Geometric matrix research for nuclear waste drum tomographic gamma scanning transmission image reconstruction*

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Abstract: A geometric matrix model of nuclear waste drums is proposed for transmission image reconstruction from tomographic gamma scans (TGS). The model assumes that rays are conical, with intensity uniformly distributed within the cone. The attenuation coefficients are centered on the voxel (cube) of the geometric center. The proposed model is verified using the EM algorithm and compared to previously reported models. The calculated results show that the model can obtain good reconstruction results even when the sample models are highly heterogeneous.

Key words: tomographic γ scanning, geometric matrix, transmission measurement, EM

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1 Introduction

The Tomographic γ Scanning (TGS) technique is a relatively new method in the field of nondestructive assays (NDA) of radioactive waste characterization. It is used in industrial Computerized Tomography (CT) imaging to increase the accuracy of attenuation corrections when the distribution of the sample medium is uneven. Thus, it improves the accuracy of the content of the non-uniform analysis of radioactive samples in γ-ray spectroscopy measurements when compared to traditional methods such as Segmented Gamma Scanning (SGS) [1–5].

Waste drums are scanned using the γ rays released by a transmission source in three dimensions, and a line attenuation matrix reconstructed using the attenuation formula in the transmission measurement. The purpose is to present a foundation to correct the data using emission measurements. The most important step in transmission image reconstruction is the construction of the geometric matrix (GM).

LANL (Los Alamos National Laboratory) presented a 'point-to-point (PP)' model to build the GM. In that model, HPGe detectors and transmission sources are considered to be points that have no geometry. The beam width is considered to be negligible in size [6]. This method is a general method that is used in commercial and research institutions to build GMs for TGS. Average track (AT) model is presented by the China Institute of atomic energy, but there is no application.

In this paper, we present a voxel-centered method to build the GM for TGS transmission measurement, and also present its validation using computer simulations and experimental measurements. The calculation results of the new method are compared with previous studies, aiming to improve the performance of TGS.

2 Theoretical basis

TGS uses a simple voxel model as a basis for image reconstruction. In TGS, we use a transmission image to build γ-ray attenuation corrections into the emission imaging problem. In the absence of attenuation, the emission problem is described by an M by N efficiency matrix, E, in which each element E_{ij} is proportional to the probability that a photon (of the correct energy) emitted from the j^{th} voxel will be detected in the i^{th} measurement. The emission image is found as the solution to the linear system [6]

$$\boldsymbol{d}=E\cdot\boldsymbol{S}, \tag{1}$$

where $\boldsymbol{d}$ is an M-vector of measurements and $\boldsymbol{S}$ is an N-vector describing the source intensity distribution. The description of the transmission problem is similar to that of the emission problem, but requires a logarithmic conversion to obtain a linear form. Let p_i equal the i^{th}

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transmission measurement,

$$p_i = Count_i / Count_{\max}, \quad (2)$$

where $Count_i$ is the photon count in the i^{th} transmission measurement and $Count_{\max}$ is the un-attenuated count for the transmission source. We define the logarithmic transmission υ_i, by the relation:

$$\upsilon_i = -\ln p_i. \quad (3)$$

With this conversion, the transmission problem can be described by an M by N thickness matrix T, where each element T is the linear thickness of the j^{th} voxel along a ray connecting the transmission source and the detector in the i^{th} measurement position. The transmission image is found as the solution of the linear system

$$\boldsymbol{\upsilon} = T \cdot \boldsymbol{u}, \quad (4)$$

where $\boldsymbol{\upsilon}$ is an M-vector of measurements and $\boldsymbol{u}$ is an N-vector of linear attenuation coefficients.

In a drum containing attenuating materials, Eq. (1) is a poor description of the emission problem. To correct for the loss of photons due to attenuation inside the drum we define an attenuation-corrected efficiency matrix, F. The elements of F are given by the relation

$$F_{i,j} = E_{i,j} A_{i,j}, \quad (5)$$

where $A_{i,j}$ is the fractional attenuation, due to the drum contents, of photons emitted from the j^{th} voxel in the i^{th} emission measurement. The attenuation corrected emission image is found as the solution of the linear system

$$\boldsymbol{d} = F \cdot \boldsymbol{S}, \quad (6)$$

where $\boldsymbol{d}$ and $\boldsymbol{S}$ have the same meanings as in Eq. (1). The values of $A_{i,j}$ are estimated from the transmission image using Beer's law:

$$A_{i,j} = \prod\nolimits_k \exp(-t_{i,j,k} \mu_k), \quad (7)$$

where the triply-indexed quantity $t_{i,j,k}$ is the linear thickness of the k^{th} absorbing voxel along a ray connecting the j^{th} emitting voxel and the detector in the i^{th} measurement position. While the table of $t_{i,j,k}$ values is constant, A depends on the drum contents and must be computed anew for each drum assayed. It is the computation of A that makes TGS image reconstructions time-consuming, even at low resolutions.

3 Transmission image reconstruction features and model

3.1 Transmission image reconstruction features

TGS transmission image reconstruction and general industrial CT image reconstruction utilize the same principle, which is to solve for the line attenuation coefficients within the sample. However, TGS transmission image reconstruction has its own characteristics, as follows.

Using multiple detector probes arranged in a matrix, CT systems have a large amount of projection data. In the TGS system, there are fewer projection data for each HPGe detector. This makes the reconstruction of the TGS system image more difficult than with CT.

CT detectors are very small, and radiation beams emitted by transmission sources are considered to be parallel beams at the detectors, such that the tracks of rays do not need to be corrected spatially. The diameter of the TGS system HPGe detector is $\geqslant$6.2 cm, which is much larger than the CT detectors. γ-ray sources relative to the detector have a larger solid angle. However, the ray beams relative to the detector cannot be treated as parallel, as there is a large cone of scattering angles in the distribution. The beam tracks were very different in the material. Therefore, this effect must be accounted for in the image reconstruction.

CT only needs to calculate the specific ray attenuation coefficients. However, various energy ray attenuation coefficients of the sample are required to be reconstructed accurately by the TGS transmission measurement.

3.2 Voxel center model

Based on utilizing TGS transmission measurements, this paper proposes a voxel center (VC) model to build the transmission geometric matrix.

We assume that rays are conical, and that the intensity of the beam is uniformly distributed within the cone. All of the attenuation coefficients are centered on the voxel (cube) at the geometric center. When a beam is transmitted through the geometric center of the voxel, the weighting factor is 1.The other weighting factors are zero. Taking a conical incident beam parallel to the x-axis as an example, the conical surface equation is as follows:

$$\sqrt{(y-\bar{y})^2+(z-\bar{z})^2} - t(x-\bar{x}) = 0. \quad (8)$$

The method is expressed by the following equation:

$$T_{i,j} = \begin{cases} 1, \sqrt{(C_y-\bar{y})^2+(C_z-\bar{z})^2} - t(C_x-\bar{x}) < 0 \\ 0, \sqrt{(C_y-\bar{y})^2+(C_z-\bar{z})^2} - t(C_x-\bar{x}) > 0 \end{cases}, \quad (9)$$

where C_x, C_y, C_z are the coordinates of the geometric center of the voxel.

This method is a simplified model of the TGS system. There are some theoretical uncertainties, but the method is very simple. Therefore, the voxels can be used to compute the geometric matrix quickly.

3.3 Transmission image reconstruction algorithm

Lange and Carson defined the image reconstruction for tomography as a maximum likelihood estimation

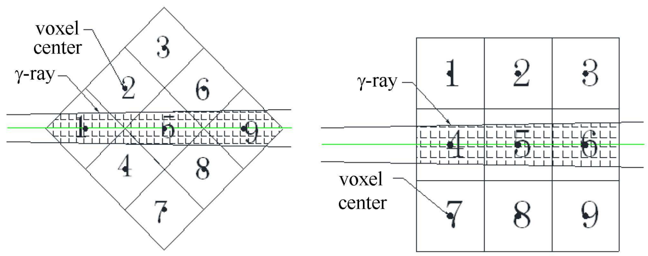

Fig. 1. Schematic of rays passing through the center of the voxel.

problem and derived the EM algorithm to obtain the maximum likelihood image estimate [7]. The EM algorithm has found extensive applications in tomography and a comprehensive description of the algorithm has been presented by Dempster. In experiments where γ-ray counts statistics are high, ignoring the true statistical nature of the observations is not a serious limitation because Poisson counting noise is only a component of the total system noise. It is precisely in the low count experiments that the EM algorithm is expected to provide the greatest improvement in the reconstruction quality. The EM algorithm is a general iterative technique for computing maximum likelihood estimates in any measurement of statistical quantities. Its use in image reconstruction for transmission tomography is only a specific application. Each iteration of the EM algorithm consists of two steps: estimation (E step) and maximization (M step). Some key elements related to the derivation and use of the E and M steps for transmission tomography are discussed here. In the estimation step (E step), the conditional expectation of the statistical model for the process of transmission of the γ-ray photons is determined on the basis of the measured data (counts data from the detector) and a parameter set (attenuation values)

$$x_j^{(k)}=x_j^{(k-1)}+\lambda_k\frac{a_{i,j}}{\sum\limits_{m=1}^{I}a_{m,j}}\sum_{i=1}^{I}\frac{b_i-A_iX^{(k-1)}}{A_iX^{(k-1)}}x_j^{(k-1)}. \quad (10)$$

4 Results and discussion

This section presents the studies of three GM results of the reconstruction, using Monte Carlo simulations and experimental verification. The VC model was verified by comparing the results of the reconstruction of the attenuation coefficients.

4.1 Experimental setup

The TGS mechanism developed by our group consists of the following modules: a level mobile/rotation platform, lifting platform detectors, a radioactive lifting platform and a transmission source shield. A diagram of the system is shown in Fig. 2. Automation of the leveling/rotation, vertical and rotational platforms are controlled by a Process Logic Controller (PLC). The system consisted of an ORTEC GEM50P4-83 detector and a transmission source.

The detector is collimated with a lead cylinder which has a square collimation window. The transmission source collimator was also made of lead. The data acquisition and analysis software platform used was γ vision32.

4.2 Verification by Monte Carlo simulation

Using a 3 × 3 single voxel sample as an example, the distance of the transmission source from the center of the sample was set to 67 cm, and the distance of the detector from the center of the sample was set to 45+16 cm. Measurement samples were filled with Acrylonitrile Butadiene Styrene plastic (ABS), with a sample volume of 5 cm×5 cm ×5 cm and a density of 1.07 g/cm^3. Three measuring points were used in the horizontal scans, set to 0 cm, 5 cm, and −5 cm. Four measurements were made for each measurement point, at sample rotation angles of 0°, 45°, 90°, 135°, for a total of 12 scan projections.

Sample model 1: the model was filled with ABS plastic, with voxels in the sample arrangement as shown in Fig. 3.

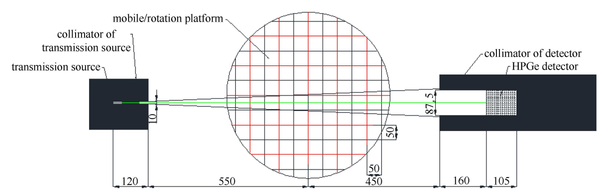

Fig. 2. Tomographic γ scanner.

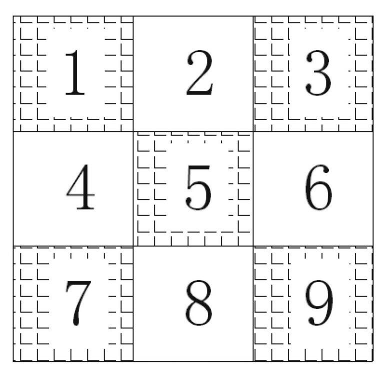

Fig. 3. Schematic of sample model 1 arrangement.

As shown in Table 1, we observe that after the reconstruction of the EM algorithm, the relative uncertainties of the AT model were the smallest. The largest value was only 1.6%, and the uncertainty is distributed more evenly. The reconstruction results of the VC model are better than the PP model; however, the fifth voxel appears to have a larger uncertainty, reaching 21%. The PP model reconstruction results had very large overall relative uncertainties. The voxel model using the average track model reconstruction has the best performance of those considered.

Sample model 2: the model was filled with ABS plastic except for the ninth voxel, which was filled with aluminum. Thus, the model sample was made more heterogeneous. Voxels were arranged as shown in Fig. 4.

Table 1. Relative uncertainties of the image reconstruction results using sample model 1.

voxel	AT model	VC model	PP model
1	−0.01422	−0.06346	−0.05799
3	−0.01094	−0.01204	−0.07002
5	−0.00875	−0.21663	−0.23523
7	−0.01532	0.041575	−0.16411
9	−0.01641	−0.01422	−0.16411

Table 2. Relative uncertainties of the image reconstruction results using sample model 2.

voxel	AT model	VC model	PP model
1	0.17943	−0.06674	−0.04376
3	0.21991	0.03392	−0.06565
5	0.03063	−0.35120	−0.26149
7	−0.36543	0.10832	−0.16521
9	−0.23674	−0.03641	−0.14910

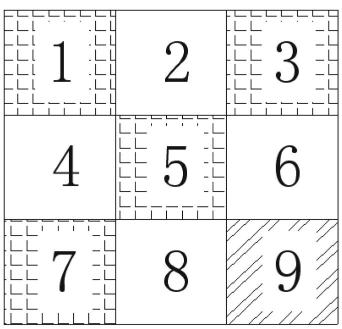

Fig. 4. Schematic of sample model 2 arrangements.

As shown in Table 2, the expected increase in heterogeneity was observed when the aluminum was added to the model samples. The three GM reconstruction results now have substantial deviations. Among them, the average track method for the aluminum line attenuation coefficient reconstruction uncertainty is 23%. The voxels at sites 1, 3, and 7 have large deviations, and overall the reconstruction effect is poorer. However, the attenuation coefficient of the VC model is good, with a small value of only 3%.The deviation of the reconstruction results in the fifth voxel is larger, however the reconstruction effect is still better than the AT model. The overall relative uncertainties of the PP model reconstruction are still large. However, the performance is more stable, and the relative uncertainty did not change significantly when the sample was made more heterogeneous.

Sample model 3: the model was filled with ABS plastic except for the third and ninth voxels, which were filled with aluminum, with the voxels in the sample arrangement as shown in Fig. 5.

As shown in Table 3, after increasing the amount of aluminum, the relative uncertainties of the reconstruction results are still large, and the attenuation coefficient is 23%. The VC model reconstruction in this case is better, with smaller relative uncertainties of only 2.7% and 14.7%. However, the deviation of the reconstruction results of the fifth voxel is 47%. The reconstruction result of the PP model reconstruction is better for voxel 3, and the deviation of voxel 5 is 30%, the same as the VC model.

Sample model 4: The model was filled with ABS plastic except for the fifth and ninth voxels, which were filled with iron and aluminum respectively, further increasing the heterogeneity of the sample. Voxels in the sample arrangement are as shown in Fig. 6.

Table 3. Relative uncertainties of image reconstruction results using sample model 3.

voxel	AT model	VC model	PP model
1	0.17615	−0.01422	−0.01204
3	0.09130	−0.02740	−0.09000
5	0.02626	−0.46608	−0.30088
7	−0.38403	0.14551	−0.18271
9	−0.23574	−0.01437	−0.16112

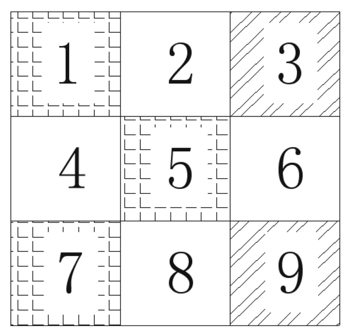

Fig. 5. Schematic of sample model 3 arrangements.

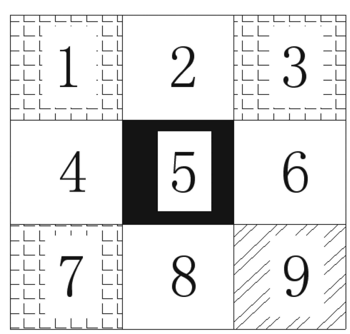

Fig. 6. Schematic of sample model 4 arrangements.

As shown in Table 4, the relative uncertainties of the voxel reconstruction by the AT model are larger (with a maximum of 43%), however for the fifth voxel, the relative uncertainty of the iron attenuation coefficient is smaller (only 1.8%). Only the relative uncertainty of the first voxel is large using the VC model. The relative uncertainties of the reconstruction of the iron and aluminum are considerably smaller.

Table 4. Relative uncertainties of the image reconstruction results using sample model 4.

voxel	AT model	VC model	PP model
1	0.136761	−0.20897	−0.03829
3	0.236324	−0.01422	−0.12254
5	−0.01801	0.017796	−0.17559
7	−0.43545	−0.00438	−0.22319
9	−0.25878	−0.04242	−0.17514

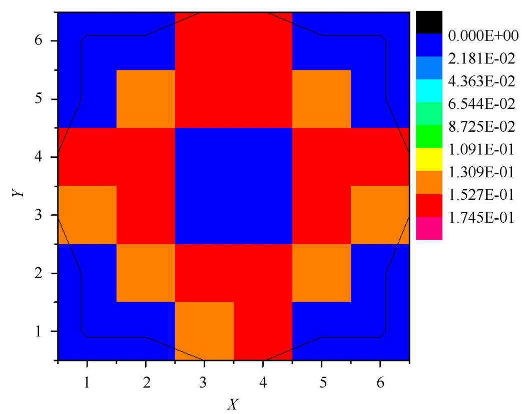

Fig. 7. The result of transmission image reconstruction.

The above four voxel model calculation results have shown that the AT model is suitable to construct small voxel attenuation coefficients. The VC model reconstruction performs well on four different voxel models. Relative uncertainties of the PP model reconstruction did not change substantially with the heterogeneity of the sample.

4.3 Experimental validation

Following the conclusions in the previous section, the geometric matrix was calculated using the VC model, and the projected value of the experimentally measured 6×6 model was reconstructed.

It is clear from Table 5 that some of the relative uncertainties of the voxel attenuation coefficients were more than 10%, but they are all located in the image edges, which do not strongly affect the image quality.

Table 5. Relative uncertainties of the experimental image reconstruction results.

voxel	relative uncertainty	voxel	relative uncertainty	voxel	relative uncertainty
1	0	13	0.1134	25	0
2	0	14	−0.0412	26	0.1238
3	0.0809	15	0	27	−0.0670
4	0.0447	16	0	28	0.0109
5	0	17	−0.0480	29	0.1422
6	0	18	0.0618	30	0
7	0	19	0.0447	31	0
8	0.1453	20	−0.0099	32	0
9	−0.0701	21	0	33	0.1140
10	0.0312	22	0	34	0.0416
11	0.1392	23	−0.0007	35	0
12	0	24	0.0502	36	0

5 Conclusion

A GM model was proposed for TGS transmission image reconstruction. The VC model was verified using the EM algorithm in four Monte Carlo sample models, as well as one experimental setup, and compared to previously reported models. The results show that the new model is less sensitive to sample heterogeneity, even when each voxel attenuation coefficient is large. We envision that our new calculation model of TGS offers significant potential to improve the performance of transmission image reconstruction.

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