# Effect of long-range correlation on scaling behavior of normalized factorial moments for the first-order phase transition *

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**Abstract:** Within the framework of Ginzburg–Landau theory, the effect of multiplicity correlation between the dynamical multiplicity fluctuations is analyzed for a first-order phase transition from quark–gluon plasma to hadrons. Normalized factorial correlators are used to study the correlated dynamical fluctuations. A scaling behavior is found among the factorial correlators, and an approximate universal exponent, which is weakly dependent on the details of the phase transition, is obtained.

**Keywords:** factorial correlators, Ginzburg–Landau model, scaling behavior

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## 1 Introduction

It is known that ultra-relativistic heavy-ion collisions are the only way to study the properties of quantum chromodynamics (QCD) under extremely high energy density in the laboratory. During such collisions, a new state of matter, quark–gluon plasma (QGP), which is theoretically predicted, might be formed with extremely high energy and matter density. Soon afterwards, the system will cool with its subsequent expansion. Eventually the temperature and energy density become low enough for the hadronization process, and a phase transition may occur from QGP to hadrons.

The quarks and gluons, however, are not detectable directly in experiments because of the color confinement of QCD. We have to search for signals of the phase transition from the final particles. Phase transitions have always been a subject of great interest in high energy physics. The critical point marks the boundary of the first and second order phase transition between hadronic and QCD matter in the QCD phase diagram. The existence of a critical point has been predicted by some lattice QCD calculations [1–3], and the possibility of observing evidence for the critical point has inspired various experiments in different laboratories [4–6] and a lot of related discussion on the possible signals [7–13]. So far, however, the order of the phase transition is still an open issue. QGP may undergo a first order or second order transition, or even a cross-over between different states with different temperature and chemical potentials. Additionally, this transition may not even be recognizable as a critical phenomenon, since hadronization takes place on the surface while the system expands.

The hadrons in the final state are strongly correlated and a variety of fluctuations appear. It is known that fluctuations are large for statistical systems near their critical points, hence the study of multiplicity fluctuations of hadrons produced in high-energy heavy-ion collisions is of importance to study the phase transition [14–18].

Ginzburg - Landau theory is a phenomenological model theory initially describing superconductors without examining their microscopic properties [19, 20]. Over the past two decades, this model has been used to study multiplicity fluctuations about first- and second-order phase transitions [21–33], and regarded as a possible means to reveal some features of phase transitions. Ref. [21–30] are examples of using this model to reveal dynamical multiplicity fluctuations in one bin. The QGP state, as observed experimentally, is strongly correlated according to experiments. Such correlations may influence the pattern of dynamical fluctuations for different parts of the phase space. Therefore, the analysis of correlation to the dynamical fluctuations is of importance. In Ref. [34] the scaling behavior among the factorial correlators is studied for a second-order phase transition from QGP to hadrons. In this article, we will try to investigate the scaling behavior of the factorial correlators of multi-

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plicity distribution within an extended Ginzburg–Landau model for a first-order phase transition for the QGP system.

This article is organized as follows. In Section 2, the normalized factorial correlators for multiplicity fluctuations are derived for a first-order phase transition within the framework of the Ginzburg–Landau model. Section 3 is devoted to our numerical results and some conclusions. In Section 4, a concise summary is presented.

## 2 Factorial correlators in the Ginzburg–Landau model for a first-order phase transition

Consider two small bins in phase space with equal size $\delta$ (it can be an interval of a one-dimensional variable, such as rapidity $\delta y$, or one in two-dimensional space, such as $\delta y\delta\eta$). Let the particle numbers in these two bins be $n_1$, $n_2$ respectively for an event. The moments of multiplicity difference distribution has been investigated in Refs. [31–33], assuming that the fluctuations in the two bins are uncorrelated.

For the purpose of measuring the correlated fluctuations in the two bins, we can write the factorial correlators, which can be defined in a similar way to Ref. [34], as

$$\begin{aligned} f_{q_1q_2} &= \langle n_1(n_1-1)\cdots(n_1-q_1+1)n_2(n_2-1)\cdots(n_2-q_2+1)\rangle \\ &= Z^{-1}\iint \mathcal{D}\phi_1\mathcal{D}\phi_2(\delta|\phi_1|)^{q_1}(\delta|\phi_2|)^{q_2}\mathrm{e}^{-F(\phi_1,\phi_2)}, \quad (1) \end{aligned}$$

where

$$Z=\iint \mathcal{D}\phi_1\mathcal{D}\phi_2\mathrm{e}^{-F(\phi_1,\phi_2)}, \tag{2}$$

where $\langle\cdots\rangle$ is the average over events, $\mathrm{e}^{-F(\phi_1,\phi_2)}$ is the dynamical factor for the process, and $\phi_1$, $\phi_2$ describe the probability for the systems in the two bins in the pure states $|\phi_1\rangle$ and $|\phi_2\rangle$ respectively. According to the Ginzburg–Landau model, the free energy function $F(\phi_1,\phi_2)$ can be written, for a first-order phase transition, as

$$\begin{aligned} F(\phi_1,\phi_2)=&\delta\sum_{i=1}^{2}(a|\phi_i|^2+b|\phi_i|^4+c|\phi_i|^6) \\ &+\lambda\delta(|\phi_1|^2-|\phi_2|^2)^2. \quad (3) \end{aligned}$$

As in Ref. [21], $a\propto(T-T_c)$ and is negative for the hadron phase, whereas $c$ is positive. $b$ is negative for the first-order transition. The last term $\lambda\delta(|\phi_1|^2-|\phi_2|^2)^2$ in Eq. (3) is introduced to parameterize the effect of correlation between particle production in the two bins. The parameter $\lambda$ is used to describe the strength of interactions in the two bins. If $\lambda>0$, the free energy $F$ is smaller when $|\phi_1|$ is closer to $|\phi_2|$ and the particle production in the two bins is positively correlated. Otherwise, particle production in the two bins is anti-correlated for $\lambda<0$. The absolute value of $\lambda$ should decrease as the distance between the two bins in a phase space increases. Furthermore, as the correlation length of the QGP system is longer near the critical point, $\lambda$ may also have a relation with the temperature departure from the critical point. Its value can then mirror the degree of separation from the critical temperature, if the distance between the two bins in phase space is fixed. This paper is confined only to $\lambda>0$ but the extension to $\lambda<0$ is obvious.

The normalized correlated factorial moments can be defined as

$$F_{q_1q_2}=f_{q_1q_2}/[(f_{1,0})^{q_1}(f_{0,1})^{q_2}]. \tag{4}$$

Since $F_{q_1q_2}$ will be constants of about 1 if there are only statistical fluctuations, the moments can be used to filter the statistical fluctuations. The so-called intermittency behavior is for a phenomenon in which $F_{q_1q_2}\propto\delta^{-\alpha_{q_1q_2}}$ with $\alpha_{q_1q_2}>0$. What is more, one can further study whether there exists a scaling law among $F_{q_1q_2}$,

$$F_{q_1q_2}\propto F_{22}^{\beta_{q_1q_2}}\ , \tag{5}$$

even when the intermittency behavior cannot be observed. If there exists no correlation between multiplicity fluctuation in the two bins, i.e. $\lambda=0$, then the factorial moments are simply

$$F_{q_1q_2}=F_{q_1}F_{q_2}, \tag{6}$$

with $F_q$ being the normalized factorial moments for multiplicity fluctuations in one bin. Then the scaling behavior among $F_{q_1q_2}$ is the same as among $F_q$. If there exists correlation between the multiplicity fluctuations in the two bins, the factorization shown by Eq. (6) is not valid, and then whether the scaling behaviors in Eq. (5) are still valid is a problem that needs to be solved.

By defining

$$J_q(z_1,z_2)=\int_0^\infty \mathrm{d}yy^q\mathrm{e}^{-y^3+z_1y+z_2y^2} \tag{7}$$

the factorial normalized moments can be finally written as

$$\begin{aligned} F_{q_1q_2}=&\frac{\int_0^\infty \mathrm{d}xx^{q_1}\mathrm{e}^{-x^3+wx+vx^2}J_{q_2}(u,v)}{\left[\int_0^\infty \mathrm{d}xx\mathrm{e}^{-x^3+wx+vx^2}J_0(u,v)\right]^{q_1}} \\ &\cdot\frac{\left[\int_0^\infty \mathrm{d}x\mathrm{e}^{-x^3+wx+vx^2}J_0(u,v)\right]^{q_1+q_2-1}}{\left[\int_0^\infty \mathrm{d}x\mathrm{e}^{-x^3+wx+vx^2}J_1(u,v)\right]^{q_2}}, \quad (8) \end{aligned}$$

where $u=w+\sqrt{w}sx, v=\sqrt{w}t$, $s=2\lambda/\sqrt{|ac|}$, $t=-(b+\lambda)/\sqrt{|ac|}$, and $w=-a\frac{\delta^{\frac{2}{3}}}{c^{\frac{1}{3}}}$. Thus $w$ can be regarded as a measure of the bin size.

Since only the first-order transition is considered in this paper, $a<0$, $b<0$, $c>0$. If one supposes $\lambda>0$, then $w>0$, $s>0$, but $t$ may be negative or positive. From the definition of $J_q$ in Eq. (7) , we can get iterative relations as follows:

$$J_2(z_1,z_2)=\frac{1}{3}+\frac{1}{3}(z_1J_0(z_1,z_2)+2z_2J_1(z_1,z_2)),$$

$$J_q(z_1,z_2)=\frac{1}{3}[(q-2)J_{q-3}(z_1,z_2)+z_1J_{q-2}(z_1,z_2)+2z_2J_{q-1}(z_1,z_2)].$$

For simplicity only the case for $q_1=q_2$ is discussed, and a more general case is left for later study.

## 3 Numerical results and discussion

From the above relations, we can calculate the normalized factorial correlators $F_{qq}$ as a function of $w$ (or the bin size resolution $\delta$) with pre-specified $s$ and $t$.

In order to get numerical results for the factorial correlators, we first fix the parameters $s$ and $t$ both equal to 0.2 and analyze the dependence of $F_{qq}$ as functions of $-\ln w$ on the bin size, with q ranging from 2 to 7. The results are shown in Fig. 1. As displayed in this figure, with the decrease of bin size $\delta$ (or parameter $w$), i.e. the increase of $-\ln w$ in the figure, $F_{qq}$ increase monotonically. This can be explained as follows. For larger bins, different dynamical fluctuations perhaps counteract each other, which renders them less observable.

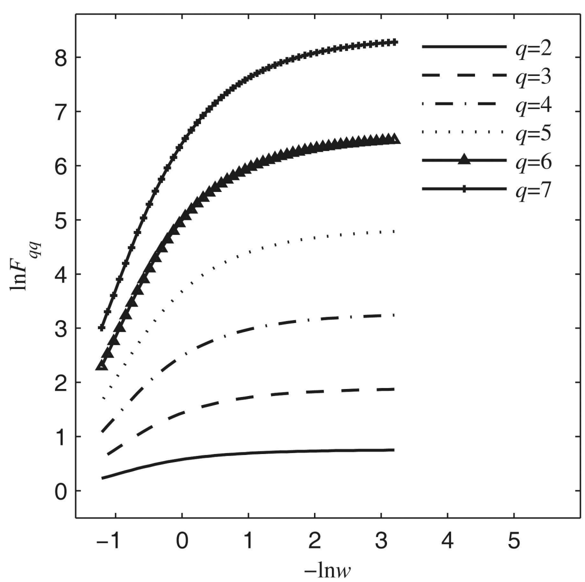


Fig. 1. The dependence of $F_{qq}$ on bin size (represented by $w$), for $q$ from 2 to 7, with parameters $s$ and $t$ fixed at 0.2.

As discussed in Ref. [21], for a self-similar dynamical process, the moments $F_{qq}$ will be a power law function of bin size $\delta$ or parameter $w$, i.e. $F_{qq}\propto w^{-\phi_q}$. It is obvious that intermittency behavior is not observed from Fig. 1, since the curves are not linear for the log–log coordinate.

The similar behaviours of $F_{qq}$ in Fig. 1 imply a quite simple relation between $F_{22}$ and $F_{qq}$. A power-law relation between $F_{22}$ and $F_{qq}$ for different values of $q$ can be reached

$$F_{qq}\propto F_{22}^{\beta_q}, \tag{9}$$

which is quite general, for Eq. (9) can still hold even if the law of intermittency is violated. The relation displayed in Eq. (9), namely the scaling behavior, can be observed in Fig. 2 for $s=0.2$, $t=0.2$ and $q$ from 3 to 7, since all the curves in the figure can be well approximated by linear lines.

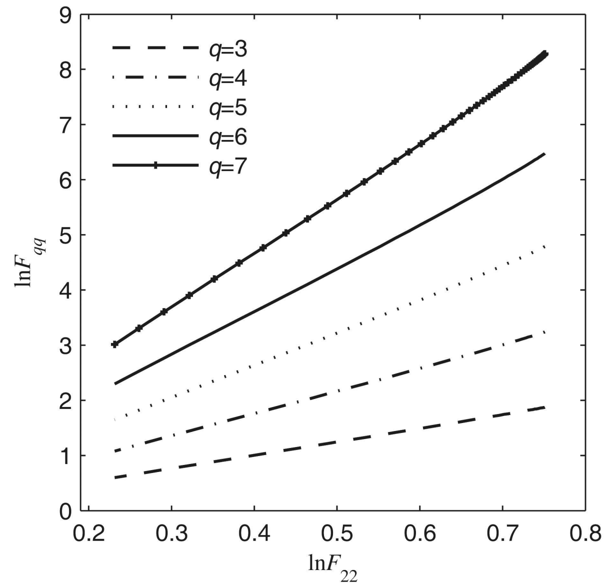


Fig. 2. Scaling behaviour between $F_{qq}$ and $F_{22}$, for $q$ from 3 to 7, and $s$ and $t$ fixed at 0.2.

The exponent $\beta_q$ is dependent on q, parameters s and t. To find an exponent that is independent of the details of our model, we can present $\beta_q$ as a function of $q$–1. The result is shown in Fig. 3 in log–log scale. Additionally, we also plot a linear fit in this picture, and immediately get

$$\beta_q\propto(q-1)^{\gamma} \tag{10}$$

with $\gamma=1.293$, which depends only on the values of parameters $s=0.2$, $t=0.2$.

This result is close to $\gamma = 1.306$ as reported in Ref. [21]. The difference between them may be ascribed to the selection of range of bin size because the value of $\gamma$ depends very weakly on the range of $-\ln w$ for the power-law fitting. If the upper limit of $-\ln w$ for the fitting is larger (corresponding to smaller bin size), then $\gamma$ will be slightly larger. In this article, the range of $-\ln w$ is from $-1.2$ to 3.2. If the lower limit is fixed, and the upper limit is extended to a higher value, for example 3.7, then $\gamma=1.303$.

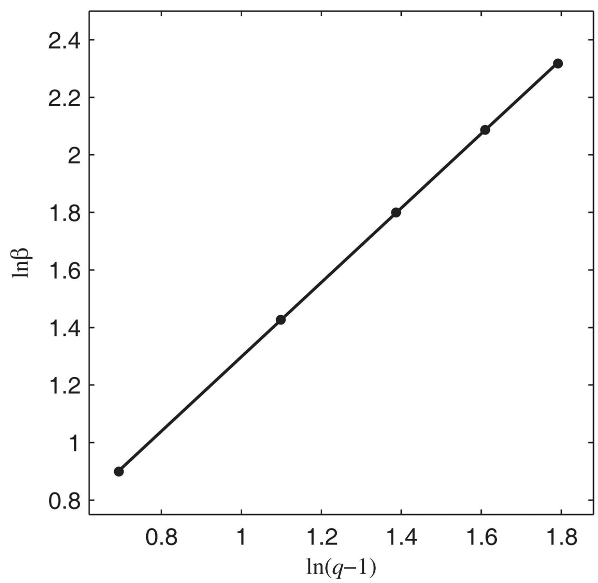


Fig. 3. Relation between $\beta_q$ and $q$, for $q$ from 3 to 7 with $s$ and $t$ fixed at 0.2. This relation can fit the scaling behavior $\beta_q \propto (q-1)^\gamma$, with $\gamma=1.293$.

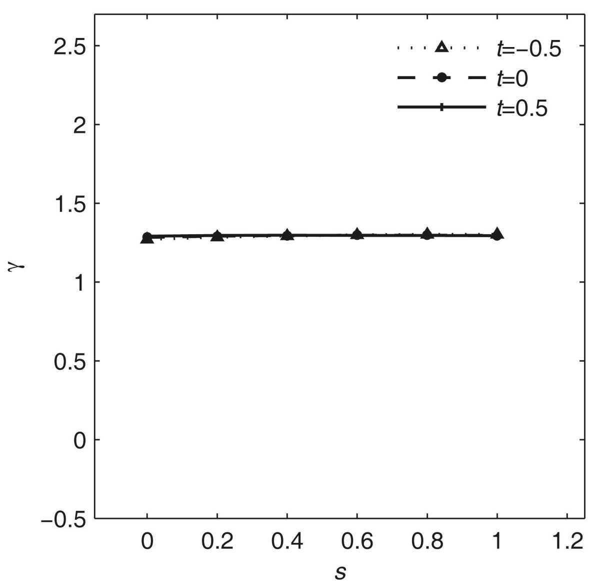


Fig. 4. Dependence of $\gamma$ on $s$ with parameter $t$ fixed at three different values −0.5, 0 and 0.5.

The next task is to study whether the same scaling behavior can be found for other values of $s$ and $t$. We have analyzed some different values for $s$ and $t$, and obtained the corresponding $\beta_q$ and $\gamma$ using the same procedure as outlined above. We come to the conclusion that the scaling relation $F_{qq} \propto F_{22}^{\beta_q}$ is valid for other values of $s$ and $t$ also.

As is depicted in Fig. 4 and Fig. 5, we also find that the exponent value $\gamma$ of Eq. (10) depends weakly on parameter $s$ in the range from 0 to 1.0 and parameter $t$ between −0.5 and 1.0. $\gamma$ is about 1.29±0.015 for other values of $s$ and $t$ in these two figures. Compared to the case of no interaction, i.e. $s=0$, it can be clearly seen that the exponent $\gamma$ is very weakly dependent on the details of interaction and only sensitive to the phase transition. Consequently, it can be regarded as a well-observable quantity to characterize the nature of phase transition.

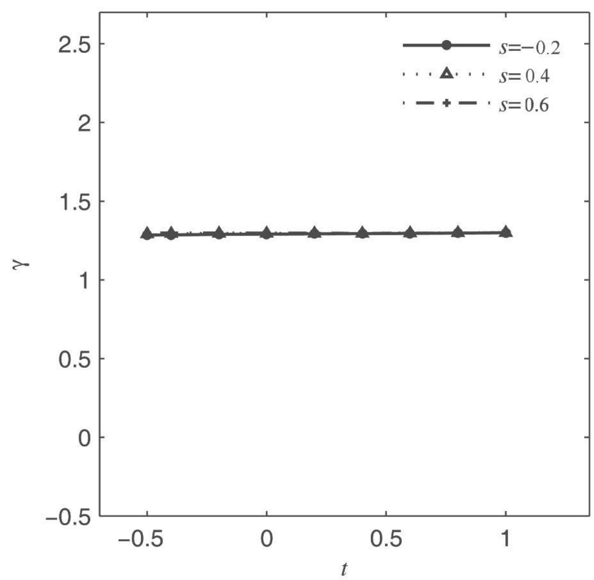


Fig. 5. Dependence of $\gamma$ on $t$ with parameter $s$ fixed at 0.2, 0.4 and 0.6.

## 4 Summary

We have studied the scaling behavior of the normalized factorial correlators for correlated multiplicity fluctuations in a first-order transition from QGP to hadrons, and found a universal scaling exponent $\gamma=1.29\pm0.015$, which is nearly independent of the dimension of phase space and the details of interactions. Therefore, it provides a practical quantity that can characterize the dynamical fluctuations during the phase transition.

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