Limiting majoron self-interactions from gravitational wave experiments*

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Abstract: We show how majoron models may be tested/limited in gravitational wave experiments. In particular, the majoron self-interaction potential may induce a first order phase transition, producing gravitational waves from bubble collisions. We dub such a new scenario the *violent majoron model*, because it would be associated with a violent phase transition in the early Universe. Sphaleron constraints can be avoided if the global $U(1)_{B-L}$ is broken at scales lower than the electroweak scale, provided that the B-L spontaneously breaking scale is lower than 10 TeV in order to satisfy the cosmological mass density bound. The possibility of a sub-electroweak phase transition is practically unconstrained by cosmological bounds and it may be detected within the sensitivity of the next generation of gravitational wave experiments: eLISA, DECIGO and BBO. We also comment on its possible detection in the next generation of electron-positron colliders, where majoron production can be observed from the Higgs portals in missing transverse energy channels.

Keywords: lepton/baryon number violation, majorons, gravitational waves, colliders

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1 Introduction

The idea that the neutrino can be identified with its anti-particle, the antineutrino ($\nu=\nu^c$), was a wonderful intuition by Ettore Majorana in '37 [1]. A Majorana mass term for the neutrino must imply a violation of the lepton number of two units, i.e. $\Delta L=2$. Nowadays, the only realistic test of Majorana's hypothesis is neutrinoless double beta decay ($0\nu2\beta$). The Majorana mass term may be originated by a spontaneous symmetry breaking of a global $U(1)_L$ or $U(1)_{B-L}$ extension of the Standard Model. This leads to the possibility that a new pseudo Nambu-Goldstone boson, dubbed the *majoron*, can be coupled with neutrinos and be emitted in the $0\nu\beta\beta$-process [2–4]. Majorons have phenomenological implications not only in $0\nu\beta\beta$ experiments, but they can be limited by astrophysical stellar cooling processes and cosmological bounds. In particular, the spontaneous symmetry breaking scale of $U(1)_L$ or $U(1)_{B-L}$ is highly constrained to be higher than the electroweak scale [6–10]. On the other hand, it has been argued that such a VEV scale cannot be higher than 10 TeV for a cosmologically consistent majoron model [11].

Despite these considerations, the majoron particle remains very elusive, with not many direct detection channels predicted. Nonetheless, the possibility of testing first order phase transitions (FOPT) in the early Universe seems to be more promising after the recent discovery of gravitational waves (GW) at LIGO [18, 19]. In particular, the next generations of interferometers like eLISA and U-DECIGO will be fundamentally important to test the gravitational signal produced by Coleman bubbles from FOPT. The production of GW from bubble collisions was first suggested in Refs. [21–25] – see also Refs. [26–30] for more analytical and numerical analysis on first order phase transitions and GW signals.

New experimental prospectives in GW experiments have motivated a *revival* of these ideas in the context of new extensions of the Standard Model [31, 34–44]. In other words, the GW data may be used to test new models of particle physics beyond the Standard Model.

With this paper we suggest to test the majoron self-interactions and the details of the spontaneous symmetry breaking mechanism of lepton symmetry from GW

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interferometers. This is still a completely open possibility, related to a FOPT of lepton symmetry in the early Universe. In particular, we show how the next generation of interferometers like (e)LISA, U-DECIGO and BBO can test the spontaneous symmetry breaking scale $V_{BL}=10\,\text{GeV}$–$10\,\text{TeV}$. We dub such a majoron particle associated with the FOPT a *violent majoron*. We will discuss how the current limits from the LHC in missing transverse energy channels do not exclude the possibility of a violent majoron. We emphasize that in the framework of the violent majoron model, the CEPC collider will be able to test an energy scale overlapping the one associated with the eLISA sensitivity for GW signals from FOPT.

2 The model

We consider the extension of the Standard Model described by the gauge groups $SU_c(3)\times SU(2)_L\times U(1)_Y\times U(1)_{B-L}$, in which the lepton and baryon numbers are promoted to a $U(1)_{B-L}$ global symmetry1). We introduce a complex scalar field coupled to neutrinos and to the Higgs boson that spontaneously breaks the $U(1)_L$ symmetry:

$$\mathcal{L}_M=fH\bar{L}\nu_R+h\sigma\bar{\nu}_R\nu_R^c+h.c.+V(\sigma,H), \quad (1)$$

where h,f are Yukawa matrices of the model, and

$$V(\sigma,H)=V_0(\sigma,H)+V_1(\sigma)+V_2(h,\sigma) \quad (2)$$

the potential, with

$$V_0(\sigma,H) = \lambda_s\left(|\sigma|^2-\frac{v_{BL}^2}{2}\right)^2+\lambda_H\left(|H|^2-\frac{v^2}{2}\right)^2 +\lambda_{sH}\left(|\sigma|^2-\frac{v_{BL}^2}{2}\right)\left(|H|^2-\frac{v^2}{2}\right), \quad (3)$$

and higher order terms

$$V_1(\sigma)=\frac{\lambda_1}{\Lambda}\sigma^5+\frac{\lambda_2}{\Lambda}\sigma^*\sigma^4+\frac{\lambda_3}{\Lambda}(\sigma^*)^2\sigma^3+\text{h.c.} \quad (4)$$

and

$$V_2(H,\sigma) = \beta_1\frac{(H^\dagger H)^2\sigma}{\Lambda}+\beta_2\frac{(H^\dagger H)\sigma^2\sigma^*}{\Lambda} +\beta_3\frac{(H^\dagger H)\sigma^3}{\Lambda}+\text{h.c.}. \quad (5)$$

In principle, the scales of new physics entering non-perturbative operators may be different from each other. For convention, we parametrize their differences in the couplings λ_i,β_i.

When σ gets a VEV, it may be decomposed in a real and a complex field:

$$\sigma=\frac{1}{\sqrt{2}}(v_{BL}+\rho+i\chi). \quad (6)$$

After the global $U(1)_L$ symmetry breaking, the RH neutrino acquires a Majorana mass term

$$M=\frac{1}{\sqrt{2}}hv_{BL} \quad (7)$$

and a Dirac mass for the LH neutrino

$$m=\frac{1}{\sqrt{2}}fv, \quad (8)$$

where $|\langle H\rangle|=v$ and $v/\sqrt{2}=174\,\text{GeV}$.

The seesaw relations are obtained for $M>>m$, namely

$$N=\nu_R+\nu_R^c+\frac{m}{M}(\nu_L+\nu_L^c), \quad (9)$$

$$\nu=\nu_L+\nu_L^c-\frac{m}{M}(\nu_R^c+\nu_R), \quad (10)$$

i.e.

$$m_N\simeq M, \qquad m_\nu\simeq\frac{m^2}{M}, \quad (11)$$

m_N standing for the mass of the right-handed neutrino N. The majoron corresponds to the pseudo-scalar field χ, which is the Nambu-Goldstone boson of the spontaneously broken $U(1)$ symmetry, while the real scalar ρ gets a mass $m_\rho\sim O(1)v_s$ once the self-coupling is assumed to be $O(1)$.

In the majoron model higher order terms, like the one entering V_1, are desired in order to induce a mass contribution that would not be allowed at perturbative level. For example, these may be induced either by gravitational effects [11] or by exotic instantons (see e.g. Ref. [12]) at lower scales than the Planck scale2).

3 Gravitational wave signal from majorons

The spontaneous symmetry breaking of the $U(1)_{B-L}$ can be catalyzed by a FOPT. This induces the generation of Coleman's bubbles expanding at high velocity, which generate a stochastic cosmological background of gravitational radiation. Gravitational waves are generated by three main processes: i) bubble-bubble collisions; ii) turbulence induced by the bubble's expansion in the plasma; iii) sound waves induced by the bubble's running in the plasma. The peak frequency of the GW signal produced by bubble collision has a frequency

$$f_{\text{collision}}\simeq3.5\times10^{-4}\left(\frac{\beta}{H_*}\right)\left(\frac{\bar{T}}{10\,\text{GeV}}\right)\left(\frac{g_*(\bar{T})}{10}\right)^{1/6}\text{mHz}, \quad (12)$$

1) The implication of the majoron in neutron-antineutron transitions was recently discussed in Refs. [14–17].

2) However, related theoretical aspects within the context of string theory are not completely understood. For instance a global $U(1)_L$ might be thought of as a local $U(1)_L$ from *flavor branes*, since in string theory no exact global $U(1)$ symmetry can arise. This possibility seems highly bounded by the weak gravity conjecture, pointed out recently in Ref. [13].

in which β is related to the size of the bubble wall and is expressed in Eq. (16), $\bar{T}$ is the temperature at the FOPT, $g_*(\bar{T})$ is the number of degrees of freedom involved, and the GW intensity is estimated as follows:

$$\Omega_{\text{collision}}(\nu_{\text{collision}})\simeq$$
$$k\mathcal{E}^2\left(\frac{\bar{H}}{\beta}\right)^2\left(\frac{\alpha}{1+\alpha}\right)^2\left(\frac{V_B^3}{0.24+V_B^3}\right)\left(\frac{10}{g_*(\bar{T})}\right). \quad (13)$$

The coefficient k introduced above has a numerical value $k\simeq 2.4\times 10^{-6}$, while

$$\mathcal{E}(\bar{T})=\left[T\frac{\mathrm{d}V_{\text{eff}}}{\mathrm{d}T}-V_{\text{eff}}(T)\right]_{T=\bar{T}}, \quad (14)$$

$$\alpha=\frac{\mathcal{E}(\bar{T})}{\rho_{\text{rad}}(\bar{T})}, \qquad \rho_{\text{rad}}=\frac{\pi^2}{30}g_*(T)T^4. \quad (15)$$

In Eq. (15) ρ_{rad} stands for the radiation energy density, while $\bar{T}\simeq v_{BL}$ is the first order phase transition temperature, defined by

$$\beta=-\left[\frac{\mathrm{d}S_E}{\mathrm{d}t}\right]_{t=\bar{t}}\simeq\left[\frac{1}{\Gamma}\frac{\mathrm{d}\Gamma}{\mathrm{d}t}\right]_{t=\bar{t}}, \quad (16)$$

in which

$$S_E(T)\simeq\frac{S_3(T)}{T}, \qquad \Gamma=\Gamma_0(T)\exp[-S_E(T)],$$

$$\Gamma_0(T)\sim T^4, \quad S_3\equiv\int \mathrm{d}^3r\left(\partial_i\sigma^\dagger\partial_i\sigma+V_{eff}(\sigma,T)\right).$$

V_B represents the velocity of the bubble. The various values of V_B will determine the amount of corrections from turbulence and sonic waves, discussed later.

The effective potential is the model dependent part of Eq.(12). In particular, the effective potential has thermal corrections which can be treated in the same approximation performed in Ref. [45]:

$$V_{\text{eff}}(s,T)\simeq CT^2(\sigma^\dagger\sigma)+V(\sigma,H), \quad (17)$$

where

$$C=\frac{1}{4}\left(\frac{m_\sigma^2}{v_{BL}^2}+\lambda_{sH}+h^2-24K_{BL}\right), \quad (18)$$

with

$$K_{BL}=(\lambda_2+\lambda_3)\frac{v_{BL}}{\Lambda}+\beta_2\frac{v_{BL}}{\Lambda}. \quad (19)$$

The case of $K_{BL}\simeq 4\times 10^{-2}$ corresponds to the one testable by eLISA, U-DECIGO and BBO. From Eq. (19), assuming[1)] $\lambda_{2,3},\beta_2=1$, $v_{BL}\simeq 1$–$100\,\text{GeV}$, and

$$\frac{1}{4}\left[\frac{m_\sigma^2}{v_{BL}^2}+\frac{1}{4}\lambda_{sH}+h^2\right]\simeq 1,$$

which corresponds to a scale of

$$\Lambda\simeq 400\ \text{GeV–}4\ \text{TeV},$$

while the GW signal is in the range 10^{-5}–10^{-3}Hz. The effect of the Higgs as a dynamical particle is suppressed as $O(v_{BL}^2/v^2)$, which is totally negligible for $v_{BL}\simeq 1$–10Gev compared to the $O(1)$ uncertanties from bubble collisions and expansions.

In principle, other contributions from turbulence and sound waves may affect the estimate of the new physics scale by a factor $O(1)$, since they will affect the power spectrum density of GW by a factor of at least $O(10)$. This may lower the scale of the new physics by a factor of 3.

Let us compare these order of magnitude semi-analytical estimations with numerical simulations. In Fig. 1, we show numerical plots for a realistic set of parameters, using the same model independent spectrum parameterization of Ref.[18]. We also consider the contribution of turbulence and shock waves as in Ref. [18]. The results are in good $O(1)$ agreement with the estimations inferred above.

3.1 LHC constraints

From the LHC, important constraints on the Higgs decay into invisible channels are derived. Let us define

$$\mu_F=\frac{\sigma^{NP}(pp\to H)}{\sigma^{SM}(pp\to H)}\frac{BR^{NP}(H\to F)}{BR^{SM}(H\to F)}, \quad (20)$$

where $F=\gamma\gamma,WW,ZZ,\tau,\tau$ label final states. In Table 1 and Fig. 1 we show the limits from various channels on Higgs decays. Comparing Eq. (20) with limits from the LHC in Fig. 2, we can set a bound on the $C_{H\chi\chi}$ parameter in the Higgs decay rate.

In particular, the model independent limit placed on the invisible decays branching ratio is (see e.g. [46])

$$\text{Br}(H\to\text{invisible})=\frac{\Gamma_{\text{inv}}}{\Gamma_{\text{inv}}+\Gamma_{\text{SM}}}<0.51(95\%\text{C.L.}), \quad (21)$$

which corresponds to the bound

$$C_{H\chi\chi}\leq 0.6. \quad (22)$$

This amounts to finding a maximal bound on the hierarchy between the v_σ and Λ scale that corresponds to the value

$$C_{H\chi\chi}\simeq C^0_{H\chi\chi}=\frac{\beta_2 v_{BL}}{\Lambda}<0.6, \quad (23)$$

where $C^0_{H\chi\chi}$ is the leading order contribution to the $C_{H\chi\chi}$, which originates from the operator in Eq. (5) parametrized by β_2. Such a constraint is easily compatible with GW signals in eLISA and cosmological

1) We chose this range of v_{BL} because this is natural in order to obtain majoron dark matter masses, without encountering sphaleron bounds, within the context of high-energy-scale baryogenesis scenarios, and undergoing majoron overproduction in the early Universe. We will discuss issues related to majoron dark matter and sphaleron bounds later on. However, in principle such an energy scale can be lowered to lower energy scales, such as KeV-ish scales. A study focusing on these space parameters would deserve a separate and new analysis, which is beyond the purposes of this paper.

bounds. For example, fixing $v_{BL}=100$GeV and $\beta_2=1$, $\Lambda>166$GeV is enough to avoid LHC constraints, while for eLISA $\Lambda\simeq 4$TeV is large enough to generate a detectable GW signal — see the previous section on gravitational waves discussed above[1)].

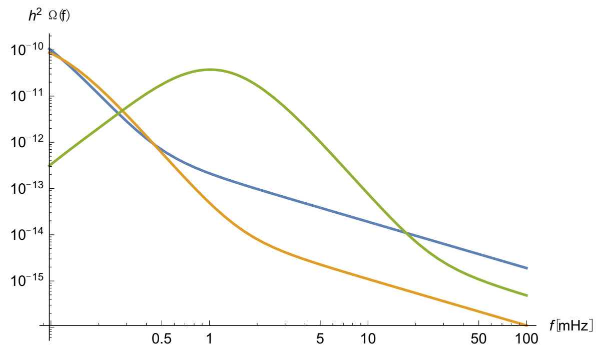

Fig. 1. (color online) The gravitational wave energy density as a function of frequency. We use the same model independent parametrization as Ref. [18]. We show three *non-runaway* bubbles cases which are compatible with the B-L first order phase transition. In blue, we consider the case of $\bar{T}=50$GeV, $\beta/\bar{H}=100$, $\alpha=0.5$, $\alpha_\infty=0.1$, $V_B=0.95$; in green $\bar{T}=20$GeV, $\beta/\bar{H}=10$, $\alpha=0.5$, $\alpha_\infty=0.1$, $V_B=0.95$; and in orange $\bar{T}=10$GeV, $\beta/\bar{H}=10$, $\alpha=0.5$, $\alpha_\infty=0.1$, $V_B=0.3$. The three cases lie in the sensitivity range of LISA [18].

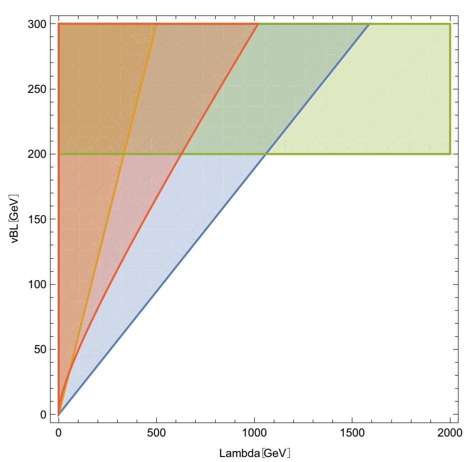

Fig. 2. (color online) Limits from the LHC and future CEPC (in brown and blue respectively), cosmological sphaleron bounds (green) and the region which will be probed by eLISA (red). The case of $\beta_2=1$ is displayed.

3.2 Electron-positron colliders and invisible Higgs decays

At the CEPC, the Higgs decay rate will be probed with a factor $O(5\text{–}10)$ sensitivity higher than LHC.

The golden channel will be the process

$$e^+e^-\to ZH\to Z\bar{b}b,$$

with a cross section

$$\sigma_{hZ\to b\bar{b}Z}=\sigma_{hZ}^{SM}\times R_{hZ}\times \mathrm{BR}(h\to b\bar{b}),$$

where R is a suppression factor related to the coupling of the Higgs boson to Z. According to Ref. [48], the number of Higgs bosons produced at center-of-mass energy $\sqrt{s}=240$GeV from the $e^+e^-\to$HZ channel will be 2×10^6, higher than the other channels. The total cross section of the process can be measured with 0.4% precision, while $\sigma_{HZ}\times Br(h\to b\bar{b})$ can reach precision 0.2% for $\sqrt{s}=240$GeV [48], while it can reach one order of magnitude more than the LHC at $\sqrt{s}=350$GeV (see Fig. 12 in Ref. [48]). In Fig. 3 we consider the constraints on Eq. (21) overimposed to current LHC constraints, cosmological bounds and future eLISA sensitivity regions.

Let us conclude this section with an important remark. We would like to stress again that in Fig. 2, β_2 is fixed to 1. However, fixing different values of the β_2 parameter will change the region plot shown. Assuming smaller values of β_2, the constraints on Λ from colliders are relaxed. Let us also note that the case $\beta_2<<1$ is still possible. This case will correspond to suppressing the invisible channels in colliders, rendering the gravitational interferometer bounds stronger than the collider bounds: the gravitational power spectrum depends on a large set of initial parameters $\lambda_{1,2,3}$, $\beta_{1,2,3}$. With $\beta_2<<\lambda_{1,2,3},\beta_1$, the GW signal is still unsuppressed. It is worth mentioning that for $\beta_2\simeq0$, invisible channels can be generated from radiative corrections, but, of course, with a strong suppression factor.

Similar constraints can be achieved from other future electron-positron colliders such as FCC-ee/eh [51, 52], ILC [49, 50], CLIC [53, 54], and LHeC [55]. From the combined statistical analysis among all different future electron-positron colliders, it is conceivable that a more precise constraint on invisible Higgs decays may be achieved.

3.3 Cosmological limits

3.3.1 Sphaleron bounds

Relating the majoron model to pre-sphaleron leptogenesis, stringent constraints are provided from washing-out processes of $(B+L)$-violating sphaleronic interactions in the Standard Model.

First of all, remember that the bound on the cosmological neutrino mass is

$$m_\nu\lesssim 50\ \mathrm{keV}\left(\frac{100\ \mathrm{GeV}}{T_{BL}}\right)^{1/2}, \tag{24}$$

1) Despite λ_{sH}, which cannot induce FOPTs, β_2 can be tested from GW and Higgs invisible channels, since it can contribute to FOPTs. Nevertheless, λ_{sH} can be “technically naturally” tuned to be very suppressed.

where T_{BL} is the temperature at which the L (or $B-L$) asymmetry is generated. The cosmological neutrino mass bound sets in turn a bound on T_{BL} from Eq. (24) that reads

$$m_\nu \lesssim 10^{-3}\,\mathrm{eV} \to T_{BL} \simeq 10^{12}\,\mathrm{GeV}. \tag{25}$$

The neutrino mass bound used in all estimations of these papers considers the cosmological and neutrinoless double beta decay bounds (see Ref. [47]). From sphaleron constraints, one can get the following bound on v_{BL}, i.e.

$$v_{BL} \lesssim \mathrm{Max}\left(200\,\mathrm{GeV}\,\lambda_1^{-1/7}(27Y)^{-4/7}U_{hl}^{-8/7}, v\right), \tag{26}$$

where

$$Y = \frac{n_\chi}{n_\gamma}, \tag{27}$$

which reduces to $Y \simeq 1/27$ [6, 7], and is compatible with big bang nucleosynthesis constraints [8, 9], with U_{hl} representing the neutrino mixing matrix. Equation (26) can be related to a bound on the Yukawa matrices:

$$\min_l \sum_i \frac{|h_{il}|^2}{f_{2i}} < 6\times 10^{-14} \times \mathrm{Max}\left(1, 0.8\lambda^{-1/7}U_{hl}^{-8/7}(2Y)^{-4/7}\right). \tag{28}$$

Equation (28) provides a very strong bound on h-couplings that reads

$$h_{il} \lesssim 10^{-7}. \tag{29}$$

This bound is strong enough to be valid even for tiny gravitationally induced (Planck scale suppressed) effects, or when the mixing U_{hl} are very small. Finally, let us comment on possible way out for this bound. The sphaleron bound can be relaxed, allowing $v_{BL} > v$, in electroweak baryogenesis scenarios and post-sphaleron baryogengesis scenarios. In this case, gravitational waves signal from the baryogengesis scenarios should be observable – as recently discussed in Ref. [34].

3.3.2 Cosmological density bound

Cosmological density constraints can be set, distinguishing the two cases:

$$(A)\, v_{BL} < v, \qquad (B)\, v_{BL} > v. \tag{30}$$

In case (A), the majoron mass is dominated by the β_1-term and casts

$$m_\chi \simeq \beta_1^{1/2}\left(\frac{v}{v_{BL}}\right)^{1/2}\mathrm{keV}. \tag{31}$$

In case (B) the mass of the majoron reads

$$m_\chi \simeq \left(\frac{25}{2}\lambda_1 + \frac{9}{2}\lambda_2 + \frac{1}{2}\lambda_3\right)^{1/2}\left(\frac{v_{BL}}{v}\right)^{3/2}\mathrm{keV}, \tag{32}$$

The cosmological density constraint on the majorons reads

$$n_\chi m_\chi < \rho_{crit}, \tag{33}$$

for $n_\chi \simeq n_\gamma$ and for $v = v_{BL}$ and $\lambda, \beta < 10^{-2}$. On the other hand, for χ decoupling sufficiently rapidly, it should be possible that $n_\chi << n_\gamma$. Consequently the constraints on χ can be weaker than the above.

Cosmological constraints that can be derived from majorons rely heavily on the assumption that the majoron is out of equilibrium, and on the decay channels that are allowed by the particular model instantiations.

If we consider massive majorons and stable LH neutrinos, limits on the Yukawa coupling h can be derived from the see-saw relation and the cosmological constraints on the neutrino mass density, i.e.

$$h \simeq \frac{\sqrt{2m_\nu M}}{v} \leqslant 10^{-6}\left(\frac{M}{\mathrm{GeV}}\right)^{1/2}, \tag{34}$$

where v is the Higgs expectation value. LH electrons and RH neutrinos are in thermal equilibrium via the interactions

$$\psi_L + h \to \nu_R + W_L, \tag{35}$$

with an interaction rate

$$\Gamma \simeq \frac{g^2 h^2}{16\pi} T. \tag{36}$$

Thermal equilibrium, as realized above, happens for

$$M \leqslant T \leqslant 10^5 M. \tag{37}$$

For $T < M$, RH neutrinos go out of thermal equilibrium, disappearing from the thermal bath. At this stage, the relevant interaction of the scalar complex field σ is with LH neutrinos, with a coupling of the order of

$$f \simeq \frac{m_\nu}{v_{BL}}. \tag{38}$$

In the case of the spontaneous symmetry breaking scale of $v_{BL} \simeq 1$–100 GeV suggested above while discussing GW signals, the limit on the majoron coupling becomes very stringent and reads

$$f \simeq 2\times(10^{-8}\text{–}10^{-10}). \tag{39}$$

Let us remark that, assuming $M > 10$ GeV or so, σ goes out of equilibrium for a temperature of about $T \simeq M$. As a consequence, the majoron density in the present Universe is

$$r_\chi = \frac{n_\chi(T_0)}{n_\gamma(T_0)} = \frac{g_*(T_0)}{g_*(T_{RH})} \simeq 0.1\text{–}0.2, \tag{40}$$

in which $g_*(T)$ represents the effective number of light particle species at a temperature T, and T_{RH} and T_0 are the decoupling temperature for the RH neutrinos and the present temperature of the Universe, respectively.

Further constraints on v_{BL} from majoron decays must be considered. The majoron decay into two neutrinos,

$$\chi \to \nu\nu, \tag{41}$$

has a decay time

$$\tau_\chi = 8\pi \left(\frac{v_{BL}}{m_\nu}\right)^2 m_\chi^{-1}. \quad (42)$$

Equation (42) constrains the majoron relic density as follows (see Ref. [11]):

$$r_\chi m_\chi \left(\frac{\tau_\chi}{\tau_U}\right)^{1/2} < 25(\Omega_0 h^2)\mathrm{eV}, \quad (43)$$

where τ_U is the age of the Universe. Equation (43) leads to

$$r_\chi \left(\frac{m_\chi}{\mathrm{keV}}\right)\left(\frac{\tau_\chi}{\mathrm{sec}}\right)^{1/2} \leqslant 10^7 \Omega_0 h^{3/2}, \quad (44)$$

where $\delta\rho/\rho \leqslant 10^{-4}$ are the initial density fluctuations.

On the other hand, the relativistic decay products of χ must be redshifted enough to maintain a matter-dominated Universe, i.e. to avoid constraints on dark radiation:

$$t_\chi n_\gamma(t_{eq}) m_\chi \left(\frac{\tau_\chi}{t_{eq}}\right)^{1/2} < \rho_m(t_{eq}), \quad (45)$$

where

$$n_\gamma(t_{eq}) = (1+z_{eq})^3 \times 422\mathrm{cm}^{-3} \quad (46)$$

and

$$\rho_M(t_{eq}) = (1+z_{eq})^3 \times 10.5(\Omega_0 h^2)\ \mathrm{KeV cm}^{-3}. \quad (47)$$

This leads to the following bound

$$m_\chi \left(\frac{v_{BL}}{v}\right)^2 \leqslant 10^6 \left(\frac{m_\nu}{25\mathrm{eV}}\right)^2 \mathrm{keV}, \quad (48)$$

leading to

$$v_{BL} < \left(\frac{m_\nu}{25\mathrm{eV}}\right)^{4/7} \times 10\ \mathrm{TeV}. \quad (49)$$

Such a bound can be generalized for higher order operators in the complex scalar sector, leading to

$$\sigma^{4+n}/\Lambda^n \rightarrow v_{BL} < 10^{10/(n+6)} \left(\frac{\Lambda}{\mathrm{GeV}}\right)^{n/(n+6)} \mathrm{GeV}. \quad (50)$$

Nonetheless, bounds on v_{BL} provided by higher n-order contributions are less stringent.

3.3.3 Dark matter

The majoron particle can provide a new candidate for dark matter [5, 11]. For the sub-electroweak phase transition considered, the majoron mass is parametrized by Eq. (31). For a $v_{BL} \simeq 10\mathrm{GeV}$ phase transition, the majoron is naturally keV-ish, while the overproduction problem is avoided. This means that the majoron can compose warm dark matter. The majoron dark matter paradigm can be tested from collider missing energy channels and from gravitational wave experiments. This certainly enforces the naturalness and phenomenological health of our proposal.

4 Conclusions and remarks

We have shown how gravitational wave experiments may provide useful information on the majoron self-interaction potential. In particular, the possibility of a first order phase transition at a scale of about 1–100 GeV is still unbounded by any cosmological limits, such as non-perturbative electroweak effects — sphalerons — and cosmological density abundance. The main message of this paper is that such a scale overlaps the sensitivity of future gravitational wave interferometers, like eLISA, U-DECIGO, and BBO. In fact, a scale of about 1–100 GeV falls into the range of frequencies around 10^{-5}–10^{-3}Hz. However, an observation of a stochastic gravitational wave signal in eLISA would imply a new physics scale of UV completion for the violent majoron model of about 3TeV or so. This means that the production of majorons in colliders may provide a complementary *test-bed* for this model. For instance, the detection of majorons in missing energy channels can be tested in the future collider CEPC from Higgs invisible decays.

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