Charged analogue of well behaved neutral spheres: an algorithmic approach

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Abstract: We establish a systematic algorithmic approach that generates new classes of solutions to the Einstein-Maxwell system in static spherically symmetric spacetime from well known uncharged solutions. A particular case is shown to satisfy all major physical features of a realistic charged star including the standard point-wise energy conditions of normal matter. The solution matches smoothly with the exterior Reissner-Nordström metric at the pressure free interface. The study, which is reported for a particular choice of gravitational potential, encourages similar approaches to study electrification of well known physically realistic uncharged models.

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1 Introduction

Solutions of the Einstein-Maxwell system in static spherically symmetric spacetime have been extensively studied in the literature with a view to determining the role played by electromagnetic fields on the stability of charged compact spheres. A physically resonable interior metric must match smoothly with the exterior Reissner-Nordström metric at the pressure free interface. It is interesting to note that, in the presence of electromagnetic fields, the collapse of a spherically symmetric distribution of matter to a point singularity may be avoided during gravitational collapse or during an accretion process onto a compact object. In this scenario, the gravitational attraction is counterbalanced by the repulsive Coulomb force in addition to the pressure gradient [1].

The analysis and determination of the maximum mass of very compact astrophysical objects has been a key issue in relativistic astrophysics for the last few decades. Mak et al [2] showed that the upper bound of the mass-to-radius ratio set by the Buchdahi limit for a static neutral sphere gets modified in the presence of charge. Weber et al [3–5] and Negreiros et al [6] have shown that electric fields generated by charge distributions of strange quark stars increase the stellar mass by up to 30% depending on the strength of the electric field. Apart from the strong bonding of particles of a quark surface, a distinction between a strange star and a normal neutron star is that strange stars, if bare, have ultra-strong electric fields. For ordinary strange matter, these are of the order of 10^{18} V/cm [7] and for color superconducting strange matter they are of the order of 10^{20} V/cm [8–10]. The influence of ultra-high electric fields on the bulk properties of compact stars was explored by Ray et al [11] and Malheiro et al [12]. These features may allow one to observationally distinguish quark stars from neutron stars. The maximum mass of a strange star is almost the same but its radius is less than a neutron star [13]. An important distinction between quark stars and conventional neutron stars is that quark stars are self-bound by the strong interaction, whereas neutron stars are bound by gravity. Hence, a signature for such quark stars is that their rotational periods are considerably shorter than those gravitationally bound neutron stars. Therefore, the study of electrically charged astrophysical compact objects with Einstein-Maxwell systems has physical significance and has attracted considerable attention.

In fact, when seeking exact solutions to describe charged compact spheres, many researchers have electrified the well known uncharged solutions found in the past. For example, Kuchowicz [14] solutions have been charged by Nduka [15] and Adler [16]; Wyman [17] solutions by Singh and Yadav [18], Nduka [19] and Klein [20]; Schwarzschild's solution by Gupta Kumar [21], Guilfoyle [22], Gupta et al [23], Florides [24] and Banerjee and Som [25]; Tolman [26] solutions by Cataldo and Mitskievic [27], Tikekar [28] and Pant and Sah [29]; Durga-

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pal and Fuloria [30] superdense star by Gupta and Maurya [31]; and Finch and Skea [32] neutron star model by Hansraj and Maharaj [33]. In this scenario the charged models of Thirukkanesh and Maharaj [34, 35], Maharaj and Thirukkanesh [36], Maurya et al. [37], Maurya and Gupta [38–41] and Gupta and Kumar [42] are notable.

In this paper we develop an algorithmic approach to generate new classes of solutions to the Einstein-Maxwell system in static spherically symmetric spacetimes by specifying a more generalized form for one of the gravitational potentials. The desirable feature is that we generate many physically admissible charged models with known uncharged solutions reviewed in Delgaty and Lake [43].

2 Field equations

To model the interior of a charged dense star, based on physical grounds we assume that the gravitational field inside a static spherically symmetric star is described by the line element

$$\mathrm{d}s^2=-\mathrm{e}^{2\nu(r)}\mathrm{d}t^2+\mathrm{e}^{2\lambda(r)}\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2) \quad (1)$$

in Schwarzschild coordinates $(x^a)=(t,r,\theta,\phi)$. We take the energy momentum tensor for an isotropic charged perfect fluid sphere to be of the form

$$T_{ij}=\mathrm{diag}(-\rho-E^2,p-E^2,p+E^2,p+E^2), \quad (2)$$

where $\rho=\rho(r)$ is the energy density, $p=p(r)$ is the isotropic pressure and $E=E(r)$ is the electric field. These quantities are measured relative to the comoving fluid velocity $u^i=\mathrm{e}^{-\nu}\delta^i_0$. For the line element (1) and matter distribution (2) the Einstein-Maxwell system of field equations can be expressed as

$$\rho+E^2=\frac{1}{r^2}\left[r(1-\mathrm{e}^{-2\lambda})\right]', \quad (3)$$

$$p-E^2=-\frac{1}{r^2}\left(1-\mathrm{e}^{-2\lambda}\right)+\frac{2\nu'}{r}\mathrm{e}^{-2\lambda}, \quad (4)$$

$$p+E^2=\mathrm{e}^{-2\lambda}\left(\nu''+\nu'^2+\frac{\nu'}{r}-\nu'\lambda'-\frac{\lambda'}{r}\right), \quad (5)$$

$$E=\frac{1}{2r^2}\int_0^r r^2\sigma\mathrm{e}^{\lambda}\mathrm{d}r=\frac{q}{r^2}, \quad (6)$$

where $\sigma=\sigma(r)$ is the proper charge density, $q=q(r)$ is the total charge within a sphere of radius r and primes denote differentiation with respect to r. In system (3)–(6), we are using units where the coupling constant $8\pi G/c^4=1$ and the speed of light $c=1$. The system of equations (3)–(6) governs the behaviour of the gravitational field for an isotropic charged perfect fluid. A different, but equivalent, form of the field equations is obtained if we introduce a new independent variable x, and define functions y and Z, as follows

$$x=Cr^2,\quad Z(x)=\mathrm{e}^{-2\lambda(r)} \text{ and } A^2y^2(x)=\mathrm{e}^{2\nu(r)}, \quad (7)$$

which was first suggested by Durgapal and Bannerji [44]. In (7), the quantities A and C are arbitrary constants. Under the transformation (7), the system (3)–(6) becomes

$$\frac{\rho}{C}+\frac{E^2}{C}=\frac{1-Z}{x}-2\dot{Z}, \quad (8)$$

$$\frac{p}{C}-\frac{E^2}{C}=4Z\frac{\dot{y}}{y}+\frac{Z-1}{x}, \quad (9)$$

$$(2x^2\dot{y}+xy)\dot{Z}+(4x^2\ddot{y}-y)Z=\left(\frac{2E^2x}{C}-1\right)y, \quad (10)$$

$$E=\frac{1}{4\sqrt{C}x}\int_0^x\sqrt{\frac{x}{Z}}\sigma\mathrm{d}x=\frac{Cq}{x}, \quad (11)$$

where dots denote differentiation with respect to the variable x.

3 Method of generating analytic solutions

The Einstein-Maxwell system (8)–(11) comprises four equations in the six unknowns Z,y,ρ,p,E and σ. Therefore we have the freedom to choose any two variables to integrate the system. In this treatment we specify the gravitational potential y and electric field intensity E on physical grounds and by integration of (10) the explicit solution of the Einstein-Maxwell system (8)–(11) then follows. We first assume a form for the gravitational potential y that generalize particular uncharged solutions found previously. The emanating solution of the field equations reduces to a linear, first order differential equation.

The metric function y is chosen as

$$y=(1+ax^n)^m, \quad (12)$$

where a,m and n are real numbers. On substituting (12) in the field equation (10) we obtain

$$\dot{Z}+\frac{[(4mn(mn-1)-1)(ax^n)^2+(4mn(n-1)-2)ax^n-1]}{x(1+ax^n)[1+(2mn+1)ax^n]}Z=\frac{\left(\frac{2E^2x}{C}-1\right)(1+ax^n)}{x[1+(2mn+1)ax^n]}, \quad (13)$$

after simplification. Using partial fractions equation (13) can be rewritten as

$$\dot{Z}+\left[-\frac{1}{x}+\frac{2n(m-1)ax^{n-1}}{(1+ax^n)}+\frac{2n[2m(n-1)+1]ax^{n-1}}{[1+(2mn+1)ax^n]}\right]Z=\frac{\left(\frac{2E^2x}{C}-1\right)(1+ax^n)}{x[1+(2mn+1)ax^n]}. \quad (14)$$

The above equation may be integrated using the relevant integrating factor and we obtain

$$Z = \frac{x}{(1+ax^n)^{2(m-1)}[1+(2mn+1)ax^n]^{\frac{2[2m(n-1)+1]}{2mn+1}}} \left[\int\left(\frac{2E^2}{Cx}-\frac{1}{x^2}\right)(1+ax^n)^{(2m-1)} [1+(2mn+1)ax^n]^{1-\frac{4m}{2mn+1}}\mathrm{d}x+B\right] \quad (15)$$

where B is an arbitrary constant. Now we are in a position to specify the electric field intensity E. In this treatment we take

$$E^2 = \frac{\alpha Cx}{(1+ax^n)^{(2m-1)}[1+(2mn+1)ax^n]^{1-\frac{4m}{2mn+1}}}, \quad (16)$$

where α is a real constant. On substituting (16) into (15) we obtain

$$Z = \frac{x}{(1+ax^n)^{2(m-1)}[1+(2mn+1)ax^n]^{\frac{2[2m(n-1)+1]}{2mn+1}}}\left[2\alpha x - \int\frac{(1+ax^n)^{(2m-1)}[1+(2mn+1)ax^n]^{1-\frac{4m}{2mn+1}}}{x^2}\mathrm{d}x+B\right]. \quad (17)$$

In principle, equation (17) is integrable if m and n are specified. However, it is possible to express the solution in terms of elementary functions for certain values of m and n only. In the following sections we consider different values of m and n and integrate (17) explicitly in terms of elementary functions.

4 Charged models

It is interesting to observe that we can generalize a number of physically reasonable neutral models reported previously by specifying values for m and n. We consider only the values of m and n for which the neutral solutions reviewed in Delgaty and Lake [43] are governed by the choice (12) with regular gravitational potentials.

4.1 Charged Tolman IV model

When $m=\frac{1}{2}$ and $n=1$, from (17) and (16) we obtain

$$Z = \frac{(1+ax)(1+Bx+2\alpha x^2)}{(1+2ax)} \quad (18)$$

$$E^2 = \alpha Cx \quad (19)$$

Therefore the line element (1) takes the form

$$\mathrm{d}s^2 = -A^2(1+aCr^2)\mathrm{d}t^2 + \frac{1+2aCr^2}{(1+aCr^2)(1+BCr^2+2\alpha C^2r^4)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2). \quad (20)$$

If we set $a=\frac{1}{D^2}, B=-\frac{1}{R^2}$ and $C=1$, then the metric (20) reduces to

$$\mathrm{d}s^2 = -A^2\left(1+\frac{r^2}{D^2}\right)\mathrm{d}t^2 + \frac{1+2\frac{r^2}{D^2}}{\left(1+\frac{r^2}{D^2}\right)\left(1-\frac{r^2}{R^2}+2\alpha r^4\right)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2). \quad (21)$$

When $E=0$ (i.e., $\alpha=0$) the line element (21) becomes

$$\mathrm{d}s^2 = -A^2\left(1+\frac{r^2}{R^2}\right)\mathrm{d}t^2 + \frac{1+2\frac{r^2}{D^2}}{\left(1+\frac{r^2}{D^2}\right)\left(1-\frac{r^2}{R^2}\right)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2). \quad (22)$$

The metric (22) correspond to the well known Tolman IV model [26]. Thus the metric (21) is a charged generalization of Tolman IV metric (22). Tolman IV gravitational potential had previously been used to study relativistic compact sphere with isotropic matter distribution by Tolman [26] which is shown to satisfy all the physical requirements [43]. Moreover, an Einstein system in static spherically symmetric spacetime modeling anisotropic strange quark matter with a linear barotropic equation of state has been studied by the authors for the Tolman IV form for one of the gravitational potentials [45].

4.2 Charged de Sitter model

When $a=-1, A=1, B=-2$ and $C=\frac{1}{R^2}$, the line element (20) becomes

$$\mathrm{d}s^2 = -A^2\left(1-\frac{r^2}{R^2}\right)\mathrm{d}t^2 + \frac{1-2\frac{r^2}{R^2}}{\left(1-\frac{r^2}{R^2}\right)\left(1-2\frac{r^2}{R^2}+2\alpha\frac{r^4}{R^4}\right)}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2), \quad (23)$$

If we set $\alpha=0$ (i.e., $E=0$) this metric reduces to the familiar de Sitter isotropic model

$$\mathrm{d}s^2 = -\left(1-\frac{r^2}{R^2}\right)\mathrm{d}t^2 + \left(1-\frac{r^2}{R^2}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2), \quad (24)$$

which models the universe as spatially flat and neglects ordinary matter, so the dynamics of the universe are dominated by the cosmological constant, interpreted to correspond to the dark energy of our universe.

4.3 Charged Einstein model

When $a=0, B=-1$ and $C=\dfrac{1}{R^2}$, from (20) we obtain

$$ds^2=-A^2dt^2+\left(1-\frac{r^2}{R^2}+2\alpha\frac{r^4}{R^4}\right)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2). \quad (25)$$

When $\alpha=0$ this metric reduces to the uncharged Einstein line element

$$ds^2=-A^2dt^2+\left(1-\frac{r^2}{R^2}\right)^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2). \quad (26)$$

The metric (26) corresponds to the well known Einstein universe, which is a matter-dominated Friedmann model with zero curvature, in which the universe will continue to expand forever with just the right amount of energy provided by the Big Bang to escape the pull of gravity and escape to infinity.

4.4 Charged Heintzmann IIa model

When $m=\dfrac{3}{2}$ and $n=1$ from (17) and (18) we obtain

$$Z=\frac{2-ax+2x(B+2\alpha x)(1+4ax)^{-1/2}}{2(1+ax)} \quad \text{and} \quad (27)$$

$$E^2=\frac{\alpha Cx\sqrt{1+4ax}}{(1+ax)^2}. \quad (28)$$

If we set the arbitrary constant $B=-\dfrac{3ac}{2}$ then equation (27) takes the form

$$Z=1-\frac{3ax}{2}\left[\frac{1+\left(c-\frac{4\alpha x}{3a}\right)(1+4ax)^{-1/2}}{1+ax}\right]. \quad (29)$$

Hence the metric (1) takes the specific form

$$\begin{aligned}ds^2 &= -A^2(1+ar^2)^3dt^2\\ &+\left(1-\frac{3ar^2}{2}\left[\frac{1+\left(c-\frac{4\alpha r^2}{3a}\right)(1+4ar^2)^{-1/2}}{1+ar^2}\right]\right)^{-1}dr^2\\ &+r^2(d\theta^2+\sin^2\theta d\phi^2), \quad (30)\end{aligned}$$

where we have set $C=1$. This reduces to the uncharged Heintzmann IIa [46] metric

$$\begin{aligned}ds^2 &= -A^2(1+ar^2)^3dt^2\\ &+\left(1-\frac{3ar^2}{2}\left[\frac{1+c(1+4ar^2)^{-1/2}}{1+ar^2}\right]\right)^{-1}dr^2\\ &+r^2(d\theta^2+\sin^2\theta d\phi^2), \quad (31)\end{aligned}$$

when $\alpha=0$.

4.5 Charged Durgapal IV model

When $m=2$ and $n=1$ from (17) and (16) we obtain

$$Z=\frac{7-10ax-a^2x^2}{7(1+ax)^2}+\frac{x(B+2\alpha x)}{(1+ax)^2(1+5ax)^{2/5}}, \quad (32)$$

$$E^2=\frac{\alpha Cx(1+5ax)^{3/5}}{(1+ax)^3}. \quad (33)$$

For this case the line element (1) takes a specific form

$$\begin{aligned}ds^2 &= -A^2(1+aCr^2)^4dt^2\\ &+\left[\frac{7-10aCr^2-a^2C^2r^4}{7(1+aCr^2)^2}\right.\\ &\left.+\frac{Cr^2(B+2\alpha Cr^2)}{(1+aCr^2)^2(1+5aCr^2)^{2/5}}\right]^{-1}dr^2\\ &+r^2(d\theta^2+\sin^2\theta d\phi^2). \quad (34)\end{aligned}$$

If we set $a=1$ and $\alpha=0$, (34) reduces to the uncharged Durgapal IV [47] metric

$$\begin{aligned}ds^2 &= -A^2(1+Cr^2)^4dt^2\\ &+\left[\frac{7-10Cr^2-C^2r^4}{7(1+Cr^2)^2}+\frac{BCr^2}{(1+Cr^2)^2(1+5Cr^2)^{2/5}}\right]^{-1}dr^2\\ &+r^2(d\theta^2+\sin^2\theta d\phi^2). \quad (35)\end{aligned}$$

4.6 Charged Durgapal V model

When $m=\dfrac{5}{2}$ and $n=1$ from (17) and (16) we obtain

$$Z=\frac{1-\frac{ax(309+54ax+8a^2x^2)}{112}+\frac{x(B+2\alpha x)}{(1+6ax)^{1/3}}}{(1+ax)^3}, \quad (36)$$

$$E^2=\frac{\alpha Cx(1+6ax)^{2/3}}{(1+ax)^4}. \quad (37)$$

For this case the line element (1) takes a specific form

$$ds^2=-A^2(1+aCr^2)^5dt^2+\left[\frac{1-\frac{aCr^2(309+54aCr^2+8a^2C^2r^4)}{112}+\frac{Cr^2(B+2\alpha Cr^2)}{(1+6aCr^2)^{1/3}}}{(1+aCr^2)^3}\right]^{-1}dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2). \quad (38)$$

It is easy to see that the above metric reduces to the uncharged Durgapal V [47] metric if we set $a=1$ and $\alpha=0$.

4.7 Charged Durgapal et al model

When $m=-\dfrac{1}{4}$ and $n=1$ from (17) and (16) we obtain

$$Z=\frac{16x(B+2\alpha x)(1+ax)^{5/2}+4(1+ax)^2(4+4ax-a^2x^2)}{(2+ax)^4}, \tag{39}$$

$$E^2=\frac{8\alpha Cx(1+ax)^{3/2}}{(2+ax)^3}. \tag{40}$$

For this case the line element (1) takes a specific form

$$\mathrm{d}s^2=-\frac{A^2}{\sqrt{1+aCr^2}}\mathrm{d}t^2+\frac{(2+aCr^2)^4}{16Cr^2(B+2\alpha Cr^2)(1+aCr^2)^{5/2}+4(1+aCr^2)^2(4+4aCr^2-a^2C^2r^4)}\mathrm{d}r^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2), \tag{41}$$

This metric reduces to the uncharged Durgapal et al [48] model if we take $a=-1$ and $\alpha=0$.

4.8 Charged Korkina and Orlyanskii model III

When $m=n=1$, from (17) and (16) we obtain

$$Z=1+(B+2\alpha x)x(1+3ax)^{-2/3}, \tag{42}$$

$$E^2=\frac{\alpha Cx(1+3ax)^{1/3}}{(1+ax)}. \tag{43}$$

In this case the line element (1) takes a specific form

$$\begin{aligned}\mathrm{d}s^2=&-A^2(1+aCr^2)^2\mathrm{d}t^2\\&+\left[1+(B+2\alpha Cr^2)Cr^2(1+3aCr^2)^{-2/3}\right]^{-1}\mathrm{d}r^2\\&+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2),\end{aligned} \tag{44}$$

If we set $B=\alpha=0$ and $C=1$, the metric (44) becomes

$$\mathrm{d}s^2=-A^2(1+ar^2)^2\mathrm{d}t^2+dr^2+r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2). \tag{45}$$

Hence, we have regained Korkina and Orlyanskii model III [49] and the metric (44) generalizes the metric obtained by Korkina and Orlyanskii.

5 Physical analysis

It is easy to show that all the gravitational potentials generated in section 4 satisfy the regularity conditions: $\mathrm{e}^{2\nu(0)}=$ constant, $\mathrm{e}^{2\lambda(0)}=1$ and $(\mathrm{e}^{2\nu(r)})'=(\mathrm{e}^{2\lambda(r)})'=0$ at the origin $r=0$. The charged solutions generated in section 4 are given in simple elementary functions that facilitate the detailed physical analysis. As an example we consider the metric obtained in (21) (i.e. Section 4.1 Charged Tolman IV model) for the study. In this case the matter variable becomes

$$\rho=\frac{3\left(\dfrac{1}{D^2}+\dfrac{1}{R^2}\right)+\dfrac{r^2}{D^2}\left(\dfrac{7}{R^2}+\dfrac{2}{D^2}+\dfrac{6r^2}{D^2R^2}\right)-\alpha r^2\left(11+30\dfrac{r^2}{D^2}+24\dfrac{r^4}{D^4}\right)}{\left(1+2\dfrac{r^2}{D^2}\right)^2}, \tag{46}$$

$$p=\frac{\left(\dfrac{1}{D^2}-\dfrac{1}{R^2}\right)-\dfrac{3r^2}{D^2R^2}+\alpha r^2\left(3+8\dfrac{r^2}{D^2}\right)}{\left(1+2\dfrac{r^2}{D^2}\right)}, \tag{47}$$

$$E^2=\frac{q^2}{r^4}=\alpha r^2, \tag{48}$$

$$\sigma^2=\frac{36\alpha\left(1+\dfrac{r^2}{D^2}\right)\left[1+r^2\left(2\alpha r^2-\dfrac{1}{R^2}\right)\right]}{\left(1+2\dfrac{r^2}{D^2}\right)}. \tag{49}$$

From (46) and (47), it is noticed that the energy density at the centre is $\rho(0)=3\left(\dfrac{1}{D^2}+\dfrac{1}{R^2}\right)$ and the isotropic pressure at the centre is $p(0)=\left(\dfrac{1}{D^2}-\dfrac{1}{R^2}\right)$. Hence, for any value of D and R the positive definiteness property is satisfied for energy density, but the property is satisfied for pressure only when $R^2>D^2$. Clearly the density and pressure are continuous functions. At the boundary of the star $r=s$ the condition $p(s)=0$ implies

$$s=\left[\frac{3-3\alpha D^2R^2-\sqrt{9+\alpha R^2(9\alpha D^4R^2+14D^2-32R^2)}}{16\alpha R^2}\right]^{1/2}. \quad (50)$$

Choosing suitable values for the parameters involved in (50), it is possible to get a positive value for radius of the star s. Since

$$\frac{\mathrm{d}\rho}{\mathrm{d}r}=-\frac{2r\left[\left(\frac{2}{D^4}+\frac{1}{D^2R^2}\right)\left(5+2\frac{r^2}{D^2}\right)+\alpha\left(11+38\frac{r^2}{D^4}+72\frac{r^4}{D^4}+48\frac{r^6}{D^6}\right)\right]}{\left(1+2\frac{r^2}{D^2}\right)^3}<0,$$

the energy density ρ is a decreasing function with increasing radius r. The derivative of the isotropic pressure

$$\frac{\mathrm{d}p}{\mathrm{d}r}=-\frac{2r\left[\frac{2}{D^4}+\frac{1}{D^2R^2}-\alpha\left(1+4\frac{r^2}{D^2}\right)\left(3+4\frac{r^2}{D^2}\right)\right]}{\left(1+2\frac{r^2}{D^2}\right)^2}<0$$

for suitable choices of parameters D, R and α. Therefore the isotropic pressure is also a decreasing function of r for suitable choices of the parameters. The electric field vanishes at the centre and increases monotonically towards the surface of the star. The charge density is regular at the centre and continuous. As it is easy to see from (6) that $E(s)=\frac{q(s)}{s^2}$, the electric field is continuous across the boundary. The interior metric (21) matches smoothly with the exterior Reissnar-Nordström metric

$$\mathrm{d}s^2=-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)\mathrm{d}t^2+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 +r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2) \quad (51)$$

across the boundary $r=s$, where M is the gravitational mass of the distribution such that

$$M=\mu(s)+\varepsilon(s), \quad (52)$$

while

$$\mu(s)=\frac{1}{2}\int_0^s\rho r^2\mathrm{d}r, \quad \varepsilon(s)=\frac{1}{2}\int_0^s r\sigma q e^{\lambda}\mathrm{d}r \quad (53)$$

$$\text{and } Q=q(s). \quad (54)$$

In the above, $\mu(s)$ is the total mass inside the sphere while $\varepsilon(s)$ is the mass equivalence of the electromagnetic energy distribution and Q is the total charge inside the sphere [24]. This imposes the conditions:

$$\left(1-\frac{2M}{s}+\frac{Q^2}{s^2}\right)=A^2\left(1+\frac{s^2}{D^2}\right) \quad (55)$$

$$\left(1-\frac{2M}{s}+\frac{Q^2}{s^2}\right)=\frac{\left(1+\frac{s^2}{D^2}\right)\left(1-\frac{s^2}{R^2}+2\alpha s^4\right)}{1+2\frac{s^2}{D^2}}. \quad (56)$$

The condition (56) is satisfied automatically due to the preposition (53) while the condition (55) restricts the constant A as

$$A^2=\frac{1-\frac{s^2}{R^2}+2\alpha s^4}{1+2\frac{s^2}{D^2}}.$$

Hence, the model satisfies all major physical requirements to describe a realistic charged star.

Now we demonstrate that the matter variables satisfy the above features throughout the interior of the star by plotting the radial dependence of physical quantities. Figures 1–9 represent respectively the energy density, the isotropic pressure, electric field, charge density, mass function, electric charge, square of speed of light, adiabatic index and energy conditions. The graphs have been plotted in geometric units for particular choice of parameters $D=1.5\,\mathrm{km}, R=2.139\,\mathrm{km}$ and $\alpha=0.01\,\mathrm{km}^{-4}$ with a stellar boundary set at $s=1\,\mathrm{km}$. Figures 1 and 2 show that the energy density and the isotropic pressure decrease continuously with increasing radius. Moreover, as seen in Figs. 1 and 2, the isotropic pressure vanishes at the boundary whereas energy density is non-zero. Figure 3 shows that the electric field is continuous and decreasing from the surface to the centre and vanishes at the centre. The charge density plotted in Fig. 4 is regular and non-zero throughout the interior of the star. Figures 5 and 6 show the mass and the electric charge profile as a function of the radius. Figure 7 shows that $0<\frac{\mathrm{d}p}{\mathrm{d}\rho}<1$ throughout the interior of the star and hence satisfies the causality condition: the speed of sound is less than the speed of light throughout the interior of the star. The combinations $\frac{\mathrm{d}p}{\mathrm{d}\rho}$ and $\frac{p}{\rho}$ are related to the speed of sound v_s as $v_s^2=\frac{\mathrm{d}p}{\mathrm{d}\rho}=\Gamma\frac{p}{\rho}$, and the bulk modulus $\kappa=\Gamma p$ (where Γ is the adiabatic index) gives a measure of the stiffness of the substance. Figure 8 illustrates that $\Gamma>4/3$ throughout the interior, which indicates that upon contraction the pressure increases more than the weight, and hence the stellar object will expand as a result, maintaining the dynamic stability of the star.

Figure 9 shows $\rho-p>0$ throughout the interior of the star, which implies that the model satisfies the dominant energy condition ($\rho \geqslant 0$ and $\rho \pm p \geqslant 0$). Moreover, the above physical analysis show that the models satisfy the null energy condition by meeting the requirement $\rho + p \geqslant 0$; weak energy conditions as $\rho \geqslant 0$ and $\rho + p \geqslant 0$; and strong energy condition because $\rho + 3p \geqslant 0$ and $\rho + p \geqslant 0$. Hence the models satisfy the standard point-wise energy condition [50] that is required by normal matter.

The above detailed study reported is for a particular metric obtained in (21). Given the simplicity of the models generated in subsequent cases in Section 4, similar physical analysis can be performed without much difficulty.

In view of comparing the model with observational data of realistic stars, values of model parameters and the relevant physical parameters were calculated by fixing radii to correspond to the strange star candidates RXJ 1856-37, Her. X-1, SAX J1808.4-3658 and 4U 1820-30 as given in Table 1, such that the pressure at the boundary $p(s) = 0$ (i.e. equation (50) satisfied). Moreover, in calculating numerical values for the physical parameters in Table 1, we choose values for parameters R and D to satisfy the physical conditions required for a realistic star as described in the beginning of this section. The calculated masses in Table 1 for the respective strange star candidates approximately coincide with the values of Tikekar and Jotania [51] and hence enable us to compare the compaction parameter (mass-to-radius ratio) in [51]: $u > 0.3$ for pulsars SAX J1808.4-3658 and 4U 1820-30 suggests they are strange stars of type I; Her X-1 and RXJ 1856-37 are of type II ($0.2 < u < 0.3$); and the u value for neutron star counterparts are considered to be still lower. Moreover, the values of central and surface densities in Table 1 are several times larger than the nuclear saturation density, which corroborates the above classifications of strange stars. The calculated mass $1.45M_{\odot}$ for radii 7.07 km corroborates the theoretical model [52] reported by analyzing pulsar SAX J1808.4-3658, which is also shown to be consistent with observational data and with the strange star models [52]. Moreover, mass-radius relation studies using different theoretical models to analyze the original experimental observations associated with cyclotron line data from the X-ray pulsar Her X-1 [53, 54], and with the X-ray burst spectra of 4U 1820-1830 [55], have shown that they are good strange star candidates.

The electric field values in Table 1 are within the upper limits reported in previous studies [56–59]. As reported by Jes Madsen [60], electron-positron pair creation in supercritical electric fields limits the net charge Q of a static, spherically symmetric strange star consisting of quark matter to $Q < 7 \times 10^{33}$ (units in fm scale), which is self-bound due to strong interactions in addition to gravity. Comparing the Q and E values in Table 1 with the calculations of Madsen [60] (i.e. to the corresponding baryon numbers ($A \sim 10^{34}$ to 10^{32})), the modeled parameter values in Table 1 may be interpreted to represent the strangelet charge of strange stars made of color superconducting strange matter.

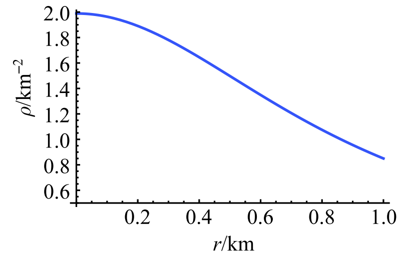

Fig. 1. (color online) Energy density.

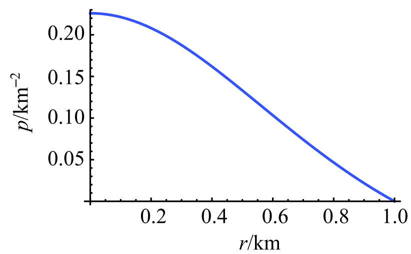

Fig. 2. (color online) Isotropic pressure.

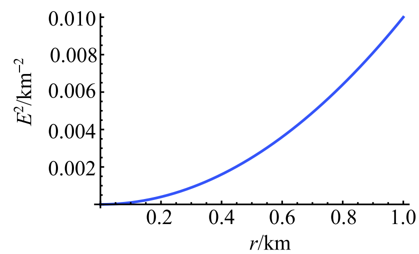

Fig. 3. (color online) Electric field.

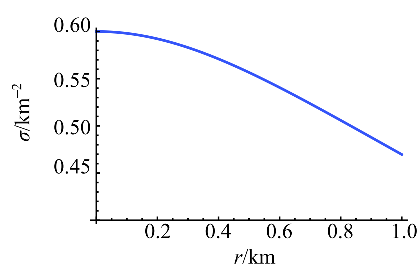

Fig. 4. (color online) Charge density.

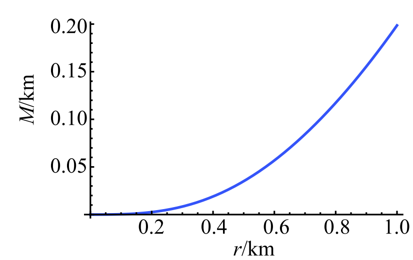

Fig. 5. (color online) Mass function.

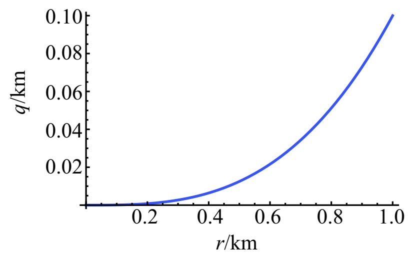

Fig. 6. (color online) Electric charge.

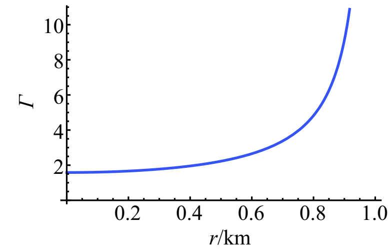

Fig. 8. (color online) Adiabatic index.

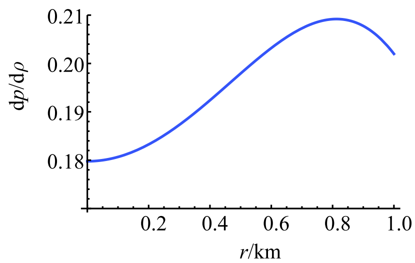

Fig. 7. (color online) Square of the speed of light.

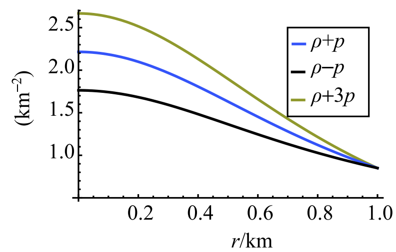

Fig. 9. (color online) Plot of radial dependence of $\rho+p$, $\rho-p$ and $\rho+3p$, which shows that the energy conditions are satisfied.

Table 1. Values of physical parameters (density at the center and surface, electric field at the surface, total charge and total mass) calculated for radii that correspond to some strange star candidates.

strange star candidates	radii (s)/ km	D/ km	R/ km	$\rho(0)$/ $(\times10^{15}\text{gcm}^{-3})$	$\rho(s)$/ $(\times10^{15}\text{gcm}^{-3})$	$E(s)$/ (Vcm^{-1})	Q/ (C)	M/ $M_\odot$
RXJ 1856-37	6	6.5	12.168	4.8926	1.4615	6.2665×10^{19}	2.5100×10^{19}	0.977
Her. X-1	6.7	7	13.401	4.1776	1.1791	6.9976×10^{19}	3.4950×10^{19}	1.179
SAX J1808.4-3658	7.07	4	12.741	11.042	1.0969	7.384×10^{19}	4.1066×10^{19}	1.449
4U 1820-30	10	2	16.769	40.777	0.5285	1.0444×10^{20}	1.1621×10^{20}	2.272

6 Conclusion

We have generated many new classes of solutions to the Einstein-Maxwell system in static spherically symmetric spacetime through a systematic algorithmic approach. The desirable feature of this approach is that well known physically reasonable uncharged models can be electrified. In this study, a particular case is shown to satisfy all major physical requirements of a realistic charged star: regularity of the gravitational potentials at the origin, positive definiteness of the energy density and the isotropic pressure at the origin, vanishing of the isotropic pressure at some finite radius, monotonic decrease of the energy density and the isotropic pressure with increasing radius, the continuity of electric field across the boundary, and the speed of sound being less than the speed of light. Moreover, the generated interior metric matches smoothly with the Reissnar-Nordström exterior metric at the boundary of the star. In addition, the model satisfies the standard point-wise energy conditions that are required by normal matter: dominant, null, weak and strong. Moreover, the generated model parameters are interpreted to represent strange quark stars with strangelet charge made of color superconducting strange matter.

In this work, we have electrified many well known uncharged models by specifying a particular form for the gravitational potential $y=(1+ax^n)^m$. Different choices of suitable y can produce many more such models. Hence this approach should encourage study of the electrification of well known neutral stars.

References

1 J. D. Bekenstein, Phys. Rev. D, **4**: 2185 (1971)
2 M. K. Mak, P. N. Dobson Jr, and T. Harko, Europhys. Lett., **55**: 310 (2001)
3 F. Weber, M. Meixner, R. P. Negreiros, and M. Malheiro, Int. J. Mod. Phys. E, **16**: 1165 (2007)
4 F. Weber, R. Negreiros, and P. Rosenfield, Astrophysics and Space Science Library, **357**: 213 (2009)
5 F. Weber, O. Hamil, K. Minura, and R. Negreiros, Int. J. Mod. Phys. D, **19**: 1427 (2010)
6 R. P. Negreiros, F. Weber, M Malheiro, and V. Usov, Phys. Rev. D, **80**: 083006 (2009)
7 C. Alcock, E. Farhi, and A. Olinto, Astrophys. J., **310**: 261 (1986)
8 V. V. Usov, Phys. Rev. D, Part. Fields, **70**: 067301 (2004)
9 V. V. Usov, T. Harko, and K. S. Cheng, ApJ, **620**: 915 (2005)
10 R. P. Negreiros, Igor N. Mishustin, S. Schramm, and F. Weber, Phys. Rev. D, **82**: 103010 (2010)
11 S. Ray, A. L. Espindola, M. Malheiro, J. P. S. Lemos, and V. T. Zanchin, Phys. Rev. D, **68**: 084004 (2003)
12 M. Malheiro, R. PicnÇo, S. Ray, José P. S. Lemos, and Vilson T. Zanchin, Int. J. Mod. Phys. D, **13**: 1375 (2004)
13 F. Weber, M. Orsaria, H. Rodrigues, and Shu-Hua Yang, *Proceedings of the International Astronomical Union Symposium S291*, Vol. 8, edited by J. van Leeuwen, (Cambridge University Press, 2012), p. 61
14 B Kuchowicz, Acta Phys. Pol., **33**: 541 (1968)
15 A. Nduka, Acta Phys. Pol. B, **8**: 75 (1977)
16 R. J. Adler, J. Math. Phys., **15**: 727 (1974)
17 M. Wyman, Phys. Rev., **75**: 1930 (1949)
18 T. Singh, and R. B. S. Yadav, Acta Phys. Pol. B, 1978, **9**: 475 (1949)
19 A. Nduka, Gen. Relativ. Gravit., **7**: 493 (1976)
20 O. Klein, Ark. Mat. Astron. Fys. A, **34**: N19 (1947)
21 Y. K. Gupta, and M. Kumar, Gen. Relativ. Gravit. **37**: 575 (2005)
22 B. S. Guilfoyle, Gen. Relativ. Gravit., **31**: 1645 (1999)
23 Y. K. Gupta, and R. S. Gupta, Acta Phys. Pol. B, **17**: 855 (1986)
24 P. S. Florides, J. Phys. A, Math. Gen., **16**: 1419 (1983)
25 A. Banerjee, and M. M. Som, Int. J. Theor. Phys., **20**: 349 (1981)
26 R. C. Tolman, Phys. Rev., **55**: 364 (1939)
27 M. Cataldo, and N. V. Mitskievic, Class. Quantum Grav., **9**: 545 (1992)
28 R. Tikekar, J. Math. Phys., **25**: 1481 (1984)
29 D. N. Pant, and A. Sah, J. Math. Phys., **20**: 2537 (1979)
30 M. C. Durgapal, and R. S. Fuloria, Gen. Relativ. Gravit., **17**: 671 (1985)
31 Y. K. Gupta, and S. K. Maurya, Astrophys. Space Sci., **331**: 135 (2011)
32 M. R. Finch, and J. E. F. Skea, Class. Quantum Grav., **6**: 467 (1989)
33 S. Hansraj, and S. D. Maharaj, Int. J. Mod. Phys. D, **15**: 1311 (2006)
34 S. Thirukkanesh, and S. D. Maharaj, Math. Meth. Appl. Sci., **32**: 684 (2009)
35 S. Thirukkanesh, and S. D. Maharaj, Class. Quantum Grav. **23**, 2697 (2006)
36 S. D. Maharaj, and S. Thirukkanesh, Nonlinear Analysis: Real World Applications, **10**: 3396 (2009)
37 S. K. Maurya, Y. K. Gupta, and Prathibha, International Journal of Modern Physics D, **20**:1289 (2011)
38 S. K. Maurya, and Y. K. Gupta, Astrophys. Space Sci., **332**: 481 (2011)
39 S. K. Maurya, and Y. K. Gupta, Astrophys Space Sci, **334**: 301 (2011)
40 S. K. Maurya, and Y. K. Gupta, Nonlinear Analysis: Real World Applications, **13**: 677 (2012)
41 S. K. Maurya, and Y. K. Gupta, Astrophys. Space Sci., **340**: 323 (2012)
42 Y. K. Gupta, and J. Kumar, Astrophys. Space Sci., **336**: 419 (2011)
43 M. S. R. Delgaty, and K. Lake, Comput. Phys. Commun., **115**: 395 (1998)
44 M. C. Durgapal, and R. Bannerji, Phys. Rev. D, **27**: 328 (1983)
45 S. Thirukkanesh, and F. C. Ragel, Astrophys. Space Sci., **352**: 743 (2014)
46 H. Heintzmann, Z. Phys., **228**: 489 (1969)
47 M. C. Durgapal, J. Phys. A, **15**: 2637 (1982)
48 M. C. Durgapal, A. K. Pande, and R. S. Phuloria, Astrophys. Space Sci., **102**: 49 (1984)
49 M. P. Korkina, and O. Yu Orlyanskii, The Ukrainian Journal of Physics, **36**: 885 (1991)
50 T. Delsate, and J. Steinhoff, Phys. Rev. Lett., **109**: 021101 (2012)
51 R. Tikekar, and K. Jotania, Pramana J. Phys., **68**: 397 (2007)
52 X. -D. Li, I. Bombaci, M. Dey, and E. P. J. van den Heuvel, Phys. Rev. Lett., **83**: 3776 (1999)
53 X. -D. Li, Z. -G. Dai, and Z. -R. Wang, Astron. Astrophys., **303**: L1 (1995)
54 M. Dey, I. Bombaci, J. Dey, S. Ray, and B. C. Samanta, Phys. Lett. B, **438**: 123 (1998)
55 I. Bombaci, Phys. Rev. C, **55**: 1587 (1997)
56 F. Weber, M. Meixner, R. P. Negreiros, and M. Malheiro, Int. J. Mod. Phys. E, **16**: 1165 (2007)
57 F. Weber, R. Negreiros, and P. Rosenfield, Astrophysics and Space Science Library, **357**: 213 (2009)
58 F. Weber, O. Hamil, K. Minura, and R. Negreiros, Int. J. Mod. Phys. D, **19**: 1427 (2010)
59 R. P. Negreiros, F. Weber, M. Malheiro, and V. Usov, Phys. Rev. D, **80**: 083006 (2009)
60 J. E Madsen, Phys. Rev. Let., **100**: 151102 (2008)