Correspondence between phenomenological and IBM-1 models of even isotopes of Yb *

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Abstract: Energy levels and the reduced probability of E2– transitions for ytterbium isotopes with proton number $Z=70$ and neutron numbers between 100 and 106 have been calculated through phenomenological (PhM) and interacting boson (IBM-1) models. The predicted low-lying levels (energies, spins and parities) and the reduced probability for E2– transitions results are reasonably consistent with the available experimental data. The predicted low-lying levels (gr–, β_1– and γ_1– band) produced in the PhM are in good agreement with the experimental data compared with those by IBM-1 for all nuclei of interest. In addition, the phenomenological model was successful in predicting the β_2–, β_3–, β_4–, γ_2– and 1^+– band while it was a failure with IBM-1. Also, the 3^+– band is predicted by the IBM-1 model for ^{172}Yb and ^{174}Yb nuclei. All calculations are compared with the available experimental data.

Key words: ytterbium (Yb), energy levels, model, even-even, isotopes, nuclei

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1 Introduction

The medium-to heavy-mass ytterbium (Yb) isotopes located in the rare-earth mass region are well-deformed nuclei that can be populated to very high spin. Much experimental information on even-odd-mass of Yb isotopes has become more abundant [1–6]. For the heavier $A=174$ to 178 nuclei [7], previous work using deep inelastic reactions and Gammasphere have begun to reveal much information about the high-spin behavior of these neutron-rich Yb isotopes. The yrast states in the well–deformed rare-earth region have been described by using the projected shell model [8–14].

Prior to the present work the level structure of the ground band state and low-lying excited states of even-even nuclei has been studied both theoretically and experimentally [15], including the decay, Coulomb excitation and various transfer reactions.

In this study, two calculations for energy levels of ^{170,172,174,176}Yb isotopes have been done by using two different models, the phenomenological model (PhM) and the interacting boson model (IBM-1). Positive parity state energies and the reduced probability of E2– transitions are calculated and compared with the available experimental data. The structure of excited levels is discussed.

2 Theoretical models

The calculations have been performed using the phenomenological and interacting boson models. In the next subsection, we will explain these models.

2.1 Phenomenological model (PhM)

To analyze the properties of low-lying positive parity states in Yb isotopes, the PhM of [16–18] is used. This model takes into account the mixing of states of the gr–, β–, γ– and $K^\pi=1^+$– band. The model Hamiltonian is chosen in the following form

$$H=H_{\rm rot}(I^2)+H^{\sigma}_{K,K'}, \quad (1)$$

where $H_{\rm rot}(I^2)$– is the rotational part of the Hamiltonian, and

$$H^{\sigma}_{K'K}(I)=\omega_K\delta_{K,K'}-\omega_{\rm rot}(I)(j_x)_{K,K'}\xi(I,K)\delta_{K,K'\pm1}, \quad (2)$$

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where ω_{K^-} is the bandhead energy of the rotational band, $\omega_{\rm rot}(I)$ is the rotational frequency of the core, and $(j_x)_{K,K'}$ is the matrix elements which describe the Coriolis mixture between rotational bands:

$$(j_x)_{gr,1}=-\sqrt{3}\cdot\tau_0,(j_x)_{\beta,1}=-\sqrt{3}\cdot\tau_1,(j_x)_{\gamma,1}=-1\cdot\tau_2$$

and

$$\xi(I,0)=1 \qquad \xi(I,2)=\sqrt{1-\frac{2}{I(I+1)}}.$$

The eigenfunction of Hamiltonian model Eq. (1) has the form

$$\begin{aligned}|IMK\rangle = & \left[\frac{2I+1}{16\pi^2}\right]^{\frac{1}{2}}\Big\{\sqrt{2}\Psi^I_{gr,K}D^I_{MK}(\theta)\\ &+\sum_{K'}\frac{\Psi^I_{K',K}}{\sqrt{1+\delta_{K',0}}}\left[D^I_{M,K'}(\theta)b^+_{K'}\right.\\ &\left.+(-1)^{I+K'}D^I_{M,-K'}(\theta)b^+_{-K'}\right]\Big\}|0\rangle, \end{aligned} \quad (3)$$

where $\Psi^I_{K',K}$ is the amplitude of the mixture of basis states.

Solving the Schrödinger equation one can determine the eigenfunctions and eigenenergies of the positive parity states.

$$\left(H^I_{K,\nu}-\varepsilon^I_\nu\right)\Psi^I_{K,\nu}=0. \quad (4)$$

The complete energy of a state is given by

$$E^I_\nu(I)=E_{\rm rot}(I)+\varepsilon^I_\nu(I). \quad (5)$$

The rotating-core energy $E_{\rm rot}(I)$ is calculated by using the Harris parameterizations [19] of the energy and the angular momentum, that is

$$E_{\rm rot}(I) = \frac{1}{2}\jmath_0\omega^2_{\rm rot}(I)+\frac{3}{4}\jmath_1\omega^4_{\rm rot}(I), \quad (6)$$

$$\sqrt{I(I+1)} = \jmath_0\omega_{\rm rot}(I)+\jmath_1\omega^3_{\rm rot}(I), \quad (7)$$

where $\jmath_0$ and $\jmath_1$ are the inertial parameters of the rotational core.

The rotational frequency of the core $\omega_{\rm rot}(I)$ is found by solving the cubic equation Eq. (7). This equation has two imaginary roots and one real root. The real root is

$$\begin{aligned}\omega_{\rm rot}(I)=&\left\{\frac{\widetilde{I}}{2\jmath_1}+\left[\left(\frac{\jmath_0}{3\jmath_1}\right)^3+\left(\frac{\widetilde{I}}{2\jmath_1}\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}}\\ &+\left\{\frac{\widetilde{I}}{2\jmath_1}-\left[\left(\frac{\jmath_0}{3\jmath_1}\right)^3+\left(\frac{\widetilde{I}}{2\jmath_1}\right)^2\right]^{\frac{1}{2}}\right\}^{\frac{1}{3}}, \end{aligned} \quad (8)$$

where $\widetilde{I}=\sqrt{I(I+1)}$. Eq. (8) gives $\omega_{\rm rot}(I)$ at the given spin I of the core.

2.2 Interacting boson model (IBM-1)

The IBM has become one of the most intensively used nuclear models, due to its ability to describe the low-lying collective properties of nuclei across an entire major shell with a Hamiltonian. In IBM-1 the spectroscopies of low-lying collective properties of even-even nuclei are described in terms of a system of interacting s bosons (L=0) and d bosons (L=2). Furthermore, the model assumes that the structure of low-lying levels is dominated by excitations among the valence particles outside major closed shells. In the particle space the number of proton bosons N_π and neutron bosons N_ϑ is counted from the nearest closed shell, and the resulting total boson number is a strictly conserved quantity. The underlying structure of the six-dimensional unitary group $SU(6)$ of the model leads to a Hamiltonian, capable of describing the three specific types of collective structures with classical geometrical analogues (vibrational [20], rotational [21], and γ– unstable [22]) and also the transitional nuclei [23] whose structures are intermediate. The IBM-1 Hamiltonian can be expressed as [22]

$$\begin{aligned}H = & \varepsilon_s(s^+\widetilde{s})+\varepsilon_d\left(d^+\widetilde{d}\right)\\ &+\sum_{L=0,2,4}\frac{1}{2}(2L+1)^{\frac{1}{2}}C_L\left[(d^+\times d^+)^{(L)}\left(\widetilde{d}\times\widetilde{d}\right)^{(L)}\right]^{(0)}\\ &+\frac{1}{2}\widetilde{\vartheta}_0\Big[(d^+\times d^+)^{(0)}(\widetilde{s}\times\widetilde{s})^{(0)}\\ &+(s^+\times s^+)^{(0)}\left(\widetilde{d}\times\widetilde{d}\right)^{(0)}\Big]^{(0)}\\ &+\frac{1}{\sqrt{2}}\widetilde{\vartheta}_2\Big[(d^+\times d^+)^{(2)}\left(\widetilde{d}\times\widetilde{s}\right)^{(2)}\\ &+(d^+\times s^+)^{(2)}\left(\widetilde{d}\times\widetilde{d}\right)^{(2)}\Big]^{(0)}\\ &+u_2\left[(d^+\times s^+)^{(2)}\left(\widetilde{d}\times\widetilde{s}\right)^{(2)}\right]^{(0)}\\ &+\frac{1}{2}u_0\left[(s^+\times s^+)^{(0)}(\widetilde{s}\times\widetilde{s})^{(0)}\right]^{(0)}, \end{aligned} \quad (9)$$

where $(s^\dagger,d^\dagger)$ and (s,d) are creation and annihilation operators for s and d bosons, respectively.

The IBM-1 Hamiltonian equation Eq. (9) can be written in the general form as [24]

$$H=\varepsilon n_d+a_0P^\dagger P+a_1LL+a_2QQ+a_3T3T3+a_4T4T4. \quad (10)$$

The full Hamiltonian H contains six adjustable parameters, where $\varepsilon=\varepsilon_d-\varepsilon_s$ is the boson energy and $Q=(d^+\times\widetilde{s}+s^+\times\widetilde{d})^2+X(d^+\times\widetilde{d})^2$; here $X=CHI$. The parameters a_0, a_1, a_2, a_3 and a_4 designate the strength of the pairing, angular momentum, quadrupole, octupole and hexadecapole interaction between the bosons. The calculations have been performed with IBM-1 for the code

and hence, no distinction is made between neutron and proton bosons taking results is given in the next section.

3 Result and discussion

In this section, the calculated results are discussed separately for low-lying states of even-even isotopes of Yb, with neutron number from 100 to 106. Our results include energy levels and the reduced probability of E2–transitions.

3.1 Energy levels

To describe the positive parity states in the PhM, we determine the model parameters via the following procedure. In accordance with [25], we suppose that, at low spins, the rotational core energy is the same as the energies of the ground band states.

Description of the parameters:
– the inertial parameters $\jmath_0$ and $\jmath_1$ of rotational core determined by Eq. (6), using the experimental energy of ground band states up to spin $I\leqslant 10\hbar$;
– headband energy of ground (gr)–, and β_1– band states taken from experimental data, because they are not perturbed by the Coriolis forces at $I=0$;
– headband energy of γ–, $K^\pi=1^+$ bands (ω_γ and ω_{1^+}) and also matrix elements $\langle K|j_x|K^{'}\pm 1\rangle$ are free parameters of the model. They have been fitted by the least squares method requiring the best agreement between the theoretical energies and experimental data. The fitting parameters of the model are given in Table 1.

Table 1. Parameters used in PhM to calculate energy of low excited states in Yb isotopes.

A	$(j_x)_{gr,1}$	$(j_x)_{\beta_1,1}$	$(j_x)_{\gamma_1,1}$
170	0.186	0.394	0.728
172	0.275	0.978	0.325
174	0.185	0.400	0.085
176	0.200	0.540	0.400

Note: where, $(j_x)_{K',K}$– are matrix elements of the Coriolis interactions and Q_0– is the intrinsic quadrupole moment of the nucleus (in fm^2 units) taken from Ref. [26].

Also, in the present work the rotational limit of IBM-1 has been applied to $^{170-176}$Yb, from the ratio $(E(4^+)/E(2^+))$ it has been found that the $^{170-176}$Yb isotopes are rotational (deformed nuclei) and these nuclei have been successfully treated as axhibiting the $SU(3)$ symmetry of IBM-1.

In IBM-1, the $(E(4^+)/E(2^+))$ ratio is 3.33, 2.5 and 2 for the $SU(3)$, $O(6)$ and $U(5)$ limits, respectively. From experimental data, the $(E(4^+)/E(2^+))$ ratios are 3.302, 3.33, 3.329 and 3.319 for $^{170-176}$Yb isotopes, respectively. Therefore, the $SU(3)$ limits have been applied to study the properties of the Yb isotopes (see Eq. (11)) and the parameters ε,a_0,a_3 and a_4 vanish ($\varepsilon=a_0=a_3=a_4=0$) for the $SU(3)$ limit.

For the analysis of excitation energies in Yb isotopes we tried to keep to the minimum number of free parameters in the Hamiltonian. The explicit expression of the Hamiltonian adopted in the calculations is [24]:

$$H=a_1L\cdot L+a_2Q\cdot Q. \tag{11}$$

In the framework of IBM-1, for the isotopic chains of Yb with $Z=70$ nuclei, the number of proton boson holes is 6, the number of neutron boson particles which varies from 9 to 11 for $^{170-174}$Yb, and the number of neutron boson holes for ^{176}Yb is 10. Table 2 shows the coefficient values which we used in IBM-1. The comparison of calculated energy levels and experimental data of low-lying states of ^{170}Yb, ^{172}Yb, ^{174}Yb and ^{176}Yb isotopes are presented in Figs. 1–4, respectively. The PhM calculations are plotted on the left of the experimental data and the IBM-1 calculations on the right for the gr–, β_1– and γ_1– band. The experimental data are taken from [26] for all isotopes of Yb and also from [27–30] for $^{170-174}$Yb and ^{176}Yb, respectively. From these figures, we can see that our calculated results (energies, spin and parity) in both models are reasonably consistent with experimental data, except that the γ_1– band energies in the IBM-1 calculations for ^{172}Yb and ^{174}Yb nuclei disagree with the experimental data. Also the phenomenological calculations are in better agreement with the experimental data than from those of IBM-1. In the high spin these figures show the difference between our calculation and the experimental data. Furthermore the phenomenological model predicts the energies, spin and parity of the β_2–, β_3–, β_4–, γ_2– and 1^+ band, as is shown in Tables 1–7, respectively. Finally, we believe that the failure to use a pairing parameter was the cause of the disagreement between the IBM-1 calculations and experimental data; this will be discussed in future studies.

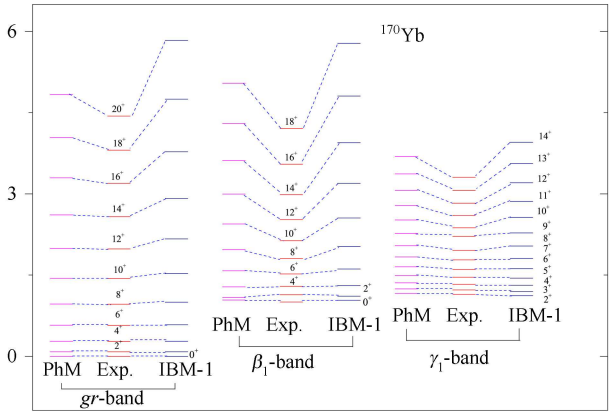

Fig. 1. (color online) Comparison of calculation energy values by PhM and IBM-1 with experimental data for ^{170}Yb.

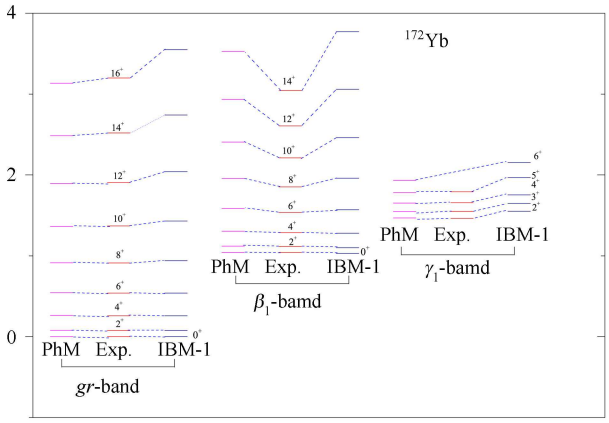

Fig. 2. (color online) Comparison of calculation energy values by PhM and IBM-1 with experimental data for ^{172}Yb.

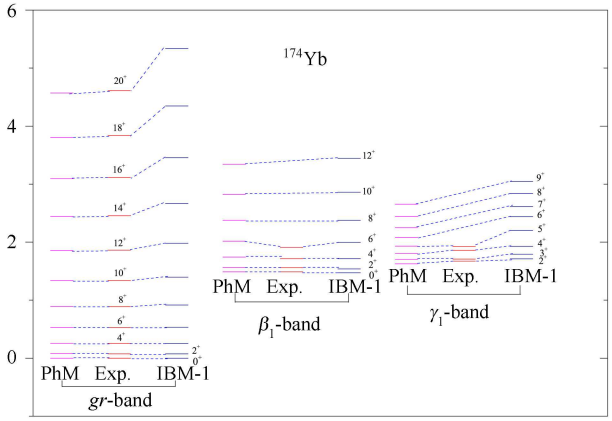

Fig. 3. (color online) Comparison of calculation energy values by PhM and IBM-1 with experimental data for ^{174}Yb.

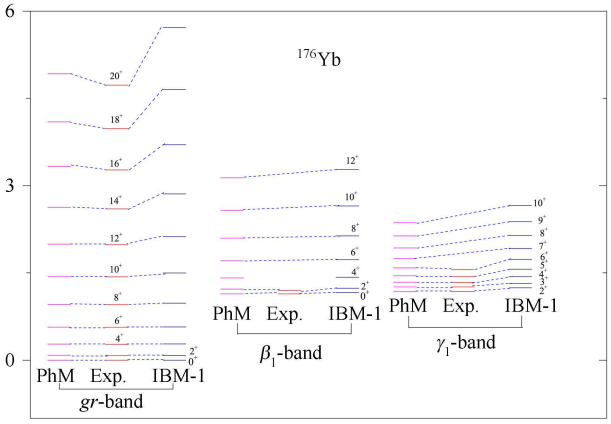

Fig. 4. (color online) Comparison of calculation energy values by PhM and IBM-1 with experimental data for ^{176}Yb.

3.2 Reduced probability of B(E2)– transitions

In the PhM, with the wave functions calculated by solving the Shrödinger equation Eq. (4), the reduced probabilities of E2– transitions between states $I_{\rm i}K_{\rm i}$ and states $I_{\rm f}K_{\rm f}$ are calculated [16, 18] as:

$$\begin{aligned} &B({\rm E2};I_{\rm i}K_{\rm i}\to I_{\rm f}K_{\rm f})\\ &=\left\{\sqrt{\frac{5}{16\pi}}eQ_0\left[\Psi^{I_{\rm f}}_{gr,gr}\Psi^{I_{\rm i}}_{gr,K_{\rm i}}C^{I_{\rm f}0}_{I_{\rm i}0;20}\right.\right.\\ &\left.+\sum_n\Psi^{I_{\rm f}}_{K_n,gr}\Psi^{I_{\rm i}}_{K_n,K_{\rm i}}C^{I_{\rm f}K_n}_{I_{\rm i}K_n;20}\right]\\ &+\sqrt{2}\left[\Psi^{I_{\rm f}}_{gr,gr}\sum_n\frac{(-1)^{K_n}m_{K_n}\Psi^{I_{\rm i}}_{K_n,K_{\rm i}}}{\sqrt{1+\delta_{K_n,0}}}C^{I_{\rm f}0}_{I_{\rm i}K_n;2-K_n}\right.\\ &\left.\left.+\Psi^{I_{\rm i}}_{gr,K_{\rm i}}\sum_n\frac{m_{K_n}\Psi^{I_{\rm f}}_{K_n,gr}}{\sqrt{1+\delta_{K_n,0}}}C^{I_{\rm f}K_n}_{I_{\rm i}K_n;2K_n}\right]\right\}^2, \end{aligned}\tag{12}$$

where $m_{K_n}=\langle gr|\hat{m}(E2)|K^\pi\rangle$ (K^π=0$^+$,2$^+$ and 1$^+$) are constants to be determined from the experimental data, Q_0 is the internal quadrupole moment of the nucleus, and $C^{I_{\rm f}K_{\rm f}}_{I_{\rm i}K_{\rm i};2(K_{\rm i}+K_{\rm f})}$ are the Clebsch-Gordon coefficients.

Table 2. Parameters used in IBM-1 to calculate energy of excited states in Yb isotopes.

A	a_1	a_2	CHI
170	0.0094	−0.0120	−1.30
172	0.0091	−0.0112	−1.20
174	0.0084	−0.0150	−1.24
176	0.0089	−0.0126	−1.30

Table 3. Values of parameters obtained from $\widehat{T}$(E2) operator in program IBMT for calculating the absolute values of B(E2).

A	$\alpha 2$	$\beta 2$
170	0.1060	−0.140
172	0.1037	−0.137
174	0.0960	−0.126
176	0.0980	−0.129

Another advantage of the interacting d- boson model is the matrix elements of the electric quadrupole operator. The reduced matrix elements of the E2 operator $\widehat{T}$(E2) have the form [20–22]

$$\begin{aligned}\widehat{T}({\rm E2}) &= \alpha 2\left[d^\dagger\widetilde{s}+s^\dagger\widetilde{d}\right]^2+\beta 2\left[d^\dagger\widetilde{d}\right]^2\\ &= \alpha 2\left\{\left[d^\dagger\widetilde{s}+s^\dagger\widetilde{d}\right]^2+\chi\left[d^\dagger\widetilde{d}\right]^2\right\}=e_BQ,\end{aligned}\tag{13}$$

here $\alpha 2$ and $\beta 2$ are two parameters and $\beta 2=\chi\alpha 2$, $\alpha 2=e_B$ (effective charge). The values of these parameters are presented in Table 3. Then the B(E2) values are given by

$$B({\rm E2};J_{\rm i}\to J_{\rm f})=\frac{1}{2J_{\rm i}+1}|\langle J_{\rm f}\|\widehat{T}(E2)\|J_{\rm i}\rangle|^2.\tag{14}$$

Table 4. Energy levels of β_2– band of Yb isotopes (in MeV).

I	^{170}Yb		^{172}Yb		^{174}Yb		^{176}Yb	
	Exp. [26, 27]	PhM	Exp. [26, 28]	PhM	Exp. [26, 29]	PhM	Exp. [26, 30]	PhM
0^+	1.228	1.228	1.404	1.404	1.885	1.885	1.518	1.518
2^+	1.306	1.313	1.476	1.483	1.958	1.962	1.610	1.601
4^+	–	1.507	1.632	1.666	2.123	2.140	–	1.792
6^+	–	1.804	–	1.947	–	2.414	–	2.086
8^+	–	2.195	–	2.317	–	2.770	–	2.476
10^+	–	2.669	–	2.769	–	3.221	–	2.954
12^+	–	3.220	–	3.295	–	3.740	–	3.512

Table 5. Energy levels of β_3– band of Yb isotopes (in MeV).

I	^{170}Yb		^{172}Yb		^{174}Yb		^{176}Yb	
	Exp. [26, 27]	PhM	Exp. [26, 28]	PhM	Exp. [26, 29]	PhM	Exp. [26, 30]	PhM
0^+	1.479	1.479	1.794	1.794	2.113	2.110	1.779	1.779
2^+	1.534	1.564	1.849	1.873	2.172	2.178	–	1.862
4^+	1.667	1.758	1.975	2.056	2.336	2.356	–	2.053
6^+	–	2.055	2.156	2.156	–	2.630	–	2.347
8^+	–	2.446	–	2.707	–	2.993	–	2.737
10^+	–	2.920	–	3.159	–	3.437	–	3.215
12^+	–	3.471	–	3.685	–	3.956	–	3.773

Table 6. Energy levels of β_4– band of Yb isotopes (in MeV).

I	^{170}Yb		^{172}Yb		^{174}Yb		^{176}Yb	
	Exp. [26, 27]	PhM	Exp. [26, 28]	PhM	Exp. [26, 29]	PhM	Exp. [26, 30]	PhM
0^+	1.894	1.894	2.821	2.821				
2^+	1.956	1.973	–	2.898				
4^+	2.100	2.156	–	3.076				
6^+	–	2.437	–	3.350				
8^+	–	2.807	–	3.713				
10^+	–	3.259	–	4.157				
12^+	–	3.785	–	4.676				

For the calculations of the absolute B(E2) values the two parameters $\alpha 2$ and $\beta 2$ of Eq. (13) are adjusted according to the experimental $B(\mathrm{E2};2^+_{gr} \to 0^+_{gr})$. Table 8 shows the values of the parameters $\alpha 2$ and $\beta 2$, obtained in the present calculations. We present our calculated results of the reduced probability of E2– transitions of both models, and the comparison of calculated values of B(E2) transitions with experimental data [31] are given in Table 9 for all nuclei of interest. In general, most of the calculated results in both models are reasonably consistent with the available experimental data, except for a few cases that deviate from the experimental data. As mentioned in Table 9 PhM calculations are better than those of IBM-1 when compared with the experimental data, except $B(\mathrm{E2};2^+_{gr}\to 0^+_{gr})$ for ^{170}Yb, ^{174}Yb and ^{176}Yb, $B(\mathrm{E2};12^+_{gr}\to 10^+_{gr})$ B(E2;) for ^{170}Yb, $B(\mathrm{E2};6^+_{gr}\to 4^+_{gr})$ for ^{172}Yb and $B(\mathrm{E2};4^+_{gr}\to 2^+_{gr})$, $B(\mathrm{E2};14^+_{gr}\to 12^+_{gr})$ for ^{174}Yb.

Table 7. Energy levels of γ_2– band of Yb isotopes (in MeV).

I	^{172}Yb		^{174}Yb	
	Exp. [26, 28]	PhM	Exp. [26, 29]	PhM
2^+	1.608	1.619	2.728	2.727
3^+	1.700	1.698	2.793	2.804
4^+	1.803	1.802	2.882	2.905
5^+	1.926	1.931	–	3.031
6^+	2.075	2.083	–	3.179
7^+	–	2.257	–	3.350
8^+	–	2.453	–	3.542
9^+	–	2.669	–	3.754
10^+	–	2.905	–	3.986
11^+	–	3.159	–	4.236
12^+	–	3.431	–	4.505
13^+	–	3.720	–	4.791
14^+	–	4.026	–	5.093

Table 8. Energy levels of 1^+– band of Yb isotopes (in MeV).

I	^{170}Yb		^{172}Yb		^{174}Yb		^{176}Yb	
	Exp. [26, 27]	PhM	Exp. [26, 28]	PhM	Exp. [26, 29]	PhM	Exp. [26, 30]	PhM
1^+	1.606	1.605	2.009	2.006	1.624	1.605	1.819	1.818
2^+	1.832	1.662	2.047	2.059	1.674	1.657	1.867	1.874
3^+	–	1.746	2.109	2.138	1.733	1.734	–	1.956
4^+	–	1.856	2.193	2.242	1.859	1.835	–	2.065
5^+	–	1.993	–	2.371	–	1.961	–	2.200
6^+	–	2.153	–	2.523	–	2.109	–	2.359
7^+	–	2.337	–	2.697	–	2.280	–	2.542
8^+	–	2.544	–	2.893	–	2.472	–	2.749
9^+	–	2.771	–	3.109	–	2.684	–	2.977
10^+	–	3.018	–	3.345	–	2.916	–	3.227
11^+	–	3.285	–	3.599	–	3.166	–	3.496
12^+	–	3.569	–	3.871	–	3.435	–	3.785
13^+	–	3.871	–	4.160	–	3.721	–	4.093
14^+	–	4.190	–	4.466	–	4.023	–	4.419

Table 9. Values of B(E2)– transitions isotopes of Yb (in W.u.).

$I_\mathrm{i}K_\mathrm{i}\to I_\mathrm{f}K_\mathrm{f}$	^{170}Yb			^{172}Yb		
	Exp. [31]	PhM	IBM-1	Exp. [31]	PhM	IBM-1
$2^+_{gr}\to 0^+_{gr}$	201(6)	216	198.543	212(2)	212	211.689
$4^+_{gr}\to 2^+_{gr}$	–	309	280.768	301(20)	303	299.697
$6^+_{gr}\to 4^+_{gr}$	–	340	303.549	320(30)	334	324.746
$8^+_{gr}\to 6^+_{gr}$	360(30)	356	309.178	400(40)	350	331.835
$10^+_{gr}\to 8^+_{gr}$	356(25)	366	306.15	375(23)	359	329.971
$12^+_{gr}\to 10^+_{gr}$	268(21)	372	296.956	430(60)	366	322.160
$14^+_{gr}\to 12^+_{gr}$	–	377	283.181	394^{+60}_{-45}	370	309.724
$16^+_{gr}\to 14^+_{gr}$	–	381	265.349	–	374	293.311
$18^+_{gr}\to 16^+_{gr}$	–	383	243.819	–	376	273.310
$20^+_{gr}\to 18^+_{gr}$	–	386	218.751	–	379	249.967
$I_\mathrm{i}K_\mathrm{i}\to I_\mathrm{f}K_\mathrm{f}$	^{174}Yb			^{176}Yb		
	Exp. [31]	PhM	IBM-1	Exp. [31]	PhM	IBM-1
$2^+_{gr}\to 0^+_{gr}$	201(7)	205	199.908	183(7)	184	182.916
$4^+_{gr}\to 2^+_{gr}$	280(9)	294	283.321	270(25)	263	258.969
$6^+_{gr}\to 4^+_{gr}$	370(50)	324	307.532	298(22)	290	280.618
$8^+_{gr}\to 6^+_{gr}$	388(21)	339	315.122	300(5)	303	286.743
$10^+_{gr}\to 8^+_{gr}$	335(22)	348	314.533	–	312	285.139
$12^+_{gr}\to 10^+_{gr}$	369(23)	354	308.624	–	317	278.384
$14^+_{gr}\to 12^+_{gr}$	320(8)	359	298.941	–	321	267.636
$16^+_{gr}\to 14^+_{gr}$	–	362	285.192	–	324	253.459
$18^+_{gr}\to 16^+_{gr}$	–	365	268.659	–	327	236.177
$20^+_{gr}\to 18^+_{gr}$	–	367	249.249	–	328	215.995

4 Summary

In this paper, energy levels and the reduced probability of E2– transitions positive parity for $^{170-176}$Yb isotopes with neutron numbers between 100 and 106 have been calculated through PhM and IBM-1 models. The predicted low-lying levels ($gr-$,β_1- and γ_1- band) by PhM are in good agreement with the experimental data as compared with those by IBM-1 for all nuclei of interest. In addition, the PhM is successful in predicting the β_2-, β_3-, β_4-, γ_2- and 1^+- band while IBM-1 fails. Also, the 3^+- band is predicted by IBM-1 for ^{172}Yb and ^{174}Yb nuclei. All calculations are compared with the available experimental data. Also, the reduced probability of E2– transitions of PhM calculations are better than those of IBM-1 when compared with the experimental data, except $B(\mathrm{E2};2^+_{gr}\rightarrow 0^+_{gr})$ for ^{170}Yb, ^{174}Yb and ^{176}Yb, $B(\mathrm{E2};6^+_{gr}\rightarrow 4^+_{gr})$ for ^{172}Yb, $B(\mathrm{E2};4^+_{gr}\rightarrow 2^+_{gr})$ and $B(\mathrm{E2};14^+_{gr}\rightarrow 12^+_{gr})$ for ^{174}Yb and also $B(\mathrm{E2};12^+_{gr}\rightarrow 10^+_{gr})$ $B(\mathrm{E2};\ )$ for ^{170}Yb.

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