Parameter design considerations for an oscillator IR-FEL *

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Abstract: An infrared oscillator FEL user facility will be built at the National Synchrotron Radiation Laboratory at in Hefei, China. In this paper, the parameter design of the oscillator FEL is discussed, and some original relevant approaches and expressions are presented. Analytic formulae are used to estimate the optical field gain and saturation power for the preliminary design. By considering both physical and technical constraints, the relation of the deflection parameter K to the undulator period is analyzed. This helps us to determine the ranges of the magnetic pole gap, the electron energy and the radiation wavelength. The relations and design of the optical resonator parameters are analyzed. Using dimensionless quantities, the interdependences between the radii of curvature of the resonator mirror and the various parameters of the optical resonator are clearly demonstrated. The effect of the parallel-plate waveguide is analyzed for the far-infrared oscillator FEL. The condition of the necessity of using a waveguide and the modified filling factor in the case of the waveguide are given, respectively.

Keywords: free electron laser, oscillator, parameters design

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1 Introduction

An infrared free electron laser (IR-FEL) user facility will be built at the National Synchrotron Radiation Laboratory (NSRL) of the University of Science and Technology of China (USTC), and will be dedicated to fundamental research in energy chemistry. The facility includes two FEL oscillators driven by a linac. The mid-infrared oscillator will generate a laser with a wavelength range of 2.5–50 μm, and the far-infrared oscillator will work in the wavelength range of 40–200 μm.

Though several very successful IR-FEL user facilities have already been developed worldwide (for example, see Refs. [1–3]), the different requirements from users and the broad wavelength range still bring us great challenges in the design of this facility. One main goal of the design and optimization for oscillator FELs is to achieve the maximum output power by maximizing the gain and minimizing the passive loss of the optical cavity for the entire operating wavelength range. Many parameters affect the performance of a FEL. They include parameters of the electron beam, of the undulator and of the optical resonator. These parameters impinge on one another, so a comprehensive consideration and optimization must be undertaken.

Compared with a single-pass high gain FEL, oscillator FELs work in multi-pass low gain mode, usually at longer wavelengths. Though the requirements of the electron beam parameters for an oscillator FEL are not as stringent as for a high gain FEL, it requires high repetition rate electron beam and a high precision optical cavity. Moreover the physical effects therein are more serious and complicated than the high gain FEL case. These effects include the diffraction effect [1,4], the pulse slippage effect [5], the effect of optical field multi-modes [6–9] and so on. In addition, the non-ideal electron beam makes it more difficult to accurately analyze and simulate the oscillator FEL performance, and the accumulation error effect in the multi-pass gain simulation (typically several hundred passes) also increases the inaccuracy of the simulation. All these place greater demands on the analysis and simulation in the design and optimization of oscillator FELs. In this paper, we give a general consideration for the parameter design of the facility. First we give a description of the optical gain and the power of the oscillator FEL, then we analyze the design and optimization of undulator parameters; next we analyze the relations and choice of the optical resonator parameters; and finally we discuss the issues of the waveguide.

2 Optical gain and power of the oscillator FEL

Usually oscillator FELs work in the low gain region

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(the interaction length is shorter than three gain lengths [10]), and the initial gain is the small signal gain:

$$g_{\rm ss} = -\left(\frac{L}{\sqrt{3}L_{\rm g}}\right)^3 \left\langle \frac{\partial}{\partial x}\sin c^2\left(\frac{x}{2}\right)\right\rangle_{\phi_0'}, \qquad (1)$$

where $L_{\rm g} = 1/2k_{\rm u}\sqrt{3}\rho$ is the power gain length; ρ is the FEL parameter, $\rho = \{\lambda_{\rm u}^2 K^2 f_{\rm c}^2 I_{\rm p}/4\pi\sigma_{\rm e} I_{\rm A}\}^{\frac{1}{3}}/2\gamma$; $k_{\rm u} = 2\pi/\lambda_{\rm u}$, $\lambda_{\rm u}$ is the undulator period; K is the dimensionless vector potential of the undulator magnetic field; $L = N\lambda_{\rm u}$ is the undulator length; $I_{\rm p}$ is the peak current of the electron beam; $I_{\rm A} = 17$ kA is the Alfven current; $\sigma_{\rm e}$ and γ are the RMS transverse size and the dimensionless energy of the electron beam, respectively; $f_{\rm c}$ is the undulator coupling factor - for a planar undulator it is the difference between two Bessel functions, $f_{\rm c} = [J,J] = J_0(\xi) - J_1(\xi)$ with $\xi = K^2/(4+2K^2)$, while for a helical undulator $f_{\rm c} = 1$; $x = \phi_0' L$; and the angular bracket represents the average over the electron's initial phase velocities (i.e. the tuning parameter) ϕ_0', which is proportional to the electron's initial energy. For a given initial energy distribution $f(\phi_0')$ of the electron beam, the small signal gain can be calculated by the following asymptotic formula [11]

$$g_{\rm ss} \approx (4\pi N\rho)^3 \sum_{k=1}^{2^n-1} \frac{2^n-k}{2^n}\frac{k}{2^{2n-1}}{\rm Im}\left[\phi\left(\frac{k}{2^n}\right)\right], \qquad (2)$$

where $\phi(t)$ is a characteristic function of the probability distribution $f(x)$. Usually the calculation to $n = 4$ is sufficient.

As the optical field intensity increases, the gain decreases. When the gain is equal to the total loss in the optical resonator, the net gain is equal to zero, then the system reaches equilibrium and the optical field reaches its maximum, namely saturation. For the given initial small signal gain $g_{\rm ss}$ and the total loss ratio in the optical resonator α, which includes the output coupling fraction and the passive loss (see Section 4), we give the calculation expression of the saturation power in the resonator [12, 13] as below

$$P_{\rm s} = \frac{1-\alpha-{\rm e}^{-g_{\rm ss}}}{\alpha}P_{\rm c}, \qquad (3)$$

where $P_{\rm c}$ is the optical power when the gain is down to about half of the initial small signal gain,

$$P_{\rm c} = \frac{g_{\rm ss}}{\beta}\rho P_{\rm e}. \qquad (4)$$

$P_{\rm e}$ is electron beam power and

$$\beta = \left(\frac{L}{\sqrt{3}L_{\rm g}}\right)^7 \left\langle \frac{1}{x^7}\{x(6-x^2)\sin x + 4(1-x^2)\cos x + \frac{3}{2}x\sin 2x + \frac{1}{2}\left(\frac{5}{2}-x^2\right)\cos 2x - \frac{21}{4}\right\}\right\rangle_{\phi_0'}. \qquad (5)$$

For a mono-energy electron beam the angular bracket parts of $g_{\rm ss}$ and β are shown in Fig. 1.

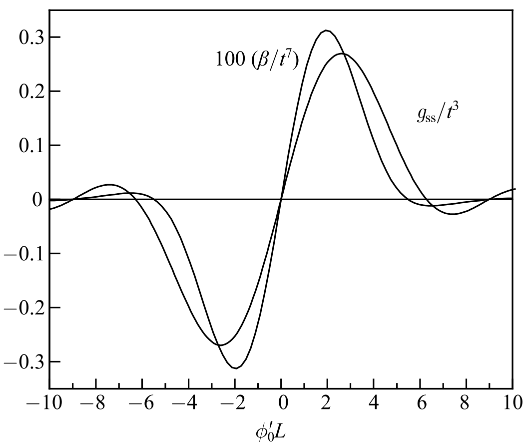

Fig. 1. For a mono-energy electron beam, the dependence of $g_{\rm ss}$ and β on the tuning parameter (Eqs.(1) and (5), $t = L/\sqrt{3}L_{\rm g}$).

The macro-pulse length of the electrons should be longer than the saturation time. It can be estimated as [12, 13]

$$T_{\rm e} > M\frac{2L_{\rm c}}{c} \approx \frac{4L_{\rm c}\ln(P_{\rm s}/P_0)}{c(g_{\rm ss}-\alpha-g_{\rm ss}\alpha)}, \qquad (6)$$

where $L_{\rm c}$ is the resonator length, M is the number of round trips of the optical pulse in the resonator needed to reach saturation, and P_0 is the initial emission power, i.e. the spontaneous emission power [14]:

$$P_0 = (2k_{\rm u}\rho L)^2 \frac{1}{N_{\rm e,s}}\rho P_{\rm e}, \qquad (7)$$

where $N_{\rm es}$ is the number of electrons per slippage distance.

3 Undulator parameter choice

The main undulator parameters include the magnetic field intensity and period. When the undulator is built, usually the period is fixed and the magnetic field intensity can be changed by changing the gap of the magnetic pole. The choice of the magnetic field intensity and period is constrained by both physics and technique.

In physics, the undulator period and deflection parameter K must satisfy the resonant relation

$$\lambda_{\rm s} = \frac{\lambda_{\rm u}}{2\gamma^2 n}\left(1+\frac{K^2}{2}\right). \qquad (8)$$

Simultaneously, K itself is a function of the undulator period, $K = 0.934\lambda_{\rm u}({\rm cm})B_{\rm u}({\rm T})$. Most often the appropriate value of K is in the range of about 1 to 3. If K is too small the radiation power will be too weak, while for large K, the harmonic component of the radiation will be too much. For a given wavelength of the radiation and period of the undulator, it has the FEL parameter $\rho \propto (K[J,J])^{2/3}/(2+K^2)^{1/2}$. Therefore we know that ρ has a maximum value, namely the gain has a maximum (Eq. (1)), when the undulator parameter K=1.556 (Fig. 2). (The optimal value of 1.2 has been given, but actually it is optimal only for the ρ with fixed radiation wavelength and electron energy. For the gain, the small signal gain of the low gain is proportional to $(\rho/\lambda_u)^3$, and the gain length of the high gain is proportional to $(\rho/\lambda_u)^{-1}$, while for fixed radiation wavelength and electron energy, the undulator period λ_u varis with K, so in this case, there is no optimal value of K for the gain, the larger the better.)

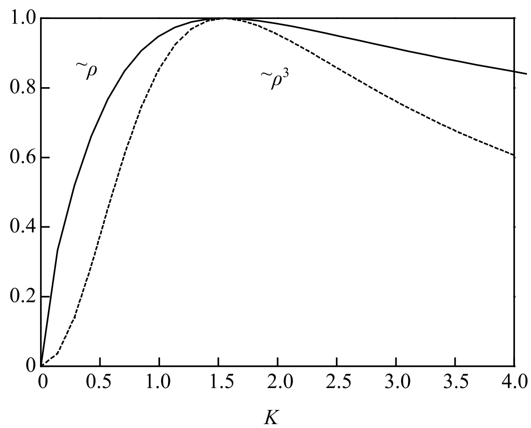

Fig. 2. The variation of FEL parameter ρ with undulator parameter K for a given radiation wavelength and undulator period.

Usually the tuning of FEL wavelength is realized by changing the intensity of the undulator magnetic field or the energy of the electron, but the former method is used more often. From the above, the value range of K should be taken around 1.556. The variation of the wavelength as the intensity of the undulator magnetic field changes is shown in Fig. 3. For the range of K =1–3, the wavelength increased by a factor of three. For our project, the tuning range of the wavelength is broad (two orders of magnitude), and adjustments of both the magnetic field and the electron energy are adopted.

In technique, the limitation of the magnetic field strength is correlated with the undulator period. In principle the undulator period should be as short as possible, so that it can have more periods in a given length. Owing to the magnetic field strength and the ratio of the magnet gap to undulator period having the relation $B_m \propto 1/\text{ch}(\pi g/\lambda_u)$, along with the decrease of the undulator period the undulator magnetic gap should also be decreased to ensure the magnetic field strength does not decrease. But the minimum gap of the undulator magnetic pole is constrained by the beam dynamic aperture and the vacuum chamber size, and the diffraction loss also must be considered for the narrower magnetic gap. Even more, with decreasing undulator period, in order to ensure the proper value of K, the magnetic field strength should not only not be decreased, but should be increased. For an undulator with a given period, the maximum achievable magnetic field strength is determined by the minimum gap of the undulator magnetic pole.

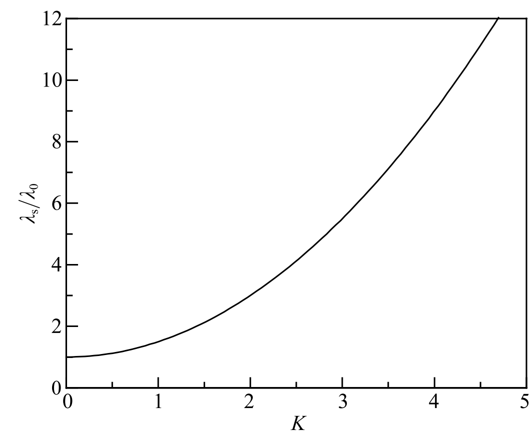

Fig. 3. The variation of the wavelength with the deflection parameter K, ($\lambda_0 = \lambda_u/2\gamma^2$).

Figure 4 gives the dependence of the undulator parameter K on the magnetic gap and period for both a hybrid permanent magnet undulator and a pure permanent magnet undulator. Their relation can be described by the empirical formula [15]

$$B_0 = a\mathrm{e}^{-\frac{g}{\lambda_u}(b-c\frac{g}{\lambda_u})}, \tag{9}$$

where a, b, and c are constants related to the remnant field strength of the permanent magnet. For a hybrid undulator these are:

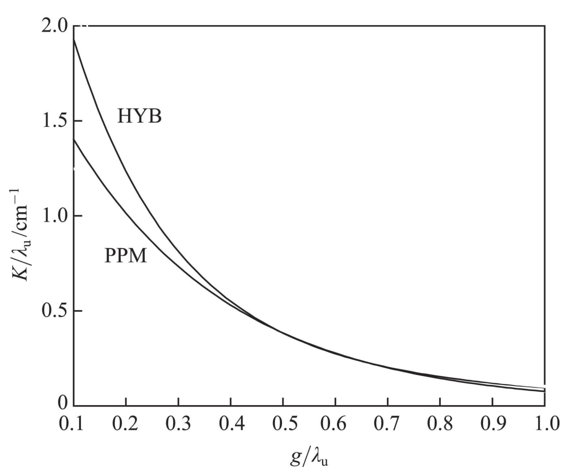

Fig. 4. The dependence of the undulator parameter K on the magnet gap and period.

$$\begin{cases} a = 0.54B_r + 2.778 \\ b = -1.95B_r + 7.225 \qquad 0.07 \leqslant g/\lambda_u \leqslant 0.7 \\ c = -1.3B_r + 2.97 \end{cases}$$

In Figure 5 we plot the relation of the deflection parameter K to the undulator period in both physics and technique. From the figure, for a given undulator period the variation range of K can be given as the magnetic field gap varies, and consequently the range of the radiation wavelength can be determined for the given electron energy or vice versa (Fig. 6). For example, if $\lambda_u = 3.8$ cm, $g_{min} = 16$ mm, the value range of K is from about 2 to about 0.3 (this is around the optimal value 1.556, as indicated by the arrow in Fig. 5). Then for the shortest wavelength, 2.5 μm in our project, the corresponding electron energy should be at least 45 MeV; while for the longest wavelength, 200 μm in this project, the corresponding electron energy is about 8 MeV (shown by the arrows in Fig. 6).

The above analysis in this section can also be applied to the high gain FEL case.

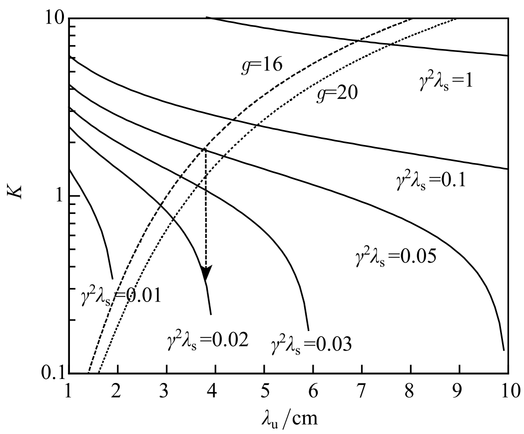

Fig. 5. The relations of the deflection parameter K to the undulator period in both physics (solid lines, Eq. (8) and technique (dotted lines, Eq. (9)).

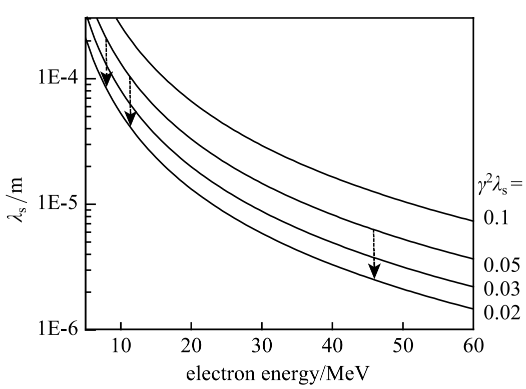

Fig. 6. The relation of the radiation wavelength and the electron energy for different given undulator parameters.

In principle, the longer the length of the undulator, the larger the FEL gain is (for an undulator length longer than three gain lengths, the initial gain will no longer be regarded as the small signal gain.). It does not follow, however, that the saturation power also increases with undulator length (from equation 3, we can also see this point). Besides, the diffraction effect of the optical beam and the slippage effect between the light pulse and the electron pulse will become serious with the increase of the undulator length, which will make the gain degrade and must be considered, especially for FELs working at longer wavelengths such as the far IR and THz bands.

From the requirement that the gain bandwidth due to the field error and the angular deflection should be smaller than the natural intrinsic bandwidth, the requirements for the peak field and the first integral of the field can be given respectively by:

$$\frac{\Delta B_m}{\bar{B}_m} < \left(1+\frac{2}{K^2}\right)\frac{1}{2N}, \quad I(\mathrm{Gs}\cdot\mathrm{m}) < 17\sqrt{\left(1+\frac{K^2}{2}\right)\frac{1}{2N}}. \tag{10}$$

For the second integral of the field, we require the transverse offset of electrons in the undulator to be smaller than the undulating amplitude of the electrons. We give the requirement for RMS second integral of the field (excluding the end parts) as

$$\sigma_{II} < \frac{\bar{B}_m}{4k_u^2}. \tag{11}$$

The effect of the phase error on the small signal gain is the same as that on the spontaneous radiation [16, 17]:

$$R \cong e^{-\sigma_\phi^2}. \tag{12}$$

The most important of the requirements is that for the second field integral (Eq. (11)).

4 Optical resonator

Optical resonators provide optical positive feedback, and establish and maintain the light oscillation. The parameters of the resonator govern the laser oscillation modes and the characteristics of the output laser beam. Careful design of the resonator parameters is needed to achieve a stable output laser beam with high power, particular pulse structure, and broadband tuning.

The length of the resonator is determined mainly by the synchronization condition between the round trip time of the optical pulse in the resonator and the electron pulse spacing. At the same time, the optical pulse number in the resonator and the practical installation space should be taken into consideration. The longer the resonator length, the more optical pulses there can be in the resonator, which can give a larger average power. The longer resonator also requires a longer electron macropulse to reach saturation (Eq. (6)). For the practical installation space, apart from the undulator, the space needs to include the bending and focusing magnets, the measurement and diagnostic equipment and so on.

The characteristic length of the diffraction effect is the Rayleigh length $Z_{\rm R}$, which is determined by the resonator length and the radii of curvature of the resonator mirror. Usually the dominant transverse mode in the resonator is the fundamental mode, the Gaussian mode. For a Gaussian beam the spot size corresponding to a field amplitude of $1/e$ of the maximum at distance Z from the light waist is

$$\omega=\omega_0\sqrt{1+{Z^2}/{Z_{\rm R}^2}}, \tag{13}$$

where $\omega_0=\sqrt{Z_{\rm R}\lambda_{\rm s}/\pi}$ is the light waist. Besides the position, the size of the light spot also varies with the wavelength (Fig.7).

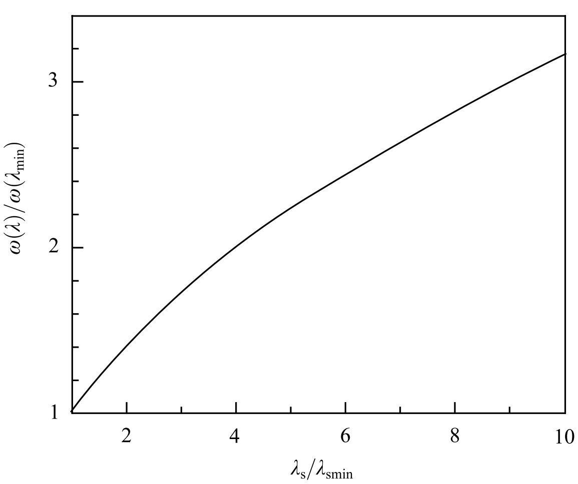

Fig. 7. The variation of the light spot size with the wavelength at a fixed position.

For an oscillator FEL, the optical beam and the electron beam should overlap to the fullest extent. Usually the cross-section of the optical beam in the undulator is larger than that of the electron beam. Unlike a conventional laser, the diffraction loss limit for FELs is mainly due to the narrower magnetic gap of the undulator. The relation of the undulator length and the Rayleigh length should be optimized.

The effect of the diffraction on the gain can be taken into account in 1D theory by introducing the filling factor:

$$F=\frac{1}{1+\overline{\omega^2}/4\sigma_{\rm e}^2}, \tag{14}$$

where ω is the radius of the optical beam, the average is over the length of the undulator, and $\sigma_{\rm e}$ is the RMS radius of the electron beam (an electron beam with axisymmetric and constant envelope is assumed). For Gaussian light with the waist at the middle of the undulator, $\overline{\omega^2}=\omega_0^2(1+L^2/12Z_{\rm R}^2)$.

If we require that the average cross-section of the optical beam in the undulator has a minimum value, then the corresponding filling factor reaches its maximum, it has [18]

$$Z_{\rm R}=L/2\sqrt{3},\quad \omega_0^2=L/\sqrt{3}k_{\rm s},\quad F_{\rm max}=\frac{1}{1+\omega_0^2/2\sigma_{\rm e}^2}. \tag{15}$$

If we require the maximum optical beam radius in the undulator $\omega_{\rm M}$ to be minimized, then it has (the light waist is at the middle of the undulator)

$$Z_{\rm R}=L/2,\quad \omega_0^2=L/k_{\rm s},\quad \omega_{\rm M\,min}=\sqrt{2}\omega_0. \tag{16}$$

Figure 8 illustrates the envelopes of the optical beam in the undulator for the two conditions. Usually $Z_{\rm R}=(1/2-1/3)L$ is taken.

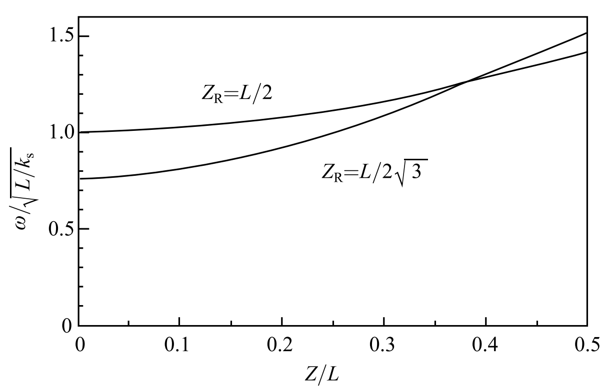

Fig. 8. The envelopes of the optical beam in the undulator for the two choices of Rayleigh length ($z=0$ at the middle of the undulator).

For a given resonator length $L_{\rm c}$, the optical beam profile in the resonator, and thus the Rayleigh length, the diameter and the position of the optical beam waist, can be determined by choosing the radii of curvature of the resonator mirror. Different radii of curvature give different types of resonator. Typical resonator structures are the confocal resonator and the concentric resonator.

A confocal resonator has a higher tolerance to misalignment of the resonator mirrors. The diffraction losses are not large, but the power density on the mirrors of the resonator is high due to the small spot radius. This means the mirrors can easily be damaged. Also, because for a larger waist radius the filling factor is small, that causes a gain decrease.

For a concentric resonator, the waist radius is smaller, providing a larger filling factor, while the larger mirror spot radius means the resonator mirrors will not easily be damaged. However, its diffraction loss is larger and the tolerance to misalignment of the resonator mirror is very low.

The angular tolerance requirement for the mirrors is given by Brau for the case of a symmetric cavity [19]:

$$\theta_{\mathrm{m}}=\sqrt{\frac{2\lambda_{\mathrm{s}}}{\pi L_{\mathrm{e}}}}(1-g)^{1/4}(1+g)^{3/4}, \tag{17}$$

where g is the stability parameter of the cavity. By parameter transformation we rewrite this angular tolerance requirement as a more simple form

$$\theta_{\mathrm{m}}=\theta_{\mathrm{f}}(1+g), \tag{18}$$

where $\theta_{\mathrm{f}}=\lambda_{\mathrm{s}}/\pi\omega_0$ is the divergence angular of the optical field in the far-field region.

The Rayleigh length, the resonator stability parameter, the angular misalignment tolerance of the mirrors and the radius ratio of the light waist to the light spot on the mirror $\omega_0/\omega_{\mathrm{m}}$, are all dependent on the radii of curvature of the mirror. Using dimensionless qualities we plot all these relations together in Fig. 9. The larger radii of curvature correspond to longer Rayleigh length and larger tolerance to angular misalignment, but also to a smaller light spot on the mirrors and consequently higher power density on the mirrors. There is a trade-off between the smaller diffraction loss, the larger tolerance to the angular misalignment and the lower power density on the mirrors. Usually an optical resonator close to a concentric resonator is adopted for oscillator FELs. For a near-concentric resonator, $g\approx -1+x$, $x\ll 1$, so it has $\theta_{\mathrm{m}}=\theta_{\mathrm{f}}x\ll\theta_{\mathrm{f}}$. For our mid-infrared oscillator, we take the ratio of the radii of curvature of the mirror to the resonator length as R/L_{c} =2.756/5.04=0.547. Then from Fig. 9, we have $Z_{\mathrm{R}}=0.15L_{\mathrm{c}}=0.77=L/3$, $\omega_0/\omega_{\mathrm{m}}=1/3.41$, $\theta_{\mathrm{m}}/\theta_{\mathrm{f}}=0.172$, $g=-0.8278$.

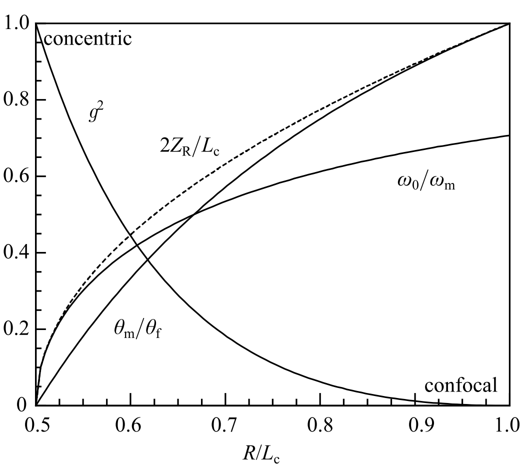

Fig. 9. The relations of the resonator parameters to the radii of curvature of the mirror (R)(ω_{m} is the light spot radius on the mirrors).

For a given intra-cavity optical power of oscillator FEL, a larger out-coupling fraction gives a larger fraction of the out-coupled power. A larger out-coupling fraction means smaller net gain, however, which leads to the optical field being saturated earlier, i.e. to lower saturation power. Therefore there exists an optimum output coupling fraction for maximum out-coupled power. We give the optimum out-coupling fraction [13]

$$\alpha_{\mathrm{oc,m}}=\frac{\sqrt{(1-\mathrm{e}^{-g_{\mathrm{ss}}})\alpha_{\mathrm{lo}}}-\alpha_{\mathrm{lo}}}{1-\alpha_{\mathrm{lo}}}, \tag{19}$$

where we denote the output coupling fraction and the passive loss (all other losses except the output coupling fraction, including the diffraction loss, the absorption of the mirrors and so on) of the cavity as α_{oc} and α_{lo}, respectively, and for the total loss in the optical cavity, $1-\alpha=(1-\alpha_{\mathrm{lo}})(1-\alpha_{\mathrm{oc}})$. The dependence of the optimum out-coupling fraction on the passive loss of the cavity for different small signal gains is presented in Fig. 10.

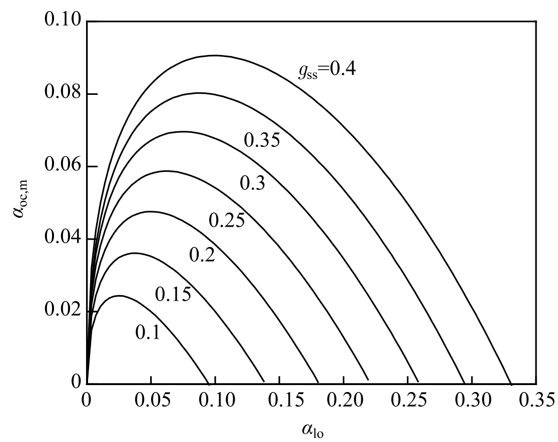

Fig. 10. The variation of optimum out-coupling fraction with passive cavity loss for different initial gain.

The corresponding maximum output power is [13]

$$P_{\mathrm{out,m}}=\left(\sqrt{1-\mathrm{e}^{-g_{\mathrm{ss}}}}-\sqrt{\alpha_{\mathrm{lo}}}\right)^2P_{\mathrm{c}}. \tag{20}$$

5 Waveguide

For a FIR-FEL, due to the relatively long wavelength, the diffraction effect becomes severe, which results in a reduction of the FEL gain and even to the FEL being unable to start oscillation. To overcome the diffraction losses, the waveguide is adapted [20–23]. The waveguide also gives rise to some new effects on the FEL performance. The most noticeable difference between the waveguide FEL and its free-space counterpart is the frequency dependence of the gain. With the waveguide the pulse slippage effect can also be reduced to some degree.

We consider a parallel-plate waveguide, which concentrates wave power along the vertical direction only, while along the horizontal direction it is similar to free-space. Thus, in the vertical plane the electric field distribution is cosine-like with zeroes at the walls, and the corresponding intensity of the light is (the fundamental mode)

$$I \propto \cos^2\left(\frac{\pi}{b}r\right), r\in[0,b/2], \tag{21}$$

where b is the vertical aperture of the waveguide. In Fig. 11, the radial distributions of the laser in the waveguide are compared with those of the Gaussian beam in free space ($I\propto \mathrm{e}^{-2r^2/\omega^2}$) for different transverse profiles. We take the condition that the laser modes in the waveguide and in free space have the same transverse size (FWHM) as the critical point for needing a waveguide or not, namely $b/2$=$2.35\omega/2$. (In comparison with Ref. [24], where the threshold value was taken at the FWHM of the Gaussian laser mode being equal to the vertical aperture of the waveguide, $2.35\omega/2$=b). Hence we give (for the case of the optical waist at the middle of the undulator),

$$b\leqslant 2.354\omega_{\mathrm{M}}=1.328\sqrt{Z_{\mathrm{R}}\lambda_{\mathrm{s}}\left(1+L^2/4Z_{\mathrm{R}}^2\right)}, \tag{22}$$

where ω_{M} is the maximum transverse profile of the optical beam in the undulator; it is at the extremity of the undulator ($z=L/2$ in Eq. (13)), and related to the wavelength.

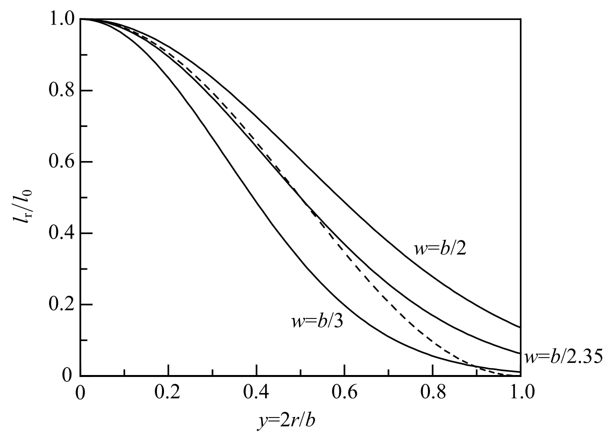

Fig. 11. The different transverse profiles of the optical mode in free space $I\propto \mathrm{e}^{-2(r/\omega)^2}$ compared with that in the waveguide (the dotted line $I\propto\cos^2(y\pi/2)$).

For the Rayleigh length $Z_{\mathrm{R}}\sim L/2$, and from Eq. (22) $b\leqslant\approx\sqrt{2L\lambda_{\mathrm{s}}}$ or

$$\frac{b}{L}\leqslant\sqrt{\frac{2\lambda_{\mathrm{s}}}{L}}. \tag{23}$$

Notice that the right side of the above inequality is the undulator radiation angle, and the physics meaning of Eq. (23) is obvious.

From Eq. (22), the cut-off wavelength is

$$\lambda_{\mathrm{sc}}=\frac{1.7636b^2}{Z_{\mathrm{R}}(1+L^2/4Z_{\mathrm{R}}^2)}. \tag{24}$$

We plot the relation of the cut-off wavelength to the aperture of the waveguide in Fig. 12. For our far-infrared oscillator the wavelength reaches 200 μm in the long wavelength region, and the vertical aperture $b=18$ mm. If the undulator length L = 2.4 m, then from Fig. 12 (the dashed line), we can know that a waveguide should be used.

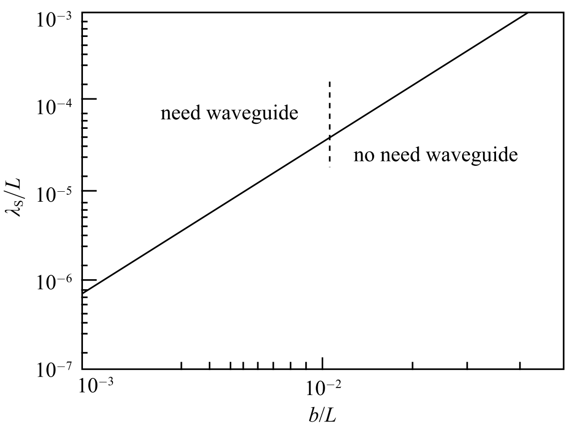

Fig. 12. The relation of the cut-off wavelength to the aperture of the waveguide.

With a parallel-plate waveguide the filling factor (Eq. (14)) now can be re-written as

$$F_{=}\frac{1}{\overline{\overline{\sqrt{1+\sigma_x^2/\sigma_{\mathrm{e}}^2}\sqrt{1+\sigma_{y,\mathrm{eff}}^2/\sigma_{\mathrm{e}}^2}}}}, \tag{25}$$

where $\sigma_{y,\mathrm{eff}}=0.2124b$ is the effective RMS transverse size of the laser mode in the waveguide (Eq. (21)), and the average is over the length of the undulator. For a parallel-plate waveguide, in the horizontal direction it is the Gaussian free-space mode

$$\overline{\sqrt{1+\sigma_x^2/\sigma_{\mathrm{e}}^2}}=\frac{1}{2}\left\{\sqrt{1+t^2+v^2t^2}+\frac{1+t^2}{vt}\mathrm{Arsh}\left(\frac{vt}{\sqrt{1+t^2}}\right)\right\}. \tag{26}$$

Here $v=L/2Z_{\mathrm{R}}, t=\omega_0/2\sigma_{\mathrm{e}}$.

Through waveguide parameter selection one may achieve zero slippage and single broad peak gain [23]. For a rectangular waveguide with a gap b, the condition is

$$b\approx\sqrt{\lambda_{\mathrm{s}}\lambda_{\mathrm{u}}}/2. \tag{27}$$

If $\lambda_{\mathrm{s}}=200$ μm, $\lambda_{\mathrm{u}}=5$ cm, then Eq. (27) gives $b\sim$1.58 mm, which is rather too small. Detailed discussions of the waveguide induced effect on slippage were given in Ref. [23].

6 Summary

The issues of parameter design for an oscillator FEL have been discussed. The estimation formulae of the gain and saturation power in the resonator have been given for preliminary design. They depend on the detuning parameters of electron beam, the total loss ratio in the optical resonator, and the undulator parameters. The design and optimization of the undulator parameters have been analyzed. For the given radiation wavelength and the undulator period we give the optimal deflection parameter K to be 1.556. Considering both physics and technical constraints, we analyzed the relation of the deflection parameter K to the undulator period. Then, for the given undulator period, the variation range of K with the magnetic field gap can be determined quickly, and consequently, the range of the radiation wavelength can be determined for the given electron energy, or vice versa. We also put forward the requirement for the second integral of the undulator field. We analyzed the relations and design of the various parameters of the optical resonator. Using dimensionless quantities, we plotted the relationship of the radii of curvature of the resonator mirror and the various optical resonator parameters together, such as the Rayleigh length, the resonator stability parameter, the angular misalignment tolerance of the mirrors and the radius ratio of the light waist to the light spot on the mirror. Thus we provided a clear and visual demonstration of their interdependence for the design and optimization of the resonator. Also a simple relation was provided for the angular tolerance requirement of the mirrors. For an oscillator FEL working at longer wavelengths, we analyzed the effect of the parallel-plate waveguide. We have given the conditions of the undulator magnetic gap and the optical wavelength for the necessity of using a waveguide, and have also given the modified filling factor in the case of the waveguide. These analyses provide a fundamental study for further more detailed simulation and more accurate optimization. They also will be helpful to gain deeper insights into FEL physics.

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