New model of kaon regeneration

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Abstract: It is shown that in the standard model of K_S^0 regeneration a system of non-coupled equations of motion is used instead of the coupled ones. A model alternative to the standard one is proposed. A calculation performed by means of the diagram technique agrees with that based on exact solution of the equations of motion.

Keywords: equations of motion, regeneration, diagram technique

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1 Introduction

The effect of kaon regeneration has been known since the 1950s. However, in the previous calculations [1–3] a system of non-coupled equations of motion was considered (see Eq. (1)) instead of the coupled equations of motion. This is a fundamental defect because it leads to a qualitative disagreement in the results. This means that the regeneration has not been described at all. The result obtained in Ref. [2] was adduced in Refs. [4, 5] and subsequent papers. In this paper we consider a model based on the exact solution of the coupled equations of motion with the potentials taken in general form. A comparison with the previous model and our calculation performed by means of the diagram technique is given as well.

Let K_L^0 fall onto the plate at t=0. Our particular interest is in the probability of finding K_S^0. Our approach is as follows. Since K^0N- and $\bar{\mathrm{K}}^0$N-interactions are known, we go to K^0,$\bar{\mathrm{K}}^0$ representation. The problem is described by coupled equations of motion for $K^0(t)$ and $\bar{K}^0(t)$. We find the corresponding solutions and revert to K_L^0,K_S^0 representation.

2 Previous calculations

In the previous calculations the starting equations are (see Eqs. (3) of Ref. [2]):

$$(\partial_x - ink)\alpha = 0,$$
$$(\partial_x - in'k)\alpha' = 0, \tag{1}$$

where n and n' are the indexes of refraction for α and α', respectively. In this equation the change of variables $\alpha,\alpha' \to \alpha_1,\alpha_2$ is performed and the effects of weak interactions are added. As a consequence of the change of variables, Eqs. (5) of Ref. [2] are coupled. The solution of Eqs. (5) in Ref. [2] gives the result in Eq. (6) of Ref. [2] (or Eq. (1) of Ref. [3]). This result is adduced in Eq. (9.32) of Ref. [4], in Eqs. (7.83)–(7.89) of Ref. [5], and subsequent papers.

We consider all the possibilities. If α and α' correspond to K_S^0 and K_L^0, α_1 and α_2 describe K^0 and $\bar{\mathrm{K}}^0$, whereas our interest is in K_S^0 and K_L^0. The indexes of refraction for K_L^0 and K_S^0 are unknown.

Let $\alpha = \mathrm{K}^0$ and $\alpha' = \bar{\mathrm{K}}^0$. Then $\alpha_1 = \mathrm{K}_S^0$ and $\alpha_2 = \mathrm{K}_L^0$. This variant follows from the initial conditions (see Ref. [2]): $\alpha_2(0) = 1$, $\alpha_1(0) = 0$. There is no off-diagonal mass $\epsilon = (m_L - m_S)/2$. Equations (1) are non-coupled. The non-coupled equations exist only for the stationary states and do not exist for K^0 and $\bar{\mathrm{K}}^0$.

In any case Eqs. (1) are unrelated to the problem. Our calculation gives the inverse $\Delta\Gamma$- and Δm-dependences (see Eq. (24) of Ref. [6]). In this paper we consider a model with the potentials taken in general form. A similar question for $\Lambda\bar{\Lambda}$ oscillations is studied in Refs. [7-10].

3 Our model

Let K_L^0 fall onto the plate at t=0. We use the model described in the second paragraph of the Introduction. In Ref. [6] the exact wave function $K_S(t)$ of K_S^0 has been calculated. The probability of finding K_S^0 or, equivalently, the probability of the $\mathrm{K}_L^0\mathrm{K}_S^0$ transition, is given by Eq. (12) of Ref. [6]:

$$|K_S(t)|^2 = \frac{1}{4}|V/p|^2 \mathrm{e}^{\mathrm{Im}Vt+2\mathrm{Im}Mt}\big[\mathrm{e}^{-\mathrm{Im}(pt)}+\mathrm{e}^{\mathrm{Im}(pt)} -\mathrm{e}^{i\mathrm{Re}(pt)}-\mathrm{e}^{-i\mathrm{Re}(pt)}\big]. \tag{2}$$

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This expression is exact; $|K_S(t{=}0)|^2{=}0$.

If Im$V{=}0$, then Im$p{=}0$ as well. In this case

$$|K_S(t)|^2{=}|V/p|^2\mathrm{e}^{-(\Gamma^a_{\mathrm{K}^0}+\Gamma^d)t}\sin^2(\mathrm{Re}(pt)/2). \quad (3)$$

This is a pure oscillation regime. Here regeneration by scattering takes place. The regeneration by absorption is described by ImV.

As in Ref. [6] we put $p\approx V+2\epsilon^2/V$ ($\epsilon=\Delta m/2$, $\Delta m=m_L-m_S$, where m_L and m_S are the masses of stationary states); $\Gamma^d_{\mathrm{K}^0}=\Gamma^d_{\bar{\mathrm{K}}^0}=\Gamma^d$, $m_{\mathrm{K}^0}=m_{\bar{\mathrm{K}}^0}=m$ (m and Γ^d are the mass and decay width, respectively, of K^0). Let us denote $\Gamma^a_{\mathrm{K}^0}$ and $\Gamma^a_{\bar{\mathrm{K}}^0}$ as widths of absorption (not decay) of K^0 and $\bar{\mathrm{K}}^0$, respectively. Then

$$2\mathrm{Im}M{=}-(\Gamma^a_{\mathrm{K}^0}+\Gamma^d), \quad (4)$$

$$\mathrm{Re}V{=}\mathrm{Re}U_{\bar{\mathrm{K}}^0}-\mathrm{Re}U_{\mathrm{K}^0}, \quad (5)$$

$$\mathrm{Im}V{=}-\frac{\Delta\Gamma}{2}, \quad (6)$$

where

$$\Delta\Gamma{=}\Gamma^a_{\bar{\mathrm{K}}^0}-\Gamma^a_{\mathrm{K}^0}. \quad (7)$$

Equation (2) gives:

$$|K_S(t)|^2{=}R\omega, \quad (8)$$

$$R{=}\frac{1}{4}|V/p|^2\mathrm{e}^{-(\Gamma^a_{\mathrm{K}^0}+\Gamma^d)t}, \quad (9)$$

$$\omega{=}\mathrm{e}^{-\Gamma(\mathrm{K}_L\to\mathrm{K}_S)t}+\mathrm{e}^{[\Gamma(\mathrm{K}_L\to\mathrm{K}_S)t-\Delta\Gamma]t}-2\mathrm{e}^{-\Delta\Gamma t/2}\cos(\mathrm{Re}(pt)), \quad (10)$$

where

$$\Gamma(\mathrm{K}_L\to\mathrm{K}_S){=}\frac{\epsilon^2}{|V|^2}\Delta\Gamma, \quad (11)$$

$$\mathrm{Re}p\approx\mathrm{Re}V+2\epsilon^2\frac{\mathrm{Re}V}{|V|^2}. \quad (12)$$

$\Gamma(\mathrm{K}_L\to\mathrm{K}_S)$ is the width of K_LK_S transition (regeneration). The value Δm is involved in $\Gamma(\mathrm{K}_L\to\mathrm{K}_S)$ and $\cos(\mathrm{Re}(pt))$.

Let

$$\Delta\Gamma t\gg 1. \quad (13)$$

In this case

$$\omega{=}\mathrm{e}^{-\Gamma(\mathrm{K}_L\to\mathrm{K}_S)t}. \quad (14)$$

The t-dependence is given by an exponential decay law. It is significant that Re$V\neq 0$, in contrast to Ref. [6]. (Note that Eq. (14) is valid if $\Delta\Gamma>2\epsilon$.)

4 Connection between models based on the diagram technique and the exact solution

The calculation presented above is cumbersome and formal, so verification is required. In Ref. [11] an approach based on perturbation theory was proposed. Regeneration followed by the decay $\mathrm{K}^0_L\to\mathrm{K}^0_S\to\pi\pi$ was considered. A similar approach is used for the $n\bar{n}$ transition in a medium followed by annihilation (see Refs. [12-15]). The process amplitude $M(\mathrm{K}^0_L\to\mathrm{K}^0_S\to\pi\pi)$ is

$$M(\mathrm{K}^0_L\to\mathrm{K}^0_S\to\pi\pi){=}\frac{\epsilon}{V}M_d(\mathrm{K}^0_S\to\pi\pi). \quad (15)$$

(See the second term of Eq. (23) of Ref. [11].) Here $M_d(\mathrm{K}^0_S\to\pi\pi)$ is the in-medium amplitude of the decay $\mathrm{K}^0_S\to\pi\pi$. The corresponding process width is

$$\Gamma(\mathrm{K}^0_L\to\mathrm{K}^0_S\to\pi\pi){=}\frac{\epsilon^2}{|V|^2}\Gamma_d(\mathrm{K}^0_S\to\pi\pi), \quad (16)$$

where $\Gamma_d(\mathrm{K}^0_S\to\pi\pi)$ is the width of the decay $\mathrm{K}^0_S\to\pi\pi$.

Consider now the connection between the models based on the diagram technique and the exact solution. In this case we write Eq. (16) in the form

$$\Gamma(\mathrm{K}^0_L\to\mathrm{K}^0_S\to\pi\pi){=}\frac{\epsilon^2}{|V|^2}\Gamma_d(\mathrm{K}^0_S\to\pi\pi)\frac{\Delta\Gamma}{\Delta\Gamma}{=}\Gamma(\mathrm{K}_L\to\mathrm{K}_S)W, \quad (17)$$

$$W{=}\frac{\Gamma_d(\mathrm{K}^0_S\to\pi\pi)}{\Delta\Gamma}, \quad (18)$$

where W is the probability of the K^0_S decay in the channel $\mathrm{K}^0_S\to\pi\pi$. The physical sense of Eq. (17) is obvious: the multistep process $\mathrm{K}^0_L\to\mathrm{K}^0_S\to\pi\pi$ involves the subprocess of K_LK_S transition (regeneration). Equation (17) is verification of the models considered above.

Due to a strong absorption of $\bar{\mathrm{K}}^0$ and zero momentum transfer in the $\mathrm{K}^0\bar{\mathrm{K}}^0$ transition vertex, the description of competition between scattering and absorption is of particular importance. In this regard the diagram technique has some advantages over the model based on the equations of motion (see Refs. [14,15]).

5 Limiting case and numerical results

Let us consider the limiting case $t\to 0$. Expanding Eq. (10) to the terms $\sim t^2$ we have

$$\omega{=}[1-\cos(\mathrm{Re}(pt))](2-\Delta\Gamma t)+\omega_1 t^2/2, \quad (19)$$

$$\omega_1{=}[\Gamma(\mathrm{K}_L\to\mathrm{K}_S)]^2+[\Gamma(\mathrm{K}_L\to\mathrm{K}_S)-\Delta\Gamma]^2-\frac{(\Delta\Gamma)^2}{2}\cos(\mathrm{Re}(pt)). \quad (20)$$

Let Re$V\neq 0$ and $\Delta\Gamma{=}0$. Then $\Gamma(\mathrm{K}_L\to\mathrm{K}_S){=}\omega_1{=}0$ and $|K_S(t)|^2$ coincides with Eq. (3). The opposite case when Re$V{=}0$ is more interesting. Then

$$\omega{=}\omega_1 t^2/2 \quad (21)$$

and

$$|K_S(t)|^2{=}\frac{1}{8}|V/p|^2\mathrm{e}^{-(\Gamma^a_{\mathrm{K}^0}+\Gamma^d)t}t^2. \quad (22)$$

Here regeneration by absorption takes place. Comparing Eq. (19) and Eq. (21) we see that Re$V\neq 0$ violates t^2-dependence.

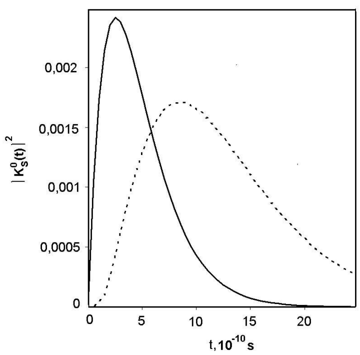

Fig. 1. Probability of finding K_S^0. The solid and dashed curves correspond to Re$V=\Delta\Gamma/2$ and calculation by means of Eq. (25), respectively.

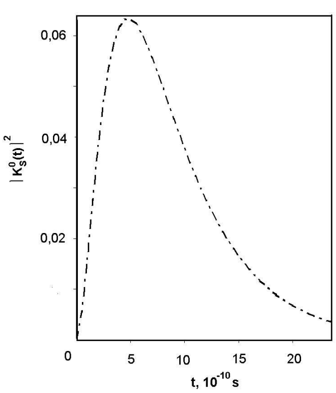

Fig. 2. Probability of finding K_S^0. The dot-dashed curve corresponds to ReV=0.

Let us revert to Eqs. (8)–(10). $\Gamma^a_{K^0}$ and $\Gamma^a_{\bar{K}^0}$ are given by standard expressions which follow from the optical theorem:

$$\Gamma^a_{K^0}=N_n v\sigma(K^0 n)+N_p v\sigma(K^0 p) \tag{23}$$

and

$$\Gamma^a_{\bar{K}^0}=N_n v\sigma(\bar{K}^0 n)+N_p v\sigma(\bar{K}^0 p), \tag{24}$$

where N_n and N_p are the number of neutrons and protons in a unit of volume, respectively; $\sigma(K^0N)$ and $\sigma(\bar{K}^0N)$ are the total cross sections of K^0N- and $\bar{K}^0$N-interactions, and v is the velocity of the K^0 meson. By way of illustration we take $\sigma(K^0n)=\sigma(K^0p)=15$ mb [16]. As in Ref. [2], we use $\sigma(K^0N)=\frac{1}{3}\sigma(\bar{K}^0N)$.

Instead of cross sections one can use the forward scattering amplitudes of kaons by the molecules of the medium. In this case

$$V=U_{\bar{K}^0}-U_{K^0}=\frac{2\pi}{m}N_m f_{21}, \tag{25}$$

$f_{21}=f-\bar{f}$. Here N_m is the number of molecules in a unit of volume, and f and $\bar{f}$ are the forward scattering amplitudes of K^0 and $\bar{K}^0$, respectively.

For a copper absorber the probability of finding K_S^0 is shown in Figs. 1 and 2. ImV is determined by Eqs. (6), (7), (23) and (24), and ReV is the parameter. The solid and dot-dashed curves correspond to $|\mathrm{Re}V|=\Delta\Gamma/2$ and ReV=0, respectively. The dashed curve corresponds to the copper plate and V defined from Eq. (25). The amplitudes f and $\bar{f}$ are taken from Ref. [17]. In the case ReV=0 only regeneration by absorption takes place. It is seen that ReV leads to the suppression of regeneration. $|K_S(t)|^2$ is about 10 times smaller than in Ref. [2] (although comparison with Ref. [2] is meaningless for the reasons given above.)

6 Conclusion

The main results of this paper are given in the abstract. The most distinctive feature of the model presented above is the inverse $\Delta\Gamma$- and Δm-dependences of the amplitude of regenerated K_S^0, or parameter regeneration r (see Eqs. (19)–(24) of Ref. [6]). The main uncertainty in the numerical results is conditioned by the uncertainty in the cross sections $\sigma(K^0N)$ and $\sigma(\bar{K}^0N)$. The same is also true for the previous results [1-5] since they have been obtained by means of the above-mentioned cross sections as well. In this connection we would like to recall that Δm is extracted from free-space oscillations without recourse to the potentials of K^0 and $\bar{K}^0$. Nevertheless, in any case the regeneration should be described correctly.

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