Next-to-leading order QCD corrections to the decay of Higgs to vector meson and Z boson[*]

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Abstract: The exclusive decay of the Higgs boson to a vector meson (J/ψ or $\Upsilon(1S)$) and Z boson is studied in this work. The decay amplitudes are separated into two parts in a gauge invariant manner. The first part comes from the direct coupling of the Higgs boson to the charm (bottom) quark and the other from the HZZ^* or the loop-induced $HZ\gamma^*$ vertexes in the standard model. While the branching ratios from the direct channel are much smaller than those of the indirect channel, their interference terms give nontrivial contributions. We further calculate the QCD radiative corrections to both channels, which reduce the total branching ratios by around 20% for both J/ψ and $\Upsilon(1S)$ production. Our results provide a possible chance to check the SM predictions of the $Hc\bar{c}(Hb\bar{b})$ coupling and to seek for hints of new physics at the High Luminosity LHC or future hadron colliders.

Keywords: standard-model Higgs bosons, perturbative calculations

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1   Introduction

The discovery of the 125 GeV Higgs-like boson by the ATLAS and CMS collaborations [1, 2] has attracted much interest from both experimental and theoretical aspects. It is most important to precisely determine the properties of this new boson to check the predictions of standard model (SM). No significant deviation from the SM has been found from measurements of the decays $H \to \gamma\gamma, ZZ$ and WW [3, 4].

In the SM, the coupling of the Higgs to the first and second generation fermions is very weak and thus difficult to measure directly. In Ref. [5], the authors point out that the $Hc\bar{c}$ coupling may be probed by the measurements of the $H \to J/\psi + \gamma$ decay mode. Their results show that the partial width via the direct decay channel, where the Higgs boson is coupled directly to the charm quark, is too small to be probed. They introduce a new mechanism where the Higgs boson decays through the loop-induced $H\gamma\gamma^*$ vertex, followed by $\gamma^* \to J/\psi$. This indirect mechanism turns out to dominate the $H \to J/\psi + \gamma$ decay, and the interference between the direct and the indirect channel may provide us with more detailed information about the $Hc\bar{c}$ coupling.

The proposed High Luminosity LHC (HL-LHC) project is designed to run at 14 TeV, with its integrated luminosity upgraded to 3000 fb^{-1}. The HL-LHC will allow us to perform precise measurements that are impossible otherwise, which motivates us to make a more detailed study of the rare decays of the SM Higgs boson. The small Yukawa coupling of the light fermions leaves space for theories of new physics beyond the SM and has attracted a lot of interest. In Refs. [6–10], the rare decay $H \to V + Z$ ($V = J/\psi$ or $\Upsilon(1S)$) has been studied. Very similar to $H \to V + \gamma$, we can separate the decay amplitudes into two parts. The first part comes from the direct coupling of Higgs boson to the charm (bottom) quark, and the other part from the HZZ^* or the loop-induced $HZ\gamma^*$ vertexes in the SM, followed by $Z^*(\gamma^*) \to V$. Since both the vector meson and Z boson can decay to a pair of charged leptons, this experimentally-clean final state may provide us a chance to probe this rare decay and seek for a hint of new physics.

In Ref. [7], the author shows that the decay rate of $H \to V + Z$ via the loop-induced $HZ\gamma^*$ vertex is comparable with the contributions from the tree level HZZ^* vertex in the SM, by analyzing this decay via the indirect channel at leading-order (LO). In Ref. [8], the authors analyze the decay $H \to V + Z$ from all channels, and interference contributions are also included. They further show that this rare decay is sensitive to the anomalous Higgs couplings originating from physics beyond the SM, by observations

of the decay of the final-state vector meson and Z boson into charged leptons. In this paper, we revisit this rare decay at LO, and evaluate its QCD corrctions at next-to-leading order (NLO) for the (un)polarized final-state vector meson and Z boson. We also carefully analyze the interference terms between the indirect channel and the direct one at both LO and NLO level.

This paper is organized as follows. In Section 2, we introduce the formalism and notations used in this work. In Section 3, we revisit the decay H→V+Z at LO, and further we calculate its QCD corrections at NLO. In Section 4, we analyze the helicity amplitudes by projecting the vector meson and Z boson to particular polarized states. We conclude our calculations in Section 5.

2   Formalism and notations

For the H → V+Z decay, there are several distinct dynamic scales: $m_H \sim m_Z > m_q$ ($m_q = m_b$ or m_c). There is another scale in heavy quarkonium production, $m_q v_r$, where v_r is the relative velocity of the heavy quark pair in the rest frame of the quakonium. For heavy quarkonium, $v_r \ll 1$ is assumed, making it a non-relativistic system. The scale $m_q v_r$ characterizes the hadronization process of the heavy quark pair to vector mesons, which is inherently nonperturbative.

We adopt the nonrelativistic QCD (NRQCD) factorization formula as described in Ref. [11]. In the NRQCD factorization framework, the production of heavy mesons can be separated into two parts. First, the constituent quarks are produced at an energy greater than the heavy quark mass and thus can be calculated perturbatively. Secondly, the quark pair binds into quarkonium at an energy much smaller than the heavy quark mass, and this process is represented by the non-perturbative long distance matrix elements (LDMEs). In this formula, the LDMEs can be expanded by the relative velocity v_r of the heavy quark pair and the short-distance coefficients are expanded perturbatively by the strong coupling α_s order by order. As a result, the production rate of the heavy quarkonium can be expressed as the sum of products of the non-perturbative LDMEs and the corresponding perturbative short-distance coefficients. We will expand the short-distance coefficients to NLO of the strong coupling α_s and the LDMEs are expanded to the lowest order of v_r throughout this work.

We define the momenta of the initial-state Higgs and the outgoing vector meson and Z boson as

$$H(p_H) \to V(p,\lambda_V) + Z(k,\lambda_Z), \qquad (1)$$

where $p=p_q+p_{\bar q}$ and k in Eq. (6) represent the momenta of the final-state vector meson and Z boson respectively, with

$$p=\left(\sqrt{m_V^2+|\boldsymbol{p}|^2},\boldsymbol{p}\right), \quad k=\left(\sqrt{m_Z^2+|\boldsymbol{p}|^2},-\boldsymbol{p}\right), \qquad (2)$$

and

$$|\boldsymbol{p}|=\frac{1}{2m_H}\sqrt{m_H^4+m_Z^4+m_V^4-2m_H^2 m_V^2-2m_H^2 m_Z^2-2m_V^2 m_Z^2}. \qquad (3)$$

m_V is the mass of the vector meson and $m_V=2m_q$ at the lowest order of v_r. $p_H=p+k$ is the momentum of the initial-state Higgs boson and $\lambda_V(\lambda_Z)=0,\pm1$ represent the helicities of the final-state vector meson (Z boson).

To project the free quark pair into our desired $^3S_1$ and color singlet quantum state, we adopt the relatively normalized spin projection operator in Eq. (A9b) of Ref. [12]:

$$\Pi_V = \frac{1}{4\sqrt{2}E_q(E_q+m_q)}(\not{p}_{\bar q}-m_q)\not{\epsilon}_V^*(p,\lambda_V)$$

$$\times(\not{p}+2E_q)(\not{p}_q+m_q)\otimes\frac{\mathbb{1}}{\sqrt{N_c}}, \qquad (4)$$

where E_q in Eq. (4) is the energy of the constituent quarks in the rest frame of the heavy quark pair. Since we expand the LDMEs to the lowest order of v_r, the relative momentum between the quark pair is thus neglected. As a result, the momenta of the constituent heavy quarks are assigned as

$$p_q=p_{\bar q}=\frac{1}{2}p, \qquad (5)$$

and $E_q=m_q$.

By Lorentz covariance, the amplitudes for H→V+Z can be decomposed as

$$\begin{aligned}\mathcal{M}_{\lambda_V\lambda_Z}[\text{H}\to\text{V+Z}] &= \mathcal{T}_{\mu\nu}\epsilon_V^{\mu*}(p,\lambda_V)\epsilon_Z^{\nu*}(k,\lambda_Z)\\ &= F_1\epsilon_V^*(p,\lambda_V)\cdot\epsilon_Z^*(k,\lambda_Z)\\ &\quad+F_2 k\cdot\epsilon_V^*(p,\lambda_V)p\cdot\epsilon_Z^*(k,\lambda_Z),\end{aligned} \qquad (6)$$

where $\epsilon_V^{\mu}(p,\lambda_V)$ and $\epsilon_Z^{\nu}(k,\lambda_Z)$ are the polarization vectors of the final-state vector meson and Z boson, and their explicit expressions will be given in Section 4. We will show in Section 4 that the helicity amplitudes in Eq. (6) are free of polar and azimuthal angles.

The decay width of H→V+Z can be expressed as

$$\Gamma(\text{H}\to\text{V+Z})=\frac{|\boldsymbol{p}|}{8\pi m_H^2}\sum_{pol}|\mathcal{M}_{\lambda_V\lambda_Z}[\text{H}\to\text{V+Z}]|^2. \qquad (7)$$

At order-v_r^0, the helicity amplitudes in Eq. (7) can be expressed as

$$\begin{aligned}\mathcal{M}_{\lambda_V\lambda_Z}[\text{H}\to\text{V+Z}] &= \sqrt{\frac{2m_V\langle\mathcal{O}_1\rangle_V}{2N_c(2E_q)^2}}\mathcal{M}_{\lambda_V\lambda_Z}[\text{H}\to q\bar q+Z]\\ &= \sqrt{\frac{\langle\mathcal{O}_1\rangle_V}{2N_c m_c}}\mathcal{M}_{\lambda_V\lambda_Z}[\text{H}\to q\bar q+Z],\end{aligned} \qquad (8)$$

where $\langle\mathcal{O}_1\rangle_V$ represents the nonperturbative LDME and we take $\langle\mathcal{O}_1\rangle_{J/\psi}=0.44$ GeV^3 and $\langle\mathcal{O}_1\rangle_{\Upsilon(1S)}=3.07$ GeV^3 for the case of J/ψ and $\Upsilon(1S)$, respectively [13].

For the phenomenological results, we take $m_H = 125.09$ GeV, $m_Z = 91.1876$ GeV, $m_W = 80.385$ GeV, and $m_t = m_t(\text{pole mass}) = 174.2$ GeV [14]. The decay width of the Higgs boson $\Gamma_H = 4.100$ MeV [15]. The electroweak coupling is parameterized as $\alpha_{G_\mu} = \frac{\sqrt{2}}{\pi}G_\mu m_W^2(1-m_W^2/m_Z^2) \approx 1/132.23$, and the Fermi coupling constant $G_\mu = 1.1663787 \times 10^{-5}$ GeV^{-2}. The strong coupling $\alpha_s(m_H/2) \approx 0.1253$, which is evaluated with RunDec [16]. We take $m_c = 1.5$ GeV and $m_b = 4.6$ GeV for the masses of the charm and bottom quarks, respectively.

3 QCD corrections to unpolarized H → V+Z

3.1 LO results revisited

Feynman diagrams for LO H → V+Z are shown in Fig. 1. The first two diagrams come from the direct channel where the charm (bottom) quark is coupled directly to the Higgs boson. The last two diagrams come from the HZZ* and the loop-induced HZγ^* vertexes in the SM, respectively.
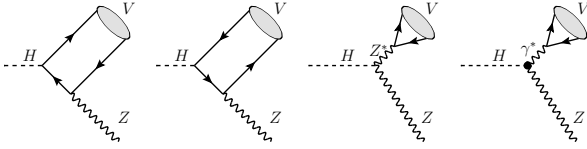

Fig. 1. Feynman diagrams for LO H → V + Z. The Feynman diagrams were generated with JaxoDraw [17].

Table 1. Branching ratios for H→V+Z at LO.

	direct	indirect	interference	total
J/ψ	8.34×10^{-10}	4.36×10^{-6}	-6.96×10^{-8}	4.30×10^{-6}
Υ(1S)	5.22×10^{-8}	2.14×10^{-5}	8.43×10^{-7}	2.23×10^{-5}

In Table 1, we list the branching ratios for H → V+Z from the direct, the indirect and the interference channels[1]. The branching ratios are rather small at LO for both J/ψ and Υ(1S). The contributions from the direct channels of J/ψ and Υ(1S) production are totally negligible, as expected. The branching ratios from the indirect channel dominate the total results and the interference branching ratios amount to -1.60% and 3.93% of the total results for J/ψ and Υ(1S) respectively.

3.2 QCD corrections

QCD radiative corrections are performed by attaching gluons to the final-state quarks as shown in Fig. 2. Note that the loop-induced HZγ^* vertex may also have QCD corrections at NLO. In Refs. [18, 19], the authors

calculate the QCD corrections to the rare decay H→Zγ at NLO and the relative corrections turn out to be rather small, around 3‰ of the LO decay width. We neglect the tiny QCD corrections to the HZγ^* vertex throughout this work and extract the parameters for the effective HZγ^* coupling by evaluating this vertex at LO.
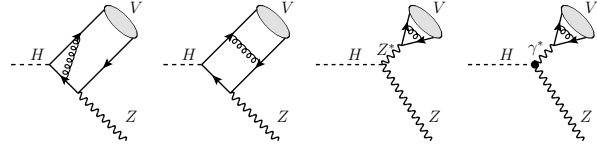

Fig. 2. Representative Feynman diagrams for QCD corrections to H→V+Z.

We choose the dimensional regulation to regularize the possible UV and IR singularities in $d = 4-2\epsilon$ dimensions. The renormalization constants Z_2 and Z_m are defined as

$$\psi_q^0 = \sqrt{Z_2}\psi_q, \quad m_q^0 = Z_m m_q, \tag{9}$$

where $\psi_q(\psi_q^0)$ and $m_q(m_q^0)$ represent the renormalized(bare) quark field and mass, respectively. In the on-shell (OS) scheme, the renormalization constants expanded to $O(\alpha_s)$ can be expressed as

$$Z_2^{\text{OS}} = 1 - C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon_{\text{UV}}} + \frac{2}{\epsilon_{\text{IR}}} - 3\gamma_E + 3\ln\frac{4\pi\mu^2}{m_q^2} + 4\right) + \mathcal{O}(\alpha_s^2), \tag{10}$$

$$Z_m^{\text{OS}} = 1 - 3C_F\frac{\alpha_s}{4\pi}\left(\frac{1}{\epsilon_{\text{UV}}} - \gamma_E + \ln\frac{4\pi\mu^2}{m_q^2} + \frac{4}{3}\right) + \mathcal{O}(\alpha_s^2). \tag{11}$$

The singularities are removed after renormalization, and the decay width for H → V+Z at NLO can be expressed as

$$\Gamma^{\text{NLO}} \equiv \Gamma^{\text{LO}}(1+2\delta_V).$$

For the QCD corrections to the direct decay channel, $\delta_{\text{J/}\psi}^{\text{dir}} = -60.98\%$ and $\delta_{\Upsilon(1S)}^{\text{dir}} = -48.88\%$ for J/ψ and Υ(1S) respectively. We can see that the results for the direct channel are greatly reduced by the NLO QCD corrections, which makes the NLO results for the direct channel of H→J/ψ+Z negative. In Ref. [20], the authors evaluated the NLO corrections to H→V+γ and the large and negative corrections to the direct channel are similar. At scale $\mu = m_H/2$, the relative corrections $\Delta\Gamma^{\text{direct}}/\Gamma^{\text{direct}}$ defined in Ref. [20] are around -85.38% and -68.38% for J/ψ and Υ(1S) respectively. These large and negative corrections at NLO may be compensated by the NNLO or the relativistic contributions. The QCD radiative corrections to the indirect channel come from the

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1) In Ref. [8], the authors list the LO results in their Table 1, and we find some disagreements with their results. The expression $m_H^2 - m_Z^2 - m_V^2$ in the denominator of Eq. (6) in Ref. [8] should be $m_H^2 + m_Z^2 - m_V^2$.

last two diagrams in Fig. 2. The results are trivial and independent of the quark species:

$$\delta_V^{\text{ind}} = -2C_F \frac{\alpha_s}{\pi} \approx -10.63\%. \qquad (12)$$

The NLO results for H→V+Z are listed in Table 2. Both of the branching ratios from the direct and the interference channels are greatly reduced, which makes these results sensitive to the strong coupling α_s. In Fig. 3, we show the dependence of the total branch ratios on the scale by sliding μ from $2m_c$ to m_H.

Table 2.   Branching ratios for H→V+Z at NLO.

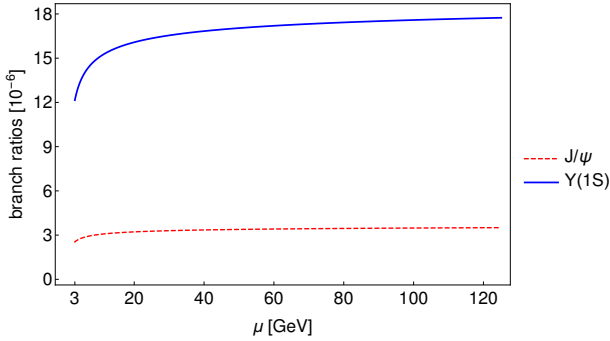|	direct	indirect	interference	total
J/ψ	-1.83×10^{-10}	3.44×10^{-6}	-1.87×10^{-8}	3.42×10^{-6}
$\Upsilon(1S)$	1.17×10^{-9}	1.69×10^{-5}	3.61×10^{-7}	1.72×10^{-5}

Fig. 3.   (color online) The dependence of the branching ratios for the decay H → V + Z on the scale μ at NLO. The scale μ stems from the QCD running coupling $\alpha_s(\mu)$, which is taken from $2m_c = 3.0$ GeV to $m_H = 125.09$ GeV. We evaluate the running coupling $\alpha_s(\mu)$ with the package RunDec and the initial value of $\alpha_s(\mu)$ is taken as $\alpha_s(m_Z)=0.1181$.

In this work, we generate the decay amplitudes with FeynArts [21], and employ the the package FeynCalc [22] to deal with the traces over the Dirac matrices. The amplitudes are further decomposed with the package Apart [23] and the IBP (integration by parts) reductions are performed with FIRE [24]. The master integrals are calculated with Package-X [25] analytically and further checked numerically with LoopTools [26].

4   Results for polarized vector meson and Z boson

It is sometimes useful to project the final-state vector meson and Z boson to particular helicity states. The corresponding polarization vectors are

$$\epsilon_V^\mu(p,\pm1) = \sqrt{\frac{1}{2}}e^{\pm i\phi}(0, \mp\cos\theta\cos\phi + i\sin\phi,$$
$$-i\cos\phi \mp \cos\theta\sin\phi, \pm\sin\theta), \qquad (13)$$

$$\epsilon_Z^\mu(k,\pm1) = \sqrt{\frac{1}{2}}e^{\mp i\phi}(0, \pm\cos\theta\cos\phi + i\sin\phi,$$
$$-i\cos\phi \pm \cos\theta\sin\phi, \mp\sin\theta), \qquad (14)$$

and

$$\epsilon_V^\mu(p,0) = \frac{\sqrt{m_V^2 + |\boldsymbol{p}|^2}}{|\boldsymbol{p}|m_V}p^\mu - \frac{m_V}{|\boldsymbol{p}|m_H}(p^\mu + k^\mu), \qquad (15)$$

$$\epsilon_Z^\mu(k,0) = \frac{\sqrt{m_Z^2 + |\boldsymbol{p}|^2}}{|\boldsymbol{p}|m_Z}k^\mu - \frac{m_Z}{|\boldsymbol{p}|m_H}(p^\mu + k^\mu), \qquad (16)$$

where we have introduced θ, ϕ as the polar and azimuthal angles of $\boldsymbol{p}$ with respect to a fixed z-axis. The *Jacob-Wick convention* [27] is adopted for the polarization vector of the final-state Z boson in Eq. (13).

Since the initial-state Higgs boson is spin-0, the helicity amplitudes must be proportional to

$$D_{\lambda_i\lambda_f}^J(\theta,\phi) = D_{00}^0(\theta,\phi) = P_0(\cos\theta) = 1, \qquad (17)$$

where $D_{\lambda_i\lambda_f}^J(\theta,\phi)$ is the *Wigner-D* matrix element in the partial wave expansion [28], and $\lambda_i = 0$, $\lambda_f = \lambda_V - \lambda_Z = 0$. Thus the helicity amplitudes defined in Eq. (6) are free of the polar and azimuthal angles θ, ϕ, and $\lambda_V = \lambda_Z = 0, \pm1$.

The numerical results for the decay of the Higgs to the polarized vector meson and Z boson are shown in Table 3. For the case of H→J/ψ+Z, the branching ratios for the longitudinally polarized J/ψ are around 62% of the total branching ratios at LO and NLO. While the branching ratios for the transversely polarized $\Upsilon(1S)$ and J/ψ are very close, the results for the longitudinally polarized $\Upsilon(1S)$ are one order of magnitude larger than those of J/ψ. As a result, the branching ratios for the longitudinally polarized $\Upsilon(1S)$ dominate the total branching ratios of the decay H→$\Upsilon(1S)$+Z at both LO and NLO.

Table 3.   The branching ratios for the polarized vector meson and Z boson for H→V+Z. The notations "T" and "L" represent the transversely and longitudinally polarized vector meson and Z boson.

	LO			NLO		
	T	L	total	T	L	total
J/ψ	1.60×10^{-6}	2.69×10^{-6}	4.30×10^{-6}	1.30×10^{-6}	2.12×10^{-6}	3.42×10^{-6}
$\Upsilon(1S)$	1.85×10^{-6}	2.05×10^{-5}	2.23×10^{-5}	1.22×10^{-6}	1.60×10^{-5}	1.72×10^{-5}

5   Conclusion

In this work, we have revisited the rare decay H→V+Z ($V =$ J/ψ or Υ(1S)) in the SM. We separated the decay amplitudes into two parts: the first part from the direct Yukawa coupling of the Higgs boson to the charm (bottom) quark, and the second from the HZZ* and the loop-induced HZγ* vertexes in the SM. We also analyzed their interference effects carefully. We took a further step by evaluating the QCD radiative corrections to both cases at NLO. The QCD corrections reduce the LO branching ratio by around 20%. Our results show that the contributions from the direct channel are negligible compared with the indirect channel, due to the tiny Yukawa coupling between the Higgs and the charm (bottom) quark. The interference branching ratios turn out to reach several percent level of the total results at both LO and NLO.

The decay of the Higgs to the polarized vector meson and Z boson was studied in Section 4. For the decay H→J/ψ+Z, the production of the longitudinally polarized J/ψ is about twice that of the transversely polarized J/ψ. For the decay H→Υ(1S)+Z, our results show that the decay to the longitudinally polarized Υ(1S) dominates.

Although the branching ratios from the indirect and interference terms give nontrivial contributions at both LO and NLO, the measurements of the Hcc̄ and Hbb̄ couplings seem to be rather difficult via the rare decay H→V+Z. The small Yukawa coupling may be enhanced by new physics beyond the SM, which provides a possible way to test the SM predictions in the projected HL-LHC or in future hadron colliders.

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