^{\nu\rho}(-2K+P)^\mu+g^{\mu\rho}(K-2P)^\nu\right],\\&V^{\beta\gamma\alpha}(K,P-K,-P)\\&= gf^{cdb}\left[g^{\beta\gamma}(2K-P)^\alpha+g^{\gamma\alpha}(2P-K)^\beta+g^{\beta\alpha}(-P-K)^\gamma\right].\end{aligned}\tag{46}$$

The four-gluon vertex is

$$\begin{aligned}V_{abcd}^{\mu\alpha\rho\sigma} &= -ig^2[f^{abe}f^{cde}(g^{\mu\rho}g^{\alpha\sigma}-g^{\mu\sigma}g^{\alpha\rho})\\&\quad+f^{ace}f^{bde}(g^{\mu\alpha}g^{\rho\sigma}-g^{\mu\sigma}g^{\alpha\rho})\\&\quad+f^{ade}f^{bce}(g^{\mu\alpha}g^{\rho\sigma}-g^{\mu\rho}g^{\alpha\sigma})].\end{aligned}\tag{47}$$

The transverse part of the gluon self energy in the HTL approximation from Eq. (35) is

$$\begin{aligned}\Pi_{\rm T}(P) &= \frac{i}{4}C_{\rm A}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\left[-8K{\cdot}P-12P^2+16\frac{n{\cdot}K}{n{\cdot}P}K{\cdot}P\right.\\&\quad-8\frac{(n{\cdot}K)^2}{(n{\cdot}P)^2}P^2+8\frac{n{\cdot}P}{n{\cdot}(K-P)}(K^2-P^2)\\&\quad\left.-8\frac{n{\cdot}P}{n{\cdot}K}(K^2-2K{\cdot}P)\right]G(K)G(K-P)\\&= \frac{1}{6}C_{\rm A}g^2T^2\left[\frac{(p_0)^2}{p^2}-\frac{p_0P^2}{2p^3}\ln\frac{p_0+p+i\epsilon}{p_0-p-i\epsilon}\right]\\&\quad+\frac{i}{4}C_{\rm A}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\left[8\frac{n{\cdot}P}{n{\cdot}(K-P)}(K^2-P^2)\right.\\&\quad\left.-8\frac{n{\cdot}P}{n{\cdot}K}(K^2-2K{\cdot}P)\right]G(K)G(K-P).\end{aligned}\tag{48}$$

The last two light cone terms in the fourth line come from the tensors of the gluon propagators $(n^\nu K^\beta+n^\beta K^\nu)/(nK)$ and $[n^\rho(K-P)^\gamma+n^\gamma(K-P)^\rho]/((n{\cdot}(K-P))$. Replacing K by $P-K$ in the first light cone term, the sum of the two light cone terms is

$$-\frac{i}{4}C_{\rm A}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}16\frac{n{\cdot}P}{n{\cdot}K}\frac{(K^2-2K{\cdot}P)}{K^2(K-P)^2}.\tag{49}$$

At zero temperature, the divergence of the integral calculation of the one-loop diagram in Fig. 2 has been renormalized successfully. Here we only consider the contribution at finite temperature. We use the ML prescription of $1/(n{\cdot}K)$ in Eq. (18). By calculating with the contour integral, we find there is no divergence in the HTL approximation, and the power of that part is of order g^3T^2, which can be ignored in the result. The proof is in the Appendix.

Similarly, the longitudinal part of the gluon self energy in the HTL approximation is obtained by

$$\begin{aligned}\frac{1}{n_P^2}\Pi_{\rm L}(P) &= \frac{i}{2}C_{\rm A}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\left[4K^2-4K{\cdot}P+2P^2+8\frac{(K{\cdot}P)^2}{P^2}-16\frac{n{\cdot}K}{n{\cdot}P}K{\cdot}P+8\frac{(n{\cdot}K)^2}{(n{\cdot}P)^2}P^2+\frac{n{\cdot}P}{n{\cdot}(K-P)}\frac{2K^2(K-P)^2}{P^2}\right.\\&\quad\left.-\frac{n{\cdot}P}{n{\cdot}K}\frac{2K^2(K-P)^2}{P^2}\right]G(K)G(K-P)=-\frac{1}{3}C_{\rm A}g^2T^2\left[\frac{P^2}{p^2}-\frac{p_0P^2}{2p^3}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\right]\end{aligned}$$

$$+\frac{i}{2}C_{\rm A}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\frac{1}{P^2}\left[2\frac{n\cdot P}{n\cdot(K-P)}-2\frac{n\cdot P}{n\cdot K}\right]. \tag{50}$$

There are two light cone terms in the third line. By calculating, the integral with the light cone terms is zero

$$\frac{i}{2}C_{\rm A}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\frac{1}{P^2}\left[2\frac{n\cdot P}{n\cdot(K-P)}-2\frac{n\cdot P}{n\cdot K}\right]=0. \tag{51}$$

During the calculation, we set the momentum $\overrightarrow{p}$ is on the positive direction of the z axis. $\overrightarrow{n}$ in Eq. (16) is in the negative direction of the z axis. The angle between $\overrightarrow{k}$ and $\overrightarrow{p}$ is θ. Here we use the Keldysh representation in the real time formalism and consider the $T>0$ contribution. In the HTL approximation, the internal momentum K is soft, and the external momentum P is hard.

3.4 Transverse and longitudinal parts of the gluon HTL self energy

Adding up the results from the longitudinal and transverse parts of the quark loop in Fig. 1, and the gluon loop and the gluon tadpole in Fig. 2 in the HTL approximation, we obtain the following expression

$$\Pi_{\rm T}(P)=\frac{1}{6}(C_{\rm A}+\frac{1}{2}N_{\rm f})g^2T^2\left[\frac{(p_0)^2}{p^2}-\frac{p_0P^2}{2p^3}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\right],$$
$$\frac{1}{n_P^2}\Pi_{\rm L}(P)=-\frac{1}{3}(C_{\rm A}+\frac{1}{2}N_{\rm f})g^2T^2\left[\frac{P^2}{p^2}-\frac{p_0P^2}{2p^3}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\right]. \tag{52}$$

The factor $\frac{1}{2}$ in the coefficient $(C_{\rm A}+\frac{1}{2}N_{\rm f})$ stems from that these integrals $\int_0^\infty kn(k){\rm d}k$ and $\int_0^\infty kf(k){\rm d}k$, which have different distribution functions. We get the same result for the transverse and longitudinal HTL gluon self energy in the light cone gauge and the Coulomb gauge, although these two gauges have different projection tensors.

In the static limit $p_0\to0$, the longitudinal and transverse parts of gluon HTL self energy in the light cone gauge reduce to

$$\begin{aligned}\lim_{p_0\to0}\frac{1}{n_P^2}\Pi_{\rm L}(P) &= \frac{1}{3}g^2T^2(C_{\rm A}+\frac{1}{2}N_{\rm f})=m_{\rm D}^2,\\ \lim_{p_0\to0}\Pi_{\rm L}(P) &= \frac{1}{6}g^2T^2(C_{\rm A}+\frac{1}{2}N_{\rm f})=\frac{1}{2}m_{\rm D}^2,\\ \lim_{p_0\to0}\Pi_{\rm T}(P) &= 0.\end{aligned} \tag{53}$$

In the static limit $p_0\to0$, the longitudinal and transverse parts of the gluon HTL self energy in the Coulomb gauge reduce to

$$\begin{aligned}\lim_{p_0\to0}\Pi_{\rm L}(P) &= \frac{1}{3}(C_{\rm A}+\frac{1}{2}N_{\rm f})g^2T^2=m_{\rm D}^2,\\ \lim_{p_0\to0}\Pi_{\rm T}(P) &= 0.\end{aligned} \tag{54}$$

We think the axial vector $n_{\rm l}^\mu=(\frac{\sqrt{2}}{2},0,0,-\frac{\sqrt{2}}{2})$ in the light cone gauge is rotated with respect to the axial vector $n_{\rm c}^\mu=(1,0,0,0)$ in the Coulomb gauge, so that it gives rise to some changes.

In the static limit, $n_\mu n_\nu\Pi^{\mu\nu}(P)$ of the gluon HTL self energy in the Coulomb gauge in Eq. (1) is

$$\begin{aligned}\lim_{p_0\to0}n_\mu n_\nu\Pi^{\mu\nu}(P) &= \lim_{p_0\to0}\Pi^{00}(P)\\ &= -\lim_{p_0\to0}\frac{L_P^{00}}{n_P^2}\Pi_{\rm L}(P)=-m_{\rm D}^2,\end{aligned} \tag{55}$$

where the axial vector $n_{\rm c}^\mu=(1,0,0,0)$ in Eq. (3).

Because the axial vector $n_{\rm l}^\mu$ in the light cone gauge is different from $n_{\rm c}^\mu$ in the Coulomb gauge, we compare $\Pi^{00}(P)$ in the light cone gauge with $\Pi^{00}(P)$ in the Coulomb gauge.

In the static limit, $\Pi^{00}(P)$ of the gluon HTL self energy in the light cone gauge in Eq. (26) is

$$\begin{aligned}\lim_{p_0\to0}\Pi^{00}(P) &= \lim_{p_0\to0}-\tilde{L}_P^{00}\frac{1}{n_P^2}\Pi_{\rm L}(P)\\ &= -\frac{g^2T^2}{3}(C_{\rm A}+\frac{1}{2}N_{\rm f})=-m_{\rm D}^2,\end{aligned} \tag{56}$$

where the axial vector $n_{\rm l}^\mu=(\frac{\sqrt{2}}{2},0,0,-\frac{\sqrt{2}}{2})$ in Eq. (16). The external momentum $P^\mu=(p_0,0,0,p_3)$, and $\overrightarrow{p}$ is on the z axis. The result is the same as that in the Coulomb gauge.

3.5 HTL resummed gluon propagator in the light cone gauge and its spectral function

The HTL resummed gluon propagator in the light cone gauge is

$$\begin{aligned}\varDelta^{\mu\nu}(P) &= \frac{\tilde{T}_P^{\mu\nu}}{-P^2+\Pi_{\rm T}(P)}+\frac{-\dfrac{n^\mu n^\nu P^2}{(n\cdot P)^2}}{-P^2+\dfrac{1}{n_P^2}\Pi_{\rm L}(P)}\\ &= \frac{-\tilde{T}_P^{\mu\nu}}{P^2-\dfrac{1}{2}m_{\rm D}^2\left[\dfrac{(p_0)^2}{p^2}-\dfrac{p_0P^2}{2p^3}\ln\dfrac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\right]}\\ &\quad+\frac{\dfrac{n^\mu n^\nu P^2}{(n\cdot P)^2}}{P^2+m_{\rm D}^2\left[\dfrac{P^2}{p^2}-\dfrac{p_0P^2}{2p^3}\ln\dfrac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\right]}.\end{aligned} \tag{57}$$

The transverse and longitudinal spectral functions are expressed as

$$\begin{aligned}\rho_{\rm T}(P) &= 2\pi Z_{\rm T}{\rm sgn}(p_0)\left[\delta(p_0-w_{\rm T})+\delta(p_0+w_{\rm T})\right]+\beta_{\rm T}(P),\\ \rho_{\rm L}(P) &= 2\pi Z_{\rm L}{\rm sgn}(p_0)\left[\delta(p_0-w_{\rm L})+\delta(p_0+w_{\rm L})\right]+\beta_{\rm L}(P).\end{aligned} \tag{58}$$

The spectral function $\rho_{\rm T/L}(P)$ is made up of the pole term and the cut term $\beta_{\mathrm{T/L}}(P)$.

For P^2 space-like, i.e $p_0^2<p^2$, the function $\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}$ generates the imaginary part,

$$\ln\frac{p_0+p\pm i\epsilon}{p_0-p\pm i\epsilon}=\ln\left|\frac{p_0+p}{p_0-p}\right|\mp i\pi\theta(p^2-p_0^2). \tag{59}$$

So the cut terms of the transverse and longitudinal part of the HTL resummed gluon propagator in the light cone gauge are obtained by

$$\beta_{\mathrm{T}}(P)=\frac{\frac{1}{2}\pi m_{\mathrm{D}}^2\frac{p_0P^2}{p^3}\theta(p^2-p_0^2)}{\left[P^2-\frac{1}{2}m_{\mathrm{D}}^2\left[\frac{(p_0)^2}{p^2}-\frac{p_0P^2}{2p^3}\ln|\frac{p_0+p}{p_0-p}|\right]\right]^2+\frac{1}{16}m_{\mathrm{D}}^4\pi^2\frac{(p_0)^2P^2P^2}{p^6}},$$

$$\beta_{\mathrm{L}}(P)=\frac{-\pi m_{\mathrm{D}}^2\frac{p_0}{p}\theta(p^2-p_0^2)}{\left[p^2+m_{\mathrm{D}}^2\left[1-\frac{p_0}{2p}\ln|\frac{p_0+p}{p_0-p}|\right]\right]^2+\frac{1}{4}m_{\mathrm{D}}^4\pi^2\frac{(p_0)^2}{p^2}}, \tag{60}$$

where $\theta(p^2-p_0^2)$ is the step function, and the Debye screening mass $m_{\mathrm{D}}^2=\frac{1}{3}(C_{\mathrm{A}}+\frac{1}{2}N_{\mathrm{f}})g^2T^2$.

Via the expression of the HTL resummed gluon propagator in the light cone gauge, the transverse dispersion relation is given by

$$\omega_{\mathrm{T}}^2-p^2-m_{\mathrm{D}}^2\left[\frac{\omega_{\mathrm{T}}^2}{p^2}-\frac{\omega_{\mathrm{T}}(\omega_{\mathrm{T}}^2-p^2)}{2p^3}\ln\frac{\omega_{\mathrm{T}}+p}{\omega_{\mathrm{T}}-p}\right]=0, \tag{61}$$

where ω_{T} is the solution of the above transverse dispersion relation.

The longitudinal dispersion relation is given by

$$p^2+m_{\mathrm{D}}^2\left[1-\frac{\omega_{\mathrm{L}}}{2p}\ln\frac{\omega_{\mathrm{L}}+p}{\omega_{\mathrm{L}}-p}\right]=0, \tag{62}$$

where ω_{L} is the solution of the above longitudinal dispersion relation.

Obviously the transverse and longitudinal parts have the same dispersion relation as that of the HTL resummed gluon propagator in the Coulomb gauge. However, these two kinds of gauge have different expressions for the transverse and longitudinal projection tensors, and then have different expressions for the HTL resummed gluon propagator. As they have the same dispersion relation, for more analyses you can refer to Ref. [32].

The residue for the transverse part is

$$\begin{aligned}Z_{\mathrm{T}} &= -\left(\left[\frac{\partial(P^2-\Pi_{\mathrm{T}})}{\partial p_0}\right]_{p_0=\omega_{\mathrm{T}}(p)}\right)^{-1}\\ &= \frac{\omega_{\mathrm{T}}(\omega_{\mathrm{T}}^2-p^2)}{m_{\mathrm{D}}^2\omega_{\mathrm{T}}^2-(\omega_{\mathrm{T}}^2-p^2)^2}.\end{aligned} \tag{63}$$

The residue for the longitudinal part is

$$\begin{aligned}Z_{\mathrm{L}} &= -\left(\left[\frac{\partial\frac{p^2}{P^2}(P^2-\frac{1}{n_p^2}\Pi_{\mathrm{L}})}{\partial p_0}\right]_{p_0=\omega_{\mathrm{L}}(p)}\right)^{-1}\\ &= \frac{\omega_{\mathrm{L}}(\omega_{\mathrm{L}}^2-p^2)}{p^2(p^2-m_{\mathrm{D}}^2-\omega_{\mathrm{L}}^2)}.\end{aligned} \tag{64}$$

More discussion of the transverse and longitudinal residues can also be found in Ref. [32] too.

Below is the proof of the reason why we can get the same dispersion relation. From the famous Ward identity,

$$P^\mu\Pi_{\mu\nu}(P)=p_0\Pi_{0\nu}-p_3\Pi_{3\nu}=0, \tag{65}$$

we can get the below relation,

$$\Pi_{3\nu}=\frac{p_0}{p_3}\Pi_{0\nu}, \tag{66}$$

where the external momentum $P^\mu=(p_0,0,0,p_3)$, $p_3>0$, and $\overrightarrow{p}$ is in the positive direction of the z axis.

Using the above relation, the longitudinal part of the gluon HTL self energy can become:

$$\begin{aligned}\tilde{L}_P^{\mu\nu}\Pi_{\mu\nu}(P) &= -\frac{n^\mu n^\nu P^2}{(n\cdot P)^2}\Pi_{\mu\nu}(P)\\ &=-\frac{n^\nu P^2}{(n\cdot P)^2}\frac{\sqrt{2}}{2}(\Pi_{0\nu}+\Pi_{3\nu})\\ &=-\frac{P^2}{(n\cdot P)^2}\frac{\sqrt{2}(p_0+p_3)}{2p_3}n^\nu\Pi_{0\nu}\\ &=-\frac{P^2}{(n\cdot P)^2}\frac{(p_0+p_3)^2}{2(p_3)^2}\Pi^{00}(P)\\ &=-\frac{P^2}{(p_3)^2}\Pi^{00}(P),\end{aligned} \tag{67}$$

where $\tilde{L}_P^{\mu\nu}$ is the longitudinal projection tensor in the light cone gauge in Eq. (28), $\Pi^{00}(P)$ is the longitudinal part of the gluon HTL self energy in the Coulomb gauge in Eq. (11), and the axial vector in the light cone gauge $n_1^\mu=(\frac{\sqrt{2}}{2},0,0,-\frac{\sqrt{2}}{2})$. The result tells us that the longitudinal part of the gluon HTL self energy in the light cone gauge is the same as that in the Coulomb gauge.

Similarly, the transverse part of the gluon HTL self energy can become

$$\frac{1}{2}\tilde{T}_P^{\mu\nu}\Pi_{\mu\nu}(P) = \frac{1}{2}\Big[g^{\mu\nu}+\frac{n^\mu n^\nu P^2}{(n\cdot P)^2}\Big]\Pi_{\mu\nu}(P) = -\frac{1}{2}m_{\rm D}^2+\frac{1}{2}\frac{P^2}{(p_3)^2}\Pi^{00}(P), \quad (68)$$

where $\tilde{T}_P^{\mu\nu}$ is the transverse projection tensor in the light cone gauge in Eq. (27). So the transverse part of the gluon HTL self energy in the light cone gauge is the same as that in the Coulomb gauge.

In the proof, we use the famous Ward identity. $\overrightarrow{p}$ is in the positive direction of the z axis, and $\overrightarrow{n}$ is in the negative direction of the z axis. We can get the same transverse and longitudinal parts of the gluon HTL self energy in the two gauges, and finally get the same dispersion relation of the transverse and longitudinal parts.

4 HTL resummed quark propagator in the light cone gauge

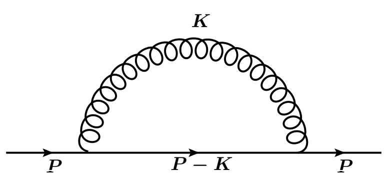

Fig. 3. The quark self energy in the light cone gauge.

The quark self energy in Fig. 3 can be expressed as

$$\Sigma(P)=iC_{\rm F}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\gamma^\mu F(P-K)\gamma^\nu d_{\mu\nu}(K)G(K), \quad (69)$$

where $C_{\rm F}=\frac{4}{3}$ is the color factor, $F(P-K)$ is the bare quark propagator, and $d_{\mu\nu}(K)G(K)$ is the bare gluon propagator.

The retarded quark self energy in the real time formalism is expressed as

$$\Sigma_{\rm R}(P) = \Sigma_{11}(P)+\Sigma_{12}(P) = iC_{\rm F}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\gamma^\mu(\not{P}-\not{K})\gamma^\nu d_{\mu\nu}(K) \times[\tilde{\Delta}_{11}(P-K)\Delta_{11}(K)-\tilde{\Delta}_{12}(P-K)\Delta_{12}(K)]. \quad (70)$$

Using the relation in Eq. (24), we get

$$\tilde{\Delta}_{11}(P-K)\Delta_{11}(K)-\tilde{\Delta}_{12}(P-K)\Delta_{12}(K) = \frac{1}{2}[\tilde{\Delta}_{\rm R}\Delta_{\rm S}+\tilde{\Delta}_{\rm R}\Delta_{\rm A}+\tilde{\Delta}_{\rm S}\Delta_{\rm R}+\tilde{\Delta}_{\rm A}\Delta_{\rm R}] = \frac{1}{2}[\tilde{\Delta}_{\rm R}\Delta_{\rm S}+\tilde{\Delta}_{\rm S}\Delta_{\rm R}], \quad (71)$$

where $\tilde{\Delta}$ and Δ respectively represent the Green functions of quark and gluon. The terms $\tilde{\Delta}_{\rm R}\Delta_{\rm A}$ and $\tilde{\Delta}_{\rm A}\Delta_{\rm R}$ are both zero temperature parts, and we neglect them here.

The quark self energy is decomposed into two parts,

$$\Sigma_{\rm R}(P)=-a(p_0,p)\not{P}-b(p_0,p)\gamma^0,$$
$$a(p_0,p)=\frac{1}{4p^2}\big[{\rm Tr}(\not{P}\Sigma_{\rm R})-p_0{\rm Tr}[\gamma_0\Sigma_{\rm R}]\big],$$
$$b(p_0,p)=\frac{1}{4p^2}\big[P^2{\rm Tr}[\gamma_0\Sigma_{\rm R}]-\gamma_0{\rm Tr}(\not{P}\Sigma_{\rm R})\big]. \quad (72)$$

With the above relation, we can do the following calculation,

$${\rm Tr}[\not{P}\Sigma_{\rm R}(P)]=iC_{\rm F}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\big[-8K\cdot P+\frac{n\cdot P}{n\cdot K}(-8K^2+16K\cdot P)\big]\frac{1}{2}\big[\tilde{\Delta}_{\rm R}(P-K)\Delta_{\rm S}(K)+\tilde{\Delta}_{\rm S}(P-K)\Delta_{\rm R}(K)\big]. \quad (73)$$

Replacing K by $P-K$ in this term $\tilde{\Delta}_{\rm S}(P-K)\Delta_{\rm R}(K)$, this expression becomes:

$$\begin{aligned}{\rm Tr}[\not{P}\Sigma_{\rm R}(P)] &= iC_{\rm F}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\Big[\Big[8(K\cdot P-P^2)+\frac{8n\cdot P}{n\cdot(K-P)}(K^2-P^2)\Big]\frac{1}{2}\tilde{\Delta}_{\rm S}(K)\Delta_{\rm R}(P-K)\\ &\quad+\Big[-8K\cdot P+\frac{n\cdot P}{n\cdot K}(-8K^2+16K\cdot P)\Big]\frac{1}{2}\tilde{\Delta}_{\rm R}(P-K)\Delta_{\rm S}(K)\Big]\\ &= iC_{\rm F}g^2\int\frac{{\rm d}^4K}{(2\pi)^4}\Big[\Big[4(K\cdot P-P^2)\tilde{\Delta}_{\rm S}(K)\Delta_{\rm R}(P-K)-4K\cdot P\tilde{\Delta}_{\rm R}(P-K)\Delta_{\rm S}(K)\Big]\\ &\quad+\Big[\frac{4n\cdot P}{n\cdot(K-P)}(K^2-P^2)\tilde{\Delta}_{\rm S}(K)\Delta_{\rm R}(P-K)+\frac{n\cdot P}{n\cdot K}(-4K^2+8K\cdot P)\tilde{\Delta}_{\rm R}(P-K)\Delta_{\rm S}(K)\Big]\Big]=4m_{\rm F}^2, \quad (74)\end{aligned}$$

In the HTL approximation, we can prove there is no spurious divergence from the light cone terms in the fourth line, and these finite terms are more power suppressed than the covariant terms, so we ignore these light cone terms.

In the same way, we can get

$$\begin{aligned}\mathrm{Tr}[\gamma_0\Sigma_{\mathrm{R}}(P)] &= C_{\mathrm{F}}g^2\int\frac{\mathrm{d}^4K}{(2\pi)^4}8[-k_0+k_0\frac{n\cdot P}{n\cdot K}-\frac{n_0}{n\cdot K}(K^2-K\cdot P)]\frac{1}{2}\left[\tilde{\Delta}_{\mathrm{R}}(P-K)\Delta_{\mathrm{S}}(K)+\tilde{\Delta}_{\mathrm{S}}(P-K)\Delta_{\mathrm{R}}(K)\right]\\
&= C_{\mathrm{F}}g^2\int\frac{\mathrm{d}^4K}{(2\pi)^4}\Big[4(k_0-p_0)\tilde{\Delta}_{\mathrm{S}}(K)\Delta_{\mathrm{R}}(P-K)-4k_0\tilde{\Delta}_{\mathrm{R}}(P-K)\Delta_{\mathrm{S}}(K)\\
&\quad+4\Big[(k_0-p_0)\frac{n\cdot P}{n\cdot(K-P)}+\frac{n_0}{n\cdot(K-P)}(K^2-K\cdot P)\Big]\tilde{\Delta}_{\mathrm{S}}(K)\Delta_{\mathrm{R}}(P-K)\\
&\quad+4\Big[k_0\frac{n\cdot P}{n\cdot K}-\frac{n_0}{n\cdot K}(K^2-K\cdot P)\Big]\tilde{\Delta}_{\mathrm{R}}(P-K)\Delta_{\mathrm{S}}(K)\Big]\\
&= 2m_{\mathrm{F}}^2\frac{1}{p}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}. \qquad (75)\end{aligned}$$

In the HTL approximation, it can be proved that there is no spurious divergence from the light cone terms in the third and fourth lines, and these finite terms are more power suppressed than the covariant terms, so we ignore these light cone terms too.

So the result shows the quark HTL self energy is the same as that of covariant gauge [40]. With the quark HTL self energy, we can derive the same quark resummed propagator.

The HTL resummed quark propagator is

$$\begin{aligned}S^*(P) &= \frac{1}{D_+(P)}\frac{\gamma_0-\hat{\vec{p}}\cdot\vec{\gamma}}{2}+\frac{1}{D_-(P)}\frac{\gamma_0+\hat{\vec{p}}\cdot\vec{\gamma}}{2},\\
D_\pm(P) &= -p_0\pm p+\frac{m_{\mathrm{F}}^2}{p}\Big[\frac{1}{2}\ln\frac{p_0+p+i\epsilon}{p_0-p+i\epsilon}\mp(\frac{p_0}{2p}\ln\frac{p_0+p+i\epsilon}{p_0-p-i\epsilon}-1)\Big], \qquad (76)\end{aligned}$$

where the effective quark mass $m_{\mathrm{F}}=g^2T^2/6$ in QCD, and $\hat{\vec{p}}=\frac{\vec{p}}{|\vec{p}|}$.

5 Damping rates of hard quarks and gluons in the light cone gauge

The damping rates of the heavy fermions have been calculate in Refs. [35, 41]. In a similar way, we use the HTL resummed gluon propagator in the light cone gauge to calculate the damping rates of the hard quarks and gluons. From the above analyses, we know the HTL resummed gluon propagator in the Coulomb gauge and the light cone gauge have the same denominator and different projection tensors in the numerator. We can prove that in the general case, using the HTL resummed gluon propagator in the two gauges gives the same result for the damping rates of hard quarks and gluons.

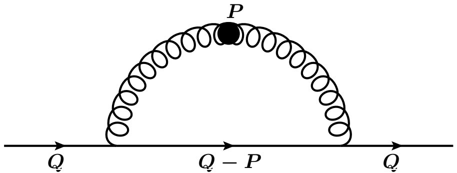

Fig. 4. The contribution to the damping rate of the hard quark in the light cone gauge.

The quark self energy in Fig. 4 is expressed as

$$\Sigma(Q)=ig^2C_{\mathrm{F}}\int\frac{\mathrm{d}^4P}{(2\pi)^4}[\gamma_\mu F(Q-P)\gamma_\nu G_{\mu\nu}(Q)], \qquad (77)$$

where $G_{\mu\nu}(Q)$ is the HTL resummed gluon propagator in the light cone gauge, and $F(Q-P)$ is the bare quark propagator.

In the real time formalism, the retarded quark self energy is expressed as

$$\begin{aligned}\Sigma_{\mathrm{R}}(Q) &= \Sigma_{11}(Q)+\Sigma_{12}(Q)\\
&= ig^2C_{\mathrm{F}}\int\frac{\mathrm{d}^4P}{(2\pi)^4}[\gamma_\mu(\not{Q}-\not{P})\gamma_\nu]\\
&\quad\times[\tilde{\Delta}_{11}(Q-P)\Delta_{11}^{\mu\nu}(P)-\tilde{\Delta}_{12}(Q-P)\Delta_{12}^{\mu\nu}(P)]. \qquad (78)\end{aligned}$$

Using the relation of the Keldysh representation in Eq. (24), we have

$$\begin{aligned}&\tilde{\Delta}_{11}(Q-P)\Delta_{11}^{\mu\nu}(P)-\tilde{\Delta}_{12}(Q-P)\Delta_{12}^{\mu\nu}(P)\\
&= \frac{1}{2}[\tilde{\Delta}_{\mathrm{R}}\Delta_{\mathrm{S}}^{\mu\nu}+\tilde{\Delta}_{\mathrm{R}}\Delta_{\mathrm{A}}^{\mu\nu}+\tilde{\Delta}_{\mathrm{S}}\Delta_{\mathrm{R}}^{\mu\nu}+\tilde{\Delta}_{\mathrm{A}}\Delta_{\mathrm{R}}^{\mu\nu}]. \qquad (79)\end{aligned}$$

By power counting, the leading contribution at finite temperature comes from the first term in the bracket $\tilde{\Delta}_{\mathrm{R}}\Delta_{\mathrm{S}}^{\mu\nu}$, which is $O(1/g^3)$. $\tilde{\Delta}_{\mathrm{R}}\Delta_{\mathrm{A}}^{\mu\nu}$ and $\tilde{\Delta}_{\mathrm{A}}\Delta_{\mathrm{R}}^{\mu\nu}$ are both $O(1/g^2)$, so we ignore them here. From the below equations, $\tilde{\Delta}_{\mathrm{S}}\Delta_{\mathrm{R}}^{\mu\nu}$ with the Fermi-Dirac distribution function is $O(g)$ more power suppressed than $\tilde{\Delta}_{\mathrm{R}}\Delta_{\mathrm{S}}^{\mu\nu}$ with the Bose-Einstein distribution function.

The internal momentum P is soft, $p_0\sim gT$. The Bose-Einstein distribution function $n_{\mathrm{B}}(p_0)$ in the term $\tilde{\Delta}_{\mathrm{R}}\Delta_{\mathrm{S}}^{\mu\nu}$ is $O(1/g)$,

$$\frac{1}{\mathrm{e}^{\frac{|p_0|}{T}}-1}\sim\frac{T}{|p_0|}\propto\frac{1}{g}. \qquad (80)$$

However, the Fermi-Dirac distribution function $f(p_0-q_0)$ in the term $\tilde{\Delta}_{\rm S}\Delta_{\rm R}^{\mu\nu}$ is $O(1)$. The external momentum Q is hard, $q_0\sim T$,

$$\frac{1}{\mathrm{e}^{\frac{|q_0-p_0|}{T}}+1}\sim O(1). \tag{81}$$

The symmetric propagator of the soft gluon in the light cone gauge is

$$\Delta_{\rm S}^{\mu\nu}(P)=-2\pi i\left[-\tilde{T}_P^{\mu\nu}\rho_{\rm T}(P)+\frac{n^\mu n^\nu P^2}{(n\cdot P)^2}\frac{p^2}{P^2}\rho_{\rm L}(P)\right][1+2n_{\rm B}(p_0)], \tag{82}$$

where we only consider the contribution at finite temperature.

Using the below equation, we can calculate the imaginary part of the quark self energy

$$\begin{aligned}\mathrm{Im}\tilde{\Delta}(Q-P) &= \mathrm{Im}\left[\frac{1}{(Q-P)^2+isgn(q_0-p_0)\epsilon}\right]\\ &= -\pi sgn(q_0-p_0)\delta[(Q-P)^2].\end{aligned} \tag{83}$$

In the integral, we use the δ function to integrate out $\cos\theta$,

$$\delta[(Q-P)^2]=\frac{1}{2pq}\delta\left[\cos\theta-\frac{p_0}{p}+\frac{P^2}{2pq}\right]\approx\frac{1}{2pq}\delta\left[\cos\theta-\frac{p_0}{p}\right], \tag{84}$$

where the term $\frac{P^2}{2pq}\sim g$, which we can ignore.

The transverse projection tensor in the light cone gauge is

$$\tilde{T}_P^{\mu\nu}=g^{\mu\nu}-\frac{n^\mu P^\nu+n^\nu P^\mu}{n\cdot P}+\frac{n^\mu n^\nu P^2}{(n\cdot P)^2}, \tag{85}$$

where the internal soft momentum $P^\mu=(p_0,0,0,p_3)$.

By calculating we can find the relation $\tilde{T}_P^{0\nu}=0,\tilde{T}_P^{3\nu}=0$, so we have

$$\tilde{T}_P^{ij}=-\delta^{ij},\qquad (i,j=1,2). \tag{86}$$

The longitudinal and transverse spectral functions in the limit $p_0\to 0$ are

$$\begin{aligned}\rho_{\rm L}(P)&\approx\frac{p_0 m_{\rm D}^2}{2p}\frac{1}{(p^2+m_{\rm D}^2)^2},\\ \rho_{\rm T}(P)&\approx\frac{p_0 p m_{\rm D}^2}{4}\frac{1}{p^6+\frac{1}{16}\pi^2 m_{\rm D}^4(p_0)^2},\end{aligned} \tag{87}$$

where we find no static magnetic screening.

With the above equations, we can calculate the imaginary part of $\mathrm{Tr}[\not{Q}\Sigma_{\rm R}(Q)]$,

$$\begin{aligned}&\mathrm{Im}\Big[\mathrm{Tr}[\not{Q}\Sigma_{\rm R}(Q)]\Big]\\ &= -4\pi^2 C_{\rm F}g^2\int\frac{\mathrm{d}^4P}{(2\pi)^4}\left[4q_\perp^2\rho_{\rm T}(P)+4\frac{(n\cdot Q)^2p^2}{(n\cdot P)^2}\rho_{\rm L}(P)\right]\\ &\quad\times n_{\rm B}(p_0)sgn(q_0-p_0)\delta[(Q-P)^2]\\ &= -\frac{1}{2\pi}C_{\rm F}g^2Tq\left[1+2\ln\frac{1}{g}\right].\end{aligned} \tag{88}$$

The damping rate for the hard quark is

$$\begin{aligned}\Gamma_q(Q) &= -\frac{1}{2q}\mathrm{Im}\big[\mathrm{Tr}[\not{Q}\Sigma_{\rm R}(Q)]\big]\\ &= \frac{1}{4\pi}C_{\rm F}g^2T[1+2\ln\frac{1}{g}].\end{aligned} \tag{89}$$

The first term in the bracket comes from the longitudinal contribution of the HTL resummed gluon propagator, and the second term comes from the transverse contribution. In the transverse part there is an IR cutoff, which results from the magnetic mass of the order $m_{\rm magn}\sim g^2T$.

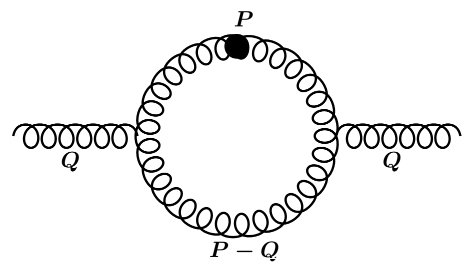

Fig. 5. The gluon loop diagram gives the leading order contribution to the damping rate of hard gluons in the light cone gauge, and the quark loop diagram is suppressed due to the Fermi-Dirac distribution function.

We can express the gluon loop in Fig. 5 as

$$\begin{aligned}\Pi_{\mu\alpha}^{ab}(Q) &= \frac{i}{2}\int\frac{\mathrm{d}^4P}{(2\pi)^4}V_{\mu\nu\rho}(Q,-P,P-Q)iG^{\nu\beta}(P)V_{\beta\gamma\alpha}\\ &\quad\times(P,Q-P,-Q)id^{\rho\gamma}(P-Q)G(P-Q),\end{aligned} \tag{90}$$

where $G^{\nu\beta}(P)$ is the HTL resummed gluon propagator in the light cone gauge in Eq. (57), $d^{\rho\gamma}(P-Q)G(P-Q)$ is the bare gluon propagator in the light cone gauge, and the three gluon vertexes $V_{\mu\nu\rho}(Q,-P,P-Q)$ and $V_{\beta\gamma\alpha}(P,Q-P,-Q)$ are given in Eq. (46).

Using the relation of the Keldysh representation in Eq. (24), we have

$$\begin{aligned}&\Delta_{11}^{\nu\beta}(P)\Delta_{11}(Q-P)-\Delta_{12}^{\nu\beta}(P)\Delta_{21}(Q-P)\\ &=\frac{1}{2}[\Delta_{\rm R}^{\nu\beta}\Delta_{\rm S}+\Delta_{\rm S}^{\nu\beta}\Delta_{\rm A}+\Delta_{\rm A}^{\nu\beta}\Delta_{\rm A}+\Delta_{\rm R}^{\nu\beta}\Delta_{\rm R}].\end{aligned} \tag{91}$$

Similarly, by power counting, the second term $\Delta_{\rm S}^{\nu\beta}\Delta_{\rm A}$ gives the leading order contribution $O(1/g^3)$ at finite temperature. $\Delta_{\rm A}^{\nu\beta}\Delta_{\rm A}$ and $\Delta_{\rm R}^{\nu\beta}\Delta_{\rm R}$ are both $O(1/g^2)$. The internal momentum P is soft, $p_0\sim gT$. The Bose-Einstein distribution function $n_{\rm B}(p_0)$ in the term $\Delta_{\rm S}^{\nu\beta}\Delta_{\rm A}$ is $O(1/g)$,

$$\frac{1}{\mathrm{e}^{\frac{|p_0|}{T}}-1}\sim\frac{T}{|p_0|}\propto\frac{1}{g}. \tag{92}$$

However, the Bose-Einstein distribution function $n_{\rm B}(p_0-q_0)$ in the term $\Delta_{\rm R}^{\nu\beta}\Delta_{\rm S}$ is $O(1)$. The external momentum Q is hard, $q_0\sim T$,

$$\frac{1}{\mathrm{e}^{\frac{|p_0-q_0|}{T}}-1}\sim O(1). \tag{93}$$

The three-gluon vertexes are simplified to

$$V_{\mu\nu\rho}(Q,-P,P-Q)\approx gf^{acd}[Q_\rho g_{\mu\nu}+Q_\mu g_{\nu\rho}-2Q_\nu g_{\rho\mu}],$$
$$V_{\beta\gamma\alpha}(P,Q-P,-Q)\approx gf^{cdb}[-Q_\alpha g_{\beta\gamma}+2Q_\beta g_{\gamma\alpha}-Q_\gamma g_{\alpha\beta}], \tag{94}$$

where the external momentum Q is hard, and the internal momentum P is soft, which is ignored in the above expression.

The numerator of the bare propagator in the light cone gauge $d^{\rho\gamma}(P-Q)$ is

$$\begin{aligned} d^{\rho\gamma}(P-Q) &= -g^{\rho\gamma}+\frac{n^\rho(P-Q)^\gamma+n^\gamma(P-Q)^\rho}{n\cdot(P-Q)} \\ &\approx -g^{\rho\gamma}+\frac{n^\rho Q^\gamma+n^\gamma Q^\rho}{n\cdot Q}. \end{aligned} \tag{95}$$

In the calculation, we have

$$Q_\rho d^{\rho\gamma}(P-Q)=\frac{n^\gamma Q^2}{n\cdot Q}=0, \tag{96}$$

where the external hard momentum Q is on shell, $Q^2=0$. We can therefore simplify the calculation.

Now we calculate the imaginary part of the transverse part $\Pi_{\mathrm{T}}(Q)$,

$$\begin{aligned} &\mathrm{Im}\Pi_{\mathrm{T}}(Q) \\ &= \frac{1}{2}\left(\delta^{ij}-\frac{q^iq^j}{q^2}\right)\mathrm{Im}\Pi^{ij}(Q) \qquad (i,j=1,2,3) \\ &= -C_{\mathrm{A}}g^2\pi^2\int\frac{\mathrm{d}^4P}{(2\pi)^4}\left[4q_\perp^2\rho_{\mathrm{T}}(P)+4\frac{(n\cdot Q)^2p^2}{(n\cdot P)^2}\rho_{\mathrm{L}}(P)\right] \\ &\quad\times n_{\mathrm{B}}(p_0)sgn(q_0-p_0)\delta[(Q-P)^2] \\ &= -\frac{1}{8\pi}C_{\mathrm{A}}g^2Tq\left[1+2\ln\frac{1}{g}\right]. \end{aligned} \tag{97}$$

The damping rate for hard gluons is

$$\varGamma_g(Q)=-\frac{1}{2q}\mathrm{Im}[\Pi_{\mathrm{T}}(\mathrm{Q})]=\frac{1}{16\pi}C_{\mathrm{A}}g^2T\left[1+2\ln\frac{1}{g}\right]. \tag{98}$$

The first term in the bracket stems from the longitudinal contribution of the HTL resummed gluon propagator, and the second term stems from the transverse contribution. For the transverse part we take an IR cutoff too.

We work out the result for the damping rates of hard quarks and gluons in the limit $p_0\to 0$. Below, we can prove the general case that the expressions for the transverse and longitudinal projections in the above calculation of the damping rates are the same in the two gauges.

Via this function $\delta[(Q-P)^2]$, we have the relation for $\cos\theta$,

$$\cos\theta\approx\frac{p_0}{p}=\frac{p_0}{|p_3|}, \tag{99}$$

where the internal soft momentum $P^\mu=(p_0,0,0,p_3)$, and the angle θ is arbitrary.

When $p_3>0$, we have

$$\cos\theta=\frac{q_3}{q}=\frac{q_3}{q_0}, \tag{100}$$

where θ is the angle between $\overrightarrow{p}$ and $\overrightarrow{q}$.

The expression for the longitudinal part in the calculation of the damping rate is

$$\begin{aligned} \frac{(n\cdot Q)^2}{(n\cdot P)^2}P^2 &= \frac{(q_0+q_3)^2}{(p_0+p_3)^2}P^2=\frac{(q_0)^2(1+\cos\theta)^2}{(p_3)^2(1+\cos\theta)^2}P^2 \\ &= \frac{(q_0)^2}{(p_3)^2}P^2=\frac{(q_0)^2}{p^2}P^2. \end{aligned} \tag{101}$$

When $p_3<0$, we have

$$\cos\theta=-\frac{q_3}{q}=-\frac{q_3}{q_0}. \tag{102}$$

The expression for the longitudinal part in the calculation is

$$\begin{aligned} \frac{(n\cdot Q)^2}{(n\cdot P)^2}P^2 &= \frac{(q_0+q_3)^2}{(p_0+p_3)^2}P^2=\frac{(q_0)^2(1-\cos\theta)^2}{(p_3)^2(1-\cos\theta)^2}P^2 \\ &= \frac{(q_0)^2}{(p_3)^2}P^2=\frac{(q_0)^2}{p^2}P^2. \end{aligned} \tag{103}$$

We have the same expression for the longitudinal projection in the two gauges.

The transverse projection tensor of the HTL resummed gluon propagator in the Coulomb gauge is

$$\delta^{ij}-\frac{p^ip^j}{p^2}. \tag{104}$$

By calculating, we have

$$\delta^{3j}-\frac{p^3p^j}{p^2}=0, \tag{105}$$

where the internal soft momentum $P^\mu=(p_0,0,0,p_3)$. So we can get the same transverse projection tensor in the two gauges.

We have a simple expression for the transverse projection tensor,

$$\delta^{ij}, \qquad (i,j=1,2). \tag{106}$$

Here we have shown that the transverse and longitudinal projection tensors in the above calculation of the damping rates are the same in the two gauges when the transverse momentum of the internal momentum P is zero. So we can get the same result for the damping rates of hard quarks and gluons in the two gauges.

6 Conclusion

In this paper, we have derived the HTL resummed gluon propagator in the light cone gauge. We obtained the transverse and longitudinal components of the gluon self energy in the light cone gauge with the HTL approximation. We showed that the quark HTL energy is independent of the light-cone gauge, and obtained the same

HTL resummed quark propagator as that in covariant gauges.

Although the derivation processes of the longitudinal and transverse components of HTL gluon self energy involve light cone terms like $1/(n{\cdot}K)$, we can prove these light cone terms are finite, more power suppressed than the covariant terms, and ignore them in the finite temperature parts. In the zero temperature parts, these light cone terms are spurious poles, and can be renormalized, which we do not consider. By calculating, we obtain the same transverse and longitudinal gluon dispersion relations in the light cone gauge and the Coulomb gauge, although in the two gauges we have different transverse and longitudinal projection tensors. The axial vector $n_1^\mu=(\frac{\sqrt{2}}{2},0,0,-\frac{\sqrt{2}}{2})$ has a longitudinal part and is rotated with respect to the axial vector $n_c^\mu=(1,0,0,0)$ in the Coulomb gauge, which brings about changes. Correspondingly, in the static limit, we compare the component $\Pi^{00}(P)$ of the gluon HTL self energy in the light cone gauge with $\Pi^{00}(P)$ in the Coulomb gauge, and find the result in the light cone gauge is the same as that in the Coulomb gauge.

We have shown the transverse and longitudinal spectral functions of the HTL resummed gluon propagator in the light cone gauge and the transverse and longitudinal dispersion relation. However, in the light cone gauge, the transverse and longitudinal projection tensors are both based on the the axial vector n_1^μ, and they have different expressions from the transverse and longitudinal projection tensors in the Coulomb gauge, so the expression for the HTL resummed gluon propagator in the light cone gauge is different from that in the Coulomb gauge.

With the HTL resummed gluon propagator in the light cone gauge, we calculated the damping rates of hard on-shell quarks and gluons in a particular limit. We have demonstrated that in the general case, we can obtain the same result for the damping rates in the two gauges. Although the expression for the HTL resummed gluon propagator in the light cone gauge is different from that in the Coulomb gauge, we find the damping rates are gauge independent.

Using those HTL resummed propagators in the light cone gauge, we can further consider corrections from soft processes for some physical quantities at high temperature in heavy ion collisions.

We are grateful to Xin-nian Wang for helpful discussions.

Appendix

At zero temperature, the kind of integral in Eq. (49) has a divergence, which has been renormalized successfully. However, in the below calculation we find that this integral has no divergence in the HTL approximation.

We use the contour integral in Fig. A1 to substitute the frequency sum of boson at finite temperature [42],

$$T\sum_{n=-\infty}^{\infty} f(p_0{=}2n\pi Ti) = \frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\mathrm{d}p_0\frac{1}{2}[f(p_0)+f(-p_0)] + \frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}\mathrm{d}p_0[f(p_0)+f(-p_0)] \times\frac{1}{\mathrm{e}^{\beta p_0}-1}. \tag{A1}$$

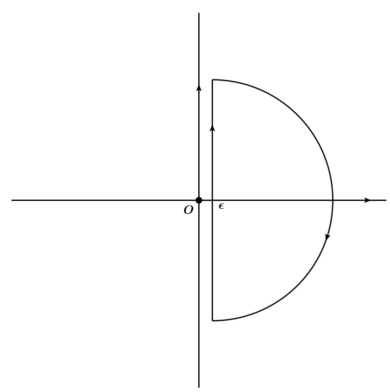

Fig. A1. The boson contour integral.

This equation has a zero temperature part and a finite temperature part, and we only consider the the finite temperature part here.

Below we prove this integral at finite temperature does not diverge in Eq. (49),

$$\frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}\mathrm{d}k_0\int\frac{\mathrm{d}^3\overrightarrow{k}}{(2\pi)^3}\frac{K^2-2K{\cdot}P}{n{\cdot}KK^2(K-P)^2}. \tag{A2}$$

where $nK=n_0k_0-\overrightarrow{n}\,\overrightarrow{k}=n(k_0+k\cos\theta)$, the angle θ is the angle between $\overrightarrow{k}$ and $\overrightarrow{p}$, and the axial vector $n_1^\mu=(\frac{\sqrt{2}}{2},0,0,-\frac{\sqrt{2}}{2})$.

When $\cos\theta>0$, using the equation of the integral contour in Eq. (A1), we have

$$\frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}\mathrm{d}k_0\int\frac{\mathrm{d}^3\overrightarrow{k}}{(2\pi)^3}\left[\frac{K^2-2K{\cdot}P}{n(k_0+k\cos\theta)K^2(K-P)^2} + \frac{K^2-2(-k_0p_0-kp\cos\theta)}{n(-k_0+k\cos\theta)K^2[K^2-2(-k_0p_0-2kp\cos\theta)+P^2]}\right] \times\frac{1}{\mathrm{e}^{\beta k_0}-1}. \tag{A3}$$

The first term in the bracket has poles at $k_0 = k$ and $k_0 = p_0 + |\overrightarrow{k} - \overrightarrow{p}|$, and the residues of the two poles do not have the term of $\frac{1}{\cos\theta-1}$, so there is no divergence at $\cos\theta=1$. The second term has poles at $k_0=k$, $k_0=k\cos\theta$ and $k_0=-p_0+|\overrightarrow{k}-\overrightarrow{p}|$. The residues with the two poles of $k_0=k$ and $k_0=k\cos\theta$ contain the terms of $\frac{1}{\cos\theta-1}$, but the sum of the two residues is finite when $\cos\theta\to1$,

$$\lim_{\cos\theta\to1}\frac{1}{(-k+k\cos\theta)2k}-\frac{1}{k^2(\cos^2\theta-1)}=\frac{1}{2k^2(\cos\theta+1)}. \quad \text{(A4)}$$

When $\cos\theta<0$, we have

$$\frac{1}{2\pi i}\int_{-i\infty+\epsilon}^{i\infty+\epsilon}\mathrm{d}k_0\int\frac{\mathrm{d}^3\overrightarrow{k}}{(2\pi)^3}\left[\frac{K^2-2K\cdot P}{n(k_0+k\cos\theta)K^2(K-P)^2}\right.$$
$$\left.+\frac{K^2-2(-k_0p_0-kp\cos\theta)}{n(-k_0+k\cos\theta)K^2[K^2-2(-k_0p_0-2kp\cos\theta)+P^2]}\right]$$
$$\times\frac{1}{\mathrm{e}^{\beta k_0}-1}. \quad \text{(A5)}$$

The second term in the bracket has poles at $k_0=k$ and $k_0=-p_0+|\overrightarrow{k}-\overrightarrow{p}|$, and the residues of the two poles do not have the term of $\frac{1}{\cos\theta+1}$, so there is no divergence at $\cos\theta=-1$. The first term has poles at $k_0=k$, $k_0=-k\cos\theta$ and $k_0=p_0+|\overrightarrow{k}-\overrightarrow{p}|$. The residues with the two poles of $k_0=k$ and $k_0=-k\cos\theta$ contain the terms of $\frac{1}{\cos\theta+1}$, but the sum of the two residues is finite when $\cos\theta\to-1$,

$$\lim_{\cos\theta\to-1}\frac{1}{k(1+\cos\theta)2k}+\frac{1}{k^2(\cos^2\theta-1)}=\lim_{\cos\theta\to-1}\frac{1}{2k^2(\cos\theta-1)}. \quad \text{(A6)}$$

Combining the situation $\cos\theta>0$ and $\cos\theta<0$, the integral at finite temperature in Eq. (49) does not diverge. The integral is $O(g^3T^2)$, which can be ignored.

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