Analysis of the Y(4220) and Y(4390) as molecular states with QCD sum rules *

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Abstract: In this article, we assign the Y(4390) and Y(4220) to be the vector molecular states $\mathrm{D\bar{D}_1(2420)}$ and $\mathrm{D^*\bar{D}_0^*(2400)}$, respectively, and study their masses and pole residues in detail with the QCD sum rules. The present calculations only favor assigning the Y(4390) to be the $\mathrm{D\bar{D}_1(1^{--})}$ molecular state.

Keywords: molecular states, QCD sum rules

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1 Introduction

In 2013, Yuan studied the cross sections of the process $\mathrm{e^+e^-\to\pi^+\pi^-h_c}$ at center-of-mass energies 3.90–4.42 GeV measured by the BESIII and the CLEO-c experiments, and observed evidence for two resonant structures, a narrow structure of mass (4216±18) MeV and width (39±32) MeV, and a possible wide structure of mass (4293±9) MeV and width (222±67) MeV [1].

In 2014, the BES collaboration searched for the production of $\mathrm{e^+e^-}\to\omega\chi_{\mathrm{cJ}}$ with $J=0,1,2$, based on data samples collected with the BESIII detector at center-of-mass energies from 4.21−4.42 GeV, and observed a resonance in the $\omega\chi_{c0}$ cross section. The measured mass and width of the resonance, Y(4230), are 4230±8±6 MeV and 38±12±2 MeV, respectively [2].

Recently, the BES collaboration measured the cross sections of the process $\mathrm{e^+e^-\to\pi^+\pi^-h_c}$ at center-of-mass energies 3.896−4.600 GeV using data samples collected with the BESIII detector, and observed two structures. The Y(4220) has mass 4218.4±4.0±0.9 MeV and width 66.0±9.0±0.4 MeV, and the Y(4390) has mass 4391.6±6.3±1.0 MeV and width 139.5±16.1±0.6 MeV [3]. The Y(4230) and Y(4220) may be the same particle. The Y(4230) has been assigned to be a vector-diquark-vector-antidiquark type vector tetraquark state [4–6] or a conventional meson ψ(4S) [7]. The near thresholds are $M_{\mathrm{D^+D_1(2420)^-}}=$ 4293 MeV, $M_{\mathrm{D^0D_1(2420)^0}}=4285$ MeV, $M_{\mathrm{D^{*+}D_0^*(2400)^-}}=$ 4361 MeV, and $M_{\mathrm{D^{*0}D_0^*(2400)^0}}=4325$ MeV [8]. It is also possible to assign the Y(4220) and Y(4390) to be the $\mathrm{D\bar{D}_1(2420)}$ or $\mathrm{D^*\bar{D}_0^*(2400)}$ molecular states.

Eleven years ago, the BaBar collaboration observed a broad resonance (Y(4260)) in the initial-state radiation process $\mathrm{e^+e^-}\to$ Y(4260) $\to\mathrm{J/\psi\pi^+\pi^-}$ in the invariant-mass spectrum of the $\mathrm{J/\psi\pi^+\pi^-}$ [9]. Later, the BaBar collaboration measured the mass and width of the Y(4260) in a more precise way [10]. The cross section rises rapidly below the peak of the Y(4260) and falls more slowly above the peak [8]. The BESIII experiment may indicate that in fact the Y(4260) consists of two peaks, a narrow peak around 4.22 GeV and a wider peak around 4.39 GeV, accounting for the asymmetry.

In Refs. [11, 12], Zhang and Huang systematically study the $\mathrm{Q\bar{q}\bar{Q}'q}$ type scalar, vector and axialvector molecular states with the QCD sum rules by calculating the operator product expansion up to the vacuum condensates of dimension 6. The predicted molecule masses $M_{\mathrm{D^*\bar{D}_0^*}}=4.26\pm0.07$ GeV and $M_{\mathrm{D\bar{D}_1}}=4.34\pm0.07$ GeV are consistent with the Y(4220) and Y(4390), respectively. However, the charge conjugations of the molecular states are not distinguished and the higher dimensional vacuum condensates are neglected. In Ref. [13], Lee, Morita and Nielsen distinguish the charge conjugations of the interpolating currents, and calculate the operator product expansion up to the vacuum condensates of dimension 6, partly including the vacuum condensates of dimension 8. They obtain the mass of the $\mathrm{D\bar{D}_1(2420)}$ molecular state with $J^{PC}=1^{-+}$, $M_{\mathrm{D\bar{D}_1}}=4.19\pm0.22$ GeV, which differs significantly from the prediction $M_{\mathrm{D\bar{D}_1}}=4.34\pm0.07$ GeV.

In Refs. [11–13], some higher dimensional vacuum condensates involving the gluon condensate, mixed condensate and four-quark condensate are neglected. The

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terms associated with $\frac{1}{T^2}$, $\frac{1}{T^4}$, $\frac{1}{T^6}$ in the QCD spectral densities manifest themselves at small values of the Borel parameter T^2, so we have to choose large values of T^2 to guarantee convergence of the operator product expansion. In the Borel windows, the higher dimensional vacuum condensates play a less important role. The higher dimensional vacuum condensates play an important role in determining the Borel windows and therefore the ground state masses and pole residues, so we should take them into account consistently.

In this article, we assign the Y(4390) and Y(4220) to be the vector molecular states $\mathrm{D\bar{D}_1(2420)}$ and $\mathrm{D^*\bar{D}_0^*(2400)}$ respectively, distinguish the charge conjugations, and construct the color singlet-singlet type currents to interpolate them. We calculate the contributions of the vacuum condensates up to dimension-10 in the operator product expansion in a consistent way, and use the energy scale formula to determine the energy scales of the QCD spectral densities [14, 15], which differs significantly from the routines taken in Refs. [11–13]. We then study the masses and pole residues in detail with the QCD sum rules.

The article is arranged as follows. We derive the QCD sum rules for the masses and pole residues of the vector molecular states in Section 2. In Section 3, we present the numerical results and discussions. Section 4 is reserved for our conclusion.

2 QCD sum rules for the vector molecular states

In the isospin limit, the quark structures of the molecular states $\mathrm{D\bar{D}_1(2420)}$ and $\mathrm{D^*\bar{D}_0^*(2400)}$ can be symbolically written as

$$\mathrm{\bar{u}d\bar{c}c}, \quad \frac{\mathrm{\bar{u}u-\bar{d}d}}{\sqrt{2}}\mathrm{\bar{c}c}, \quad \mathrm{\bar{d}u\bar{c}c}, \quad \frac{\mathrm{\bar{u}u+\bar{d}d}}{\sqrt{2}}\mathrm{\bar{c}c}. \tag{1}$$

The isospin triplet $\mathrm{\bar{u}d\bar{c}c}$, $\frac{\mathrm{\bar{u}u-\bar{d}d}}{\sqrt{2}}\mathrm{\bar{c}c}$, $\mathrm{\bar{d}u\bar{c}c}$ and the isospin singlet $\frac{\mathrm{\bar{u}u+\bar{d}d}}{\sqrt{2}}\mathrm{\bar{c}c}$ have degenerate masses. In this article, we take the isospin limit and study the masses of the charged partners of the Y(4220) and Y(4390) for simplicity.

In the following, we write down the two-point correlation functions $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

$$\Pi_{\mu\nu}(p) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}p\cdot x}\langle 0|T\left\{J_\mu(x)J_\nu^\dagger(0)\right\}|0\rangle, \tag{2}$$

where $J_\mu(x)=J_\mu^1(x), J_\mu^2(x), J_\mu^3(x), J_\mu^4(x)$,

$$\begin{aligned}
J_\mu^1(x) &= \frac{1}{\sqrt{2}}\{\bar{u}(x)i\gamma_5c(x)\bar{c}(x)\gamma_\mu\gamma_5d(x) \\
&\quad -\bar{u}(x)\gamma_\mu\gamma_5c(x)\bar{c}(x)i\gamma_5d(x)\}, \\
J_\mu^2(x) &= \frac{1}{\sqrt{2}}\{\bar{u}(x)i\gamma_5c(x)\bar{c}(x)\gamma_\mu\gamma_5d(x) \\
&\quad +\bar{u}(x)\gamma_\mu\gamma_5c(x)\bar{c}(x)i\gamma_5d(x)\}, \\
J_\mu^3(x) &= \frac{1}{\sqrt{2}}\{\bar{u}(x)c(x)\bar{c}(x)\gamma_\mu d(x)+\bar{u}(x)\gamma_\mu c(x)\bar{c}(x)d(x)\}, \\
J_\mu^4(x) &= \frac{1}{\sqrt{2}}\{\bar{u}(x)c(x)\bar{c}(x)\gamma_\mu d(x)-\bar{u}(x)\gamma_\mu c(x)\bar{c}(x)d(x)\},
\end{aligned} \tag{3}$$

Under charge conjugation transform $\widehat{C}$, the currents $J_\mu(x)$ have the properties,

$$\begin{aligned}
\widehat{C}J_\mu^{1/3}(x)\widehat{C}^{-1} &= -J_\mu^{1/3}(x)|_{u\leftrightarrow d}, \\
\widehat{C}J_\mu^{2/4}(x)\widehat{C}^{-1} &= +J_\mu^{2/4}(x)|_{u\leftrightarrow d}.
\end{aligned} \tag{4}$$

The charge conjugations of the molecular states Y(4220) and Y(4390) are unknown. If the decays take place through

$$\mathrm{Y(4220/4390)} \to \rho h_c \to h_c\pi^+\pi^-, \tag{5}$$

the charge conjugation is positive; on the other hand, if the decays take place through

$$\mathrm{Y(4220/4390)} \to Z_c^\pm(4025)\pi^\mp \to h_c\pi^+\pi^-, \tag{6}$$

the charge conjugation is negative, where we assume that there is a relative S-wave between the intermediate mesons ρh_c or $Z_c^\pm(4025)\pi^\mp$. The decay

$$\mathrm{Y(4230)} \to \omega\chi_{c0}, \tag{7}$$

has been observed [2]. If the Y(4220) and Y(4230) are the same particle, the Y(4220) may have the quantum numbers $J^{PC}=1^{--}$.

On the phenomenological side, we insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J_\mu(x)$ into the correlation functions $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation [16–18]. After isolating the ground state contributions of the vector molecular states, we get the following results:

$$\Pi_{\mu\nu}(p) = \frac{\lambda_Y^2}{M_Y^2-p^2}\left(-g_{\mu\nu}+\frac{p_\mu p_\nu}{p^2}\right)+\cdots, \tag{8}$$

where the pole residues λ_Y are defined by $\langle 0|J_\mu(0)|Y(p)\rangle = \lambda_Y\varepsilon_\mu$, and the ε_μ are the polarization vectors of the vector molecular states.

In the following, we perform Fierz re-arrangement for the currents J_μ both in color space and Dirac-spinor space to obtain the results:

$$
\begin{aligned}
J_\mu^1 = & \frac{1}{2\sqrt{2}}\Big\{\frac{1}{3}i\bar{u}\gamma_\mu d\bar{c}c-\frac{1}{3}i\bar{u}d\bar{c}\gamma_\mu c-\frac{1}{3}\bar{u}\gamma^\beta\gamma_5 d\bar{c}\sigma_{\mu\beta}\gamma_5 c+\frac{1}{3}\bar{u}\sigma_{\mu\beta}\gamma_5 d\bar{c}\gamma^\beta\gamma_5 c \\
& +\frac{1}{2}i\bar{u}\gamma_\mu\lambda^a d\bar{c}\lambda^a c-\frac{1}{2}i\bar{u}\lambda^a d\bar{c}\gamma_\mu\lambda^a c-\frac{1}{2}\bar{u}\gamma^\beta\gamma_5\lambda^a d\bar{c}\sigma_{\mu\beta}\gamma_5\lambda^a c+\frac{1}{2}\bar{u}\sigma_{\mu\beta}\gamma_5\lambda^a d\bar{c}\gamma^\beta\gamma_5\lambda^a c\Big\}, \\
J_\mu^2 = & \frac{1}{2\sqrt{2}}\Big\{\frac{1}{3}\bar{u}\sigma_{\mu\beta} d\bar{c}\gamma^\beta c+\frac{1}{3}\bar{u}\gamma^\beta d\bar{c}\sigma_{\mu\beta} c-\frac{1}{3}\bar{u}i\gamma_5 d\bar{c}\gamma_\mu\gamma_5 c-\frac{1}{3}\bar{u}\gamma_\mu\gamma_5 d\bar{c}i\gamma_5 c \\
& +\frac{1}{2}\bar{u}\sigma_{\mu\beta}\lambda^a d\bar{c}\gamma^\beta\lambda^a c+\frac{1}{2}\bar{u}\gamma^\beta\lambda^a d\bar{c}\sigma_{\mu\beta}\lambda^a c-\frac{1}{2}\bar{u}i\gamma_5\lambda^a d\bar{c}\gamma_\mu\gamma_5\lambda^a c-\frac{1}{2}\bar{u}\gamma_\mu\gamma_5\lambda^a d\bar{c}i\gamma_5\lambda^a c\Big\}, \\
J_\mu^3 = & \frac{1}{2\sqrt{2}}\Big\{-\frac{1}{3}\bar{u}\gamma_\mu d\bar{c}c-\frac{1}{3}\bar{u}d\bar{c}\gamma_\mu c-\frac{1}{3}i\bar{u}\gamma^\beta\gamma_5 d\bar{c}\sigma_{\mu\beta}\gamma_5 c-\frac{1}{3}i\bar{u}\sigma_{\mu\beta}\gamma_5 d\bar{c}\gamma^\beta\gamma_5 c \\
& -\frac{1}{2}\bar{u}\gamma_\mu\lambda^a d\bar{c}\lambda^a c-\frac{1}{2}\bar{u}\lambda^a d\bar{c}\gamma_\mu\lambda^a c-\frac{1}{2}i\bar{u}\gamma^\beta\gamma_5\lambda^a d\bar{c}\sigma_{\mu\beta}\gamma_5\lambda^a c-\frac{1}{2}i\bar{u}\sigma_{\mu\beta}\gamma_5\lambda^a d\bar{c}\gamma^\beta\gamma_5\lambda^a c\Big\}, \\
J_\mu^4 = & \frac{1}{2\sqrt{2}}\Big\{-\frac{1}{3}i\bar{u}\sigma_{\mu\beta} d\bar{c}\gamma^\beta c+\frac{1}{3}i\bar{u}\gamma^\beta d\bar{c}\sigma_{\mu\beta} c+\frac{1}{3}i\bar{u}i\gamma_5 d\bar{c}\gamma_\mu\gamma_5 c-\frac{1}{3}i\bar{u}\gamma_\mu\gamma_5 d\bar{c}i\gamma_5 c \\
& -\frac{1}{2}i\bar{u}\sigma_{\mu\beta}\lambda^a d\bar{c}\gamma^\beta\lambda^a c+\frac{1}{2}i\bar{u}\gamma^\beta\lambda^a d\bar{c}\sigma_{\mu\beta}\lambda^a c+\frac{1}{2}i\bar{u}i\gamma_5\lambda^a d\bar{c}\gamma_\mu\gamma_5\lambda^a c-\frac{1}{2}i\bar{u}\gamma_\mu\gamma_5\lambda^a d\bar{c}i\gamma_5\lambda^a c\Big\}. \qquad (9)
\end{aligned}
$$

The components $\bar{u}\Gamma d\bar{c}\Gamma' c$ and $\bar{u}\Gamma\lambda^a d\bar{c}\Gamma'\lambda^a c$ potentially couple to a series of charmonium-light-meson pairs or charmonium-like molecular states or charmonium-like molecule-like states, where $\Gamma,\Gamma' = 1,\gamma_\mu,\gamma_\mu\gamma_5,i\gamma_5,\sigma_{\mu\beta},\sigma_{\mu\beta}\gamma_5$. For example, the current J_μ^1 potentially couples to the meson pairs through its components,

$$
\begin{aligned}
\bar{u}\gamma_\mu d\bar{c}c &\propto \chi_{c0}\rho^-,\cdots, \\
\bar{u}d\bar{c}\gamma_\mu c &\propto \mathrm{J}/\psi a_0^-(980),\cdots, \\
\bar{u}\gamma^\beta\gamma_5 d\bar{c}\sigma_{\mu\beta}\gamma_5 c &\propto \mathrm{J}/\psi a_1^-(1260),\ \mathrm{J}/\psi\pi^-, \\
& \quad h_c a_1^-(1260),\ h_c\pi^-,\cdots, \\
\bar{u}\sigma_{\mu\beta}\gamma_5 d\bar{c}\gamma^\beta\gamma_5 c &\propto \eta_c\rho^-,`\chi_{c1}\rho^-,\ \eta_c h_1^-(1170), \\
& \quad \chi_{c1}h_1^-(1170),\cdots. \qquad (10)
\end{aligned}
$$

We cannot distinguish those contributions to study them exclusively, and assume that the currents $\bar{u}\Gamma d\bar{c}\Gamma' c$ and $\bar{u}\Gamma\lambda^a d\bar{c}\Gamma'\lambda^a c$ couple to a particular resonance Y, which is a special superposition of the scattering states, molecular states and molecule-like states, and embodies the net effect. Some meson pairs (in other words, its components) such as $\chi_{c0}\rho$, $\mathrm{J}/\psi a_0(980)$, $\mathrm{J}/\psi\pi$, $\cdots$ lie below the Y, so the Y can decay to those meson pairs easily through the fall-apart mechanism. Although the rearrangements in color space and Dirac-spinor space are highly non-trivial, the decays contribute a finite width to the Y.

In the following, we study the contributions of the intermediate meson-loops to the correlation function $\Pi_{\mu\nu}(p)$ for the current $J_\mu^1(x)$ as an example. The current $J_\mu^1(x)$ has nonvanishing couplings with the scattering states $\mathrm{J}/\psi a_0(980)$, $\chi_{c0}\rho$, etc.

$$
\Pi_{\mu\nu}(p) = -\frac{\widehat{\lambda}_Y^2}{p^2-\widehat{M}_Y^2-\Sigma_{\mathrm{J}/\psi a_0(980)}(p)-\Sigma_{\chi_{c0}\rho}(p)+\cdots} \times\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+\cdots, \qquad (11)
$$

where $\widehat{\lambda}_Y$ and $\widehat{M}_Y$ are bare quantities to absorb the divergences in the self-energies $\Sigma_{\mathrm{J}/\psi a_0(980)}(p)$, $\Sigma_{\chi_{c0}\rho}(p)$, etc. The renormalized self-energies contribute a finite imaginary part to modify the dispersion relation,

$$
\Pi_{\mu\nu}(p) = -\frac{\lambda_Y^2}{p^2-M_Y^2+\mathrm{i}\sqrt{p^2}\Gamma(p^2)}\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+\cdots. \qquad (12)
$$

The physical widths $\Gamma_{Y(4220)}=66.0\pm9.0\pm0.4$ MeV and $M_{Y(4390)}=139.5\pm16.1\pm0.6$ MeV [3] are not large, and the finite width effects can be absorbed into the pole residues λ_Y. In previous works, we observed that the effects of the finite widths, such as $\Gamma_{X(4500)}=92\pm21^{+21}_{-20}$ MeV, $\Gamma_{X(4700)}=120\pm31^{+42}_{-33}$ MeV, $\Gamma_{Z_c(4200)}=370^{+70+70}_{-70-132}$ MeV, can be safely absorbed into the pole residues λ_{X/Z_c} [19, 20]. In this article, we take the zero width approximation, and expect that the predicted masses are reasonable.

We carry out the operator product expansion in a consistent way, and obtain the QCD spectral densities through dispersion relation. We then take the quark-hadron duality below the continuum thresholds s_0 and perform Borel transform with respect to the variable $P^2=-p^2$ to obtain the following QCD sum rules,

$$
\lambda_Y^2\exp\left(-\frac{M_Y^2}{T^2}\right)=\int_{4m_c^2}^{s_0}\mathrm{d}s\,\rho(s)\exp\left(-\frac{s}{T^2}\right), \qquad (13)
$$

where $\rho(s)=\rho_1(s),\rho_2(s),\rho_3(s),\rho_4(s)$,

$$
\begin{aligned}
\rho_1(s) &= \rho(s,r)|_{r=1}, \\
\rho_2(s) &= \rho(s,r)|_{r=-1}, \\
\rho_3(s) &= \rho(s,r)|_{r=1,m_c\to-m_c}, \\
\rho_4(s) &= \rho(s,r)|_{r=-1,m_c\to-m_c}. \qquad (14)
\end{aligned}
$$

The explicit expressions of the QCD spectral densities $\rho(s,r)$ are given in the Appendix. In this article, we carry out the operator product expansion for the vacuum condensates up to dimension-10 and assume vacuum saturation for the higher dimension vacuum condensates. The condensates $\langle g_s^3GGG\rangle$, $\left\langle\frac{\alpha_sGG}{\pi}\right\rangle^2$, $\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\langle\bar{q}g_s\sigma Gq\rangle$ have dimensions 6, 8, 9 respectively, but they are the vacuum expectations of the operators of the order $\mathcal{O}(\alpha_s^{3/2})$, $\mathcal{O}(\alpha_s^2)$, $\mathcal{O}(\alpha_s^{3/2})$ respectively, and are discarded. We take the truncations $n\leqslant 10$ and $k\leqslant 1$ in a consistent way, and operators of orders $\mathcal{O}(\alpha_s^k)$ with $k>1$ are discarded [14, 15, 21–23]. Furthermore, the numerical values of the condensates $\langle g_s^3GGG\rangle$, $\left\langle\frac{\alpha_sGG}{\pi}\right\rangle^2$, $\left\langle\frac{\alpha_sGG}{\pi}\right\rangle\langle\bar{q}g_s\sigma Gq\rangle$ are very small, and can safely be neglected.

We derive Eq. (13) with respect to $\tau=\frac{1}{T^2}$, and eliminate the pole residues λ_Y to obtain the QCD sum rules for the masses,

$$M_Y^2=\frac{\int_{4m_c^2}^{s_0}\mathrm{d}s\left(-\frac{\mathrm{d}}{\mathrm{d}\tau}\right)\rho(s)\mathrm{e}^{-\tau s}}{\int_{4m_c^2}^{s_0}\mathrm{d}s\rho(s)\mathrm{e}^{-\tau s}}. \tag{15}$$

3 Numerical results and discussion

The vacuum condensates are taken to be the standard values $\langle\bar{q}q\rangle=-(0.24\pm0.01\,\mathrm{GeV})^3$, $\langle\bar{q}g_s\sigma Gq\rangle=m_0^2\langle\bar{q}q\rangle$, $m_0^2=(0.8\pm0.1)\ \mathrm{GeV}^2$, $\left\langle\frac{\alpha_sGG}{\pi}\right\rangle=(0.33\ \mathrm{GeV})^4$ at the energy scale $\mu=1\,\mathrm{GeV}$ [16–18, 24]. The quark condensate and mixed quark condensate evolve with the renormalization group equation, $\langle\bar{q}q\rangle(\mu)=\langle\bar{q}q\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{4}{9}}$, and $\langle\bar{q}g_s\sigma Gq\rangle(\mu)=\langle\bar{q}g_s\sigma Gq\rangle(Q)\left[\frac{\alpha_s(Q)}{\alpha_s(\mu)}\right]^{\frac{2}{27}}$. In this article, we take the $\overline{MS}$ mass $m_c(m_c)=(1.275\pm0.025)$ GeV from the Particle Data Group [8] and take into account the energy-scale dependence of the $\overline{MS}$ mass,

$$\begin{aligned} m_c(\mu) &= m_c(m_c)\left[\frac{\alpha_s(\mu)}{\alpha_s(m_c)}\right]^{\frac{12}{25}}, \\ \alpha_s(\mu) &= \frac{1}{b_0t}\left[1-\frac{b_1}{b_0^2}\frac{\log t}{t}+\frac{b_1^2(\log^2t-\log t-1)+b_0b_2}{b_0^4t^2}\right], \end{aligned} \tag{16}$$

where

$$t=\log\frac{\mu^2}{\Lambda^2},\quad b_0=\frac{33-2n_f}{12\pi},\quad b_1=\frac{153-19n_f}{24\pi^2},$$

$$b_2=\frac{2857-\frac{5033}{9}n_f+\frac{325}{27}n_f^2}{128\pi^3},$$

and $\Lambda=213$ MeV, 296 MeV and 339 MeV for the number of flavors $n_f=5$, 4 and 3, respectively [8].

The hidden charm (or hidden bottom) four-quark systems $\mathrm{Qq\bar{Q}\bar{q}'}$ could be described by a double-well potential in the heavy quark limit. The heavy quark Q serves as one static well potential and combines with the light antiquark $\bar{\mathrm{q}}'$ to form a heavy meson-like state or correlation (not a physical meson) in color singlet. The heavy antiquark $\bar{\mathrm{Q}}$ serves as the other static well potential and combines with the light quark state q to form another heavy meson-like state or correlation (not a physical meson) in the color singlet. The two meson-like states (not two physical mesons) combine together to form a physical molecular state. Then the double heavy molecular state Y is characterized by the effective heavy quark mass $\mathbb{M}_\mathrm{Q}$ and the virtuality $V=\sqrt{M_Y^2-(2\mathbb{M}_\mathrm{Q})^2}$ [14, 15]. It is natural to choose the energy scales of the QCD spectral densities as $\mu=V$, which works well in the QCD sum rules for the molecular states. In Ref. [14], we obtained the optimal value $\mathbb{M}_\mathrm{c}=1.84$ GeV. Recently, we re-checked the numerical calculations and corrected a small error involving the mixed condensates. After the small error was corrected, the Borel windows are modified slightly and the predictions are also improved slightly, but the conclusions survive. In this article, we choose the updated value $\mathbb{M}_\mathrm{c}=1.85$ GeV.

In the scenario of molecular states, we study the color singlet-singlet type and octet-octet type scalar, axial-vector and tensor hadronic molecular states with the QCD sum rules in a systematic way [14, 15], and tentatively assign the $X(3872)$, $Z_\mathrm{c}(3900/3885)$, Y(4140), $Z_\mathrm{c}(4020/4025)$ and $Z_b(10610/10650)$ to be the molecular states:

$$\begin{aligned} X(3872) &= \frac{1}{\sqrt{2}}\left(D\overline{D}^*-D^*\overline{D}\right)\ (\text{with } 1^{++}), \\ Z_\mathrm{c}(3900/3885) &= \frac{1}{\sqrt{2}}\left(D\overline{D}^*+D^*\overline{D}\right)\ (\text{with } 1^{+-}), \\ Z_\mathrm{c}(4020/4025) &= D^*\overline{D}^*\ (\text{with } 1^{+-} \text{ or } 2^{++}), \\ Y(4140) &= D_s^*\overline{D}_s^*\ (\text{with } 0^{++}), \\ Z_b(10610) &= \frac{1}{\sqrt{2}}\left(B\overline{B}^*+B^*\overline{B}\right)\ (\text{with } 1^{+-}), \\ Z_b(10650) &= B^*\overline{B}^*\ (\text{with } 1^{+-}). \end{aligned} \tag{17}$$

Now we search for the Borel parameters T^2 and continuum threshold parameters s_0 to satisfy the following four criteria:

- Pole dominance on the phenomenological side;
- Convergence of the operator product expansion;
- Appearance of the Borel platforms;
- Satisfaction of the energy scale formula.

The resulting Borel parameters, continuum threshold parameters, pole contributions and energy scales are shown explicitly in Table 1. From the Table, we can see that the central values of the pole contributions are larger than 50%, so the pole dominance condition can be satisfied. In the Borel windows, the contributions from the vacuum condensates D_i of dimension i are

$$\begin{aligned}
D\bar{D}_1(1^{--}) :\ & D_0=(94-95)\%, D_3=0\%, D_4\ll 1\%, D_5=(20-24)\%, \\
& D_6=-(12-17)\%, D_7=-(1-2)\%, -D_8<1\%, D_{10}\ll 1\%, \\
D\bar{D}_1(1^{-+}) :\ & D_0=(122-127)\%, D_3=-(24-27)\%, D_4=-1\%, D_5=(24-29)\%, \\
& D_6=-(19-27)\%, D_7=-(2-3)\%, D_8\leq 1\%, D_{10}\ll 1\%, \\
D^*\bar{D}_0^*(1^{--}) :\ & D_0=(118-122)\%, D_3=0\%, D_4\ll 1\%, D_5=-(13-16)\%, \\
& D_6=-(5-7)\%, D_7<1\%, -D_8<1\%, D_{10}\ll 1\%, \\
D^*\bar{D}_0^*(1^{-+}) :\ & D_0=(111-116)\%, D_3=(19-22)\%, D_4=-1\%, D_5=-(18-22)\%, \\
& D_6=-(13-18)\%, D_7=(1-2)\%, D_8<1\%, D_{10}\ll 1\%, \qquad (18)
\end{aligned}$$

where $i=0$, 3, 4, 5, 6, 7, 8, 10. The operator product expansion well convergent. In the QCD sum rules for the hidden charm tetraquark states and molecular states, the operator product expansion converges slowly, and we have to increase the Borel parameters to large values. Larger Borel parameters lead to smaller pole contributions on the hadron side. So in the QCD sum rules for the hidden charm tetraquark states and molecular states, the Borel windows are rather small, $T^2_{max}-T^2_{min}\approx 0.4$ GeV^2, while the lower bounds of the pole contributions are about $(40-45)\%$. From Table 1, the threshold parameters and the predicted masses satisfy the relation $\sqrt{s_0}=M_Y+(0.4\sim0.6)$ GeV. Naively, we expect that the energy gap between the ground state and the first radial excited state is about $0.4\sim0.6$ GeV, so the present predictions are reasonable. Although the lower bounds of the pole contributions are less than 50%, the contaminations of the radial excited states and continuum states are expected to be excluded by the continuum threshold parameter s_0.

We take into account all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the vector molecular states, which are shown explicitly in Figs. 1–2 and Table 1. In Figs. 1–2, we plot the masses and pole residues with variations of the Borel parameters at much larger intervals than the Borel windows shown in Table 1. From the figures, we can see that there indeed appear platforms in the Borel windows. Furthermore, from Table 1, we can see that the energy scale formula is well satisfied. Now the four criteria of the QCD sum rules are all satisfied, so we expect to make reasonable predictions.

The prediction $M_{\mathrm{D\bar{D}_1(1^{--})}}=4.36\pm0.08$ GeV is consistent with the experimental data $M_{\mathrm{Y(4390)}}=4391.6\pm6.3\pm1.0$ MeV within uncertainties [3], while the predictions $M_{\mathrm{D\bar{D}_1(1^{-+})}}=4.60\pm0.08$ GeV, $M_{\mathrm{D^*\bar{D}_0^*(1^{--})}}=4.78\pm0.07$ GeV and $M_{\mathrm{D^*\bar{D}_0^*(1^{-+})}}=4.73\pm0.07$ GeV are much larger than the upper bounds of the experimental data, $M_{\mathrm{Y(4390)}}=4218.4\pm4.0\pm0.9$ MeV and $M_{\mathrm{Y(4390)}}=4391.6\pm6.3\pm1.0$ MeV [3]. Moreover, they are much larger than the near thresholds $M_{\mathrm{D^+D_1(2420)^-}}=4293$ MeV, $M_{\mathrm{D^0D_1(2420)^0}}=4285$ MeV, $M_{\mathrm{D^{*+}D_0^*(2400)^-}}=4361$ MeV, $M_{\mathrm{D^{*0}D_0^*(2400)^0}}=4325$ MeV [8]. The present predictions only favor assigning the Y(4390) to be the $\mathrm{D\bar{D}_1}(1^{--})$ molecular state.

In Refs. [11, 12], Zhang and Huang do not distinguish the charge conjugations and obtain the masses $M_{\mathrm{D^*\bar{D}_0^*}}=4.26\pm0.07$ GeV and $M_{\mathrm{D\bar{D}_1}}=4.34\pm0.07$ GeV. In Ref. [13], Lee, Morita and Nielsen distinguish the charge conjugations and obtain the mass $M_{\mathrm{D\bar{D}_1(1^{-+})}}=4.19\pm0.22$ GeV. In this article, we distinguish the charge conjugations of the currents, calculate the contributions of the vacuum condensates up to dimension-10 in the operator product expansion in a consistent way, with the intervals of the vacuum condensates much larger than the ones in Refs. [11–13]. Moreover, we use the energy scale formula to determine the energy scales of the QCD spectral densities, which worked well in our previous works [14, 15]. We obtain the predictions $M_{\mathrm{D\bar{D}_1(1^{--})}}=4.36\pm0.08$ GeV, $M_{\mathrm{D\bar{D}_1(1^{-+})}}=4.60\pm0.08$ GeV, $M_{\mathrm{D^*\bar{D}_0^*(1^{--})}}=4.78\pm0.07$ GeV and $M_{\mathrm{D^*\bar{D}_0^*(1^{-+})}}=4.73\pm0.07$ GeV, which differ significantly from the results in Refs.[11–13], changing the conclusion.

Table 1. The Borel parameters, continuum threshold parameters, pole contributions, energy scales, masses and pole residues of the vector molecular states.

	T^2/GeV^2	$\sqrt{s_0}/\mathrm{GeV}$	pole(%)	μ/GeV	M_Y/GeV	$\lambda_Y(10^{-2}/\mathrm{GeV}^5)$
$\mathrm{D\bar{D}_1}$ (1^{--})	3.2–3.6	4.9±0.1	45–65	2.3	4.36±0.08	3.97±0.54
$\mathrm{D\bar{D}_1}$ (1^{-+})	3.5–3.9	5.1±0.1	44–63	2.7	4.60±0.08	5.26±0.65
$\mathrm{D^*\bar{D}_0^*}$ (1^{--})	4.0–4.4	5.3±0.1	44–61	3.0	4.78±0.07	7.56±0.84
$\mathrm{D^*\bar{D}_0^*}$ (1^{-+})	3.8–4.2	5.2±0.1	44–61	2.9	4.73±0.07	6.83±0.84

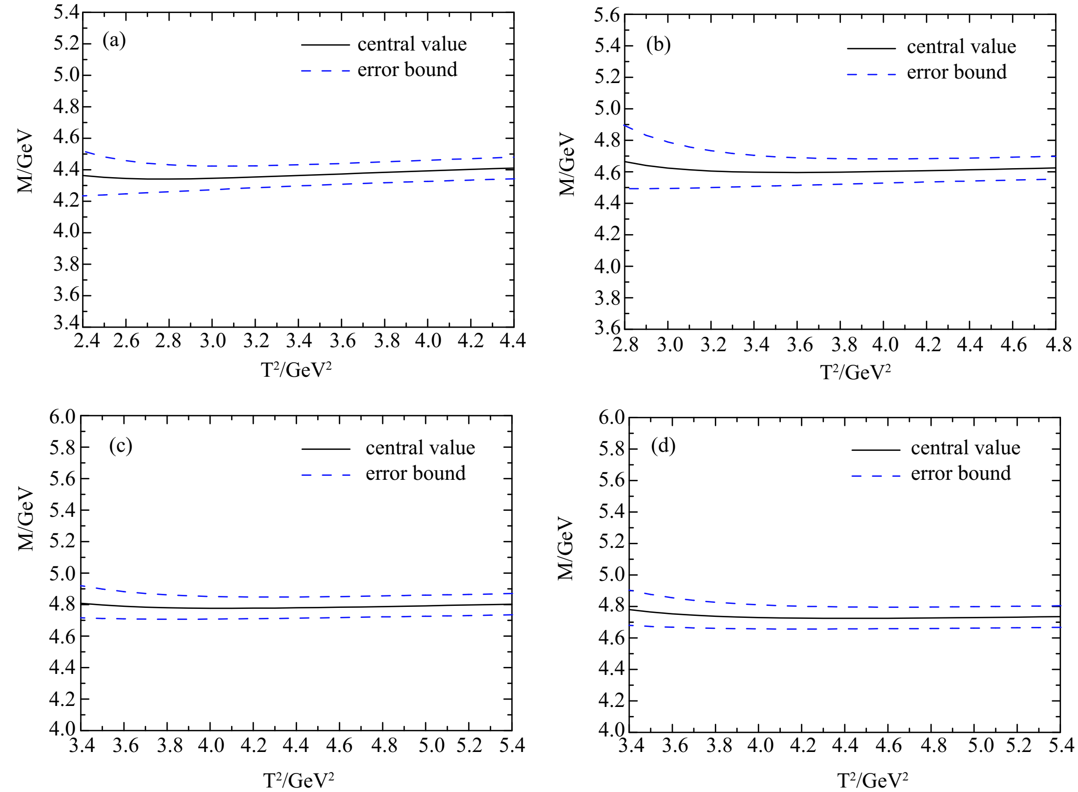

Fig. 1. (color online) The masses with variations of the Borel parameters T^2, where (a), (b), (c) and (d) denote the molecular states $D\bar{D}_1(1^{--})$, $D\bar{D}_1(1^{-+})$, $D^*\bar{D}_0^*(1^{--})$ and $D^*\bar{D}_0^*(1^{-+})$, respectively.

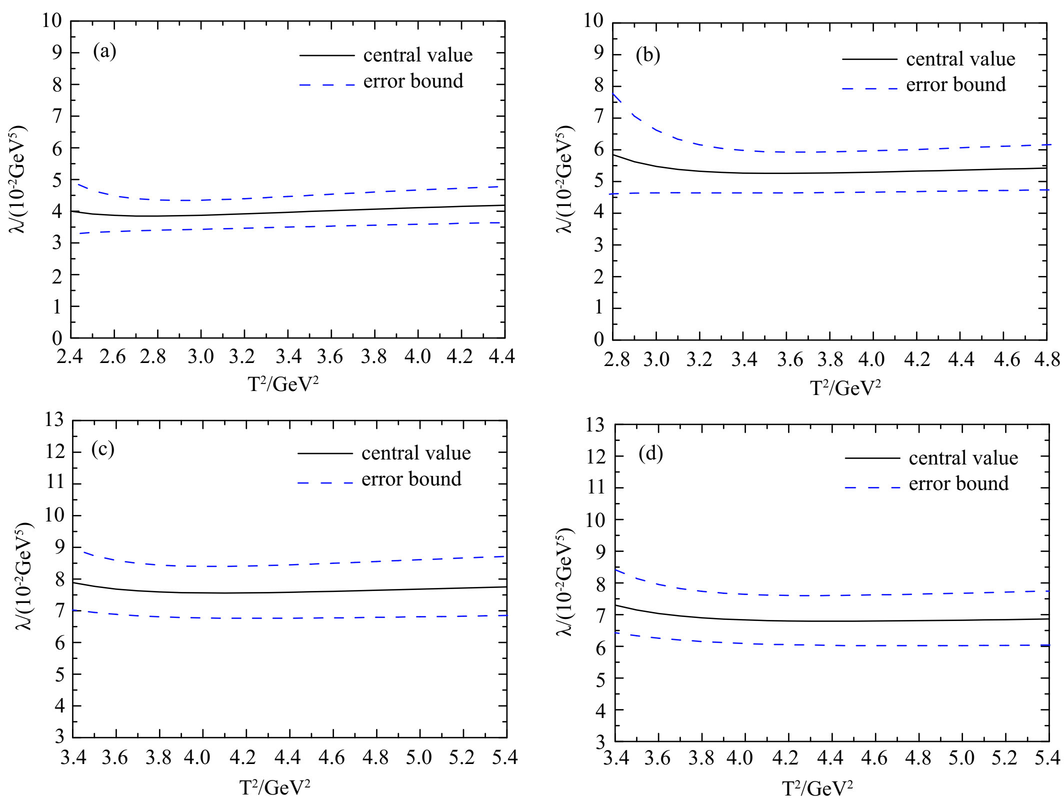

Fig. 2. (color online) The pole residues with variations of the Borel parameters T^2, where (a), (b), (c) and (d) denote the molecular states $D\bar{D}_1(1^{--})$, $D\bar{D}_1(1^{-+})$, $D^*\bar{D}_0^*(1^{--})$ and $D^*\bar{D}_0^*(1^{-+})$, respectively.

In 2007, the Belle collaboration measured the cross section for the process $e^+e^- \to \pi^+\pi^-\psi'$, and observed two structures Y(4360) and Y(4660) in the $\pi^+\pi^-\psi'$ mass spectrum, at (4361±9±9)MeV with a width of (74±15±10)MeV, and (4664±11±5)MeV with a width of (48±15±3) MeV, respectively [25]. The quantum numbers of the Y(4360) and Y(4660) are $J^{PC}=1^{--}$ [8]. The Y(4390) and Y(4360) have analogous masses and widths, so they may be the same particle, the $D\bar{D}_1(1^{--})$ molecular state. The main decay modes of the $D\bar{D}_1(1^{--})$ molecular state are $DD^*\pi$ [26, 27], so it is important to search for the decay modes $DD^*\pi$ to diagnose the nature of the Y(4260) and Y(4390).

4 Conclusion

In this article, we assign the Y(4390) and Y(4220) to be the vector molecular states $D\bar{D}_1(2420)$ and $D^*\bar{D}_0^*(2400)$, respectively, distinguish the charge conjugations,and construct the color singlet-singlet type currents to interpolate them. We calculate the contributions of the vacuum condensates up to dimension-10 in the operator product expansion in a consistent way, use the energy scale formula to determine the energy scales of the QCD spectral densities, and study the masses and pole residues in detail with the QCD sum rules. The present predictions only favor assigning the Y(4390) to be the $D\bar{D}_1(1^{--})$ molecular state.

Appendix A

The explicit expression for the QCD spectral density $\rho(s,r)$ is:

$$
\begin{aligned}
\rho(s,r) =\; & \frac{1}{4096\pi^6}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\, yz(1-y-z)^3\left(s-\overline{m}_c^2\right)^2\left(35s^2-26s\overline{m}_c^2+3\overline{m}_c^4\right) \\
&+r\frac{m_c^2}{2048\pi^6}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(1-y-z)^3\left(s-\overline{m}_c^2\right)^3 \\
&+(1-r)\frac{3m_c\langle\bar{q}q\rangle}{256\pi^4}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(y+z)(1-y-z)\left(s-\overline{m}_c^2\right)^2 \\
&-\frac{m_c^2}{3072\pi^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\left(\frac{y}{z^2}+\frac{z}{y^2}\right)(1-y-z)^3\left\{8s-3\overline{m}_c^2+s^2\delta\left(s-\overline{m}_c^2\right)\right\} \\
&-r\frac{m_c^4}{6144\pi^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\left(\frac{1}{y^3}+\frac{1}{z^3}\right)(1-y-z)^3 \\
&+r\frac{m_c^2}{2048\pi^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\left(\frac{1}{y^2}+\frac{1}{z^2}\right)(1-y-z)^3\left(s-\overline{m}_c^2\right) \\
&+\frac{1}{1024\pi^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(y+z)(1-y-z)^2 s\left(5s-4\overline{m}_c^2\right) \\
&+r\frac{3m_c^2}{2048\pi^4}\langle\frac{\alpha_sGG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\left(\frac{1}{y}+\frac{1}{z}\right)(1-y-z)^2\left(s-\overline{m}_c^2\right) \\
&-(1-r)\frac{3m_c\langle\bar{q}g_s\sigma Gq\rangle}{512\pi^4}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(y+z)\left(s-\overline{m}_c^2\right) \\
&-\frac{3m_c\langle\bar{q}g_s\sigma Gq\rangle}{256\pi^4}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\left(\frac{y}{z}+\frac{z}{y}\right)(1-y-z)\left(2s-\overline{m}_c^2\right) \\
&-r\frac{3m_c\langle\bar{q}g_s\sigma Gq\rangle}{256\pi^4}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(1-y-z)\left(s-\overline{m}_c^2\right) \\
&+r\frac{\langle\bar{q}q\rangle^2}{32\pi^2}\int_{y_i}^{y_f}\mathrm{d}y\, y(1-y)\left(s-\widetilde{m}_c^2-2r\widetilde{m}_c^2\right) \\
&+\frac{g_s^2\langle\bar{q}q\rangle^2}{864\pi^4}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\, yz\left\{8s-3\overline{m}_c^2+r\frac{m_c^2}{2yz}+s^2\delta\left(s-\overline{m}_c^2\right)\right\} \\
&-\frac{g_s^2\langle\bar{q}q\rangle^2}{1728\pi^4}\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(1-y-z)\left\{3\left(\frac{y}{z}+\frac{z}{y}\right)\left(7s-4\overline{m}_c^2\right)+\left(\frac{y}{z^2}+\frac{z}{y^2}\right)m_c^2\right. \\
&\left.\left[7+5s\delta\left(s-\overline{m}_c^2\right)\right]-(y+z)\left(4s-3\overline{m}_c^2\right)+r\left(\frac{1}{y}+\frac{1}{z}\right)\frac{3m_c^2}{2}\right\}
\end{aligned}
$$

$$
\begin{aligned}
&-(1-r)\frac{m_c^3\langle\bar{q}q\rangle}{768\pi^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(y+z)(1-y-z)\left(\frac{1}{y^3}+\frac{1}{z^3}\right)\delta\left(s-\overline{m}_c^2\right)\\
&+(1-r)\frac{m_c\langle\bar{q}q\rangle}{256\pi^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\,(1-y-z)\left(\frac{z}{y^2}+\frac{y}{z^2}\right)\\
&+\frac{3m_c\langle\bar{q}q\rangle}{128\pi^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\left\{1+\frac{4s}{9}\delta\left(s-\overline{m}_c^2\right)\right\}\\
&-r\frac{m_c\langle\bar{q}q\rangle}{256\pi^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\int_{z_i}^{1-y}\mathrm{d}z\left\{\left(\frac{y}{z}+\frac{z}{y}\right)-\frac{r(1-r)}{6}\right\}\\
&+\frac{m_c^2\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{32\pi^2}\int_{y_i}^{y_f}\mathrm{d}y\left(1+\frac{s}{T^2}\right)\delta\left(s-\widetilde{m}_c^2\right)\\
&-r\frac{3\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{64\pi^2}\int_{y_i}^{y_f}\mathrm{d}y\,y(1-y)\left\{1+\frac{s}{3}\delta\left(s-\widetilde{m}_c^2\right)\right\}\\
&+r\frac{\langle\bar{q}q\rangle\langle\bar{q}g_s\sigma Gq\rangle}{64\pi^2}\int_{y_i}^{y_f}\mathrm{d}y\left\{\frac{1}{2}-rs\delta\left(s-\widetilde{m}_c^2\right)\right\}\\
&+r\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{256\pi^2}\int_{y_i}^{y_f}\mathrm{d}y\,y(1-y)\left(3+\frac{2s}{T^2}+\frac{s^2}{2T^4}-r\frac{s^3}{T^6}\right)\delta\left(s-\widetilde{m}_c^2\right)\\
&+\frac{m_c^4\langle\bar{q}q\rangle^2}{288T^4}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\left\{\frac{1}{y^3}+\frac{1}{(1-y)^3}\right\}\delta\left(s-\widetilde{m}_c^2\right)\\
&-r\frac{m_c^2\langle\bar{q}q\rangle^2}{576T^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\left\{\frac{1-y}{y^2}+\frac{y}{(1-y)^2}\right\}\delta\left(s-\widetilde{m}_c^2\right)\\
&-\frac{m_c^2\langle\bar{q}q\rangle^2}{96T^2}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\left\{\frac{1}{y^2}+\frac{1}{(1-y)^2}\right\}\delta\left(s-\widetilde{m}_c^2\right)\\
&-r\frac{\langle\bar{q}g_s\sigma Gq\rangle^2}{512\pi^2}\int_{y_i}^{y_f}\mathrm{d}y\left(\frac{26}{27}+\frac{s}{T^2}-r\frac{2s^2}{T^4}\right)\delta\left(s-\widetilde{m}_c^2\right)\\
&+r\frac{\langle\bar{q}q\rangle^2}{96}\langle\frac{\alpha_s GG}{\pi}\rangle\int_{y_i}^{y_f}\mathrm{d}y\,y(1-y)\left(1+\frac{2s}{3T^2}+\frac{s^2}{6T^4}-r\frac{s^3}{3T^6}\right)\delta\left(s-\widetilde{m}_c^2\right), \qquad \text{(A1)}
\end{aligned}
$$

where $y_f=\frac{1+\sqrt{1-4m_c^2/s}}{2}$, $y_i=\frac{1-\sqrt{1-4m_c^2/s}}{2}$, $z_i=\frac{ym_c^2}{ys-m_c^2}$, $\overline{m}_c^2=\frac{(y+z)m_c^2}{yz}$, $\widetilde{m}_c^2=\frac{m_c^2}{y(1-y)}$, and $\int_{y_i}^{y_f}\mathrm{d}y\to\int_0^1\mathrm{d}y$, $\int_{z_i}^{1-y}\mathrm{d}z\to\int_0^{1-y}\mathrm{d}z$ when the δ functions $\delta\left(s-\overline{m}_c^2\right)$ and $\delta\left(s-\widetilde{m}_c^2\right)$ appear.

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