

# Low-lying $1/2^-$ hidden strange pentaquark states in the constituent quark model<sup>\*</sup>

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**Abstract:** We investigate the spectrum of the low-lying  $1/2^-$  hidden strange pentaquark states, employing the constituent quark model, and looking at two ways within that model of mediating the hyperfine interaction between quarks – Goldstone boson exchange and one gluon exchange. Numerical results show that the lowest  $1/2^-$  hidden strange pentaquark state in the Goldstone boson exchange model lies at  $\sim 1570$  MeV, so this pentaquark configuration may form a notable component in  $S_{11}(1535)$  if the Goldstone boson exchange model is applied. This is consistent with the prediction that  $S_{11}(1535)$  couples very strongly to strangeness channels.

**Keywords:** quark model, pentaquark states, baryon resonances

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## 1 Introduction

The recent observation of two hidden charm exotic resonances in  $\Lambda_b \rightarrow J/\psi K^- p$  decay [1] has triggered lots of theoretical investigations of the pentaquark states, especially the doubly-heavy hadronic molecules [2–6], although the narrow structure at  $\sim 4.45$  GeV may be interpreted as a kinematical effect of the rescattering from  $\chi_{c1} p \rightarrow J/\psi p$  [7]. For recent reviews, see Refs. [8, 9]. Pentaquark states lying at  $\sim 4$  GeV, as dynamically generated nucleon and  $\Lambda$  resonances in charmed meson and baryon interactions, were first predicted at the beginning of this decade [10], then systematically studied using the constituent quark model with three different kinds of hyperfine interactions [11]. The hidden strange baryon-meson bound states were investigated very recently [12]. One bound state with  $J^P = 1/2^-$  dominated by a  $N\eta'$  component and another bound state with  $J^P = 3/2^-$  dominated by a  $N\phi$  component were found. Generally, it is worth studying the  $qqqQ\bar{Q}$  (with  $q = u, d$ ;  $Q = s, c$ ) configurations, since these configurations may play very special roles in the properties of nucleon resonances.

The structure of the nucleon excitation  $S_{11}(1535)$ , however, is still enigmatic, because of its sizable  $N\eta$  decay branch [13] and strong couplings to strangeness channels, such as  $K\Lambda$  [14] and  $N\phi$  [15]. In the traditional  $qqq$  constituent quark model, this resonance is expected to be the first orbitally excited state of the nucleon [16–18], while within the framework of unitarized coupled-

channel chiral perturbation theory,  $S_{11}(1535)$  is indicated to be a dynamically generated state [19–21], which is not a three-quark resonance but rather generated by strong channel couplings, with a dominant  $K\Sigma-K\Lambda$  component in its wave function [19]. In Ref. [14],  $S_{11}(1535)$  was first proposed to be an admixture of the traditional three-quark and  $qqqs\bar{s}$  pentaquark states with a sizable mixing angle. The  $qqqs\bar{s}$  components in  $S_{11}(1535)$  may lead to the strong couplings between  $S_{11}(1535)$  and strangeness channels, and the amazing mass ordering of  $S_{01}(1405)$ ,  $P_{11}(1440)$  and  $S_{11}(1535)$ . In fact, the first theoretical attempt to study the  $qqqs\bar{s}$  components in the nucleon was made by investigations of the strangeness magnetic moment and spin of the nucleon [22–26]. Then, the roles of the lowest  $qqqs\bar{s}$  component in the radiative and strong decays of  $S_{11}(1535)$  were explicitly investigated in Refs. [27–29], and the probability of the lowest  $qqqs\bar{s}$  component in the wave function of  $S_{11}(1535)$  predicted to be  $\sim 20\%–30\%$ . In addition, another  $S_{11}$  resonance lying at  $\sim 1730$  MeV, which should couple strongly to the  $N\eta$  channel but hardly at all to the  $N\pi$  channel, was proposed for interpretation of the  $\eta$  photoproduction data [30]. This resonance will be a  $qqqs\bar{s}$  state if it really exists.

Moreover, the low-lying  $sss\bar{q}$  states with negative parity and the newly observed  $\Omega_c^0$  resonances as pentaquark states were investigated in Refs. [31, 32]. It is very interesting that the lowest obtained  $sss\bar{q}$  state in Ref. [31] should be lower than the first orbitally excited  $\Omega$

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state predicted by traditional three-quark models. Analogously, in the present work we study the spectrum of the low-lying  $J^P = 1/2^-$  hidden strange pentaquark states, employing the constituent quark model, to investigate whether the  $qqqs\bar{s}$  states could form sizable components in the  $S_{11}$  nucleon resonances.

This manuscript has four sections. Section 2 describes the theoretical approach for the present work, our numerical results are presented in Section 3, and a brief summarizing discussion is given in Section 4.

## 2 Theoretical approach

The constituent quark model for pentaquark configurations was recently developed in Refs. [31, 32]. Here, we briefly present the key ingredients of the model.

The Hamiltonian for a five-light-quark system can be written as

$$H = \sum_{i=1}^5 \left( m_i + \frac{\vec{p}_i^2}{2m_i} \right) + \sum_{i < j} V_{\text{conf}}(r_{ij}) + H_{\text{hyp}}, \quad (1)$$

where  $V_{\text{conf}}(r_{ij})$  denotes the quark confinement potential, and  $H_{\text{hyp}}$  is the hyperfine interaction between quarks, which is often treated as a perturbation. Because all four quarks and the antiquark in the  $qqqs\bar{s}$  pentaquark system studied here are in their ground states, we take  $V_{\text{conf}}(r_{ij})$  to be the harmonic oscillator quark confinement potential,

$$V_{\text{conf}}(r_{ij}) = -\frac{3}{8} \lambda_i^C \cdot \lambda_j^C [C(\vec{r}_i - \vec{r}_j)^2 + V_0], \quad (2)$$

where  $\lambda_i^C$  denotes the Gell-Mann matrix in  $SU(3)$  color space. To calculate the matrix elements of  $\lambda_i^C \lambda_j^C$ , we need the explicit wave functions of the pentaquark states. As discussed explicitly in Ref. [33], there should be five different  $qqqs\bar{s}$  pentaquark configurations which have the quantum number  $J^P = 1/2^-$ . We show these configurations in Table 1. We have denoted the orbital, flavor, spin and color wave functions of the four-quark subsystem in the pentaquark system by Young tableaux.

Table 1. The  $qqqs\bar{s}$  configurations studied here.

$N_{\text{con}}$	FS
1	$qqqs([4]_X [211]_C [31]_{FS} [211]_F [22]_S) \otimes \bar{s}$
2	$qqqs([4]_X [211]_C [31]_{FS} [211]_F [31]_S) \otimes \bar{s}$
3	$qqqs([4]_X [211]_C [31]_{FS} [22]_F [31]_S) \otimes \bar{s}$
4	$qqqs([4]_X [211]_C [31]_{FS} [31]_F [22]_S) \otimes \bar{s}$
5	$qqqs([4]_X [211]_C [31]_{FS} [31]_F [31]_S) \otimes \bar{s}$

The general wave function for these five configurations can be expressed as

$$\begin{aligned} \psi_{i,s}^{(i)} = & \sum_{a,b,c} \sum_{Y,T_z} \sum_{S_z,t_z} C_{[31]_a [211]_a}^{[1^4]} C_{[F^{(i)}]_b [S^{(i)}]_c}^{[31]_a} \\ & \times [F^{(i)}]_{b,Y,T_z} [S^{(i)}]_{c,S_z} [211; C]_a \langle Y, T, T_z, y, \bar{t}, t_z | 1, 1/2, t \rangle \\ & \times \langle S, S_z, 1/2, s_z | 1/2, s \rangle \bar{\chi}_{y,t_z} \bar{\xi}_{s_z} \varphi_{[5]}, \end{aligned} \quad (3)$$

where the coefficients  $C_{[...][...]}^{[...]}$  are Clebsch-Gordan coefficients of the  $S_4$  permutation group. As we can see in the above equation, the color symmetry of the four-quark subsystem is  $[211]_C$ , which in fact has three orthogonal wave functions, and the matrix elements of the color operator  $\vec{\lambda}_i^C \cdot \vec{\lambda}_j^C$  in these color wave functions for different quark-quark pairs may be different. As an example, here we show the matrix elements of  $\vec{\lambda}_1^C \cdot \vec{\lambda}_2^C$  as a matrix:

$$\langle [211]_C | \lambda_1^C \cdot \lambda_2^C | [211]_C \rangle = \begin{pmatrix} 4/3 & 0 & 0 \\ 0 & -8/3 & 0 \\ 0 & 0 & -8/3 \end{pmatrix}. \quad (4)$$

Accordingly, the matrix element of  $\vec{\lambda}_1^C \cdot \vec{\lambda}_2^C$  in the color singlet  $[211]_C \otimes [\bar{1}]_C$  leads to:

$$\langle \psi_{i,s}^{(i)} | \vec{\lambda}_1^C \cdot \vec{\lambda}_2^C | \psi_{i,s}^{(i)} \rangle = -4/3. \quad (5)$$

The wave function (3) is completely antisymmetric with respect to the four-quark subsystem. In other words, changing the ordering of any two of the four quarks only results in a negative sign for the wave function, therefore, the matrix elements  $\langle \vec{\lambda}_i^C \cdot \vec{\lambda}_j^C \rangle$  for all the quark-quark pairs must be the same value,  $-4/3$ . In the case of  $j=5$ , namely, the  $j$ th quark is the antiquark, one has to re-define the color operator in Eq. (2) by the following replacement:

$$\vec{\lambda}_j^C \rightarrow -\vec{\lambda}_j^{C*}, \quad (6)$$

giving the same matrix elements  $\langle \vec{\lambda}_i^C \cdot \vec{\lambda}_5^C \rangle = -4/3$ . The quark confinement potential  $V_{\text{conf}}(r_{ij})$  reduces to

$$V_{\text{conf}}(r_{ij}) = \frac{1}{2} [C(\vec{r}_i - \vec{r}_j)^2 + V_0]. \quad (7)$$

It is convenient to define the Jacobi coordinates for the five-quark system as follows:

$$\begin{aligned} \vec{\xi}_i &= \frac{1}{\sqrt{i+i^2}} \left( \sum_{j=1}^i \vec{r}_j - i\vec{r}_{i+1} \right), \quad i=1, \dots, 4, \\ \vec{R}_{\text{cm}} &= \frac{1}{5} \sum_{i=1}^5 \vec{r}_i. \end{aligned} \quad (8)$$

Taking the Jacobi coordinates, and removing contributions from the motion of center of mass, the five coupled harmonic oscillators in Eq. (1) reduce to four decoupled ones, and the Hamiltonian for the five-quark system can be rewritten as follows:

$$H = \sum_{i=1}^5 m_i + \sum_{i=1}^4 \left( \frac{\vec{p}_i^2}{2m_i} + \frac{5C}{2} \vec{\xi}_i^2 \right) + 5V_0 + H_{\text{hyp}}, \quad (9)$$

where  $\vec{p}_i$  is the corresponding momentum operator of the Jacobi coordinate  $\vec{\xi}_i$ . Once the hyperfine interaction between quarks is taken to be a perturbation, the Hamiltonian (9) will result in a degenerate energy  $E_0$  for all five

pentaquark configurations in Table 1,

$$E_0 = \sum_{i=1}^5 m_i + 5V_0 + 6\omega, \quad (10)$$

where  $\omega = \sqrt{5C/m}$  is the harmonic oscillator parameter.

The above Hamiltonian is for a five-light-quark system. Because of the existence of the  $s\bar{s}$  pairs in the states we are studying, here we take the  $SU(3)$  breaking corrections to Eq. (9) to be [34]

$$H' = \frac{m - m_s}{m} \sum_{i=1}^4 \frac{\vec{p}_i^2 + \vec{p}_5^2}{2m_s} \delta_{is}, \quad (11)$$

where  $\delta_{is}$  is a flavor dependent operator acting on the  $i$ th quark with eigenvalue 1 for a strange quark and 0 for light quarks. Explicit calculations of the matrix elements of  $H'$  in all five configurations given in Table 1 lead to the same result

$$\langle H' \rangle = \frac{6}{5} \left( \frac{m}{m_s} - 1 \right) \omega. \quad (12)$$

Finally, we take the hyperfine interaction between quarks  $H_{\text{hyp}}$  in Eq. (9) to be either the widely accepted one gluon exchange (OGE) interaction [35–38] or the Goldstone boson exchange (GBE) interaction for multi-quark states [31, 32]. These interactions can be written as

$$H_{\text{hyp}}^{\text{OGE}} = - \sum_{i,j} C_{i,j} \vec{\lambda}_i^C \cdot \vec{\lambda}_j^C \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (13)$$

$$H_{\text{hyp}}^{\text{GBE}} = - \sum_{i,j} C_{i,j}^M \vec{\lambda}_i^F \cdot \vec{\lambda}_j^F \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (14)$$

respectively.

In addition, the favored channels of the pentaquark configurations given in Table 1, which can be obtained by analyzing the flavor structure of these configurations, are of course very interesting. In both the first two configurations, the flavor states of the four-quark subsystem are  $[211]_F$ , which indicates that these two configurations should couple strongly to the  $K\Lambda$  and  $\eta N$  channels. The third configuration, in which the flavor state of the four-quark subsystem is  $[22]_F$ , should couple strongly to the  $\eta N$  and  $K\Sigma$  channels. Finally, the configurations in which the four-quark flavor states are  $[31]_F$ , namely the last two in Table 1, should favor the  $\eta N$  channel.

### 3 Numerical results

Before moving to our numerical results, we have to discuss the parameters in the present model explicitly. As shown in Section 2, the parameters are the constituent quark masses, harmonic oscillator parameter, and the coupling strength constants in the OGE and GBE models. Values for all the parameters are taken empirically [31, 37–39], as shown in Table 2. Note that the

oscillator parameter in Ref. [31] is for the strange quark system, while the present one is for the light quark system. The coupling strength constants for K meson and  $s\bar{s}$  exchanges in the GBE model can be obtained by the following relations:

$$C^K = \frac{m}{m_s} C^\pi, \quad C^{s\bar{s}} = \left( \frac{m}{m_s} \right)^2 C^\pi. \quad (15)$$

We give the explicit numerical results in the following two subsections.

#### 3.1 Numerical results without configuration mixing

With the parameters listed in Table 2, explicit calculations of the matrix elements of the Hamiltonian (9) lead to the following results:

$$\mathcal{E}_{\text{OGE}} = \begin{pmatrix} 1962.4 & 209.7 & -41.6 & -21.3 & 19.6 \\ 209.7 & 1816.7 & 11.0 & -45.2 & -7.8 \\ -41.6 & 11.0 & 1885.8 & -32.3 & -29.0 \\ 21.3 & -45.3 & -32.3 & 2141.6 & -114.5 \\ 19.6 & -7.8 & -29.0 & -114.5 & 2249.3 \end{pmatrix}, \quad (16)$$

$$\mathcal{E}_{\text{GBE}} = \begin{pmatrix} 1579.9 & 0 & 0 & -32.9 & 0 \\ 0 & 1632.3 & -35.8 & 0 & 25.3 \\ 0 & -35.8 & 1701.7 & 0 & 10.3 \\ -32.9 & 0 & 0 & 1704.1 & 0 \\ 0 & 25.3 & 10.3 & 0 & 1771.1 \end{pmatrix} \quad (17)$$

in units of MeV, for the OGE and GBE models, respectively.

Table 2. Values for the parameters in the present work (in units of MeV).

OGE	$m$	340	$m_s$	460	$\omega$	225	$V_0$	-208
	$C_{q\bar{q}}$	18.3	$C_{q\bar{s}}$	11.2	$C_{s\bar{s}}$	6.8	$C_{q\bar{q}}$	29.8
	$C_{q\bar{s}}$	18.4	$C_{s\bar{s}}$	8.6				
GBE	$m$	340	$m_s$	460	$\omega$	225	$V_0$	-269
	$C^\pi$	21						

As we can see in the matrices (16) and (17), the numerical results for the two hyperfine interaction models are very different. Firstly, for the diagonal terms, energies in the OGE model are several hundred MeV higher than those in the GBE model. Furthermore, the diagonal energy splitting in the GBE model is much smaller than that in the OGE model. Secondly, all the non-diagonal terms in Eq. (16) are non-zero, while 12 of those in Eq. (17) vanish. This difference is because the quark-antiquark hyperfine interactions mediated by OGE are strong, but those mediated by GBE are assumed to be very small and are already included in the quark-quark interactions [18]. If one neglects the quark-antiquark

interactions in the OGE model, then 12 non-diagonal terms in Eq. (16) will also vanish.

Compared to the results obtained in Ref. [39], the diagonal numerical results in Eq. (17) are somewhat lower. This is because we have taken into account the flavor  $SU(3)$  breaking effects in the present work, as explained in Section 2. As discussed in the above paragraph, 8 of the 20 non-diagonal matrix elements are non-zero. As we know, all the non-diagonal matrix elements are from the hyperfine interaction between quarks, and are in fact not negligible. If the  $SU(3)$  breaking effects are not taken into account, namely, the coupling strength constants for different meson exchanges are taken to be the same, all the non-diagonal matrix elements of the flavor-dependent operator in GBE will vanish, and almost all the non-diagonal matrix elements in Eq. (17) should be zero. In other words, flavor  $SU(3)$  symmetry breaking in the GBE interaction should lead to non-negligible configuration mixing effects, which have not been considered in Ref. [39].

### 3.2 Numerical results with configuration mixing corrections

Diagonalization of Eqs. (16) and (17) leads to the energies of the physical states and the configuration mixing coefficients given in Table 3. The lowest state in the GBE model lies at  $\sim 1572$  MeV, which is very close to the mass of  $S_{11}(1535)$ . Accordingly, this state may form a notable component in  $S_{11}(1535)$ . The next-to-lowest state in the GBE model lies at  $\sim 1612$  MeV, so it may be a component of  $S_{11}(1650)$ . However, the spin configuration for the three quark component of  $S_{11}(1650)$  is the completely symmetric  $[3]_S$  in the traditional constituent

quark model, which may weaken the coupling between the traditional three-quark component of  $S_{11}(1650)$  and the pentaquark states obtained here. Consequently, we conclude that the qqqs $\bar{s}$  pentaquark states in the GBE model may take larger probabilities in  $S_{11}(1535)$  than in  $S_{11}(1650)$ .

The other three pentaquark states obtained with the GBE model lie in the range 1700–1800 MeV. As we know, there is no well established  $S_{11}$  nucleon resonance in this energy range in either the traditional three-quark theoretical picture or in hadronic experiments. However, in Ref. [30], another  $S_{11}$  nucleon resonance lying at  $\sim 1730$  MeV has previously been proposed to fit the  $\eta$  photoproduction data. It is claimed that the coupling of this new  $S_{11}$  resonance to the  $N\pi$  channel should be very weak, but its coupling to channels with strangeness should be strong. Consequently, one may conclude that the proposed third  $S_{11}$  resonance may be dominated by the three higher states obtained in the present GBE model, if it really exists.

Moreover, as we can see in Table 3, the first two states obtained in the GBE model are dominated by the first and second configurations shown in Table 1, respectively. Therefore, as mentioned in Section 2, these two states should couple strongly to the  $K\Lambda$  and  $\eta N$  channels, which is consistent with the previous predictions. The last three states obtained in the GBE model are dominated by configurations |4>, |3> and |5> shown in Table 1, respectively. So if the suggested  $S_{11}(1730)$  is dominated by these states, then  $S_{11}(1730)$  should couple strongly to the  $\eta N$  channel, which is consistent with the prediction in Ref. [30].

Table 3. The masses of the uud $\bar{s}$  pentaquark states studied, and the configuration mixing coefficients. The numbers in the second row are the energies for the five pentaquark states in unit of MeV, and the numbers in columns 2-11 are the corresponding mixing coefficients for the configurations in Table 1. The left and right panels are results obtained with the OGE and GBE models, respectively.

	OGE					GBE				
	1661	1874	2069	2124	2327	1572	1612	1712	1717	1776
1)	0.577	0.049	0.434	0.678	-0.129	0.971	0.000	-0.241	0.000	0.000
2)	-0.804	0.176	0.204	0.523	-0.088	0.000	0.908	0.000	0.388	-0.159
3)	0.135	0.951	-0.278	-0.001	0.025	-0.000	0.384	-0.000	-0.921	-0.061
4)	-0.049	0.208	0.738	-0.342	0.541	0.241	0.000	0.971	-0.000	0.000
5)	-0.033	0.138	0.386	-0.386	-0.826	0.000	-0.170	0.000	-0.006	-0.985

As shown in the left panel of Table 3, the obtained states in the OGE model lie in a wide range, from  $\sim 1660$  MeV to  $\sim 2300$  MeV. Mixing between the five pentaquark configurations given in Table 1 is very strong, so we can only conclude that all the states obtained in the OGE model should couple strongly to the strangeness channels (we cannot predict exactly which channels). As

we can see, the energy of the lowest state in the GBE model is very close to the mass of  $S_{11}(1650)$ . However,  $S_{11}(1650)$  does not couple strongly to the strangeness channels. As we know, the partial decay width of  $S_{11}(1650)$  to  $K\Lambda$  is less than that to  $N\pi$  by two orders of magnitude. Therefore, in general, it is unreasonable to say the strangeness pentaquark configurations take large

probabilities in  $S_{11}(1650)$ . Just as we have discussed above, the completely symmetric spin configuration  $[3]_S$  may weaken the coupling between the three- and five-quark components in  $S_{11}(1650)$ , so the lowest state in the present OGE model may be not a dominant component of  $S_{11}(1650)$ . One may also try to analyze the probability of this strangeness state in  $S_{11}(1535)$  by considering the coupling between three- and five-quark components, although it is  $\sim 100$  MeV higher. For the other four states obtained, three are higher than 2 GeV, but the experimental data for the  $1/2^-$  nucleon resonances above 2 GeV is still poor [13]. One may look for these resonances in hadronic reactions with strangeness final states.

Finally, it is very difficult for us to say which hyperfine interaction model is more reasonable, since both the two models can reproduce the spectrum of baryon resonances below 2 GeV very well, and neither of the two models is perfect. This is why we make the calculations using both the models. One may examine these models by fitting the strong or electromagnetic decay data of nucleon resonances.

## 4 Summary

In this manuscript, we have investigated the spectrum of the low-lying  $1/2^-$  hidden strange pentaquark states, within the constituent quark model. The hyperfine interaction between quarks was considered to be mediated by either Goldstone boson exchange or one gluon exchange.

Our numerical results show that the lowest state in the GBE model lies at  $\sim 1570$  MeV, which is very close to the mass of the nucleon resonance  $S_{11}(1535)$ . This indicates that the lowest strangeness pentaquark state may form a notable component in  $S_{11}(1535)$ . This is consistent with the large partial width of  $S_{11}(1535) \rightarrow \eta N$  and the strong couplings between  $S_{11}(1535)$  and the chan-

nels with strangeness, such as  $K\Lambda$  and  $\phi N$ . Three of the other states obtained in the GBE model lie in the range  $\sim 1700$ – $1800$  MeV, which may correspond for the third  $S_{11}$  resonance proposed by Saghai and Li [30] to fit the  $\eta$  photoproduction data, since it is claimed that the proposed  $S_{11}(1730)$  must couple strongly to the  $\eta N$  channel, and very weakly to the  $\pi N$  channel.

The numerical results obtained in the OGE model show very strong mixing between different pentaquark configurations, so we can only conclude that all the states obtained in the OGE model should couple strongly to the strangeness channels (we cannot predict the exact channels). The resulting hidden strange pentaquark states lie in the energy range  $\sim 1660$  MeV to  $\sim 2300$  MeV. Three of the five states obtained are higher than 2 GeV, but since the experimental data for nucleon resonances above 2 GeV is still poor, we cannot come to a solid conclusion from these numerical results. One can try to search for these kinds of states in hadronic reactions with strangeness final states, such as kaon and  $\Lambda$  resonances.

It is of course also very interesting to study the pentaquark states discussed here in the meson-baryon molecular picture. To do this, one has to rewrite the wave functions in the basis of (3,2) clustering type rather than the (4,1) basis used here. Then, diagonalization of the energy matrices will lead to explicit mixing between the meson-baryon molecular components whose color wave functions must be  $\mathbf{1}_{5q}^C = \mathbf{1}_{qqq}^C \otimes \mathbf{1}_{q\bar{q}}^C$ , and the pure compact five-quark components whose color wave functions are  $\mathbf{1}_{5q}^C = \mathbf{8}_{qqq}^C \otimes \mathbf{8}_{q\bar{q}}^C$ , in each physical state. These kinds of investigations are now in progress.

Finally, it is difficult for us to say which hyperfine model is more reasonable, OGE or GBE, since both models can reproduce the baryon spectrum below 2 GeV very well. One may further examine these two models by fitting the experimental data for the electromagnetic and strong decays of baryon resonances.

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