

Band head spin determination of triaxial superdeformed bands in $^{163,164,165}\text{Lu}$ through two-parameter formulae

Honey Sharma H. M. Mittal¹⁾

Dr. B.R. Ambedkar National Institute of Technology, Jalandhar, 144011, India

Abstract: The two-parameter formulae, i.e. the nuclear softness formula and the power index formula, have been used to obtain the band head spin (I_0) of the triaxial superdeformed (SD) bands in $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$, in the $A \sim 160$ mass region. The least squares fitting approach is used. The values of the root mean square (RMS) deviation among the computed and the measured experimental transition energies are obtained by calculating the model parameters. Whenever accurate spins are available, superb agreement is shown between the determined and the measured experimental transition energies. In comparison to the power index formula, the values of band head spin (I_0) of the triaxial SD bands in $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ obtained by the nuclear softness formula are closer to the experimental data. The lowest RMS deviation is also achieved by the nuclear softness formula. Hence, the nuclear softness formula works well for obtaining the band head spin (I_0) for the triaxial SD bands in $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ in the $A \sim 160$ mass region. The dynamic moment of inertia against $\hbar\omega$ is also studied.

Keywords: band head spin, nuclear softness formula, power index formula, triaxial superdeformed bands

PACS: 21.10.Hw, 21.60.-n, 27.70.+q **DOI:** 10.1088/1674-1137/42/11/114102

1 Introduction

Deviations from the axial symmetry of atomic nuclei has been a prominent topic of nuclear structure studies. Most of the known nuclei have been found to be deformed. Many of them can be explained by axial symmetry, while some are proposed to have triaxial shapes. Triaxial motion is a crucial problem in nuclear structure physics and plays an important role in various nuclear phenomena. For example, triaxiality has been adopted to describe the signature splitting in rotational bands in the $A \sim 130$ mass region [1], probable chiral band doublets in some odd-odd nuclei [2], and fast decay from isomers [3].

Total Routhian calculation [4, 5] has indicated that stable triaxial deformation should arise in the superdeformed (SD) well with precise energy minima occurring up to large rotational frequency in nuclei with $Z = 72$ and $N = 94$. Therefore, the structures located in these minima are generally called the triaxial SD bands. By using the Ultimate Cranker (UC) model [6, 7], Petersen et al. [4] determined that single particle shell gaps occur at high deformation for $Z = 72$ and $N = 94$. Therefore, the neutron shell gaps occurring at high deformation were found to be linked with triaxial deformation of $\gamma \sim 20^\circ$. It was further suggested in Ref. [4] that the

existence of shell gaps ensures the occurrence of triaxial SD minima in the total potential energy surfaces of nuclei near $N=94$. Thus, the majority of the experimental investigations for triaxial SD bands have focused on the $N=92, 94$ regions. Experimentally, SD structures have been confirmed in $^{163-165}\text{Lu}$ [8–10] and ^{168}Hf [11] by the calculation of transitional quadrupole moments. Superb candidates for SD bands also occur in $^{161,162,167}\text{Lu}$ [12, 13] and ^{170}Hf [14].

All these candidates are based on the excitation of the $i_{13/2}$ proton. The affirmation of triaxiality for these bands has been found on the basis of UC calculations. However, Bohr and Mottelson [15] predicted excited triaxial SD bands and linking transitions among the cascades in ^{163}Lu [16–18], ^{165}Lu [19] and ^{167}Lu [13]. Therefore, it has been reported [15] to be constant with the behavior of wobbling excitations [20, 21] appearing from the rotation of triaxial nuclei. The investigations of the first and second phonon wobbling bands in $^{161,163,165,167}\text{Lu}$ and ^{167}Ta have shown the wobbling mode to be a universal phenomenon in the $A \sim 160$ mass region and yield the strongest signature for the stable triaxial shape. Therefore, these odd- A Lu nuclei are excellent examples of stable triaxiality. Large transition quadrupole moments of these wobbling bands, analogous to strong deformation of about $\varepsilon_2 \sim 0.4$, were obtained from lifetime

Received 24 May 2018, Revised 2 August 2018, Published online 13 October 2018

1) E-mail: mittalhm@nitj.ac.in

©2018 Chinese Physical Society and the Institute of High Energy Physics of the Chinese Academy of Sciences and the Institute of Modern Physics of the Chinese Academy of Sciences and IOP Publishing Ltd

measurements. Hence, these wobbling bands are proofs of stable SD triaxial shapes in the $A \sim 160$ mass region.

To obtain reliable and exact spins of SD bands, various theoretical models have been employed, like Harris expansion, Bohr-Mottelson expansion, the three-parameter model, the Lipas-Ejiri relation, the VMI-inspired IBM model, the $SU_q(2)$ model, etc. [22–27]. The VMI model was used by Jain et al. [28] for the calculation of band head spin of triaxial SD bands in ^{164}Lu .

The applicability of the VMI model in triaxial SD bands motivates us to calculate the band head spin of triaxial SD bands in $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ in the $A \sim 160$ mass region by using the two-parameter formulae, i.e. the nuclear softness formula and the power index formula. This study may show whether or not the nuclear softness formula and the power index formula are valid in the triaxial SD bands of the $A \sim 160$ mass region. The nuclear softness formula has already proved its validity in the $A \sim 60\text{--}80$ [29] and 190 [30] mass regions, and the power index formula has been shown to be valid for SD bands in $^{132}\text{Ce}(1)$ [31], ^{133}Pr [32] and Hg isotopes [33]. However, one can also check which of the two formulae works better to study the SD spectroscopy of triaxial bands in $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ in the $A \sim 160$ mass region.

2 Formalism

2.1 Nuclear softness formula

The nuclear softness formula was proposed by Gupta [34]. In the nuclear softness formula, the transition energy level of ground state bands in even-even nuclei is studied by applying the modification of moment of inertia with spin (I).

The energy formula is given by

$$E = \frac{\hbar^2}{2\mathfrak{I}} I(I+1). \quad (1)$$

The experimental values are always higher than the energies determined by the rigid rotor formula. The moment of inertia with angular momentum was modified to account for this effect. So, Eq. (1) is changed to

$$E_I = \frac{\hbar^2}{2\mathfrak{I}_I} I(I+1). \quad (2)$$

By executing a Taylor series expansion of \mathfrak{I}_I on $I=0$ about its ground state value \mathfrak{I}_0 , we get

$$E_I = \frac{\hbar^2}{2} \left[\left(\frac{1}{\mathfrak{I}_0} - \left(\frac{1}{\mathfrak{I}_I^2} \frac{\partial \mathfrak{I}_I}{\partial I} \right) \right)_{I=0} I + \left[\frac{2}{\mathfrak{I}_I^3} \left(\frac{\partial \mathfrak{I}_I}{\partial I} \right)^2 - \frac{1}{\mathfrak{I}_I^2} \frac{\partial^2 \mathfrak{I}_I}{\partial I^2} \right]_{I=0} \frac{I^2}{2!} + \dots \right] I(I+1). \quad (3)$$

The increase in spin approaches the moment of inertia to a rigid rotor value. The nuclear softness parameter (σ)

interprets the rigidity of a nucleus. Therefore, Eq. (3) may be written as

$$E_I = \frac{\hbar^2 I(I+1)}{2\mathfrak{I}_0} \frac{1}{(1+\sigma_1 I)} \times \left(1 - \frac{\sigma_2 I^2}{(1+\sigma_1 I + \sigma_2 I^2)} - \frac{\sigma_3 I^3}{(1+\sigma_1 I + \sigma_3 I^3)} + \dots \right), \quad (4)$$

where

$$\sigma_1 = \frac{1}{\mathfrak{I}_0} \frac{\Delta \mathfrak{I}_0}{\Delta I}, \quad \sigma_2 = \frac{1}{2! \mathfrak{I}_0} \frac{\partial^2 \mathfrak{I}_0}{\partial I^2}, \quad \sigma_3 = \frac{1}{3! \mathfrak{I}_0} \frac{\partial^3 \mathfrak{I}_0}{\partial I^3} \dots \quad (5)$$

are the nuclear softness parameters of first σ_1 , second σ_2 and third σ_3 order respectively. Limiting only up to the $\sigma_1 = \sigma$ terms i.e. $\sigma_{2,3,\dots} = 0$, we get

$$E = \frac{\hbar^2}{2\mathfrak{I}_0} \times \frac{I(I+1)}{(1+\sigma I)}, \quad (6)$$

where \mathfrak{I}_0 and σ are the model parameters calculated by the technique of least squares fit. The only known descriptive data regarding SD bands are their intraband transition energies and intensities,

$$E_\gamma(I) = E(I) - E(I-2). \quad (7)$$

Using Eqs. (6) and (7) we get

$$E_\gamma(I) = \frac{\hbar^2}{2\mathfrak{I}_0} \times \left[\frac{I(I+1)}{(1+\sigma I)} - \frac{(I-2)(I-1)}{1+\sigma(I-2)} \right]. \quad (8)$$

2.2 Power index formula

To determine the ground band transition level energies of a soft rotor, a simple term was recommended by Gupta et al. [35]. This was termed the power index formula. In the power index formula, the geometric mean of two values was taken into the account instead of the arithmetic mean,

$$E(I) = aI^b, \quad (9)$$

where a and b represent the model parameters obtained by least squares fitting. E_γ energies and intensities are the only vital data available to describe the nature of SD bands. So, we have

$$E_\gamma(I) = E(I) - E(I-2). \quad (10)$$

Using Eqs. (9) and (10),

$$E_\gamma(I) = a \left(I^b - (I-2)^b \right). \quad (11)$$

2.3 Dynamic moment of inertia $J^{(2)}$

Whenever particular spins are assigned to the SD bands, the dynamic moment of inertia $J^{(2)}$ can be estimated by employing the determined transition energies [36],

$$J^{(2)} = 4000 / [E_\gamma(I+2) - E_\gamma(I)]. \quad (12)$$

3 Results and discussion

The two-parameter formulae, viz. nuclear softness formula and power index formula, are applied to achieve the band head spin of $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$, $^{165}\text{Lu}(1,2,3)$ triaxial SD bands in the $A \sim 160$ mass region. The observed experimental E_γ transition energies displayed by the SD bands are $E_\gamma(I_0+2n)$, $E_\gamma(I_0+2n-2)$, The cascade of observed experimental transition energies is fitted through the least squares fitting approach. All the available data for $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ have been taken from the SD tables given by Singh et al. [37] and continuously updated National Nuclear Data Center site [38]. For ^{164}Lu , we have considered only first three triaxial SD bands, i.e. $^{164}\text{Lu}(1,2,3)$, in our study and neglected the other five triaxial SD bands i.e. $^{164}\text{Lu}(4,5,6,7,8)$, because the estimated spins are not available for these bands, which would be required for the least squares fitting technique. The least squares fitting approach is used to fit the data in Eq. (8) and Eq. (11).

A comparison is made between the computed and the measured experimental transition energies. The values of RMS deviation of the computed and the measured transition energies show the dependence upon the specific band head spin. Whenever relevant spins are assigned, a superb agreement is observed among the computed and the observed experimental transition energies. However, a minor divergence of ± 1 in the values of band head spin (I_0) displays a considerable amount of change in the RMS deviation values. The RMS deviation may be given as

$$\chi = \left[\frac{1}{n} \sum_{n=1}^n \left(\frac{E_\gamma^{\text{cal}}(I_i) - E_\gamma^{\text{exp}}(I_i)}{E_\gamma^{\text{exp}}(I_i)} \right)^2 \right]^{1/2}, \quad (13)$$

where n is the total energy transition levels engaged in the fitting scheme.

The band head spin of $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$,

and $^{165}\text{Lu}(1,2,3)$ triaxial SD bands in the $A \sim 160$ mass region obtained by the nuclear softness formula, the power index formula, and a comparison with the experimental data, are given in Table 1. The nuclear softness formula coincides really well with the experimental data. At specific band head spins, the minimal RMS deviation values are also attained by the nuclear softness formula.

As the nuclear softness formula works on the idea of the modification of moment of inertia with spin in a very simplified and generalized manner, the obtained fit of energy level is better than the power index formula. The model parameters (σ , J_0) and (a , b) assessed from the technique of least square fitting employing the nuclear softness formula and the power index formula are given in Table 2. In the nuclear softness formula, the model parameter sigma (σ) provides information about the rigidity of the triaxial SD nuclei. This means that the larger the deformation, the smaller the sigma (σ) and the higher the rigidity.

The model parameter J_0 in the nuclear softness formula is used to characterize the triaxial SD rotational bands and also depends upon the intrinsic structure of the triaxial SD rotational bands. J_0 also proves to be useful for the spin proposition. In the power index formula, the index (b) suggests the degree of deformation. Any change in the index (b) provides information on structural change at a given spin.

The inverse of scaling coefficient (a) corresponds to the moment of inertia and also characterizes the nuclei. An explanatory example of least squares fitting of $^{163}\text{Lu}(1)$ triaxial SD bands using the nuclear softness formula is shown in Table 3. Subsequently, the computed and the measured transition energies are also in accordance with each other (see Table 3). From Table 3, the transition energies are replicated extremely well at $I_0=4.5$ for $^{163}\text{Lu}(1)$. However, if $I_0=3.5$ or 5.5 , a large difference is seen in the values of RMS deviation. Thus, both values are ignored. Further support in this regard

Table 1. The band head spin (I_0) obtained for $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$, and $^{165}\text{Lu}(1,2,3)$ triaxial SD bands by using the power index formula and nuclear softness formula. Here 1, 2, 3 and 4 in parentheses represent bands 1, 2, 3 and 4.

SD band	$E_\gamma(I_0+2 \rightarrow I_0)$	power index formula	nuclear softness formula	Ref. [38]
$^{163}\text{Lu}(1)$	196.7	11.5	4.5	6.5
$^{163}\text{Lu}(2)$	407	23.5	13.5	13.5
$^{163}\text{Lu}(3)$	505.5	21.5	15.5	16.5
$^{163}\text{Lu}(4)$	702.2	17.5	23.5	23.5
$^{164}\text{Lu}(1)$	374	11	13	14
$^{164}\text{Lu}(2)$	354	8	14	13
$^{164}\text{Lu}(3)$	536	21	18	18
$^{165}\text{Lu}(1)$	445	8.5	11.5	12.5
$^{165}\text{Lu}(2)$	539	25.5	17.5	17.5
$^{165}\text{Lu}(3)$	661	16.5	20.5	20.5

Table 2. Parameters obtained from least squares fitting of $^{163,164,165}\text{Lu}$ isotopes in the $A \sim 160$ mass region using the nuclear softness formula and power index formula. χ is the error in the calculation.

$E_\gamma^{\text{exp}}(I)$	TSD bands	nuclear softness formula		power index formula		χ
		σ	J_0	a	b	
196.7	$^{163}\text{Lu}(1)$	2.84×10^{-5}	58.2	3.32	2.14	0.015
407	$^{163}\text{Lu}(2)$	3.98×10^{-6}	69.9	8.09	1.96	0.00826
505.5	$^{163}\text{Lu}(3)$	1.61×10^{-5}	63.8	11.62	1.88	0.00106
702.2	$^{163}\text{Lu}(4)$	3.04×10^{-6}	70.6	19.99	1.78	0.000135
374	$^{164}\text{Lu}(1)$	1.29×10^{-5}	76.7	8.87	1.95	0.00874
354	$^{164}\text{Lu}(2)$	-1.45×10^{-5}	82.2	15.53	1.83	0.0255
536	$^{164}\text{Lu}(3)$	1.15×10^{-5}	68.5	5.73	2.02	0.00702
445	$^{165}\text{Lu}(1)$	5.18×10^{-5}	51.9	26.9	1.70	0.00126
539	$^{165}\text{Lu}(2)$	-3.23×10^{-5}	110.5	8.50	1.95	0.00277
661	$^{165}\text{Lu}(3)$	2.60×10^{-5}	62.6	21.27	1.75	0.00151

Table 3. Spin determination of the $^{163}\text{Lu}(1)$ triaxial SD band using the nuclear softness formula. I_0 corresponds to band head spin. $\delta = E_\gamma^{\text{exp}}(I) - E_\gamma^{\text{cal}}(I)$, where E_γ is in keV.

$E_\gamma^{\text{exp}}(I)$	$I_0=3.5$			$I_0=4.5$			$I_0=5.5$		
	I	$E_\gamma^{\text{cal}}(I)$	δ	I	$E_\gamma^{\text{cal}}(I)$	δ	I	$E_\gamma^{\text{cal}}(I)$	δ
196.7	5.5	181.6	15.1	6.5	191.7	5.0	7.5	217.6	-20.9
263.3	7.5	251.3	12	8.5	258.8	4.5	9.5	278.4	-15.1
314.8	9.5	319.3	-4.5	10.5	324.6	-9.8	11.5	338.6	-23.8
386.3	11.5	385.6	0.7	12.5	389.1	-2.8	13.5	398.1	-11.8
450.3	13.5	450.4	-0.1	14.5	452.2	-1.9	15.5	457.0	-6.7
515.3	15.5	513.7	1.6	16.5	514.2	1.1	17.5	515.4	-0.1
578.6	17.5	575.6	3.0	18.5	574.9	3.7	19.5	573.1	5.5
638.9	19.5	636.0	2.9	20.5	634.4	4.5	21.5	630.3	8.6
696.9	21.5	695	1.9	22.5	692.8	4.1	23.5	686.9	10
752.6	23.5	752.7	-0.1	24.5	750.1	2.5	25.5	743	9.6
805.5	25.5	809.1	-3.6	26.5	806.2	-0.7	27.5	798.5	7.0
857.7	27.5	864.2	-6.5	28.5	861.4	-3.7	29.5	853.4	4.3
909.7	29.5	918.2	-8.5	30.5	915.4	-5.7	31.5	907.8	1.9
962.5	31.5	970.9	-8.4	32.5	968.5	-6.0	33.5	961.7	0.8
1016.5	33.5	1022.5	-6.0	34.5	1020.6	-4.1	35.5	1015.0	1.5
1071.5	35.5	1073.0	-1.5	36.5	1071.7	-0.2	37.5	1067.8	3.7
1126.2	37.5	1122.4	3.8	38.5	1121.8	4.4	39.5	1120.1	6.1
1179.3	39.5	1170.7	8.6	40.5	1171.1	8.2	41.5	1171.9	7.4
1227	41.5	1218.0	9.0	42.5	1219.5	7.5	43.5	1223.2	3.8
1269	43.5	1264.3	4.7	44.5	1266.9	2.1	45.5	1274.0	-5.0
1303.5	45.5	1309.7	-6.2	46.5	1313.6	-10.1	47.5	1324.3	-20.8
χ	0.0227412			0.0152136			0.0477193		

is provided by the χ plot showing the band head spin of $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$, and $^{165}\text{Lu}(1,2,3)$ triaxial SD bands in the $A \sim 160$ mass region using the nuclear softness formula (see Figs. 1 and 2). Hence, the nuclear softness formula works well for triaxial SD bands in $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$, and $^{165}\text{Lu}(1,2,3)$ in the $A \sim 160$ mass region for achieving the band head spin (I_0). From the above discussion one can also say that the nuclear softness formula is an effective means of studying SD spectroscopy of triaxial SD bands in $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$.

At specific band head spins the calculated transition

energies obtained from the nuclear softness formula and the power index formula are used to determine $J^{(2)}$ by employing Eq. (12). A comparison of the computed and the observed experimental outcomes of $J^{(2)}$ is displayed in Fig. 3. It is observed from Fig. 3 that the nuclear softness formula and power index formula agree well with the experimental data for $J^{(2)}$ from 0.1 to 0.65 $\hbar\omega$ for $^{163}\text{Lu}(1)$, 0.26 to 0.5 $\hbar\omega$ for $^{164}\text{Lu}(3)$, 0.24 to 0.46 $\hbar\omega$ for $^{165}\text{Lu}(1)$, and 0.35 to 0.55 $\hbar\omega$ for $^{165}\text{Lu}(3)$. The nuclear softness formula agrees well with the experimental data for $J^{(2)}$ from 0.2 to 0.6 $\hbar\omega$ for $^{163}\text{Lu}(2)$, 0.25 to 0.5 $\hbar\omega$ for $^{163}\text{Lu}(3)$, 0.35 to 0.5 $\hbar\omega$ for $^{163}\text{Lu}(4)$, and 0.27 to

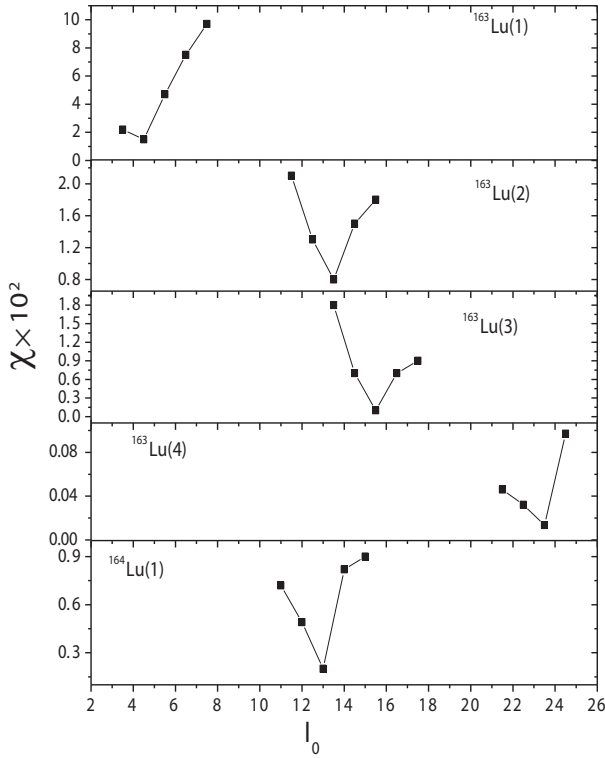


Fig. 1. χ plot to obtain I_0 for triaxial SD bands in $^{163}\text{Lu}(1, 2, 3, 4)$ and $^{164}\text{Lu}(1)$ isotopes in the $A \sim 160$ mass region.

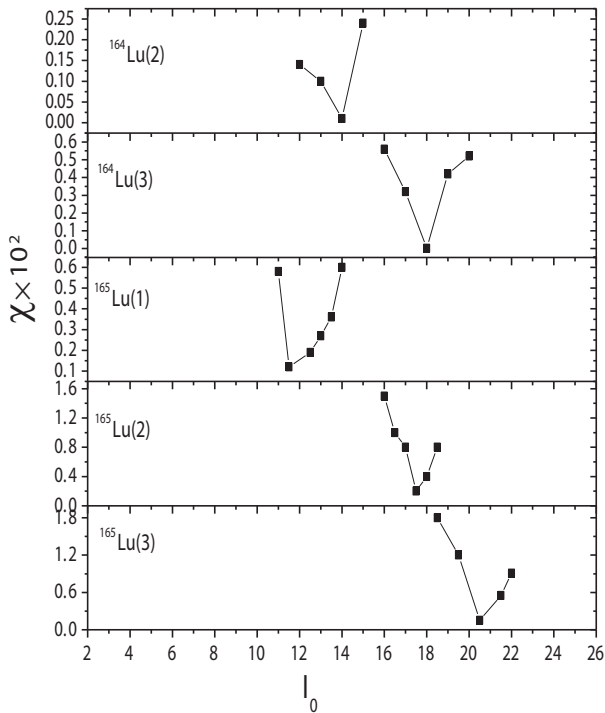


Fig. 2. χ plot to obtain I_0 for triaxial SD bands in $^{164}\text{Lu}(2, 3)$ and $^{165}\text{Lu}(1, 2, 3)$ isotopes in the $A \sim 160$ mass region.

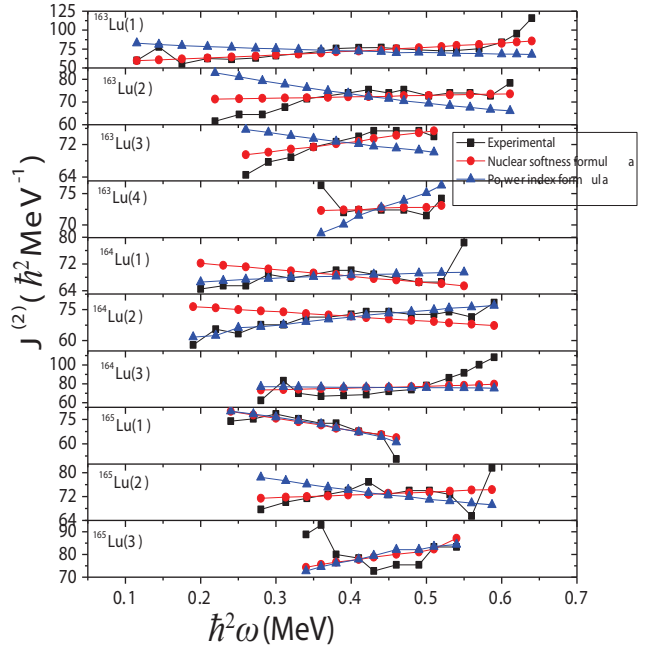


Fig. 3. (color online) Variation of calculated results of $J^{(2)}$ with $\hbar\omega$ for triaxial SD bands in $^{163}\text{Lu}(1, 2, 3, 4)$, $^{164}\text{Lu}(1, 2, 3)$ and $^{165}\text{Lu}(1, 2, 3)$ isotopes in the $A \sim 160$ mass region, and comparison with experimental data.

0.54 $\hbar\omega$ for $^{165}\text{Lu}(2)$. The power index formula agrees well with the experimental data for $J^{(2)}$ from 0.2 to 0.55 and 0.2 to 0.6 $\hbar\omega$ for $^{164}\text{Lu}(1,2)$.

4 Conclusion

We have evaluated the band head spins of $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ triaxial SD bands in the $A \sim 160$ mass region by using the nuclear softness formula and the power index formula. The values of band head spin (I_0) of $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ triaxial SD bands obtained by the nuclear softness formula are much nearer to the experimental data than those from the power index formula. The nuclear softness formula also gives a better fit for energy than the power index formula. The least RMS deviation is also achieved by the nuclear softness formula.

The computed and the observed transition energies of $^{163}\text{Lu}(1)$ triaxial SD bands are also in accordance with each other. Hence, the nuclear softness formula works more efficiently than the power index formula for achieving the band head spin (I_0) for $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ triaxial SD bands in the $A \sim 160$ mass region. This means the nuclear softness formula is an effective means of studying SD spectroscopy of triaxial SD bands in these nuclei.

$J^{(2)}$ versus $\hbar\omega$ was also studied for $^{163}\text{Lu}(1,2,3,4)$, $^{164}\text{Lu}(1,2,3)$ and $^{165}\text{Lu}(1,2,3)$ in the $A \sim 160$ mass re-

gion. It is observed that the nuclear softness formula and power index agree well with the experimental data of $J^{(2)}$ from 0.1 to 0.65 $\hbar\omega$ for $^{163}\text{Lu}(1)$, 0.26 to 0.5 $\hbar\omega$ for $^{164}\text{Lu}(3)$, 0.24 to 0.46 $\hbar\omega$ for $^{165}\text{Lu}(1)$, and 0.35 to 0.55 $\hbar\omega$ for $^{165}\text{Lu}(3)$. The nuclear softness formula agrees well

with the experimental data for $J^{(2)}$ from 0.2 to 0.6 $\hbar\omega$ for $^{163}\text{Lu}(2)$, 0.25 to 0.5 $\hbar\omega$ for $^{163}\text{Lu}(3)$, 0.35 to 0.5 $\hbar\omega$ for $^{163}\text{Lu}(4)$, and 0.27 to 0.54 $\hbar\omega$ for $^{165}\text{Lu}(2)$. The power index formula agrees well with the experimental data for $J^{(2)}$ from 0.2 to 0.55 and 0.2 to 0.6 $\hbar\omega$ for $^{164}\text{Lu}(1,2)$.

References

- 1 Y. S. Chen, S. Frauendorf, and G. A. Leander, Phys. Rev. C, **28**: 2437 (1983)
- 2 S. Frauendorf and J. Meng, Nucl. Phys. A, **617**: 131-147 (1997)
- 3 B. Crowell, P. Chowdhury, S. J. Freeman et al, Phys. Rev. Lett., **72**: 1164-1167 (1994)
- 4 H. S. Petersen, R. Bengtsson, R. A. Bark et al, Nucl. Phys. A, **594**: 175-202 (1995)
- 5 T. Bengtsson, www.matfys.lth.se/~ragnar/TSD.html
- 6 T. Bengtsson, Nucl. Phys. A, **496**: 56 (1989)
- 7 T. Bengtsson, Nucl. Phys. A, **512**: 124 (1990)
- 8 W. Schmitz, H. Hubel, C. X. Yang et al, Phys. Lett. B, **303**: 230 (1993)
- 9 G. Schonwaber, H. Hubel, G. B. Hagemann et al, Eur. Phys. J. A, **13**: 291 (2002)
- 10 G. Schonwaber, H. Hubel, G. B. Hagemann et al, Eur. Phys. J. A, **15**: 435-437 (2002)
- 11 H. Amro, P. G. Varrette, W. C. Ma et al, Phys. Lett. B, **506**: 39-44 (2001)
- 12 C.X. Yang, X. G. Wu, H. Zheng et al, Eur. Phys. J. A, **1**: 237 (1998)
- 13 H. Amro, W. C. Ma, G. B. Hagemann et al, Phys. Lett. B, **553**: 197 (2003)
- 14 A. Neuber, H. Hubel, G. B. Hagemann et al, Eur. Phys. J. A, **15**: 439(2002)
- 15 A. Bohr, B.R. Mottelson, Nuclear Structure, Vol. 2: (New York: Benjamin, 1975)
- 16 S. W. Odegard, G. B. Hagemann, D. R. Jensen et al, Phys. Rev. Lett., **86**: 5866 (2001)
- 17 D. R. Jensen, G. B. Hagemann, I. Hamamoto et al, Nucl. Phys. A, **703**: 3 (2002)
- 18 D.R. Jensen, G. B. Hagemann, I. Hamamoto et al, Phys. Rev. Lett., **89**: 142503 (2002)
- 19 G. Schonwaber, H. Hubel, G. B. Hagemann et al, Phys. Lett. B, **552**: 9 (2003)
- 20 I. Hamamoto, Phys. Rev. C, **65**: 044305 (2002)
- 21 M. Matsuzaki, Y. Shimizu, and K. Matsuyanagi, Phys. Rev. C, **65**: 041303(R) (2002)
- 22 J. A. Becker, E. A. Henry, A. Kuhnert et al, Phys. Rev. C, **46**: 889 (1992)
- 23 J. Meng, C. S. Wu and J. Y. Zeng, Phys. Rev. C, **44**: 2545 (1991)
- 24 C. S. Wu, J. Y. Zeng, Z. Xing et al, Phys. Rev. C, **45**: 261 (1992)
- 25 S. X. Liu and J. Y. Zeng, Phys. Rev. C, **58**: 3266 (1998)
- 26 D. Bonatsos, S. B. Drenska, P. P. Raychev et al, J. Phys. G: Nucl. Part. Phys., **17**: L67 (1991)
- 27 Y. Liu, J. G. Song, H. Z. Sun et al, J. Phys. G: Nucl. Part. Phys., **24**: 117 (1998)
- 28 P. Jain, V. S. Uma and A. Geol, Proceeding of DAE- BRNS Symp. on Nucl. Phys., **61**: 298-299 (2016)
- 29 H. Sharma and H. M. Mittal, Int. J. Mod. Phys E, **26**: 1750074 (2017)
- 30 A. Dadwal, H. M. Mittal, Eur. Phys. J. A, **53**: 2 (2017)
- 31 H. Sharma and H. M. Mittal, Chinese Physics C, **41**: 124105 (2017)
- 32 H. Sharma and H. M. Mittal, Mod. Phys. Lett. A, **33**: 1850048 (2018)
- 33 H. Sharma and H. M. Mittal, Chinese Physics C, **42**: 054104 (2018)
- 34 R. K. Gupta, Phys. Lett. B, **36**: 173 (1971)
- 35 J. B. Gupta, A. K. Kavathekar, and R. Sharma, Phys. Scr., **51**: 316 (1995)
- 36 X. L Han and C. L. Wu, At. Data and Nucl. Data Tables, **73**: 43 (1999)
- 37 B. Singh, R. Zywina, and R. B. Firestone, Nuclear Data Sheets, **97**: 241-592 (2002)
- 38 <http://www.nndc.bnl.gov/>