The breaking of flavor democracy in the quark sector *

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Abstract: The democracy of quark flavors is a well-motivated flavor symmetry, but it must be properly broken in order to explain the observed quark mass spectrum and flavor mixing pattern. We reconstruct the texture of flavor democracy breaking and evaluate its strength in a novel way, by assuming a parallelism between the $Q=+2/3$ and $Q=-1/3$ quark sectors and using a nontrivial parametrization of the flavor mixing matrix. Some phenomenological implications of such democratic quark mass matrices, including their variations in the hierarchy basis and their evolution from the electroweak scale to a super-high energy scale, are also discussed.

Keywords: flavor democracy, quark mass, flavor mixing, CP violation

PACS: 12.15.Ff, 12.39.-x, 11.10.Hi **DOI:** 10.1088/1674-1137/41/9/093104

1 Introduction

In the standard electroweak model the origin of quark masses is attributed to the Yukawa interactions and the Higgs mechanism. However, the model gives no quantitative prediction for the structures of the Yukawa coupling matrices $Y_{+2/3}$ and $Y_{-1/3}$ in the $Q=+2/3$ and $Q=-1/3$ quark sectors, respectively. That is why there is no explanation of the observed strong hierarchies of quark masses, namely $m_{\rm u}/m_{\rm c}\sim m_{\rm c}/m_{\rm t}\sim\lambda^4$ and $m_{\rm d}/m_{\rm s}\sim m_{\rm s}/m_{\rm b}\sim\lambda^2$ with $\lambda\simeq 0.2$ [1], within the standard model. In other words, why are the three eigenvalues of the Yukawa coupling matrix $Y_{+2/3}$ or $Y_{-1/3}$ (i.e., $f_\alpha=m_\alpha/v$ with $v\simeq 174$ GeV being the vacuum expectation value and α running over u, c and t for $Y_{+2/3}$ or d, s and b for $Y_{-1/3}$) so different in magnitude? This remains a highly puzzling question.

As first pointed out by Harari, Haut and Weyers in 1978 [2], it should be very natural to conjecture that quark fields of the same electric charge initially have identical Yukawa interactions with the Higgs field, namely,

$$Y_Q^{(0)}=\frac{C_Q^{(0)}}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}, \qquad (1)$$

where $C_Q^{(0)}$ is a dimensionless coefficient, and $Q=+2/3$ for the up-quark sector or $Q=-1/3$ for the down-quark sector. Such a form of $Y_Q^{(0)}$ means that the corresponding quark mass matrix $M_Q^{(0)}$ must have the same "flavor democracy",

$$M_Q^{(0)}=\frac{m_3}{3}\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}, \qquad (2)$$

where $m_3\equiv vC_Q^{(0)}$, equal to the top-quark mass $m_{\rm t}$ for $Q=+2/3$ or the bottom-quark mass $m_{\rm b}$ for $Q=-1/3$. The corresponding quark mass term can be written as

$$\frac{m_3}{3}\sum_\alpha\sum_\beta\overline{\alpha_{\rm L}}\ \beta_{\rm R}+{\rm h.c.}, \qquad (3)$$

and it is completely invariant under the permutation of all the three left-handed quark fields and all the three right-handed quark fields, where $\alpha,\beta=u,c,t$ for $Q=+2/3$ or $\alpha,\beta=d,s,b$ for $Q=-1/3$. That is to say, the flavor democracy of $Y_Q^{(0)}$ or $M_Q^{(0)}$ implies that the quark mass term in Eq. (3) has exact $S(3)_{\rm L}\times S(3)_{\rm R}$ symmetry. This symmetry must be broken, since two of the three eigenvalues of M_Q are vanishing. The breaking of this flavor

Received 4 May 2017

* Supported by National Natural Science Foundation of China (11375207) and National Basic Research Program of China (2013CB834300)

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democracy leads to the flavor mixing effects between the two quark sectors [3–5].

How to break the democracy of quark flavors and to what extent to break it are two highly nontrivial questions for model building in this regard [6]. In the present work we are going to address ourselves to these two questions by assuming a structural parallelism between the mass matrices of $Q=+2/3$ and $Q=-1/3$ quarks. Such a phenomenological assumption makes sense if the generation of quark masses in the two sectors is governed by the same dynamics, and combining it with a nontrivial parametrization of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix proposed by Fritzsch and Xing [7] allows one to figure out the texture and strength of flavor democracy breaking in each quark sector in terms of the observed values of quark masses and flavor mixing parameters. Some interesting implications of such flavor-democratized quark mass matrices, including their variations in the hierarchy basis and their evolution with the energy scales, are also discussed.

2 Flavor democracy breaking

Let us begin with diagonalizing the flavor-democratized quark mass matrix $M_Q^{(0)}$ as follows:

$$V_0^\dagger M_Q^{(0)} V_0 = m_3 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

where

$$V_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{5}$$

We therefore arrive at $m_1 = m_2 = 0$, which is qualitatively consistent with the experimental facts $m_{\rm u}, m_{\rm c} \ll m_{\rm t}$ or $m_{\rm d}, m_{\rm s} \ll m_{\rm b}$. However, there is no flavor mixing in this special case, because the resulting CKM matrix $V = V_0^\dagger V_0 = \mathbf{1}$ is an identity matrix.

The realistic CKM quark mixing matrix

$$V = V_{+2/3}^\dagger V_{-1/3} = (V_0 V_{+2/3})^\dagger (V_0 V_{-1/3}) \tag{6}$$

measures a mismatch between the diagonalization of the $Q=+2/3$ quark mass matrix $M_{+2/3}$ and that of the $Q=-1/3$ quark mass matrix $M_{-1/3}$, and thus it provides a natural description of the observed phenomena of quark flavor mixing. Notice that $M_{+2/3}$ and $M_{-1/3}$ can always be arranged to be Hermitian, thanks to a proper choice of the flavor basis in the standard model or its extensions which have no flavor-changing right-handed currents [8]. So let us simply focus on Hermitian quark mass matrices in the following and take into account the corresponding flavor democracy in such a basis, namely,

$$(V_0 V_Q)^\dagger M_Q (V_0 V_Q) = \widehat{M}_Q \equiv \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \tag{7}$$

where $m_1 = \pm m_{\rm u}$, $m_2 = \pm m_{\rm c}$ and $m_3 = m_{\rm t}$ for $Q=+2/3$, or $m_1 = \pm m_{\rm d}$, $m_2 = \pm m_{\rm s}$ and $m_3 = m_{\rm b}$ for $Q=-1/3$. Here the sign ambiguity of m_1 or m_2 is attributed to the fact that the eigenvalues of the Hermitian matrix M_Q can be either positive or negative under the above unitary transformation. To reconstruct the pattern of M_Q in terms of V_0, V_Q and $\widehat{M}_Q$, however, one must specify the form of V_Q with the help of the parameters of V.

We find that the most suitable parametrization of the CKM matrix V for our purpose is the one advocated by two of us in Ref. [7]:

$$V = \begin{pmatrix} \sin\theta_{\rm u}\sin\theta_{\rm d}\cos\theta + \cos\theta_{\rm u}\cos\theta_{\rm d}{\rm e}^{-{\rm i}\phi} & \sin\theta_{\rm u}\cos\theta_{\rm d}\cos\theta - \cos\theta_{\rm u}\sin\theta_{\rm d}{\rm e}^{-{\rm i}\phi} & \sin\theta_{\rm u}\sin\theta \\ \cos\theta_{\rm u}\sin\theta_{\rm d}\cos\theta - \sin\theta_{\rm u}\cos\theta_{\rm d}{\rm e}^{-{\rm i}\phi} & \cos\theta_{\rm u}\cos\theta_{\rm d}\cos\theta + \sin\theta_{\rm u}\sin\theta_{\rm d}{\rm e}^{-{\rm i}\phi} & \cos\theta_{\rm u}\sin\theta \\ -\sin\theta_{\rm d}\sin\theta & -\cos\theta_{\rm d}\sin\theta & \cos\theta \end{pmatrix} \tag{8}$$

with the subscripts "u" and "d" denoting "up" ($Q=+2/3$) and "down" ($Q=-1/3$), respectively. The reason is simply that this form of V can be decomposed into $V_{+2/3}$ and $V_{-1/3}$ in an exactly *parallel* way as follows:

$$\begin{aligned} V_{+2/3} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\left(+\frac{2}{3}\theta\right) & -\sin\left(+\frac{2}{3}\theta\right) \\ 0 & \sin\left(+\frac{2}{3}\theta\right) & \cos\left(+\frac{2}{3}\theta\right) \end{pmatrix} \begin{pmatrix} \exp\left(+{\rm i}\frac{2}{3}\phi\right) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_{\rm u} & -\sin\theta_{\rm u} & 0 \\ \sin\theta_{\rm u} & \cos\theta_{\rm u} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ V_{-1/3} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\left(-\frac{1}{3}\theta\right) & -\sin\left(-\frac{1}{3}\theta\right) \\ 0 & \sin\left(-\frac{1}{3}\theta\right) & \cos\left(-\frac{1}{3}\theta\right) \end{pmatrix} \begin{pmatrix} \exp\left(-{\rm i}\frac{1}{3}\phi\right) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos\theta_{\rm d} & -\sin\theta_{\rm d} & 0 \\ \sin\theta_{\rm d} & \cos\theta_{\rm d} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{9}$$

Since all four parameters in this parametrization of V can be determined to a good degree of accuracy by using current experimental data, one may therefore fix the patterns of $V_{+2/3}$ and $V_{-1/3}$. Of course, the decomposition made in Eq. (9) depends also on a purely phenomenological assumption: the up- and down-type components of the flavor mixing angle θ are demanded to be proportional to the corresponding charges of these two quark sectors, and so are the components of the CP-violating phase ϕ. Such an assumption is another reflection of the up-down parallelism, which has been taken as the main guiding principle of our treatment, although it is very hard to argue any potential connection between the quark mass textures and the quark charges at this stage[1)]. One is certainly allowed to try some other possibilities of decomposing V into $V_{+2/3}$ and $V_{-1/3}$ [5], but the key point should be the same as ours — to minimize, within reason, the number of free parameters, at least at the phenomenological level.

Given Eqs. (7) and (9), we are now in a position to reconstruct the quark mass matrices $M_{+2/3}$ and $M_{-1/3}$ based on the flavor democracy. The texture of M_Q can be expressed as

$$M_Q = A_Q^2 M_Q^{(0)} + M_Q^{(1)} + M_Q^{(2)}, \qquad (10)$$

where $A_Q = -\sin(Q\theta)/\sqrt{2} + \cos(Q\theta)$, $M_Q^{(0)}$ has been defined in Eq. (2), and

$$\begin{aligned} M_Q^{(1)} &= C_Q^{(11)} \begin{pmatrix} 1 & 1 & -r_Q \\ 1 & 1 & -r_Q \\ -r_Q & -r_Q & r_Q^2 \end{pmatrix} + C_Q^{(12)} \begin{pmatrix} 0 & 0 & r_Q \\ 0 & 0 & r_Q \\ r_Q & r_Q & 2+r_Q \end{pmatrix}, \\ M_Q^{(2)} &= C_Q^{(21)} \left[\cos(Q\phi) \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} + \mathrm{i}\sin(Q\phi) \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \right] \\ &\quad + C_Q^{(22)} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} + C_Q^{(23)} \left[\cos(Q\phi) \begin{pmatrix} 2 & 0 & 1 \\ 0 & -2 & -1 \\ 1 & -1 & 0 \end{pmatrix} - \mathrm{i}\sin(Q\phi) \begin{pmatrix} 0 & -2 & -1 \\ 2 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \right], \end{aligned} \qquad (11)$$

in which $r_Q = 2A_Q/B_Q$ with $B_Q = \sqrt{2}\sin(Q\theta) + \cos(Q\theta)$, and

$$\begin{aligned} C_Q^{(11)} &= \frac{1}{6}\left(m_1 \sin^2\theta_{\rm q} + m_2 \cos^2\theta_{\rm q}\right) B_Q^2, \\ C_Q^{(12)} &= \frac{1}{2\sqrt{2}} m_3 \sin(Q\theta) B_Q, \\ C_Q^{(21)} &= \frac{1}{2\sqrt{3}} (m_1 - m_2) \cos(Q\theta) \sin 2\theta_{\rm q}, \\ C_Q^{(22)} &= \frac{1}{2}\left(m_1 \cos^2\theta_{\rm q} + m_2 \sin^2\theta_{\rm q}\right), \\ C_Q^{(23)} &= \frac{1}{2\sqrt{6}} (m_1 - m_2) \sin(Q\theta) \sin 2\theta_{\rm q} \end{aligned} \qquad (12)$$

with ${\rm q} = {\rm u}$ for $Q = +2/3$ and ${\rm q} = {\rm d}$ for $Q = -1/3$. The matrices $M_Q^{(0)}$, $M_Q^{(1)}$ and $M_Q^{(2)}$ perform the $S(3)_{\rm L} \times S(3)_{\rm R}$, $S(2)_{\rm L} \times S(2)_{\rm R}$ and $S(1)_{\rm L} \times S(1)_{\rm R}$ flavor symmetries, respectively. Among the five coefficients of $M_Q^{(1)}$ and $M_Q^{(2)}$ in Eq. (12), $C_Q^{(12)}$ is proportional to $m_3 \sin(Q\theta)$ and the others are all dominated by the terms proportional to m_2. Hence their ratios to the coefficient of $M_Q^{(0)}$ (i.e., $m_3/3$) are suppressed at the levels of $\sin(Q\theta)$ and m_2/m_3, respectively. Because $\theta \sim \lambda^2$ [7] and $|m_2/m_3| \sim \lambda^4$ (for $Q = +2/3$) or λ^2 (for $Q = -1/3$) [1], the relevant suppression is at least at the percent level. In other words, the strength of flavor democracy breaking must be at or below the percent level.

To see this point more clearly, let us take account of the strong quark mass hierarchy and the smallness of the three flavor mixing angles to make a reasonable analytical approximation for the expression of M_Q in Eq. (10). Then we arrive at

$$\begin{aligned} M_Q \simeq \frac{1}{3} m_3 \Bigg\{ & \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \left[\frac{1}{2}\frac{m_2}{m_3} \begin{pmatrix} 1 & 1 & -r \\ 1 & 1 & -r \\ -r & -r & r^2 \end{pmatrix} + \frac{3\sqrt{2}}{4} Q\theta \begin{pmatrix} 0 & 0 & r \\ 0 & 0 & r \\ r & r & 2+r \end{pmatrix} \right] \\ & -\sqrt{3}\,\theta_{\rm q} \frac{m_2}{m_3} \left[\cos(Q\phi) \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} + \mathrm{i}\sin(Q\phi) \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \right] \end{aligned}$$

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1) However, it has been argued that the origin of some differences between the up- and down-quark sectors might simply represent a difference between their charges in a dynamical model which can explain the observed family structure, rather than a fundamental difference between the two sectors [9].

$$-\frac{\sqrt{6}}{2}Q\theta\theta_{\rm q}\frac{m_2}{m_3}\left[\cos(Q\phi)\begin{pmatrix}2 & 0 & 1\\ 0 & -2 & -1\\ 1 & -1 & 0\end{pmatrix}-{\rm i}\sin(Q\phi)\begin{pmatrix}0 & -2 & -1\\ 2 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$
$$+\frac{3}{2}\left(\frac{m_1}{m_3}+\theta_{\rm q}^2\frac{m_2}{m_3}\right)\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}\Bigg\}, \tag{13}$$

in which the subscript of r_Q has been omitted. In fact, $r_Q\simeq 2\text{–}3\sqrt{2}\,Q\theta$ is not very sensitive to the value of Q due to the smallness of θ. The result in Eq. (13) shows a hierarchical chain of flavor democracy breaking in the quark sector. First, the $S(3)_{\rm L}\times S(3)_{\rm R}$ symmetry is broken down to the $S(2)_{\rm L}\times S(2)_{\rm R}$ symmetry, and the strength of this effect is characterized by the small quantities m_2/m_3 and θ. Second, the $S(2)_{\rm L}\times S(2)_{\rm R}$ symmetry is further broken down to $S(1)_{\rm L}\times S(1)_{\rm R}$, and the corresponding effect is further suppressed because it is characterized by the much smaller quantities $\theta_{\rm q}m_2/m_3$, $\theta\theta_{\rm q}m_2/m_3$, $\theta_{\rm q}^2m_2/m_3$ and m_1/m_3. In particular, the CP-violating phase ϕ comes in at the second symmetry-breaking stage and hence the effect of CP violation is strongly suppressed.

We proceed to evaluate the strength of flavor democracy breaking in a numerical way. To do so, we make use of the central values of six quark masses renormalized to the electroweak scale characterized by the Z-boson mass [1]:

$$\begin{aligned} &m_{\rm u}\simeq 1.38~{\rm MeV}, \quad m_{\rm c}\simeq 638~{\rm MeV}, \quad m_{\rm t}\simeq 172.1~{\rm GeV};\\ &m_{\rm d}\simeq 2.82~{\rm MeV}, \quad m_{\rm s}\simeq 57~{\rm MeV}, \quad m_{\rm b}\simeq 2.86~{\rm GeV}. \end{aligned} \tag{14}$$

The values of the flavor mixing parameters $\theta_{\rm u}$, $\theta_{\rm d}$, θ and ϕ can be obtained by establishing their relations with the well-known Wolfenstein parameters [10], whose values have been determined to an impressively good degree of accuracy [11, 12]:

$$\begin{aligned} &\theta_{\rm u}\simeq\arctan\left(\lambda\sqrt{\overline{\rho}^2+\overline{\eta}^2}\right)\simeq 0.086,\\ &\theta_{\rm d}\simeq\arctan\left(2\lambda\sqrt{\frac{(1-\overline{\rho})^2+\overline{\eta}^2}{\left[\lambda^2(1-2\overline{\rho})-2\right]^2+4\lambda^4\overline{\eta}^2}}\right)\simeq 0.206,\\ &\theta\simeq\arcsin\left(A\lambda^2\sqrt{1+\lambda^2(\overline{\rho}^2+\overline{\eta}^2)}\right)\simeq 0.042,\\ &\phi\simeq\arccos\left(\frac{\sin^2\theta_{\rm u}\cos^2\theta_{\rm d}\cos^2\theta+\cos^2\theta_{\rm u}\sin^2\theta_{\rm d}-\lambda^2}{2\sin\theta_{\rm u}\cos\theta_{\rm u}\sin\theta_{\rm d}\cos\theta_{\rm d}\cos\theta}\right)\simeq 1.636, \end{aligned} \tag{15}$$

where the best-fit values $A\simeq 0.825$, $\lambda\simeq 0.2251$, $\overline{\rho}\simeq 0.160$ and $\overline{\eta}\simeq 0.350$ [12] have been input. Namely, we have

$$\theta_{\rm u}\simeq 4.951^\circ, \quad \theta_{\rm d}\simeq 11.772^\circ, \quad \theta\simeq 2.405^\circ, \quad \phi\simeq 93.730^\circ, \tag{16}$$

implying $\theta_{\rm u}\sim 2\lambda^2$, $\theta_{\rm d}\sim\lambda$ and $\theta\sim\lambda^2$ in terms of the expansion parameter $\lambda\simeq 0.2$. The fact that ϕ is very close to $\pi/2$ proves to be quite suggestive in quark flavor phenomenology, as already discussed in Ref. [13].

With the help of the central values of six quark masses and four flavor mixing parameters given in Eqs. (14) and (16), one may start from Eq. (10) to numerically calculate the elements of $M_{+2/3}$ and $M_{-1/3}$ for two typical possibilities:

(a) $(m_1,m_2) = (-m_{\rm u},+m_{\rm c})$ for $Q = +2/3$ and $(-m_{\rm d},+m_{\rm s})$ for $Q=-1/3$, leading to

$$\begin{aligned} M_{+2/3} \simeq\ & 55.07~{\rm GeV}\times\left\{\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\left[3.2\times10^{-2}\begin{pmatrix}0&0&1.89\\0&0&1.89\\1.89&1.89&3.89\end{pmatrix}-2.07\times10^{-3}\begin{pmatrix}-1&-1&1.89\\-1&-1&1.89\\1.89&1.89&-3.56\end{pmatrix}\right]\right.\\ &-\left[2.65\times10^{-4}\begin{pmatrix}1&0&-1\\0&-1&1\\-1&1&0\end{pmatrix}-3.03\times10^{-5}\begin{pmatrix}1&-1&0\\-1&1&0\\0&0&0\end{pmatrix}+5.24\times10^{-6}\begin{pmatrix}2&0&1\\0&-2&-1\\1&-1&0\end{pmatrix}\right]\\ &\left.-{\rm i}\left[5.09\times10^{-4}\begin{pmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{pmatrix}-1.01\times10^{-5}\begin{pmatrix}0&-2&-1\\2&0&1\\1&-1&0\end{pmatrix}\right]\right\}, \end{aligned} \tag{17}$$

and

$$M_{-1/3} \simeq 0.97~{\rm GeV}\times\left\{\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}-\left[1.43\times10^{-2}\begin{pmatrix}0&0&2.06\\0&0&2.06\\2.06&2.06&4.06\end{pmatrix}+8.98\times10^{-3}\begin{pmatrix}-1&-1&2.06\\-1&-1&2.06\\2.06&2.06&-4.25\end{pmatrix}\right]\right.$$

$$
-\left[6.08\times10^{-3}\begin{pmatrix}1&0&-1\\0&-1&1\\-1&1&0\end{pmatrix}+1.63\times10^{-4}\begin{pmatrix}1&-1&0\\-1&1&0\\0&0&0\end{pmatrix}-6.02\times10^{-5}\begin{pmatrix}2&0&1\\0&-2&-1\\1&-1&0\end{pmatrix}\right]
$$

$$
\left.+\,\mathrm{i}\left[3.69\times10^{-3}\begin{pmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{pmatrix}+3.65\times10^{-5}\begin{pmatrix}0&-2&-1\\2&0&1\\1&-1&0\end{pmatrix}\right]\right\}; \tag{18}
$$

(b) $(m_1,m_2)=(+m_{\rm u},+m_{\rm c})$ for $Q=+2/3$ and $(+m_{\rm d},+m_{\rm s})$ for $Q=-1/3$, leading to

$$
M_{+2/3}\simeq 55.07\ \mathrm{GeV}\times\left\{\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\left[3.2\times10^{-2}\begin{pmatrix}0&0&1.89\\0&0&1.89\\1.89&1.89&3.89\end{pmatrix}-2.07\times10^{-3}\begin{pmatrix}-1&-1&1.89\\-1&-1&1.89\\1.89&1.89&-3.56\end{pmatrix}\right]\right.
$$

$$
-\left[2.64\times10^{-4}\begin{pmatrix}1&0&-1\\0&-1&1\\-1&1&0\end{pmatrix}-5.52\times10^{-5}\begin{pmatrix}1&-1&0\\-1&1&0\\0&0&0\end{pmatrix}+5.22\times10^{-6}\begin{pmatrix}2&0&1\\0&-2&-1\\1&-1&0\end{pmatrix}\right]
$$

$$
\left.-\,\mathrm{i}\left[5.06\times10^{-4}\begin{pmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{pmatrix}-1.00\times10^{-5}\begin{pmatrix}0&-2&-1\\2&0&1\\1&-1&0\end{pmatrix}\right]\right\}, \tag{19}
$$

and

$$
M_{-1/3}\simeq 0.97\ \mathrm{GeV}\times\left\{\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}-\left[1.43\times10^{-2}\begin{pmatrix}0&0&2.06\\0&0&2.06\\2.06&2.06&4.06\end{pmatrix}+9.01\times10^{-3}\begin{pmatrix}-1&-1&2.06\\-1&-1&2.06\\2.06&2.06&-4.25\end{pmatrix}\right]\right.
$$

$$
-\left[5.51\times10^{-3}\begin{pmatrix}1&0&-1\\0&-1&1\\-1&1&0\end{pmatrix}-2.62\times10^{-3}\begin{pmatrix}1&-1&0\\-1&1&0\\0&0&0\end{pmatrix}-5.45\times10^{-5}\begin{pmatrix}2&0&1\\0&-2&-1\\1&-1&0\end{pmatrix}\right]
$$

$$
\left.+\,\mathrm{i}\left[3.34\times10^{-3}\begin{pmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{pmatrix}+3.31\times10^{-5}\begin{pmatrix}0&-2&-1\\2&0&1\\1&-1&0\end{pmatrix}\right]\right\}. \tag{20}
$$

Some comments on the implications of these results are in order.

1) The other two possibilities, corresponding to $(m_1,m_2)=(+m_{\rm u},-m_{\rm c})$ and $(-m_{\rm u},-m_{\rm c})$ in the $Q=+2/3$ quark sector or $(m_1,m_2)=(+m_{\rm d},-m_{\rm s})$ and $(-m_{\rm d},-m_{\rm s})$ in the $Q=-1/3$ quark sector, are numerically found to be very similar to cases (a) and (b) shown above. Hence they will not be discussed separately.

2) The $S(2)_{\rm L}\times S(2)_{\rm R}$ terms of M_Q are not sensitive to the sign ambiguities of m_1 and m_2, but the latter can affect the $S(1)_{\rm L}\times S(1)_{\rm R}$ terms of M_Q to some extent. In other words, a specific model-building exercise should take into account the fine structure of M_Q which is associated with both the lightest quark mass and the CP-violating phase in each quark sector.

3) It is always possible to combine the two $S(2)_{\rm L}\times S(2)_{\rm R}$ terms of M_Q, and such a combination does not violate the $S(2)_{\rm L}\times S(2)_{\rm R}$ symmetry. Since the coefficients of five $S(1)_{\rm L}\times S(1)_{\rm R}$ terms are very different in magnitude, it is reasonable to neglect the most strongly suppressed ones when building a phenomenologically viable quark mass model. In particular, Eqs. (17)—(20) suggest that $C_Q^{(22)}\simeq0$ and $C_Q^{(23)}\simeq0$ should be two good approximations, which can also be observed from their analytical expressions in Eq. (12) or (13) by considering $|m_1|\ll|m_2|\ll m_3$ and the smallness of θ and $\theta_{\rm q}$. In this situation the analytical approximation of M_Q in Eq. (13) is further simplified to

$$
M_Q\simeq\frac{1}{3}m_3\left\{\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}+\left[\frac{1}{2}\frac{m_2}{m_3}\begin{pmatrix}1&1&-2\\1&1&-2\\-2&-2&4\end{pmatrix}+\frac{3\sqrt{2}}{4}Q\theta\begin{pmatrix}0&0&2\\0&0&2\\2&2&4\end{pmatrix}\right]\right.
$$

$$-\sqrt{3}\,\theta_{\rm q}\frac{m_2}{m_3}\left[\cos(Q\phi)\begin{pmatrix}1 & 0 & -1\\ 0 & -1 & 1\\ -1 & 1 & 0\end{pmatrix}+{\rm i}\sin(Q\phi)\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]\Bigg\}, \tag{21}$$

where $r\simeq 2$ has been taken into account.

In short, the strength of $S(3)_{\rm L}\times S(3)_{\rm R}\to S(2)_{\rm L}\times S(2)_{\rm R}$ breaking is at the percent level for both up- and down-quark sectors, while the effects of $S(2)_{\rm L}\times S(2)_{\rm R}\to S(1)_{\rm L}\times S(1)_{\rm R}$ breaking are at the percent and ten percent levels for the up- and down-quark sectors, respectively.

3 On the hierarchy basis

It is sometimes convenient to ascribe the hierarchy of the quark mass spectrum directly to the hierarchy of the corresponding quark mass matrix. In the latter basis, which is usually referred to as the hierarchy basis, the quark mass matrix $M^\prime_Q$ is related to its democratic counterpart M_Q via the following transformation:

$$M^\prime_Q=V_0^\dagger M_Q V_0, \tag{22}$$

where V_0 and M_Q have been given in Eqs. (5) and (10), respectively. To be explicit, we obtain

$$M^\prime_Q=\begin{pmatrix}2C_Q^{(22)} & \sqrt{3}\,C_Q^{(21)}{\rm e}^{{\rm i}Q\phi} & \sqrt{6}\,C_Q^{(23)}{\rm e}^{{\rm i}Q\phi}\\ \sqrt{3}\,C_Q^{(21)}{\rm e}^{-{\rm i}Q\phi} & X_Q & Y_Q\\ \sqrt{6}\,C_Q^{(23)}{\rm e}^{-{\rm i}Q\phi} & Y_Q & Z_Q\end{pmatrix}, \tag{23}$$

where

$$\begin{aligned}X_Q&=\frac{2}{3}\left[\left(r_Q+1\right)^2C_Q^{(11)}-\left(r_Q-2\right)C_Q^{(12)}\right],\\ Y_Q&=-\frac{\sqrt{2}}{3}\left(r_Q+1\right)\left[\left(r_Q-2\right)C_Q^{(11)}+2C_Q^{(12)}\right],\\ Z_Q&=\frac{1}{3}\left[\left(r_Q-2\right)^2C_Q^{(11)}+\left(5\,r_Q+2\right)C_Q^{(12)}\right]+A_Q^2m_3\,.\end{aligned} \tag{24}$$

The exact expression of $M^\prime_Q$ in Eq. (23) can be simplified, if the analytical approximation made in Eq. (13) for M_Q is taken into account. In this case,

$$M^\prime_Q\simeq\begin{pmatrix}m_1+\theta_{\rm q}^2m_2 & -\theta_{\rm q}m_2{\rm e}^{{\rm i}Q\phi} & -Q\theta\theta_{\rm q}m_2{\rm e}^{{\rm i}Q\phi}\\ -\theta_{\rm q}m_2{\rm e}^{-{\rm i}Q\phi} & m_2+Q^2\theta^2m_3 & -Q\theta m_3\\ -Q\theta\theta_{\rm q}m_2{\rm e}^{-{\rm i}Q\phi} & -Q\theta m_3 & m_3\end{pmatrix}. \tag{25}$$

The hierarchical structure of $M^\prime_Q$ is therefore determined by the hierarchy $|m_1|\ll|m_2|\ll m_3$ and the smallness of θ and $\theta_{\rm q}$.

Corresponding to the numerical illustration of M_Q in Eqs. (17)—(20), the results of $M^\prime_Q$ with the same inputs are give below.

(a) $(m_1,m_2)=(-m_{\rm u},+m_{\rm c})$ for $Q=+2/3$ and $(-m_{\rm d},+m_{\rm s})$ for $Q=-1/3$, leading to

$$M^\prime_{+2/3}\simeq\begin{pmatrix}3.337 & -54.695{\rm e}^{1.091{\rm i}} & -1.532{\rm e}^{1.091{\rm i}}\\ -54.695{\rm e}^{-1.091{\rm i}} & 767.678 & -4798.559\\ -1.532{\rm e}^{-1.091{\rm i}} & -4798.559 & 171965.605\end{pmatrix}{\rm MeV}, \tag{26}$$

and

$$M^\prime_{-1/3}\simeq\begin{pmatrix}-0.317 & -11.976{\rm e}^{-0.545{\rm i}} & 0.168{\rm e}^{-0.545{\rm i}}\\ -11.976{\rm e}^{0.545{\rm i}} & 55.047 & 39.272\\ 0.168{\rm e}^{0.545i} & 39.272 & 2859.450\end{pmatrix}{\rm MeV}; \tag{27}$$

(b) $(m_1,m_2)=(+m_{\rm u},+m_{\rm c})$ for $Q=+2/3$ and $(+m_{\rm d},+m_{\rm s})$ for $Q=-1/3$, leading to

$$M^\prime_{+2/3}\simeq\begin{pmatrix}6.077 & -54.458{\rm e}^{1.091{\rm i}} & -1.525{\rm e}^{1.091{\rm i}}\\ -54.458{\rm e}^{-1.091{\rm i}} & 767.698 & -4798.559\\ -1.525{\rm e}^{-1.091{\rm i}} & -4798.559 & 171965.605\end{pmatrix}{\rm MeV}, \tag{28}$$

and

$$M^\prime_{-1/3}\simeq\begin{pmatrix}5.087 & -10.847{\rm e}^{-0.545{\rm i}} & 0.152{\rm e}^{-0.545{\rm i}}\\ -10.847{\rm e}^{0.545{\rm i}} & 55.283 & 39.269\\ 0.152{\rm e}^{0.545i} & 39.269 & 2859.450\end{pmatrix}{\rm MeV}. \tag{29}$$

One can see that the sign ambiguities of m_1 and m_2 mainly affect the magnitude of the (1,1) element of M'_Q. The smallness of this matrix element is especially guaranteed if m_1 and m_2 take the opposite signs, as numerically shown in Eqs. (26) and (27).

In the hierarchy basis the language of texture "zeros" has proved to be very useful in establishing some experimentally testable relations between the ratios of quark masses and the flavor mixing angles [14, 15]. Those zeros dynamically mean that the corresponding matrix elements are sufficiently suppressed as compared with their neighboring counterparts, and this kind of suppression may reasonably arise from an underlying flavor symmetry [16]. In this sense Eqs. (26)—(29) motivate us to conjecture the well-known four-zero textures of Hermitian quark mass matrices [17] as the fairest extension of the original Fritzsch ansatz which contains six texture zeros [15]:

$$M'_Q = \begin{pmatrix} \mathbf{0} & \diamondsuit_Q & \mathbf{0} \\ \diamondsuit^*_Q & \heartsuit_Q & \triangle_Q \\ \mathbf{0} & \triangle^*_Q & \square_Q \end{pmatrix}, \tag{30}$$

where the relevant symbols denote the nonzero matrix elements. In fact, the pattern of M_Q with an approximate flavor democracy obtained in Eq. (21) just leads us to the four-zero textures of M'_Q in the hierarchy basis, if one takes $r \simeq 2 - 3\sqrt{2}\, Q\theta$ instead of $r \simeq 2$:

$$M'_Q \simeq \begin{pmatrix} \mathbf{0} & -\theta_q m_2 e^{iQ\phi} & \mathbf{0} \\ -\theta_q m_2 e^{-iQ\phi} & m_2 + Q^2\theta^2 m_3 & -Q\theta m_3 \\ \mathbf{0} & -Q\theta m_3 & m_3 \end{pmatrix}, \tag{31}$$

which can also be read off from Eq. (25) if similar approximations are made. As pointed out in Refs. [18, 19], current experimental data require that the (2,2) and (2,3) elements of $M'_{-1/3}$ be comparable in magnitude. In any case the pattern of M_Q in Eq. (21) or the texture of M'_Q in Eq. (31) can be very helpful for building a viable quark mass model.

4 On the scale dependence

In the above discussions we have restricted ourselves to the quark mass matrices at the electroweak scale characterized by $\mu = M_Z$. Since the flavor democracy might be realized at a much higher energy scale M_X, where a kind of fundamental new physics may occur, it makes sense to study the scale dependence of M_Q by means of the one-loop renormalization-group equations (RGEs) for the Yukawa coupling matrices and the CKM flavor mixing matrix [20]. For the sake of simplicity, here we work in the framework of the minimal supersymmetric standard model (MSSM) and calculate the relevant RGEs by taking account of the strong hierarchies of charged fermion masses and that of the CKM parameters. The approximate analytical results turn out to be [21]

$$\begin{aligned} m_t(M_Z) &\simeq m_t(M_X)(\zeta_u \xi_t^6 \xi_b), \\ m_b(M_Z) &\simeq m_b(M_X)(\zeta_d \xi_t \xi_b^6 \xi_\tau); \end{aligned} \tag{32}$$

and

$$\begin{aligned} \frac{m_u(M_X)}{m_t(M_X)} &\simeq \frac{m_u(M_Z)}{m_t(M_Z)}(\xi_t^3 \xi_b), \\ \frac{m_c(M_X)}{m_t(M_X)} &\simeq \frac{m_c(M_Z)}{m_t(M_Z)}(\xi_t^3 \xi_b), \\ \frac{m_d(M_X)}{m_b(M_X)} &\simeq \frac{m_d(M_Z)}{m_b(M_Z)}(\xi_t \xi_b^3), \\ \frac{m_s(M_X)}{m_b(M_X)} &\simeq \frac{m_s(M_Z)}{m_b(M_Z)}(\xi_t \xi_b^3); \end{aligned} \tag{33}$$

and

$$\begin{aligned} &\theta_u(M_X) \simeq \theta_u(M_Z), \quad \theta_d(M_X) \simeq \theta_d(M_Z), \\ &\theta(M_X) \simeq \theta(M_Z)(\xi_t \xi_b), \quad \phi(M_X) \simeq \phi(M_Z), \end{aligned} \tag{34}$$

where

$$\begin{aligned} \zeta_q &\equiv \exp\left[\frac{1}{2}\int_0^{\ln(M_X/M_Z)} \sum_{i=1}^{3} \frac{c_i^q g_i^2(0)}{8\pi^2 - b_i g_i^2(0)\chi} d\chi\right], \\ \xi_\alpha &\equiv \exp\left[-\frac{1}{16\pi^2}\int_0^{\ln(M_X/M_Z)} f_\alpha^2(\chi) d\chi\right] \end{aligned} \tag{35}$$

with $q = u$ or d, $\alpha = t$, b or τ, and $\chi = \ln(\mu/M_Z)$. In Eq. (35) c_i^q and b_i are the model-dependent coefficients whose values can be found in Ref. [20].

With the help of Eqs. (13) and (32)–(34), one can then express the democratic quark mass matrices at M_X by using the quark masses and flavor mixing parameters at M_Z and taking into account their RGE evolution effects:

$$\begin{aligned} M_{+2/3}(M_X) \simeq \frac{m_t}{3\zeta_u \xi_t^6 \xi_b} \Bigg\{ &\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \xi_t \xi_b \left[\frac{1}{2}\xi_t^2 \frac{m_c}{m_t} \begin{pmatrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & 4 \end{pmatrix} + \frac{\sqrt{2}}{2}\theta \begin{pmatrix} 0 & 0 & 2 \\ 0 & 0 & 2 \\ 2 & 2 & 4 \end{pmatrix} \right] \\ &- \sqrt{3}\xi_t^3 \xi_b \theta_u \frac{m_c}{m_t} \left[\cos\left(+\frac{2}{3}\phi\right) \begin{pmatrix} 1 & 0 & -1 \\ 0 & -1 & 1 \\ -1 & 1 & 0 \end{pmatrix} + i\sin\left(+\frac{2}{3}\phi\right) \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} \right] \end{aligned}$$

$$-\frac{\sqrt{6}}{3}\xi_{\rm t}^4\xi_{\rm b}^2\theta\theta_{\rm u}\frac{m_{\rm c}}{m_{\rm t}}\left[\cos\left(+\frac{2}{3}\phi\right)\begin{pmatrix}2 & 0 & 1\\ 0 & -2 & -1\\ 1 & -1 & 0\end{pmatrix}-{\rm i}\sin\left(+\frac{2}{3}\phi\right)\begin{pmatrix}0 & -2 & -1\\ 2 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$

$$\left.+\frac{3}{2}\xi_{\rm t}^3\xi_{\rm b}\left(\frac{m_{\rm u}}{m_{\rm t}}+\theta_{\rm u}^2\frac{m_{\rm c}}{m_{\rm t}}\right)\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}\right\}, \tag{36}$$

and

$$M_{-1/3}(M_X)\simeq\frac{m_{\rm b}}{3\zeta_{\rm d}\xi_{\rm t}\xi_{\rm b}^6\xi_\tau}\left\{\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}+\xi_{\rm t}\xi_{\rm b}\left[\frac{1}{2}\xi_{\rm b}^2\frac{m_{\rm s}}{m_{\rm b}}\begin{pmatrix}1 & 1 & -2\\ 1 & 1 & -2\\ -2 & -2 & 4\end{pmatrix}-\frac{\sqrt{2}}{4}\theta\begin{pmatrix}0 & 0 & 2\\ 0 & 0 & 2\\ 2 & 2 & 4\end{pmatrix}\right]\right.$$

$$-\sqrt{3}\xi_{\rm t}\xi_{\rm b}^3\theta_{\rm d}\frac{m_{\rm s}}{m_{\rm b}}\left[\cos\left(-\frac{1}{3}\phi\right)\begin{pmatrix}1 & 0 & -1\\ 0 & -1 & 1\\ -1 & 1 & 0\end{pmatrix}+{\rm i}\sin\left(-\frac{1}{3}\phi\right)\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$

$$+\frac{\sqrt{6}}{6}\xi_{\rm t}^2\xi_{\rm b}^4\theta\theta_{\rm d}\frac{m_{\rm s}}{m_{\rm b}}\left[\cos\left(-\frac{1}{3}\phi\right)\begin{pmatrix}2 & 0 & 1\\ 0 & -2 & -1\\ 1 & -1 & 0\end{pmatrix}-{\rm i}\sin\left(-\frac{1}{3}\phi\right)\begin{pmatrix}0 & -2 & -1\\ 2 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$

$$\left.+\frac{3}{2}\xi_{\rm t}\xi_{\rm b}^3\left(\frac{m_{\rm d}}{m_{\rm b}}+\theta_{\rm d}^2\frac{m_{\rm s}}{m_{\rm b}}\right)\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}\right\}, \tag{37}$$

where $r_Q\simeq 2$ has been taken. Typically taking $M_X=10^{16}$ GeV, $M_{\rm Z}=91.187$ GeV and $\tan\beta_{\rm MSSM}=10$ for illustration, we numerically obtain $\zeta_{\rm u}\simeq 3.47$, $\zeta_{\rm d}\simeq 3.38$, $\xi_{\rm t}\simeq 0.854$, $\xi_{\rm b}\simeq 0.997$ and $\xi_\tau\simeq 0.998$ from the one-loop RGEs [21]. In this case the expressions of $M_{+2/3}$ and $M_{-1/3}$ at M_X turn out to be

$$M_{+2/3}(M_X)\simeq 0.75\cdot\frac{1}{3}m_{\rm t}\left\{\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}+0.85\left[0.73\cdot\frac{1}{2}\frac{m_{\rm c}}{m_{\rm t}}\begin{pmatrix}1 & 1 & -2\\ 1 & 1 & -2\\ -2 & -2 & 4\end{pmatrix}+\frac{\sqrt{2}}{2}\theta\begin{pmatrix}0 & 0 & 2\\ 0 & 0 & 2\\ 2 & 2 & 4\end{pmatrix}\right]\right.$$

$$-0.62\cdot\sqrt{3}\theta_{\rm u}\frac{m_{\rm c}}{m_{\rm t}}\left[\cos\left(+\frac{2}{3}\phi\right)\begin{pmatrix}1 & 0 & -1\\ 0 & -1 & 1\\ -1 & 1 & 0\end{pmatrix}+{\rm i}\sin\left(+\frac{2}{3}\phi\right)\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$

$$-0.53\cdot\frac{\sqrt{6}}{3}\theta\theta_{\rm u}\frac{m_{\rm c}}{m_{\rm t}}\left[\cos\left(+\frac{2}{3}\phi\right)\begin{pmatrix}2 & 0 & 1\\ 0 & -2 & -1\\ 1 & -1 & 0\end{pmatrix}-{\rm i}\sin\left(+\frac{2}{3}\phi\right)\begin{pmatrix}0 & -2 & -1\\ 2 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$

$$\left.+0.62\cdot\frac{3}{2}\left(\frac{m_{\rm u}}{m_{\rm t}}+\theta_{\rm u}^2\frac{m_{\rm c}}{m_{\rm t}}\right)\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}\right\}, \tag{38}$$

and

$$M_{-1/3}(M_X)\simeq 0.35\cdot\frac{1}{3}m_{\rm b}\left\{\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}+0.85\left[1.00\cdot\frac{1}{2}\frac{m_{\rm s}}{m_{\rm b}}\begin{pmatrix}1 & 1 & -2\\ 1 & 1 & -2\\ -2 & -2 & 4\end{pmatrix}-\frac{\sqrt{2}}{4}\theta\begin{pmatrix}0 & 0 & 2\\ 0 & 0 & 2\\ 2 & 2 & 4\end{pmatrix}\right]\right.$$

$$-0.85\cdot\sqrt{3}\theta_{\rm d}\frac{m_{\rm s}}{m_{\rm b}}\left[\cos\left(-\frac{1}{3}\phi\right)\begin{pmatrix}1 & 0 & -1\\ 0 & -1 & 1\\ -1 & 1 & 0\end{pmatrix}+{\rm i}\sin\left(-\frac{1}{3}\phi\right)\begin{pmatrix}0 & 1 & -1\\ -1 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$

$$+0.72\cdot\frac{\sqrt{6}}{6}\theta\theta_{\rm d}\frac{m_{\rm s}}{m_{\rm b}}\left[\cos\left(-\frac{1}{3}\phi\right)\begin{pmatrix}2 & 0 & 1\\ 0 & -2 & -1\\ 1 & -1 & 0\end{pmatrix}-{\rm i}\sin\left(-\frac{1}{3}\phi\right)\begin{pmatrix}0 & -2 & -1\\ 2 & 0 & 1\\ 1 & -1 & 0\end{pmatrix}\right]$$
$$+0.85\cdot\frac{3}{2}\left(\frac{m_{\rm d}}{m_{\rm b}}+\theta_{\rm d}^2\frac{m_{\rm s}}{m_{\rm b}}\right)\begin{pmatrix}1 & -1 & 0\\ -1 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}\Bigg\}, \quad (39)$$

from which one can clearly see the RGE-induced corrections to the relevant terms in each quark sector. Hence such quantum effects should not be ignored when building a specific quark mass model based on the flavor democracy at M_X and confronting its predictions with the experimental data at $M_{\rm Z}$.

At this point it is worth mentioning that the approximate four-zero textures of $M_{+2/3}$ and $M_{-1/3}$ in the hierarchy basis are essentially stable against the RGE running effects. Here the stability of the texture zeros means that the (1,1), (1,3) and (3,1) elements of each quark mass matrix at M_X remain strongly suppressed in magnitude as compared with their neighboring counterparts, and thus it is a reasonable approximation to take them to be vanishing at any energy scale between $M_{\rm Z}$ and M_X from a phenomenological point of view [19]. Such an observation makes sense because the four-zero textures of Hermitian quark mass matrices or their variations are especially favored by current experimental data and deserve some special attention in the model-building exercises.

5 Summary

It has been known for quite a long time that the democracy of quark flavors is one of the well-motivated flavor symmetries for building a viable quark mass model, but how to break this symmetry and to what extent to break it are highly nontrivial. To minimize the number of free parameters, in this work we have assumed structural parallelism between $Q=+2/3$ and $Q=-1/3$ quark sectors, and proposed a novel way to reconstruct the texture of flavor democracy breaking and evaluate its strength in each sector with the help of the Fritzsch-Xing parametrization of the CKM flavor mixing matrix. Some phenomenological implications of such flavor-democratized quark mass matrices, in particular their variations with possible texture zeros in the hierarchy basis and their RGE evolution from the electroweak scale to a super-high energy scale, have also been discussed. We hope that this kind of study will be useful to more deeply explore the underlying correlation between the quark flavor structures and the observed quark mass spectrum and flavor mixing pattern.

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