Polarization of protons in the optical model

Yu. A. Berezhnoy[1;1)] V. P. Mikhailyuk[2;2)]

[1] Karazin Kharkov National University, Kharkov 61077, Ukraine
[2] Institute for Nuclear Research, Kiev 03680, Ukraine

Abstract: A development of the optical model for the description of hadron-nucleus scattering is proposed. When describing the behaviour of observables for elastic proton scattering from ^{40}Ca nuclei at the energy of 200 MeV the second Born approximation is used. Analytical expressions for the scattering amplitudes as well as for the differential cross section and polarization observables were obtained. The observables calculated in this approach are in reasonable agreement with the available experimental data.

Keywords: optical model, Born approximation, intermediate energies, elastic scattering, polarization observables

PACS: 204.10.Ht, 25.45.De **DOI:** 10.1088/1674-1137/41/2/024102

1 Introduction

The development of the optical model [1–6] for the investigation of hadron scattering by nuclei is an important fundamental problem of nuclear physics.

In the optical model, hadron-nucleus scattering is considered by analogy with scattering of a wave of light by a liquid spherical drop, which is characterized by certain values of refraction and absorption indices. The particle-nucleus scattering in this case is described by a complex potential, whose real part determines the scattered wave refraction in the nuclear matter, and its imaginary part characterizes absorption of the scattered particles by the nucleus.

The essence of the optical model lies in the fact that the multiparticle interaction of a projectile with individual nucleons of the nucleus or with other particles, which can exist in the nucleus, is replaced by an effective two-particle complex potential, i.e. the complicated multi-particle problem is reduced to a simple two-particle problem. This approach greatly simplifies the calculations of the scattering observables and finds good agreement with the experimental data, as well as with a number of more important fundamental physical arguments.

Taking into account that the optical model is a powerful tool for explaining and interpreting a large number of experimental data over a wide range of energies, an important conclusion is that the concept of nuclear matter is entirely realistic. This matter is characterized by certain values of refraction and absorption coefficients for each wavelength of the scattered hadron, i.e its complex potential is equivalent to the complex refractive coefficient of the nuclear matter, and the imaginary part of such a potential describes the absorption properties of the target nucleus. In this case absorption of incident particles should be considered as their elimination from the elastic channel to various inelastic ones.

If the energy of the incident hadron is sufficiently high, the hadron-nucleus scattering amplitudes can be considered in the Born approximation (BA). Note that when considering the scattering of protons by zero-spin nuclei, it is necessary to add the spin-orbit part to the optical potential, which is, by analogy with the shell model, usually chosen in the Thomas form.

However, it turns out that the scattering amplitude calculated in the 1st BA with the Hermitian potential is real. As a result, in this approach the polarization of the nucleons from nuclei is equal to zero. Therefore, when calculating the hadron-nucleus polarization observables at least the 2nd BA should be used.

In this paper, analytical expressions for the amplitudes and the observables for elastic scattering of protons by ^{40}Ca nuclei at the energy of 200 MeV with the 2nd BA taken into account are obtained.

In Section 2 we describe the theoretical formalism, and in Section 3 we present the results of calculations and discussion.

2 Theoretical formalism

Generally, the scattering amplitude of particles from nuclei can be presented as a series in powers of perturbation

Received 20 May 2016

1) E-mail: yuberezhnoy@karazin.ua
2) E-mail: mikhailyuk@kinr.kiev.ua

$$F(\boldsymbol{k},\boldsymbol{k}')=\sum_{n=1}^{\infty}f^{(n)}(\boldsymbol{k},\boldsymbol{k}'). \tag{1}$$

Retaining in (1) only the first two terms in the Born approximation (BA) for the amplitudes $f^{(n)}(\boldsymbol{k},\boldsymbol{k}')$ we have

$$f^{(1)}(\boldsymbol{k},\boldsymbol{k}')=-\frac{m}{2\pi\hbar^2}\int \mathrm{d}^3r\mathrm{e}^{-\mathrm{i}\boldsymbol{k}'\boldsymbol{r}}U(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}}, \tag{2}$$

$$f^{(2)}(\boldsymbol{k},\boldsymbol{k}')=\left(\frac{m}{2\pi\hbar^2}\right)^2\int \mathrm{d}^3r\mathrm{d}^3r'\mathrm{e}^{-\mathrm{i}\boldsymbol{k}'\boldsymbol{r}}U(\boldsymbol{r}') \times\frac{\mathrm{e}^{\mathrm{i}k|\boldsymbol{r}-\boldsymbol{r}'|}}{|\boldsymbol{r}-\boldsymbol{r}'|}U(\boldsymbol{r})\mathrm{e}^{\mathrm{i}\boldsymbol{k}\boldsymbol{r}}, \tag{3}$$

where $m=m_1m_2/(m_1+m_2)$ is the reduced mass of the colliding particles.

In this approach we do not take into account the Coulomb interaction, because at the energy considered the contribution of this interaction is not very significant, but the theoretical calculations become noticeably complicated. Therefore, the optical potential $U(\boldsymbol{r})$ with the spin-orbit part taken into account is

$$U(\boldsymbol{r})=U_1(r)+U_2(r)(\boldsymbol{\sigma}\boldsymbol{l}). \tag{4}$$

In this formula the radial parts of such a potential are chosen in the form

$$U_1(r)=-V_0\left\{g_{\mathrm{v}}(r)+\mathrm{i}\varsigma g_{\mathrm{w}}(r)\right\}, \tag{5}$$

$$U_2(r)=\frac{aV_{\mathrm{s}}(1+\mathrm{i}\varsigma_{\mathrm{s}})}{r}\left\{\frac{\mathrm{d}g_{\mathrm{s}}(r)}{\mathrm{d}r}+\gamma R_{\mathrm{s}}\frac{\mathrm{d}^2g_{\mathrm{s}}(r)}{\mathrm{d}r^2}\right\}, \tag{6}$$

where $\varsigma=W_0/V_0$, $\varsigma_{\mathrm{s}}=W_{\mathrm{s}}/V_{\mathrm{s}}$, and values $V_{\mathrm{j}},W_{\mathrm{j}}$ denote the strengths of the real, imaginary and spin-orbit parts of the potential $U(\boldsymbol{r})$, respectively. The dimensionless parameter $\gamma=\Delta R/R$ in (6) determines the relative change of the radius of the real part of the optical potential $U(\boldsymbol{r})$ due to allowance for the spin-orbit interaction, and constant a characterizes the magnitude of the spin-orbit interaction. Note that the conventional pion Compton wave-length factor $(\hbar/m_\pi c)^2=2$, usually used in similar calculations, is included in the value of the constant a.

In (5), (6) values $g_{\mathrm{j}}(r)$ are chosen in Woods-Saxon form taking into account the differences between the parameters for real, imaginary and spin-orbit parts:

$$g_{\mathrm{j}}(r)=1/\left\{1+\exp(r-R_{\mathrm{j}})/d_{\mathrm{j}}\right\},\quad \mathrm{j}=\mathrm{v},\mathrm{w},\mathrm{s} \tag{7}$$

When determining the form of the spin-orbit part of potential (4) the following arguments are used.

Usually in the optical model the spin-orbit part of the potential $U(\boldsymbol{r})$ is assumed to be real, and its radial shape is proportional to the density gradient. Thus in (6) the parameters ς_s and γ should be considered as equal to zero.

However, as was mentioned in [7, 8], for parallel and antiparallel vectors $\boldsymbol{l}$ and $\boldsymbol{\sigma}$ the real part of the optical potential has different radii and depths. Therefore, following the approach proposed in [7, 8], we include in the calculations the second derivative in the potential (6).

Performing integration in (2) and taking into account relations (5)–(7), for the scattering amplitude $f^{(1)}(\boldsymbol{k},\boldsymbol{k}')$ we have

$$f^{(1)}(\boldsymbol{q})=f_{\mathrm{c}}(q)+f_{\mathrm{s}}(q)(\boldsymbol{\sigma}\boldsymbol{n}), \tag{8}$$

$$f_{\mathrm{c}}(q)=-\frac{2mV_0}{\hbar^2q}\left\{R_{\mathrm{v}}A_{\mathrm{v}}(q)+\mathrm{i}\varsigma R_{\mathrm{w}}A_{\mathrm{w}}(q)\right\}, \tag{9}$$

$$f_{\mathrm{s}}(q)=-\mathrm{i}\frac{2mR_{\mathrm{s}}V_{\mathrm{s}}}{\hbar^2q}(1+\mathrm{i}\varsigma_{\mathrm{s}})ak^2\sin\theta \times\left\{(1-\gamma)A_{\mathrm{s}}(q)+\gamma B(q)\right\}, \tag{10}$$

where $\boldsymbol{n}=[\boldsymbol{k},\boldsymbol{k}']/|[\boldsymbol{k},\boldsymbol{k}']|$, $\boldsymbol{q}=\boldsymbol{k}-\boldsymbol{k}'$, $|\boldsymbol{q}|\approx 2k\sin(\theta/2)$.

Here amplitudes $A_{\mathrm{j}}(q)$ and $B(q)$ are equal

$$A_{\mathrm{j}}(q)=\frac{\mathrm{d}}{\mathrm{d}q}\tilde{A}_{\mathrm{j}}(q),\quad B(q)=\frac{\mathrm{d}}{\mathrm{d}q}\tilde{B}(q), \tag{11}$$

$$\tilde{A}_{\mathrm{j}}(q)=F_{d_{\mathrm{j}}}(q)j_0(qR_{\mathrm{j}}),\quad \tilde{B}(q)=qR_{\mathrm{s}}F_{d_{\mathrm{s}}}(q)j_1(qR_{\mathrm{s}}). \tag{12}$$

In these formulae $j_n(x)$ are the spherical Bessel functions, and the damping factor $F_{d_{\mathrm{j}}}(q)$ has the form

$$F_{d_{\mathrm{j}}}(q)=\frac{\pi qd_{\mathrm{j}}}{\sinh(\pi qd_{\mathrm{j}})}. \tag{13}$$

Performing integration in (3), for the scattering amplitude $f^{(2)}(\boldsymbol{k},\boldsymbol{k}')$ we obtain

$$f^{(2)}(\boldsymbol{q})=f_{\mathrm{cc}}(q)+f_{\mathrm{ss}}(q)+2f_{\mathrm{cs}}(q)(\boldsymbol{\sigma}\boldsymbol{n}), \tag{14}$$

$$f_{\mathrm{cc}}(q)=\frac{mV_0}{2k^2\hbar^2}\left\{\mathcal{F}_1^{(\mathrm{l})}(k)f_{\mathrm{c}}(q)+q\mathcal{F}_2^{(\mathrm{l})}(k)\frac{\mathrm{d}f_{\mathrm{c}}(q)}{\mathrm{d}q}\right\}, \tag{15}$$

$$\mathcal{F}_j^{(\mathrm{l})}(k)=\mathcal{F}_j^{(\mathrm{v})}(k)+\mathrm{i}\varsigma\mathcal{F}_j^{(\mathrm{w})}(k),\quad j=1,2, \tag{16}$$

$$\mathcal{F}_1^{(\mathrm{l})}(k)=1+2\mathrm{i}kR_{\mathrm{l}}-\mathrm{e}^{2\mathrm{i}kR_{\mathrm{l}}}F_{d_{\mathrm{l}}}(2k), \tag{17}$$

$$\mathcal{F}_2^{(\mathrm{l})}(k)=1+\mathrm{e}^{\mathrm{i}kR_{\mathrm{l}}}F_{d_{\mathrm{l}}}(2k)-2j_0(kR_{\mathrm{l}})\mathrm{e}^{\mathrm{i}kR_{\mathrm{l}}}, \tag{18}$$

$$\begin{aligned}f_{\mathrm{ss}}(q)&\\ =kR_{\mathrm{s}}\cos^2\theta&\left(\frac{2maV_{\mathrm{s}}}{\hbar^2}(1+\mathrm{i}\varsigma_{\mathrm{s}})\right)^2\left\{k\frac{\mathrm{d}}{\mathrm{d}k'}-\frac{\mathrm{d}}{\mathrm{d}k'}k'\right\}\frac{\mathrm{d}}{\mathrm{d}k}\\ &\times\left[\left\{(1-\gamma)\tilde{A}_{\mathrm{s}}(q)+\gamma\tilde{B}(q)\right\}\left(1-\gamma R_{\mathrm{s}}\frac{\mathrm{d}}{\mathrm{d}R_{\mathrm{s}}}\right)\right.\\ &\times j_0(k'R_{\mathrm{s}})\mathrm{e}^{\mathrm{i}kR_{\mathrm{s}}}-\frac{k\cos\theta-k'}{q}\frac{\mathrm{d}}{\mathrm{d}k'}\\ &\times\left\{(1-\gamma)A_{\mathrm{s}}(q)+\gamma B(q)\right\}\\ &\left.\times\frac{\mathrm{d}}{\mathrm{d}R_{\mathrm{s}}}\left(1-\gamma R_{\mathrm{s}}\frac{\mathrm{d}}{\mathrm{d}R_{\mathrm{s}}}\right)\frac{1}{R_{\mathrm{s}}}j_0(k'R_{\mathrm{s}})\left(\mathrm{e}^{\mathrm{i}kR_{\mathrm{s}}-1}\right)\right],\end{aligned} \tag{19}$$

where $l=v,w$.

Note that the shape of the amplitude $f_{cs}(q)$ in (14) is the same as that given in (15)–(18), but in these formulae the amplitude $f_c(q)$ (9) should be replaced by $f_s(q)$ (10).

We also emphasize that when calculating integral (3) we used the approximation $d/R \ll 1$ as well as an expansion of the potentials $U_i(r)$ $(i=1,2)$ into the series up to the first significant terms

$$U(|\boldsymbol{u}+\boldsymbol{r}'|) \simeq U(u) + \frac{\boldsymbol{u}\boldsymbol{r}'}{u}\frac{\mathrm{d}U(u)}{\mathrm{d}u}, \tag{20}$$

where $|\boldsymbol{u}+\boldsymbol{r}'| \simeq u + \boldsymbol{u}\boldsymbol{r}'/u, \quad \boldsymbol{u}=\boldsymbol{r}-\boldsymbol{r}'$.

This expansion can be justified by the fact that the Green's function $G_0^{(+)}(|\boldsymbol{r}-\boldsymbol{r}'|)$ in (3) grows sharply at $|\boldsymbol{r}| \approx |\boldsymbol{r}|'$.

Finally, for the amplitude (1) we have

$$F(\boldsymbol{k},\boldsymbol{k}') = F_c(q) + F_s(q)(\boldsymbol{\sigma}\boldsymbol{n}), \tag{21}$$

where

$$F_c(q) = f_c(q) + f_{cc}(q) + f_{ss}(q), \quad F_s(q) = f_s(q) + 2f_{cs}(q). \tag{22}$$

3 Results and discussion

A complete description of the elastic proton scattering by zero-spin nuclei requires measuring three independent observables as functions of the scattering angle [9]. This complete set of values most frequently includes the differential cross section $\sigma(q) \equiv \mathrm{d}\sigma/\mathrm{d}\Omega$ and polarization (analyzing power) $P(q)$, as well as one of the additional independent polarization observables, namely, the spin-rotation function $Q(q)$.

The relations between these quantities and the amplitudes $F_c(q)$ and $F_s(q)$ are

$$\sigma(q) = |F_c(q)|^2 + |F_s(q)|^2, \tag{23}$$

$$P(q)\sigma(q) = 2\mathrm{Re}\left(F_c(q)F_s^*(q)\right), \tag{24}$$

$$Q(q)\sigma(q) = 2\mathrm{Im}\left(F_c(q)F_s^*(q)\right). \tag{25}$$

The analytical expressions for the observables in (23)–(25) can easily be obtained in the 1st BA. Retaining in (21), (22) only the terms associated with the 1st BA and assuming for simplicity in (9) and (12) $R_v = R_w = R_s \equiv R, d_v = d_w = d_s \equiv d$, we have

$$\sigma(q) = R^2\left(\frac{2mV_0}{\hbar^2 q}\right)^2 \sigma_1(q), \tag{26}$$

$$\sigma_1(q) = (1+\varsigma)^2 A^2(q) + a^2k^4\sin^2\theta\left\{(1-\varsigma)A(q)+\gamma B(q)\right\}^2, \tag{27}$$

$$P(q)\sigma_1(q) = \varsigma 2ak^2\sin\theta A(q)\left\{(1-\varsigma)A(q)+\gamma B(q)\right\}, \tag{28}$$

$$Q(q)\sigma_1(q) = -2ak^2\sin\theta A(q)\left\{(1-\varsigma)A(q)+\gamma B(q)\right\}. \tag{29}$$

As can be seen from the above formulae, the polarization, calculated in the 1st BA with the Hermitian potential ($\zeta=0$), is equal to zero. Therefore, when calculating the polarization observables for the scattering of particles from nuclei using BA, we include in the calculations the 2nd BA.

Using relations (21), (22), the complete set of observables for the elastic scattering of protons by ^{40}Ca nuclei at the energy of 200 MeV are calculated.

The results of such calculations are given in Figs. 1 and 2, and the obtained values of the fitting parameters of the optical potential are shown in Table 1.

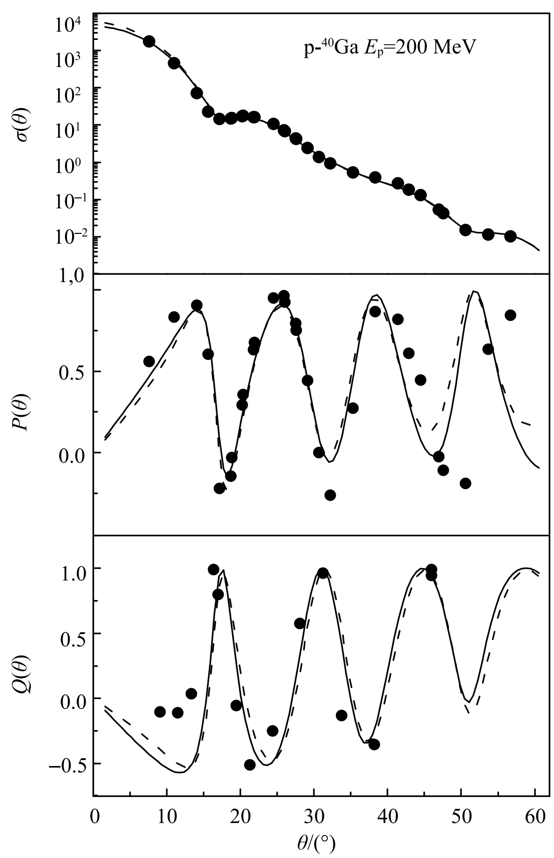

Fig. 1. Differential cross section $\sigma(\theta) \equiv \mathrm{d}\sigma/\mathrm{d}\Omega$ (mb/sr), polarization $P(\theta)$ and spin-rotation function $Q(\theta)$ for 200 MeV proton elastic scattering on ^{40}Ca nuclei, with 2nd BA. The experimental data are taken from [10]. For description of the curves see the text.

In Fig. 1 we present the results of the calculations, performed using the 2nd BA. In this Figure, the solid curves are calculated using the set of parameters 2 from Table 1, and the dashed curves are calculated with the set of parameters 3 from the same table.

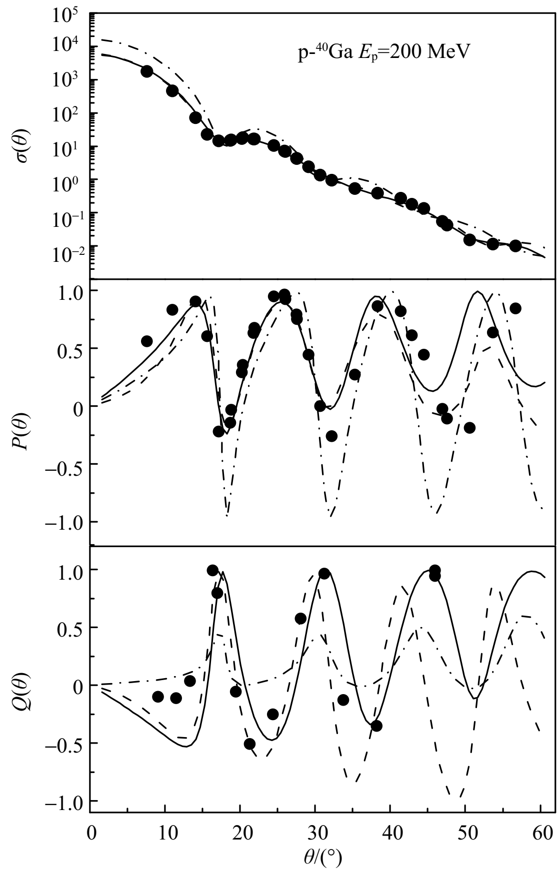

Fig. 2. The same as in Fig. 1 but with 1st and 2nd BA. For description of the curves see the text.

Note that all the parameters in Table 1 were obtained from the best fitting of the existing experimental data using FUMILI code. In Table 1, set 1 corresponds to the results obtained in the 1st BA, and sets 2 and 3 to the results obtained in the 2nd BA. We emphasize that the values of these parameters should not be the same as those obtained from the numerical solution of the Schrödinger equation. At the same time, the values of such parameters must not differ significantly from the parameters which can be obtained from the numerical solution of the Schrödinger equation. In our calculations, when obtaining the parameters of the optical potential used, we focused on the analytic relations, presented in [11–13] for 180 MeV proton energy. As Table 1 shows, the values of such parameters of the optical potential coincide with those presented in these papers.

In this paper two alternative sets of the optical potential parameters in the 2nd BA (Table 1, sets 2 and 3) and the set of parameters for such a potential in the 1st BA (Table 1, set 1) were obtained. The ambiguity of the optical potential parameters is a well-known fact. Therefore, justification, or the uniqueness of the parameters obtained in this study cannot be a subject for consideration. The main result of this work is obtaining analytical expressions for the scattering observables that cannot be done in the numerical solution of the Schrödinger equation. The presence of large number of the parameters of the optical potential does not contradict the generally accepted approaches. The attempts of many authors to reduce the number of these parameters, unfortunately, do not eliminate the problem of such ambiguity.

In Fig. 2 we present the comparison of the results obtained with the 2nd and 1st BA taken into account. In this figure the solid curves were calculated in the 2nd BA using parameter set 3 from Table 1, and the dashed curves were calculated in the 1st BA using parameter set 1 from the Table 1. The dot-dashed curves in this figure were calculated in the 1st BA using parameter set 3, obtained for the 2nd BA, i.e. calculated without any fitting parameters.

As Fig. 2 shows, using the 2nd BA allows us to describe the available experimental data more precisely as compared with those in the 1st BA.

Finally, we consider the conditions of applicability of the BA used. Usually, when determining these conditions, only the central part of the potential (4) is used. In such an approach the general formula for determining these conditions is

$$\frac{m}{2\pi\hbar^2}\left|\int \mathrm{d}^3 r \frac{U(\boldsymbol{r})}{r}\mathrm{e}^{(\mathrm{i}kr+\mathrm{i}\boldsymbol{kr})}\right| \ll 1. \tag{30}$$

Using the approximation $kR \gg 1$ ($R = 1.34A^{1/3}$) and allowing the relation (30), we have

$$\begin{gathered} kR \approx 14.5, \quad V_0 \ll V^{(\mathrm{lim})}, \\ V^{(\mathrm{lim})} = \frac{\hbar^2}{mR^2}kR \approx 29.5\ \mathrm{MeV}. \end{gathered} \tag{31}$$

Table 1 shows that in such an approach the conditions of applicability of the BA used are fulfilled.

Table 1. Parameters of the optical potential.

N	V_0/MeV	W_0/MeV	d_v/fm	R_v/fm	d_w/fm	R_w/fm	V_s/MeV	W_s/MeV	d_s/fm	R_s/fm	a/fm^2	γ
1	11.388	4.553	0.526	4.964	0.296	3.950	7.343	−4.833	0.820	4.944	0.322	0.294
2	12.813	13.351	0.506	4.789	0.532	4.459	5.763	−5.108	0.715	4.305	0.654	0.127
3	14.511	19.637	0.512	4.628	0.588	4.440	4.923	−5.111	0.736	4.391	0.582	0.164

4 Summary and conclusions

Investigation of the interaction of intermediate energy particles with nuclei has attracted great interest during many decades of the development of nuclear physics. This interest is motivated by the possibility of studying the basic properties and structure of the colliding nuclei as well as the mechanism of their interaction.

Various approaches are used for investigation of such processes. For example, the multiple proton scattering theory with "elementary" nucleon–nucleon amplitudes included in the calculations was used in [14], and the optical model with various microscopic optical potentials was applied in [15]. In [16] we introduced the approach based on the α-cluster model with dispersion, which allows us to describe a large amount of the experimental data for the elastic and inelastic scattering of protons by light nuclei from ^{9}Be to ^{24}Mg.

In the present paper analytical expressions for the amplitudes and polarization observables for the elastic scattering of particles from zero-spin nuclei were obtained. When calculating such observables the 2nd BA is used. Using of such approximation is caused by the fact that the scattering amplitude, calculated in the 1st BA with the Hermitian potential, is real. As a result, in this approach the polarization of the nucleons from nuclei is equal to zero. Therefore, when calculating the polarization observables for the scattering of particles from nuclei using BA, we included in the calculations the 2nd BA.

The limits of applicability of the BA used are also discussed. It was shown that in such an approach the conditions of applicability of the BA used are fulfilled.

The comparison between theoretical predictions and experimental data for proton elastic scattering from ^{40}Ca nuclei at 200 MeV energy were given both in the 1st and 2nd BA. The results obtained show that the calculations performed in the 2nd BA allow us to describe the available experimental data quite well and more precisely than those obtained in the 1st BA.

The parameters of the optical potential used were obtained from the best fitting of the existing experimental data, and its values do not differ significantly from those obtained in the numerical solution of the Schrödinger equation.

References

1 H. Feshbach, C. E. Porter, V. F. Weiskopf, Phys. Rev., **90**: 166–167 (1953)
2 H. Feshbach, C. E. Porter, V. F. Weisskopf, Phys. Rev., **96**: 448–464 (1954)
3 H. Feshbach, Ann. Phys. (New York), **5**: 357–390 (1958)
4 P. E. Hodgson, Nuclear reactions and nuclear structure, eds. W. Marshall and D.H. Wilinson (Clarendon Rress - Oxford, 1971), p. 661
5 H. Feshbach, Ann. Phys. (New York), **281**: 519–546 (2000)
6 R. D. Woods, D. S. Saxon, Phys. Rev., **95**: 577–578 (1954)
7 A. Dar, B. Kozlowsky Phys. Lett., **20**: 314–317 (1966)
8 S. Varma, Nucl. Phys. A, **97**: 282–288 (1967)
9 L. Wolfenstein, Annu. Rev. Nucl. Sci., **6**: 43–76 (1956)
10 H. Seifert, J. J. Kelly, A. E. Feldman et al, Phys. Rev. C, **47**: 1615–1635 (1993)
11 W. T. H. van Oers, Phys. Rev. C, **3**: 1550–1559 (1971)
12 A. Nadasen, P. Schwandt, P. P. Singh et al, Phys. Rev. C, **23**: 1023–1043 (1981)
13 P. Schwandt, H. O. Meyer, W. W. Jacobs et al, Phys. Rev. C, **26**: 55–64 (1982)
14 G. D. Alkhazov, S. L. Belostotsky, A. A. Vorobyov, Phys. Rep., **42**: 89–144 (1978)
15 L. Ray, G. W. Hoffmann, W. R. Coker, Phys. Rep., **212**: 223–328 (1992)
16 Yu. A. Berezhnoy, V. P. Mikhailyuk, V. V. Pilipenko, Int. J. Mod. Phys. E, **24**: 1530004-1–1530004-89 (2015)