

Producing terahertz coherent synchrotron radiation at the Hefei Light Source^{*}

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Abstract: This paper theoretically proves that an electron storage ring can generate coherent radiation in the THz region using a quick kicker magnet and an AC sextupole magnet. When the vertical chromaticity is modulated by the AC sextupole magnet, the vertical beam collective motion excited by the kicker produces a wavy spatial structure after a number of longitudinal oscillation periods. The radiation spectral distribution was calculated from the wavy bunch parameters at the Hefei Light Source (HLS). When the electron energy is reduced to 400 MeV, extremely strong coherent synchrotron radiation (CSR) at 0.115 THz should be produced.

Key words: CSR, THz, storage ring

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1 Introduction

Synchrotron radiation is coherent when the wavelength of the radiation can be compared with the bunch length. In a storage ring, the bunch length is typically tens of picoseconds (ps). Therefore, the coherent radiation has a spectrum mainly in the short-wave radio and microwave regions, which can be suppressed by metallic shields [1]. For ultrashort bunches, the coherent synchrotron radiation (CSR) can be extended to the THz range. A THz wave is an electromagnetic wave within the band of frequencies from 0.1 to 10 THz. Terahertz radiation occupies the middle ground between microwaves and infrared light waves in the electromagnetic spectrum. It has many excellent features that can be widely used in research.

Compared with other methods, the THz radiation from a storage ring is brilliant, broadband and stable [2]. In order to obtain a short-pulse electron beam bunch, the common approach is quasi-isochronous operation, by which the momentum compaction factor of the storage ring is reduced to nearly zero. There are many storage ring light sources worldwide which apply this method to supply stable THz or sub-THz radiation [2–8]. However in these cases, the intensity is limited by a low electron beam instability threshold, while the generation of shorter wavelength radiation requires extreme stability for the ring [9]. Ref. [10] proposes a simpler method for

generating coherent radiation in the THz region from an electron storage ring with a quick kicker magnet and an AC sextupole magnet. When the vertical chromaticity is modulated by the AC sextupole magnet, the vertical beam collective motion excited by the kicker produces a wavy spatial structure after a number of longitudinal oscillation periods. The modulated electron bunch can produce CSR in the THz region. This narrow bandwidth radiation is extremely strong, can be tuned by controlling the ring parameters, and is easy to generate.

This paper investigates the theoretical possibility of applying the above method to the Hefei Light Source (HLS), and calculates the radiation spectral distribution from the wavy bunch in detail. Based on the results, suitable parameters are chosen for the kicker and the AC sextupole magnet.

2 Production of bunch structure

2.1 Bunch modulation

After being excited by the quick kicker magnet, the vertical displacement can be expressed by:

$$y = y_{\text{beta}} + y_{\text{kicker}}. \quad (1)$$

Here y_{beta} is betatron motion and y_{kicker} is produced by the vertical kicker magnet.

The kick angle is denoted by θ . If the electron bunch is excited by the kicker at time $t = 0$, then the vertical

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displacement produced by the vertical kicker magnet is:

$$\begin{bmatrix} y \\ y' \end{bmatrix}_{\text{kicker}} = \begin{bmatrix} 0 \\ \theta \end{bmatrix}. \quad (2)$$

In a storage ring, the transfer matrix [11] can be expressed by:

$$M(s_2/s_1) = \begin{bmatrix} \sqrt{\frac{\beta_2}{\beta_1}}(\cos\Delta\psi + \alpha_1\sin\Delta\psi) & \sqrt{\beta_1\beta_2}\sin\Delta\psi \\ -\frac{(1+\alpha_1\alpha_2)\sin\Delta\psi + (\alpha_2 - \alpha_1)\cos\Delta\psi}{\sqrt{\beta_1\beta_2}} & \sqrt{\frac{\beta_1}{\beta_2}}(\cos\Delta\psi - \alpha_2\sin\Delta\psi) \end{bmatrix}.$$

Here α_1, β_1 are the Twiss parameters at position s_1 and α_2, β_2 are the Twiss parameters at position s_2 . $\Delta\psi$ is the phase difference from s_1 to s_2 .

Therefore, after being excited by the kicker, the displacement at any point is expressed by:

$$y_{\text{kicker}} = M(s_2/s_1) \begin{bmatrix} y \\ y' \end{bmatrix}_{\text{kicker},1} = \theta \sqrt{\beta_1\beta_2} \sin\Delta\psi. \quad (3)$$

Here s_1 is the kicker location, namely the excited point. Only vertical coherent oscillation is considered. The oscillation equation is expressed by:

$$y = y_0 \sin\Delta\psi. \quad (4)$$

After modulation, the vertical chromaticity, whose frequency is ω , is expressed by:

$$\xi_y = \xi_0 + \xi_1 \sin\omega t. \quad (5)$$

Here ξ_0 and ξ_1 are the amplitudes of the DC (off-set) and AC (modulation) components. When $\omega = \omega_s$, where ω_s is the angular frequency of the synchrotron oscillation, the chromaticity produces a betatron phase shift [10] which is given by:

$$\delta\psi = \frac{2\pi}{T_{\text{rev}}} \left[\varepsilon \left(\frac{\xi_0}{\omega_s} + \frac{\xi_1}{2} t \right) \sin\omega_s t + \tau \left(\xi_0 \frac{\cos\omega_s t - 1}{\alpha_p} + \xi_1 \frac{\omega_s t \cos\omega_s t - \sin\omega_s t}{2\alpha_p} \right) \right]. \quad (6)$$

Here T_{rev} is the electron cyclotron period, τ is longitudinal time displacement, ε is the energy spread and α_p is the momentum compaction factor.

When $\omega_s t = n\pi, n=0,1,2,\dots$:

$$\delta\psi = \omega_\psi \tau, \quad \omega_\psi = -\frac{2\pi}{T_{\text{rev}}} \frac{1}{\alpha_p} \left[(\pm 1 + 1)\xi_0 \pm \frac{1}{2} n\pi \xi_1 \right]. \quad (7)$$

This indicates that the vertical collective oscillation has nothing to do with the energy spread at these moments. In other words, the electron bunch produces a periodic structure along the longitudinal axis. The distribution of the bunch in the τ - y plane is shown in Fig. 1. As n increases, the bunch repetition frequency along the longitudinal axis is higher, which produces higher CSR frequency.

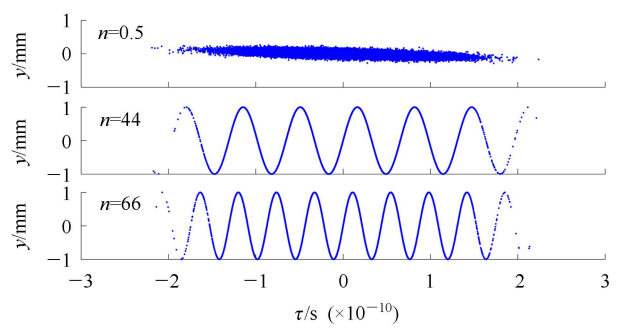


Fig. 1. (color online) Distribution of the bunch in the τ - y plane at different moments. Here y means the vertical displacement, and τ means the longitudinal time displacement. The phase of the reference electron is $\psi_{y0} = 0$. When n is an integer, the bunch produces a periodic structure along the longitudinal axis.

2.2 Radiation excitation

For convenience, Eq. (6) is rewritten as follows:

$$\delta\psi = A(t)\varepsilon + B(t)\tau. \quad (8)$$

Assume that an electron, whose longitudinal parameters are (ε, τ) , emits a quantum of energy ε_N at time $t = t_N$, which causes an extra phase shift:

$$\Delta(\delta\psi) = -A(t_N)\varepsilon_N. \quad (9)$$

The vertical collective motion then becomes:

$$\begin{aligned} y &= y_0 \sin[\Phi_{y0} + \delta\Phi_y + \Delta(\delta\Phi_y)] \\ &= y_0 \sin(\Phi_{y0} + \delta\Phi_y) \cos[\Delta(\delta\Phi_y)] \\ &\quad + y_0 \cos(\Phi_{y0} + \delta\Phi_y) \sin[\Delta(\delta\Phi_y)]. \end{aligned} \quad (10)$$

The expected phase shift variance [10] produced by random radiation is given by:

$$\sigma_{\Psi N}^2 = \langle \varepsilon_N^2 \rangle \int_0^t A^2(t_N) dt_N. \quad (11)$$

$\langle \varepsilon_N^2 \rangle = \frac{4\sigma_\varepsilon^2}{\tau_L}$, τ_L is longitudinal damping time. Substituting this into Eq. (11), we then get:

$$\begin{aligned} \sigma_{\Psi N}^2 = & \left(\frac{2\pi}{T_{\text{rev}}} \right)^2 \frac{\sigma_{\varepsilon}^2}{\tau_L \omega_s^3} \left\{ \xi_0^2 (2\omega_s t - \sin 2\omega_s t) \cos^2 \omega_s t \right. \\ & + \xi_0 \xi_1 \left(\omega_s^2 t^2 \cos^2 \omega_s t - \frac{1}{4} \omega_s t \sin 4\omega_s t \right) + \left(\frac{\xi_1}{2} \right)^2 \\ & \left. \times \left[\frac{2}{3} \omega_s^3 t^3 - \omega_s t + \sin 2\omega_s t \left(\frac{1}{2} + \omega_s^2 t^2 \sin^2 \omega_s t \right) \right] \right\}. \end{aligned} \quad (12)$$

Assuming that the extra phase shift follows a Gaussian distribution, then the bunch average position is:

$$\langle y \rangle = y_0 \sin(\Phi_{y0} + \delta\Phi_y) \exp\left(-\frac{\sigma_{\Psi N}^2}{2}\right). \quad (13)$$

where $\langle \rangle$ stands for the average over the extra phase shift. As time increases, $\sigma_{\Psi N}^2$ becomes greater, and the electron bunch becomes incoherent. Meanwhile, the vertical collective oscillation amplitude becomes smaller. In principle, ω_{ψ} should be as large as possible to gain higher THz CSR, and $\sigma_{\Psi N}^2$ should be as small as possible to maintain better coherence.

3 Radiation

3.1 Single electron

The far-field radiation generated by a single electron [12] is:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \int_{-\infty}^{\infty} \vec{n} \times (\vec{n} \times \vec{\beta}) e^{j\omega[t - \vec{n} \cdot \vec{r}(t)/c]} dt \right|^2. \quad (14)$$

where \vec{n} is the unit vector from the electron to the observation point, $\vec{r}(t)$ stands for the position of the electron and $\vec{\beta}$ is the velocity relative to the light speed.

We define:

$$\begin{aligned} \vec{A}(\omega) = & \int_{-\infty}^{\infty} \vec{n} \times (\vec{n} \times \vec{\beta}) e^{j\omega[t - \vec{n} \cdot \vec{r}(t)/c]} dt \\ \approx & \frac{1}{\sqrt{3}} \left[-\vec{e}_{\parallel} \left(\frac{1}{\gamma^2} + \theta^2 \right) K_{\frac{2}{3}}(\xi) \right. \\ & \left. + \vec{e}_{\perp} \theta \left(\frac{1}{\gamma^2} + \theta^2 \right) K_{\frac{1}{3}}(\xi) \right]. \end{aligned} \quad (15)$$

where

$$\xi = \frac{\omega \rho}{3c} \left(\frac{1}{\gamma^2} + \theta^2 \right)^{\frac{3}{2}}. \quad (16)$$

\vec{e}_{\parallel} is the unit vector in the y direction, corresponding to horizontal polarization, and $\vec{e}_{\perp} = \vec{n} \times \vec{e}_{\parallel}$ is the orthogonal polarization vector corresponding approximately to

vertical polarization. Then

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left| \vec{A}(\omega) \right|^2. \quad (17)$$

In the electron bunch, $\vec{r}(t)$ is the reference electron position at time t , \vec{R} is the vector from the reference electron to the observation point and θ is the angle between \vec{n} and the horizontal. Consider an arbitrary electron in the bunch:

$$\vec{r}'(t) - \vec{r}(t) = x\vec{e}_x + y\vec{e}_y + c\tau\vec{e}_s, \quad (18)$$

where x , y , $c\tau$ and θ are first order small quantities. Ignoring higher-order small quantities:

$$\vec{n} = \cos\theta\vec{e}_s + \sin\theta\vec{e}_y,$$

$$\vec{n}' = \frac{\vec{R} - \vec{r}'}{|\vec{R} - \vec{r}'|} \approx \vec{n}, \quad (19)$$

$$\vec{n}' \cdot \vec{r}'(t) - \vec{n} \cdot \vec{r}(t) = c\tau \cos\theta + y \sin\theta \approx c\tau.$$

So:

$$\begin{aligned} \vec{A}'(\omega) = & \int_{-\infty}^{\infty} \vec{n}' \times (\vec{n}' \times \vec{\beta}) e^{j\omega[t - \vec{n}' \cdot \vec{r}'(t)/c]} dt \\ = & \vec{A}(\omega) e^{-j\omega\tau}. \end{aligned} \quad (20)$$

3.2 Coherent radiation

The total radiation spectrum of the bunch [13] is:

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} = & \frac{e^2 \omega^2}{4\pi^2 c} \sum_m^{N_e} \vec{A}(\omega) e^{-j\omega\tau_m} \cdot \sum_n^{N_e} \vec{A}^*(\omega) e^{j\omega\tau_n} \\ = & \frac{e^2 \omega^2}{4\pi^2 c} |\vec{A}(\omega)|^2 \sum_m^{N_e} \left[1 + \sum_{n \neq m}^{N_e} e^{j\omega(\tau_n - \tau_m)} \right] \\ = & \frac{e^2 \omega^2}{4\pi^2 c} |\vec{A}(\omega)|^2 \left[N_e + \sum_m^{N_e} \sum_{n \neq m}^{N_e} e^{j\omega(\tau_n - \tau_m)} \right] \\ = & \frac{e^2 \omega^2}{4\pi^2 c} |\vec{A}(\omega)|^2 [N_e + N_e(N_e - 1) \langle e^{j\omega(\tau_n - \tau_m)} \rangle]. \end{aligned} \quad (21)$$

where N_e is the number of electrons in the bunch, and $\langle \rangle$ stands for the average for all electrons.

Assuming that the electrons follow a Gaussian distribution in the longitudinal direction,

$$\Phi(\tau) = \frac{N_e}{\sqrt{2\pi}\sigma_{\tau}} \exp\left(-\frac{\tau^2}{2\sigma_{\tau}^2}\right)$$

where σ_{τ} is the natural bunch length. Then

$$\begin{aligned} \langle e^{j\omega(\tau_n - \tau_m)} \rangle = & \iint d\tau_n d\tau_m \frac{\Phi(\tau_n)}{N_e} \frac{\Phi(\tau_m)}{N_e} e^{j\omega(\tau_n - \tau_m)} \\ = & \exp(-\omega^2 \sigma_{\tau}^2). \end{aligned} \quad (22)$$

The total spectrum is then:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} |\vec{A}(\omega)|^2 [N_e + N_e(N_e - 1) \exp(-\omega^2 \sigma_\tau^2)]. \quad (23)$$

3.3 Radiation from the modulated bunch

Defining the azimuth of the observation point relative to the reference electron as θ_0 , for an electron having vertical displacement y :

$$\theta = \theta_0 - \frac{y}{R}. \quad (24)$$

Because the observation point is far enough away from the electron, θ is a small quantity:

$$\vec{A}(\omega) = \vec{A}_0(\omega) + \frac{\partial \vec{A}_0(\omega)}{\partial \theta} \left(-\frac{y}{R} \right). \quad (25)$$

For convenience, this can be written:

$$\vec{A}(\omega) = \vec{A}_0 + \vec{B}_0 \left(-\frac{y}{R} \right). \quad (26)$$

Substituting Eq. (26) into Eq. (21):

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} = & \frac{e^2 \omega^2}{4\pi^2 c} \sum_m^{N_e} \left(\vec{A}_0 \cdot \vec{A}_0^* - \vec{A}_0 \cdot \vec{B}_0^* \frac{y_m}{R} - \vec{A}_0^* \cdot \vec{B}_0 \frac{y_m}{R} \right. \\ & + \vec{B}_0 \cdot \vec{B}_0^* \frac{y_m^2}{R^2} \left. \right) + \frac{e^2 \omega^2}{4\pi^2 c} \sum_m^{N_e} \sum_{n \neq m}^{N_e} \left[\vec{A}_0 \cdot \vec{A}_0^* e^{j\omega(\tau_n - \tau_m)} \right. \\ & - \vec{A}_0 \cdot \vec{B}_0^* \frac{y_n}{R} e^{j\omega(\tau_n - \tau_m)} - \vec{A}_0^* \cdot \vec{B}_0 \frac{y_m}{R} e^{j\omega(\tau_n - \tau_m)} \\ & \left. + \vec{B}_0 \cdot \vec{B}_0^* \frac{y_m \cdot y_n}{R^2} e^{j\omega(\tau_n - \tau_m)} \right]. \quad (27) \end{aligned}$$

Integrating all the modulated bunches, we then get:

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} = & N_e \left\{ p_0 - \frac{2y_0 p_1}{R} \sin \psi_0 e^{-\frac{\omega_\psi^2 \sigma_\tau^2}{2}} \right. \\ & \left. + \frac{y_0^2 p_2}{2R^2} (1 - \cos 2\psi_0 e^{-2\omega_\psi^2 \sigma_\tau^2}) \right\} \\ & + N_e(N_e - 1) e^{-\omega^2 \sigma_\tau^2} \left\{ p_0 \right. \\ & - p_1 \frac{y_0}{R} \cdot 2 \sin \psi_0 \cosh \omega_\psi \omega \sigma_\tau^2 \cdot e^{-\frac{\omega_\psi^2 \sigma_\tau^2}{2}} \\ & \left. + p_2 \frac{y_0^2}{R^2} \cdot \frac{\cosh 2\omega_\psi \omega \sigma_\tau^2 - \cos 2\psi_0}{2} \cdot e^{-\omega_\psi^2 \sigma_\tau^2} \right\}, \quad (28) \end{aligned}$$

where:

$$\begin{aligned} p_0 &= \frac{e^2 \omega^2}{4\pi^2 c} |\vec{A}_0(\omega)|^2, \\ p_1 &= \frac{e^2 \omega^2}{4\pi^2 c} \vec{A}_0 \cdot \vec{B}_0^*, \\ &= \frac{e^2 \omega^2}{4\pi^2 c} \vec{B}_0 \cdot \vec{A}_0^*, \\ p_2 &= \frac{e^2 \omega^2}{4\pi^2 c} \vec{B}_0 \cdot \vec{B}_0^*. \end{aligned} \quad (29)$$

When $\omega_\psi \sigma_\tau \gg 1$, Eq. (28) is simplified as:

$$\begin{aligned} \frac{d^2 I}{d\omega d\Omega} = & N_e p_0 [1 + e^{-\omega^2 \sigma_\tau^2} (N_e - 1)] \\ & + \frac{N_e p_2 y_0^2}{2R^2} [1 + e^{-(\omega - \omega_\psi)^2 \sigma_\tau^2} (N_e - 1)/2]. \end{aligned} \quad (30)$$

The first term of Eq. (30) is radiation from the bunch without modulation. The second term is CSR produced by the spatial structure within the bunch. The central frequency of the CSR is ω_ψ . The radiation spectrum is shown in Fig. 2. In the neighborhood of ω_ψ , CSR is much larger than ordinary bend radiation.

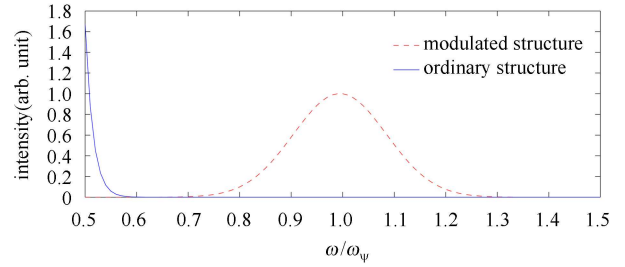


Fig. 2. (color online) The radiation spectrum produced by the modulated electron bunch. In the neighborhood of ω_ψ , the second term of Eq. (30) is much larger than the first term.

4 Application to the HLS

The required parameters for the HLS are as shown in Table 1. At an electron energy of 800 MeV, the natural bunch length $\sigma_\tau = 50$ ps, and the coherent frequency (normal CSR) is 0.003 THz, which lies in the microwave range and can be suppressed by metallic shields. After deflection by the kicker magnet and modulation by the AC sextupole, the center frequency of CSR is $2\pi \times 0.026$ THz when $t = 24\pi/\omega_s$, and $\sigma_{\psi N}^2 = 1.5$. The complete results are shown in Fig. 3.

It can be concluded that the CSR frequency does not reach the THz region. When the electron energy is reduced to 400 MeV, however, Fig. 4 was obtained.

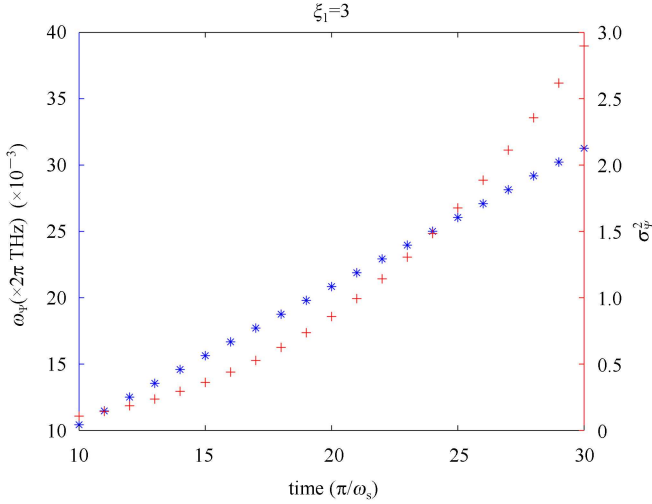


Fig. 3. (color online) When the stored electron energy is 800 MeV, the center frequency and the bunch coherence vary with time. As time increases, the center frequency becomes higher while the bunch coherence becomes weaker.

Table 1. HLS parameters.

	800	400
electron energy/MeV	800	400
momentum compaction factor (α_p)	0.0205	0.0205
revolution frequency (f_{rev})/MHz	4.533	4.533
curvature of radius of bending magnet (ρ)/m	2.16451	2.16451
natural energy spread (σ_e)	0.00047	0.00024
longitudinal damping time (τ_L)/ms	10.8	86.7
synchrotron oscillation frequency (ω_s)/MHz	0.193	0.273
DC chromaticity (ξ_0)	0	0
AC chromaticity (ξ_1)	3	5
number of electrons per bunch (N_e)	2.58×10^{11}	2.58×10^{11}
vertical oscillation amplitude (y_0)/mm	5	5

From Fig. 4, it can be seen that as chromaticity increases, the center frequency of CSR can reach the THz region faster, whereas the bunch coherence decreases faster. Considering both aspects, the following parameters were chosen: $\xi_1 = 5$, $\sigma_r = 18$ ps, $\omega_\psi = 2\pi \times 0.115$ THz at $t_n = 66\pi/\omega_s$. From further examination of Fig. 4, the following can be concluded.

- 1) At $t=0$, the bunch is excited by the kicker.
- 2) When $t_n = n\pi/\omega_s$, $n = 58, 59, \dots, 84$, $\omega_\psi \in [0.10, 0.15]$ THz, $\sigma_{\psi N}^2 \in [0.5, 2]$, the bunch produces CSR in the THz region.
- 3) When $t_n \geq 120\pi/\omega_s$, $\sigma_{\psi N}^2 \geq 5.8$, the bunch is no longer coherent. When $e^{-\sigma_{\psi N}^2/2} \leq 5\%$, the vertical collective oscillation can be ignored.

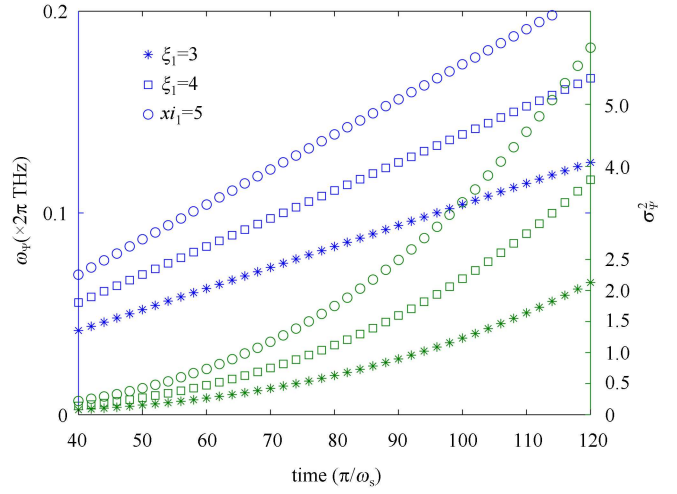


Fig. 4. (color online) When the stored electron energy is 400 MeV, with different chromaticity, the center frequency and the bunch coherence vary with time. As time increases, the center frequency becomes higher while the bunch coherence becomes weaker.

The frequency of the quick kicker magnet should therefore satisfy:

$$f_{\text{kicker}} \leq \frac{\omega_s}{120\pi} = 724 \text{ Hz.} \quad (31)$$

A lower frequency can reduce the repetition frequency of CSR, so we hope that the kicker has as high a frequency as possible.

Taking into account the dynamic aperture of the storage ring, at the position where the kicker is located, we choose the amplitude of vertical collective oscillation $y_0 = 5$ mm. The storage ring can tolerate this kick. $\beta_y \approx 10$ m, so the kick-angle is:

$$\theta = \frac{y_0}{\beta_y} = 0.5 \text{ mrad.} \quad (32)$$

When the stored energy is 400 MeV and the AC chromaticity is $\xi_1 = 5$, the CSR spectrum is as shown in Fig. 5. As shown in the figure, FWHM = 0.0138 THz.

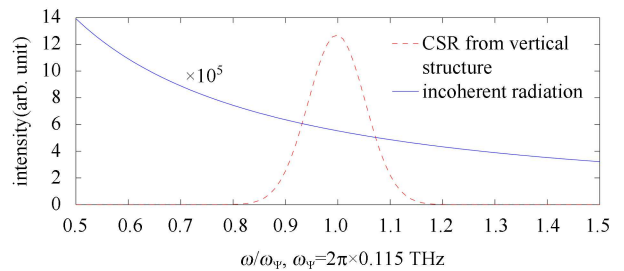


Fig. 5. (color online) Radiation spectrum of the HLS in the frequency neighborhood of $\omega_\psi = 2\pi \times 0.115$ THz when the stored energy is 400 MeV. The observation point lies in the orbital plane.

5 Conclusion

A stable, high flux CSR at the HLS can be produced when the stored electron energy is 400 MeV. This has important implications for expanding the scope of appli-

cation of the HLS. However, CSR may cause great disturbance to the storage ring. Moreover, introduction of an AC sextupole magnet can lead to nonlinear effects, which may affect the dynamic aperture of the beam. These considerations need further research.

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