

S -wave resonance contributions to $B_{(s)}^0 \rightarrow \eta_c(2S)\pi^+\pi^-$ in the perturbative QCD factorization approach*

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Abstract: By employing the perturbative QCD (PQCD) factorization approach, we study the quasi-two-body $B_{(s)}^0 \rightarrow \eta_c(2S)\pi^+\pi^-$ decays, where the pion pair comes from the S -wave resonance $f_0(X)$. The Breit–Wigner formula for the $f_0(500)$ and $f_0(1500)$ resonances and the Flatté model for the $f_0(980)$ resonance are adopted to parameterize the time-like scalar form factors in the two-pion distribution amplitudes. As a comparison, Bugg’s model is also used for the wide $f_0(500)$ in this work. For decay rates, we found the following PQCD predictions: (a) $\mathcal{B}(B_s^0 \rightarrow \eta_c(2S)f_0(X)[\pi^+\pi^-]_s) = (2.67_{-1.08}^{+1.78}) \times 10^{-5}$ when the contributions from $f_0(980)$ and $f_0(1500)$ are all taken into account; (b) $\mathcal{B}(B^0 \rightarrow \eta_c(2S)f_0(500)[\pi^+\pi^-]_s) = (1.40_{-0.56}^{+0.92}) \times 10^{-6}$ in the Breit-Wigner model and $(1.53_{-0.61}^{+0.97}) \times 10^{-6}$ in Bugg’s model.

Keywords: PQCD factorization approach, two-pion distribution amplitudes, quasi-two-body decay

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1 Introduction

The study of three-body hadronic B meson decays can help us understand the standard model and search for the possible effects of new physics. Experimentally, quite a number of channels have been measured by collaborations like BaBar [1–6], Belle [7–10] and LHCb [11–21]. Theoretically, there are several approaches used in this field, for instance, QCD factorization [22–38], the perturbative QCD (PQCD) approach [39–47], and some methods based on symmetry principles [48–60]. The aim of those studies is to understand the resonant and non-resonant contributions, as well as the final state interactions (FSIs) [37, 58] in three-body B decays. Studying these decays is still at an early stage, however, for both theoretical studies and experimental measurements.

The PQCD factorization approach is one of the major theoretical frameworks to deal with two-body hadronic B meson decays [61, 62]. Very recently, some three-body hadronic B meson decays have been studied by employing the PQCD factorization approach, for example in

Refs. [39–47]. For the cases of three-body decays, however, the previous PQCD approach [61, 62] should be modified by introducing the two-meson distribution amplitudes [63–66] to describe the selected pair of final state mesons. This is due to the reason discussed in [61, 62]: the contribution from the direct evaluation of hard b-quark decay kernels containing two virtual gluons is generally power suppressed, and the dominant contribution comes most possibly from the region where the two energetic light mesons are almost collimated with each other with an invariant mass below $O(\bar{\Lambda}m_B)$ ($\bar{\Lambda} = m_B - m_b$, the B meson and b quark mass difference). Then, the typical PQCD factorization formula with the crucial nonperturbative input of two-hadron distribution amplitudes for a $B \rightarrow h_1 h_2 h_3$ decay amplitude can be written symbolically in the form

$$\mathcal{A} = \phi_B \otimes H \otimes \phi_{h_1 h_2} \otimes \phi_{h_3}. \quad (1)$$

Here the hard kernel $H(x_i, b_i, t)$ contains the contributions from one hard gluon exchange diagrams only, the nonperturbative inputs $\phi_B(x, b)$, $\phi_{h_1 h_2}(z, \omega)$, $\phi_{h_3}(x_3, b_3)$

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are the distribution amplitudes for the B meson, the h_1 - h_2 pair and the h_3 meson respectively, while the symbols \otimes mean the convolution integration over the variables of the momentum fractions (x, z, x_3) and the conjugate space coordinates b_i of k_{iT} . With the help of the two-pion distribution amplitudes, many studies have been done for quasi-two-body decays, and the parameters in the S -wave and P -wave two-pion distribution amplitudes have been fixed in Refs. [42, 43]. Based on these works, we have studied the S -wave resonance contributions to the decays $B_{(s)}^0 \rightarrow \eta_c \pi^+ \pi^-$ [44], $B_s^0 \rightarrow \psi(2S) \pi^+ \pi^-$ [45], and the P -wave resonance ($\rho(770)$) contributions to $B_{(s)}^0 \rightarrow (D/P)\rho \rightarrow (D/P)\pi\pi$ decays [46, 47], where D represents the charmed D mesons and P stands for the light pseudoscalar mesons π, K, η or η' .

Up to now, several decay modes of the B and B_s mesons to the charmonium state plus pion pair, like $B^0 \rightarrow J/\psi \pi^+ \pi^-$ [1, 16–18], $B_s^0 \rightarrow J/\psi \pi^+ \pi^-$ [14, 15], $B_{(s)}^0 \rightarrow \psi(2S) \pi^+ \pi^-$ [20] and $B_s^0 \rightarrow \eta_c \pi^+ \pi^-$ [21], have been measured by the BaBar and LHCb Collaborations. With the ongoing running of the LHCb experiment, more data of such B/ B_s decays with the inclusion of various excited charmonium states ($\eta_c(2S)$ etc.) will be collected. It is therefore interesting to study such decay modes theoretically. In this work, we will study the S -wave resonance contributions to $B_{(s)}^0 \rightarrow \eta_c(2S) f_0(X) \rightarrow \eta_c(2S) \pi^+ \pi^-$ decays and give our predictions for the branching fractions of the considered decay modes.

This paper is organized as follows. In Section 2, we give a brief introduction to the theoretical framework. The numerical values, some discussions and the conclusions will be given in the last two sections.

2 Theoretical framework

In $B_{(s)}^0 \rightarrow \eta_c(2S) \pi^+ \pi^-$ decays, by using the light-cone coordinates and in the rest frame of the $B_{(s)}^0$ meson, the momentum of $B_{(s)}^0$, the pion pair and $\eta_c(2S)$ can be chosen as

$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, 0_T), \quad p = p_1 + p_2 = \frac{m_B}{\sqrt{2}}(1 - r^2, \eta, 0_T),$$

$$p_3 = \frac{m_B}{\sqrt{2}}(r^2, 1 - \eta, 0_T), \quad (2)$$

where $\eta = \omega^2 / [(1 - r^2)m_B^2]$, $r = m_{\eta_c(2S)} / m_B$ and $\omega^2 = p^2$ means the squared invariant mass of the pion pair. The momenta for the spectators in the $B_{(s)}^0$ meson, the pion pair, and the $\eta_c(2S)$ meson read as

$$k_B = \left(0, \frac{m_B}{\sqrt{2}} x_B, k_{BT} \right),$$

$$k = \left(\frac{m_B}{\sqrt{2}} z(1 - r^2), 0, k_T \right),$$

$$k_3 = \left(\frac{m_B}{\sqrt{2}} r^2 x_3, \frac{m_B}{\sqrt{2}} (1 - \eta) x_3, k_{3T} \right), \quad (3)$$

where the momentum fractions x_B, z , and x_3 run from zero to unity.

The S -wave two-pion distribution amplitudes can be written as [42, 67]

$$\Phi_{\pi\pi}^{S\text{-wave}} = \frac{1}{\sqrt{2N_c}} \left[\not{p} / \Phi_{\nu\nu=-}^{I=0}(z, \zeta, \omega^2) + \omega \Phi_s^{I=0}(z, \zeta, \omega^2) + \omega (\not{n}_+ \not{n}_- - 1) \Phi_{\nu\nu=+}^{I=0}(z, \zeta, \omega^2) \right], \quad (4)$$

with $n_+ = (1, 0, \mathbf{0}_T)$, $n_- = (0, 1, \mathbf{0}_T)$ and the π^+ meson momentum fraction $\zeta = p_1^+ / p^+$. Their asymptotic forms are parameterized as [42]

$$\Phi_{\nu\nu=-}^{I=0} = \frac{9F_s(\omega^2)}{\sqrt{2N_c}} a_2^{I=0} z(1-z)(1-2z),$$

$$\Phi_s^{I=0} = \frac{F_s(\omega^2)}{2\sqrt{2N_c}},$$

$$\Phi_{\nu\nu=+}^{I=0} = \frac{F_s(\omega^2)}{2\sqrt{2N_c}} (1-2z), \quad (5)$$

with the time-like scalar form factor $F_s(\omega^2)$ and the Gegenbauer coefficient $a_2^{I=0} = 0.2 \pm 0.2$.

The expressions of the time-like scalar form factor $F_s(\omega^2)$ associated with the $s\bar{s}$ component of both $f_0(980)$ and $f_0(1500)$, and $d\bar{d}$ component of $f_0(500)$ can be found in Ref. [42]. Following the LHCb Collaboration [14–17], the Breit–Wigner (BW) formula for the $f_0(500)$ and $f_0(1500)$ resonances will be used to parameterize the time-like scalar form factors in the two-pion distribution amplitudes, which include both the resonant and non-resonant contributions of the $\pi\pi$ pair. For $f_0(980)$, however, the Flatté model [68] will be used, since $f_0(980)$ is close to the $K\bar{K}$ threshold and the BW formula does not work well for this meson [68, 69]. We know that there is some dispute about the nature of the $f_0(500)$ meson due to its wide shape. Following the same treatment of $f_0(500)$ as LHCb Collaboration [19], here we also parameterize its contribution to the scalar form factor in the Bugg resonant line-shape [69]

$$R_{f_0(500)}(s) = m_r \Gamma_1(s) \left[m_r^2 - s - g_1^2 \frac{s - s_A}{m_r^2 - s_A} [j_1(s) - j_1(m_r^2)] - i m_r \sum_{i=1}^4 \Gamma_i(s) \right]^{-1}, \quad (6)$$

with the following relevant parameters

$$m_r \Gamma_1(s) = g_1^2 \frac{s - s_A}{m_r^2 - s_A} \rho_1(s),$$

$$g_1^2(s) = m_r (b_1 + b_2 s) \exp(-(s - m_r^2)/A),$$

$$j_1(s) = \frac{1}{\pi} \left[2 + \rho_1 \ln \left(\frac{1 - \rho_1}{1 + \rho_1} \right) \right],$$

$$m_r \Gamma_2(s) = 0.6 g_1^2(s) (s/m_r^2) \exp(-\alpha |s - 4m_K^2|) \rho_2(s),$$

$$m_r \Gamma_3(s) = 0.2 g_1^2(s) (s/m_r^2) \exp(-\alpha |s - 4m_\eta^2|) \rho_3(s),$$

$$m_r \Gamma_4(s) = m_r g_{4\pi} \rho_{4\pi}(s) / \rho_{4\pi}(m_r^2),$$

$$\rho_{4\pi}(s) = 1 / [1 + \exp(7.082 - 2.845s)]. \quad (7)$$

In the numerical calculation, we set $m_r = 0.953$ GeV, $s_A = 0.41$ m_π^2 , $b_1 = 1.302$ GeV, $b_2 = 0.340$ GeV^{-1} , $A = 2.426$ GeV^2 and $g_{4\pi} = 0.011$ GeV [69]. The phase-space factors of the decay channels $\pi\pi$, KK and $\eta\eta$ are defined as $\rho_i(s) = \sqrt{1 - 4m_i^2/s}$ with $i=1,2,3$ for π, K and η respectively. It is worth mentioning that another description of the pion-pion form factors was introduced in Refs. [70, 71].

For the $B_{(s)}^0$ mesons, we use the same distribution amplitudes $\phi_B(x, b)$ in the b space as used for example in Ref. [44],

$$\Phi_B = \frac{i}{\sqrt{2N_c}} (\not{p}_B + m_B) \gamma_5 \phi_B(\mathbf{k}_1). \quad (8)$$

The distribution amplitude is chosen as

$$\phi_B(x, b) = N_B x^2 (1-x)^2 \exp\left[-\frac{M_B^2 x^2}{2\omega_B^2} - \frac{1}{2}(\omega_B b)^2\right]. \quad (9)$$

In the numerical calculation, we also use the shape parameter $\omega_B = 0.40 \pm 0.04$ GeV with $f_B = 0.19$ GeV for B^0 decays, and $\omega_{B_s} = 0.50 \pm 0.05$ GeV with $f_{B_s} = 0.236$ GeV for B_s^0 decays [44].

As the first radial excitation of the η_c charmonium ground state, $\eta_c(2S)$ was first observed by the Belle Collaboration in B decays [72, 73]. The harmonic-oscillator wave function with the principal quantum number $n=2$

and the orbital angular momentum $l=0$ is defined as [74]

$$\langle \eta_c(2S) | \bar{c}(z)_\alpha c(0)_\beta | 0 \rangle =$$

$$-\frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{ixp_3 \cdot z} [(\gamma_5 \not{p}_3)_{\alpha\beta} \psi^v(x, b) + m(\gamma_5)_{\alpha\beta} \psi^s(x, b)]. \quad (10)$$

The asymptotic models for the twist-2 distribution amplitudes ψ^v , and the twist-3 distribution amplitudes ψ^s for the radially excited $\eta_c(2S)$ is parameterized as [75]

$$\Psi^v(x, b) = \frac{f_{\eta_c(2S)}}{2\sqrt{2N_c}} N^v x \bar{x} \mathcal{T}(x) e^{-x\bar{x} \frac{m_c}{w} [w^2 b^2 + (\frac{x-\bar{x}}{2x\bar{x}})^2]},$$

$$\Psi^s(x, b) = \frac{f_{\eta_c(2S)}}{2\sqrt{2N_c}} N^s \mathcal{T}(x) e^{-x\bar{x} \frac{m_c}{w} [w^2 b^2 + (\frac{x-\bar{x}}{2x\bar{x}})^2]}, \quad (11)$$

with the function $\mathcal{T}(x) = 1 - 4b^2 m_c w x \bar{x} + m_c (x - \bar{x})^2 / (w x \bar{x})$ and the same normalization conditions as the $B_{(s)}^0$ mesons: $\int_0^1 \Psi^i(x, b=0) dx = f_{\eta_c(2S)} / (2\sqrt{6})$. We also choose $f_{\eta_c(2S)} = 0.243_{-0.111}^{+0.079}$ GeV and $w = 0.2 \pm 0.1$ GeV as in Ref. [75].

In the PQCD factorization approach, there are four kinds of emission Feynman diagram for the $B_{(s)}^0 \rightarrow \eta_c(2S) \pi^+ \pi^-$, as illustrated in Fig. 1, where (a) and (b) are factorizable diagrams, while (c) and (d) are the non-factorizable ones. We will use F^{LL}, F^{LR}, F^{SP} and M^{LL}, M^{LR}, M^{SP} to describe the contributions of the factorizable (Fig. 1(a) and 1(b)) and non-factorizable (Fig. 1(c) and 1(d)) emission diagrams with the $(V-A)(V-A), (V-A)(V+A),$ and $(S-P)(S+P)$ currents, respectively. The total decay amplitudes for the considered decays can therefore be written as

$$\mathcal{A}(B_{(s)}^0 \rightarrow \eta_c(2S) \pi^+ \pi^-) = V_{cb}^* V_{cd(cs)} \left[\left(C_1 + \frac{C_2}{3} \right) F^{LL} + C_2 M^{LL} \right] - V_{tb}^* V_{td(ts)} \left[\left(C_3 + \frac{C_4}{3} + C_9 + \frac{C_{10}}{3} \right) F^{LL} \right.$$

$$\left. + \left(C_5 + \frac{C_6}{3} + C_7 + \frac{C_8}{3} \right) F^{LR} + (C_4 + C_{10}) M^{LL} + (C_6 + C_8) M^{SP} \right], \quad (12)$$

where $C_i(\mu)$ ($i=1, \dots, 10$) are Wilson coefficients at the renormalization scale μ . For simplicity, we denote the distribution amplitudes $\Phi_{\nu\nu=+}^{I=0}(z, \zeta, \omega^2)$ [$\Phi_s^{I=0}(z, \zeta, \omega^2)$,

$\Phi_{\nu\nu=+}^{I=0}(z, \zeta, \omega^2)$] by ϕ_0 (ϕ_s, ϕ_σ) below. From Fig. 1(a) and 1(b), we find

$$F^{LL} = 8\pi C_F m_B^4 f_{\eta_c(2S)} \int_0^1 dx_B dz \int_0^\infty b_B db_B b db \phi_B(x_B, b_B)$$

$$\times \left\{ \left[\sqrt{\eta(1-r^2)} \left[((1-2z)(1-\eta) + r^2(1+2z(1-\eta))) (\phi_s - \phi_\sigma) + 2(1-\eta)(1-2(1-r^2)z) \phi_\sigma \right] \right. \right.$$

$$\left. + [(1+z)(1-\eta) - r^2(1-2\eta+2z(1-\eta))] \phi_0 \right] E_e(t_a) h_a(x_B, z, b_B, b) + \left[2\sqrt{\eta(1-r^2)} \right.$$

$$\left. \times [1-\eta-r^2(1+x_B-2\eta)] \phi_s + (1-r^2)[r^2(x_B-\eta) - (1-\eta)\eta] \phi_0 \right] E_e(t_b) h_b(x_B, z, b_B, b) \left. \right\}, \quad (13)$$

$$F^{LR} = -F^{LL}, \quad (14)$$

with a color factor $C_F = 4/3$. The explicit expressions of the hard functions h_a and h_b , the evolution factors $E_e(t_i)$, including the Sudakov exponents and the hard scales (t_a, t_b) , can be found for example in Ref. [42]. Fol-

lowing the same procedure, one can obtain the explicit expressions for decay amplitude M^{LL} , M^{LR} and M^{SP} from the evaluation of Fig. 1(c) and 1(d).

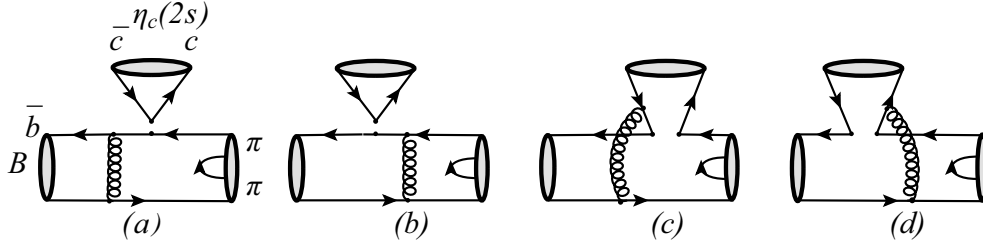


Fig. 1. Typical Feynman diagrams contributing to the three-body decays $B_s^0 \rightarrow \eta_c(2S)\pi^+\pi^-$.

3 Numerical results

In the numerical calculations, the following input parameters are used implicitly. The QCD scale, masses and decay constants are in units of GeV [76]:

$$\begin{aligned} \Lambda_{\overline{MS}}^{(f=4)} &= 0.25, & m_{B_s^0} &= 5.367, & m_{B^0} &= 5.280, \\ M_{\eta_c(2S)} &= 3.639; & m_{\pi^\pm} &= 0.140, & m_{\pi^0} &= 0.135, \\ m_c &= 1.27, & \tau_{B^0} &= 1.520 \text{ ps}, & \tau_{B_s^0} &= 1.510 \text{ ps}. \end{aligned} \quad (15)$$

The Wolfenstein parameters for the CKM matrix elements read as [76]:

$$\begin{aligned} \lambda &= 0.22506 \pm 0.00050, & A &= 0.811 \pm 0.026 \\ \bar{\rho} &= 0.124_{-0.018}^{+0.019}, & \bar{\eta} &= 0.356 \pm 0.011. \end{aligned} \quad (16)$$

The differential branching ratio for the $B_{(s)}^0 \rightarrow \eta_c(2S)\pi^+\pi^-$ decay can be written as [42]:

$$\frac{d\mathcal{B}}{d\omega} = \tau_B \frac{\omega |\vec{p}_1| |\vec{p}_3|}{4(2\pi)^3 m_B^3} |\mathcal{A}|^2, \quad (17)$$

with the $B_{(s)}^0$ meson mean lifetime τ_B . The kinematic variables $|\vec{p}_1|$ and $|\vec{p}_3|$ denote the magnitudes of the π^+ and $\eta_c(2S)$ momenta in the center-of-mass frame of the pion pair,

$$\begin{aligned} |\vec{p}_1| &= \frac{1}{2} \sqrt{\omega^2 - 4m_{\pi^\pm}^2}, \\ |\vec{p}_3| &= \frac{1}{2\omega} \sqrt{[m_B^2 - (\omega + m_{\eta_c(2S)})^2][m_B^2 - (\omega - m_{\eta_c(2S)})^2]}. \end{aligned} \quad (18)$$

From our numerical calculations, we find the following results:

1) In Fig. 2(a), we show the differential branching ratios $d\mathcal{B}/d\omega$ for the $B_s^0 \rightarrow \eta_c(2S)\pi^+\pi^-$ decay, where the solid curve and the dotted curve show the contributions from $f_0(980)$ and $f_0(1500)$, respectively. In Fig. 2(b), we show the ω -dependence of the differential decay rate $d\mathcal{B}/d\omega$ when the BW model (solid curve) and Bugg's model (dotted curve) are employed. The allowed region of ω is $4m_{\pi^\pm}^2 \leq \omega^2 \leq (M_B - m_{\eta_c(2S)})^2$.

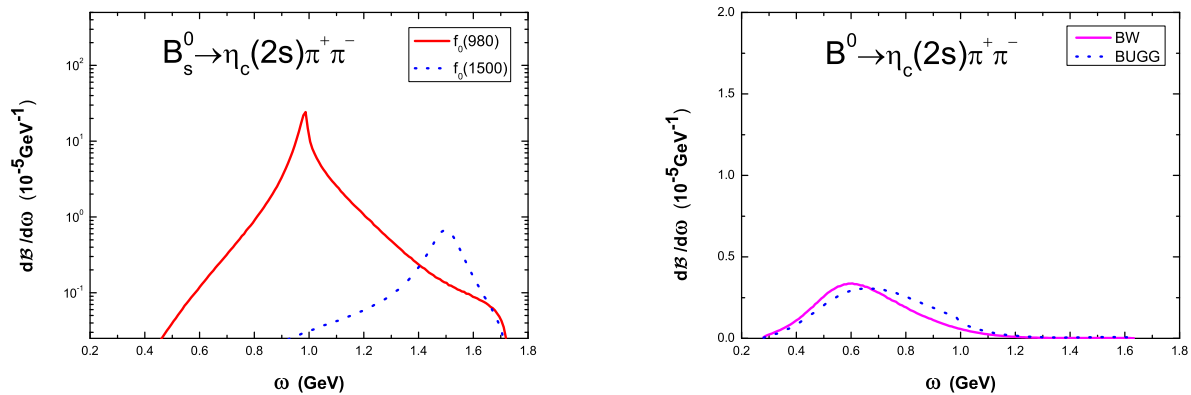


Fig. 2. The ω -dependence of $d\mathcal{B}/d\omega$ for (a) the contribution from the resonances $f_0(980)$ and $f_0(1500)$ for the $B_s^0 \rightarrow \eta_c(2S)\pi^+\pi^-$ decay; and (b) the contribution from $f_0(500)$ for the $B^0 \rightarrow \eta_c(2S)\pi^+\pi^-$ decay.

2) For the decays $B_s^0 \rightarrow \eta_c(2S)f_0(X) \rightarrow \eta_c(2S)\pi^+\pi^-$, when the contributions from $f_0(980)$ and $f_0(1500)$ are included respectively, the PQCD predictions for the

branching ratios $\mathcal{B}(B_s^0 \rightarrow \eta_c(2S)f_0(X) \rightarrow \eta_c(2S)\pi^+\pi^-)$ are of the form

$$\begin{aligned} \mathcal{B}(B_s^0 \rightarrow \eta_c(2S)f_0(980)[f_0(980) \rightarrow \pi^+\pi^-]) &= (2.19_{-0.55}^{+0.69}(\omega_{B_s^0})_{-0.42}^{+0.50}(a_2)_{-0.45}^{+1.05}(w)_{-0.26}^{+0.36}(f_{\eta_c(2S)})) \times 10^{-5}, \\ \mathcal{B}(B_s^0 \rightarrow \eta_c(2S)f_0(1500)[f_0(1500) \rightarrow \pi^+\pi^-]) &= (1.31_{-0.12}^{+0.08}(\omega_{B_s^0})_{-0.31}^{+0.39}(a_2)_{-0.56}^{+0.62}(w)_{-0.50}^{+0.77}(f_{\eta_c(2S)})) \times 10^{-6}, \end{aligned} \quad (19)$$

where the first two errors come from the uncertainty $\omega_{B_s} = 0.50 \pm 0.05$ GeV and $a_2^{T=0} = 0.2 \pm 0.2$, and the last two errors are from $w = 0.2 \pm 0.1$ GeV and $f_{\eta_c(2S)} = 0.243_{-0.111}^{+0.079}$ GeV (the parameters in the wave function of $\eta_c(2S)$). The errors from the uncertainties of other input parameters, for instance the CKM matrix elements, are very small and have been neglected.

By taking into account the S -wave contributions from $f_0(980)$ and $f_0(1500)$ simultaneously, we find the PQCD prediction for the total branching ratio:

$$\begin{aligned} \mathcal{B}(B_s^0 \rightarrow \eta_c(2S)(\pi^+\pi^-)_S) \\ = (2.67_{-0.62}^{+0.74}(\omega_{B_s^0})_{-0.54}^{+0.61}(a_2)_{-0.60}^{+1.43}(w)_{-0.36}^{+0.47}(f_{\eta_c(2S)})) \times 10^{-5}. \end{aligned} \quad (20)$$

It is easy to see that the dominant contribution comes from the resonance $f_0(980)$ (82.0%), while the constructive interference between $f_0(980)$ and $f_0(1500)$ provides $\sim 13\%$ enhancement to the total decay rate. This can be approximately seen from Fig. 2(a). When compared with the previous study for $B_s^0 \rightarrow \eta_c(\pi^+\pi^-)_s$ in Ref. [44], we find that $\mathcal{B}(B \rightarrow \eta_c(2S)[\pi^+\pi^-]_s) : \mathcal{B}(B \rightarrow \eta_c[\pi^+\pi^-]_s) \approx 1:2$.

3) For the $B^0 \rightarrow \eta_c(2S)f_0(500) \rightarrow \eta_c(2S)\pi^+\pi^-$ decay, the PQCD predictions based on the BW model or Bugg's model for the parametrization of the wide $f_0(500)$ are the following:

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow \eta_c(2S)f_0(500)[f_0(500) \rightarrow \pi^+\pi^-])_{\text{(BW)}} \\ = 1.40_{-0.56}^{+0.92} \times 10^{-6} \end{aligned} \quad (21)$$

$$\begin{aligned} \mathcal{B}(B^0 \rightarrow \eta_c(2S)f_0(500)[f_0(500) \rightarrow \pi^+\pi^-])_{\text{(Bugg)}} \\ = 1.53_{-0.61}^{+0.97} \times 10^{-6}, \end{aligned} \quad (22)$$

where the major errors have been added in quadrature.

One can see easily that the PQCD predictions obtained by employing the BW model or Bugg's model are very similar, with a difference of only about 10%.

4) Based on our previous studies of the quasi-two-body B meson decays involving the ρ meson [43], we know that the main contribution lies in the region around the pole mass of the ρ resonance. Because $\Gamma_{\eta_c(2S)} \approx 11.3$ MeV, which is much narrower than $\Gamma_\rho \approx 149$ MeV, it is reasonable for us to assume that the possible effect due to the narrow width of $\eta_c(2S)$ is very small and can safely be neglected.

4 Summary

In summary, we have studied the quasi-two-body $B_{(s)}^0 \rightarrow \eta_c(2S)(\pi^+\pi^-)_S$ decays in the PQCD factorization approach by introducing the S -wave two-pion distribution amplitudes. For the $B_s^0 \rightarrow \eta_c(2S)f_0(X) \rightarrow \eta_c(2S)\pi^+\pi^-$ decay, the contributions from the S -wave resonances $f_0(980)$ and $f_0(1500)$ were taken into account, but the $f_0(980)$ provides the dominant contribution to the PQCD prediction: $\mathcal{B}(B_s^0 \rightarrow \eta_c(2S)(\pi^+\pi^-)_S) = (2.67_{-1.08}^{+1.78}) \times 10^{-5}$. For the $B^0 \rightarrow \eta_c(2S)f_0(X) \rightarrow \eta_c(2S)\pi^+\pi^-$ decay, the contribution from $f_0(500)$ was taken into account, and the PQCD prediction for its decay rate is $(1.40_{-0.56}^{+0.92}) \times 10^{-6}$ in the BW model or $(1.53_{-0.61}^{+0.97}) \times 10^{-6}$ in Bugg's model. These PQCD predictions for the branching ratios of the decays considered can be measured and tested at the LHCb and/or Belle-II experiments in the near future.

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