Unified Hamiltonian model for mesons and baryons *

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Abstract: A new Hamiltonian model is introduced to study the spectrum of light hadrons. It combines relativistic field theory with elements of the constituent quark model. In addition to the standard linear confining and pseudoscalar meson exchange interactions with predetermined parameters, an additional interaction with different covariant spin structures is examined. Using a large scale Monte Carlo variational procedure, the resulting model Hamiltonian provides a very good, unified description of the light quark baryon (both octet and decuplet) and meson spectra.

Key words: hadron spectroscopy, Hamiltonian model, meson, baryon

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1 Introduction

Although QCD is widely accepted as the fundamental theory for strong interactions, it is extremely challenging to calculate the observed hadron spectrum directly from the QCD Lagrangian. Due to non-Abelian and non-perturbative aspects, one method is lattice QCD, which provides reasonably well described ground states [1–3] and some success for excited states [4–6]. However for light hadrons such as the pion and even the proton, accurate predictions are still not possible. Consequently there are many phenomenological models and effective theories, such as QCD sum rules [7–11], NRQCD [12–14], chiral perturbation theory [15–19] along with potential models [20–22]. Typical potential models utilize a Cornell type interaction, having linear confinement supplemented with the usual Coulomb potential governing short-distance behavior. A representative example is the constituent model detailed in Ref. [23] which, with an additional spin dependent interaction, obtained a good description of the light meson spectrum. This model has also been applied to baryons [24] with similar success but requires different model parameters. To describe the interaction between color singlet objects, potential models have been extended by including a meson-exchange interaction, such as the chiral $SU(3)$ quark potential model [25–28], which gave a good description of the baryon interaction.

A more theoretical and less phenomenological approach is the Coulomb gauge model, which has been successfully applied to mesons [29–31], glueballs [32, 33], hybrids [34–36] and tetraquark states [37–39]. The predicted results are consistent with both lattice simulations and experimental data. Different from the above potential models, the Coulomb gauge approach entails relativistic field theory and is formulated in the same mathematical framework as the exact QCD Hamiltonian in the Coulomb gauge. Further, it contains no free model parameters as it only utilizes the known current quark masses and two dynamical constants, the string tension σ and the QCD coupling constant α_s, that are predetermined from the literature. While this model provides a reasonable hadron description it does not simultaneously reproduce both the meson and baryon spectra with the same overall accuracy as the multi-parameter pure meson or baryon models mentioned above.

The purpose of the current work is to provide a unified model that can accurately reproduce both meson and baryon masses with the same set of model parameters which to date has not been achieved. The motivation is to develop a robust framework for reliably predicting and understanding more exotic systems, such as light and heavy tetraquark states which are of intense interest. Building on the attractive theoretical features of the Coulomb gauge model and phenomenological successes of constituent quark models, a unified Hamiltonian appr-

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oach has been developed that combines relativity, field theory and elements of the constituent quark model with, most significantly, a single set of parameters that can simultaneosuly describe both meson and baryon masses.

This paper is organized into six sections. In Section 2, the unified Hamiltonion is described and then the meson and baryon model wave functions are detailed in Section 3 and Section 4, respectively. Section 5 presents numerical results and highlights the accurate hadron descriptions. Finally, conclusions are summarized in Section 6.

2 Model Hamiltonian

The model Hamiltonian is

$$H_{\rm t} = H_{\rm kine}+H_{I0}+H_{I1}+H_{\rm ch}, \qquad (1)$$

where $H_{\rm kine}$ is the relativistic kinetic energy

$$H_{\rm kine}=\int {\rm d}\boldsymbol{x}\Psi^{\dagger}(\boldsymbol{x})(-{\rm i}\alpha\cdot\nabla+\beta m)\Psi(\boldsymbol{x}), \qquad (2)$$

and H_{I0}, H_{I1}, and $H_{\rm ch}$ are the interactions detailed below. Similar to the Coulomb gauge model [29, 31], H_{I0} is the confining interaction

$$H_{I0} = -\frac{1}{2}\int {\rm d}\boldsymbol{x}{\rm d}\boldsymbol{y}[\bar{\Psi}(\boldsymbol{x})\gamma^0 T^{\rm a}\Psi(\boldsymbol{x})]V_0(|\boldsymbol{x}-\boldsymbol{y}|) \times[\bar{\Psi}(\boldsymbol{y})\gamma^0 T^{\rm a}\Psi(\boldsymbol{y})], \qquad (3)$$

where $T^{\rm a}=\frac{\lambda^{\rm a}}{2}$ are the color $SU(3)$ group generators and $V_0(|\boldsymbol{x}-\boldsymbol{y}|)$ is a Cornell type potential

$$V_0(|\boldsymbol{x}-\boldsymbol{y}|)=\frac{C}{(2\pi)^3}-\frac{\alpha_{\rm s}}{|\boldsymbol{x}-\boldsymbol{y}|}+\sigma|\boldsymbol{x}-\boldsymbol{y}|. \qquad (4)$$

Following constituent quark models a constant energy C is introduced and σ, $\alpha_{\rm s}$ are the same as in the Coulomb gauge model. This is a "charge-charge" color interaction. Performing a Fourier transformation, the potential in momentum space is

$$\hat{V}_0(|\boldsymbol{q}|)=C\delta^3(\boldsymbol{q})-\frac{4\pi\alpha_{\rm s}}{q^2}-\frac{8\pi\sigma}{q^4}+\delta^3(\boldsymbol{q})\int {\rm d}\boldsymbol{q}'\frac{8\pi\sigma}{q'^4}. \qquad (5)$$

The last term is to satisfy the condition that at $r=0$, the confining potential is zero [40]. It is also important to deal with the divergence of the integral with linear confining potential at zero momentum transfer.

The interaction between two colored objects can have other forms, for example, "current-current" interaction, "spin-spin" interaction, etc. In particular, to account for hadron spin splittings, a hyperfine type interaction H_{I1} is included with structure

$$H_{I1}=-\frac{1}{2}\int {\rm d}\boldsymbol{x}{\rm d}\boldsymbol{y}[\bar{\Psi}(\boldsymbol{x})\Gamma T^{\rm a}\Psi(\boldsymbol{x})]V_1(|\boldsymbol{x}-\boldsymbol{y}|)[\bar{\Psi}(\boldsymbol{y})\Gamma T^{\rm a}\Psi(\boldsymbol{y})]. \qquad (6)$$

The Γ matrix can be 1, $\vec{\gamma}$, γ_5, $\vec{\gamma}\gamma_0$, $\gamma_5\gamma_0$, $\vec{\gamma}\gamma_5$. The potential $V_1(|\boldsymbol{x}-\boldsymbol{y}|)$ is taken to be similar to V_0 with linear and Coulomb terms

$$V_1(|\boldsymbol{x}-\boldsymbol{y}|)=-\frac{\alpha_1}{|\boldsymbol{x}-\boldsymbol{y}|}+\sigma_1|\boldsymbol{x}-\boldsymbol{y}|. \qquad (7)$$

This interaction will be used to reproduce the πρ splitting which is large due to the small π mass governed by chiral symmetry as documented in Ref. [29], which uses a Random Phase Approximation diagonalization to obtain a light chiral pion. Here a light pion mass is obtained entirely via spin-splitting similar to the constituent treatment of Ref. [23].

The above interaction is between two colored objects. To describe interacting color singlet hadrons a pseudoscalar meson exchange interaction $H_{\rm ch}$ is also included using the quark-meson Lagrangian

$$\mathcal{L}_{\rm ch}=-g_{\rm ch}\overline{\psi}\left({\rm i}\gamma_5\sum_{a=1}^{8}\lambda_a\pi_a\right)\psi. \qquad (8)$$

Here λ_a are the $SU_{\rm f}(3)$ generators and π_a are the pseudoscalar meson fields. The coupling constant $g_{\rm ch}$ is determined from the NNπ interaction [27]

$$\frac{g^2_{\rm ch}}{4\pi}=\frac{9}{25}\frac{m^2_{\rm u}}{m^2_{\rm N}}\frac{g^2_{\rm NN\pi}}{4\pi}, \qquad (9)$$

where $\frac{g^2_{\rm NN\pi}}{4\pi}=13.67$ [41]. The constituent quark mass $m_{\rm u}$ is chosen to be 220 MeV [23, 24]. The Goldstone field is

$$\sum_{a=1}^{8}\lambda_a\pi_a=\begin{pmatrix} \pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0+\frac{1}{\sqrt{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta \end{pmatrix}. \qquad (10)$$

From the quark-meson interaction the one-meson exchange potential can be extracted. For example, the one-pion exchange potential between two color singlets is

$$H^{\pi}_{\rm ch}=\frac{1}{2}\int {\rm d}\boldsymbol{x}{\rm d}\boldsymbol{y}[\bar{\Psi}(\boldsymbol{x})\gamma^5\Psi(\boldsymbol{x})]V^{\pi}_{\rm ch}(|\boldsymbol{x}-\boldsymbol{y}|)[\bar{\Psi}(\boldsymbol{y})\gamma^5\Psi(\boldsymbol{y})], \qquad (11)$$

where $V^{\pi}_{\rm ch}(|\boldsymbol{x}-\boldsymbol{y}|)$ is the Fourier transformation of $\hat{V}^{\pi}_{\rm ch}(\boldsymbol{q})$

$$\hat{V}^{\pi}_{\rm ch}(\boldsymbol{q})=\frac{g^2_{\rm ch}}{\boldsymbol{q}^2+m^2_{\pi}}. \qquad (12)$$

The pseudoscalar meson mass in the meson exchange potential is chosen to be the experimental value. In the previous quark model, there is a cut-off in the form factor of the meson exchange interaction [27]. This reflects the non-point structure of the meson. Here we treat the pion as a point particle. The meson exchange potential is derived directly from the meson free propagator. We work in momentum space and there is no divergence due to the variational Gauss function.

In the above equations, the quark field operators can be expanded as:

$$\Psi(\boldsymbol{x}) = \int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}[u_\lambda(\boldsymbol{k})b_{\lambda\mathcal{C}}(\boldsymbol{k})+v_\lambda(-\boldsymbol{k})d^\dagger_{\lambda\mathcal{C}}(-\boldsymbol{k})]\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\boldsymbol{\epsilon}_{\mathcal{C}}, \tag{13}$$

$$\bar{\Psi}(\boldsymbol{x}) = \int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}[\bar{u}_\lambda(\boldsymbol{k})b^\dagger_{\lambda\mathcal{C}}(\boldsymbol{k})+\bar{v}_\lambda(-\boldsymbol{k})d_{\lambda\mathcal{C}}(-\boldsymbol{k})]\mathrm{e}^{\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}\bar{\boldsymbol{\epsilon}}_{\mathcal{C}}, \tag{14}$$

where the Dirac spinors are

$$u_\lambda(\boldsymbol{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{\dfrac{\omega+m}{\omega}}\chi_\lambda\\ \sqrt{\dfrac{\omega-m}{\omega}}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}\chi_\lambda\end{pmatrix}, \tag{15}$$

$$v_\lambda(\boldsymbol{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix}\sqrt{\dfrac{\omega-m}{\omega}}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}\chi_\lambda\\ \sqrt{\dfrac{\omega+m}{\omega}}\chi_\lambda\end{pmatrix}, \tag{16}$$

and $\omega=\sqrt{m^2+\boldsymbol{k}^2}$. The spinors χ_λ are fermions,

$$\chi_+=\begin{pmatrix}1\\0\end{pmatrix},\quad \chi_-=\begin{pmatrix}0\\1\end{pmatrix}, \tag{17}$$

and anti-fermions,

$$\chi_+=\begin{pmatrix}0\\1\end{pmatrix},\quad \chi_-=\begin{pmatrix}1\\0\end{pmatrix}. \tag{18}$$

3 Mesons

In the center of momentum the meson state $|q\bar{q}\rangle$ is given by

$$|\Psi^{J^{PC}}\rangle=\sum_{\mathcal{C}_i\lambda_i}\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\Psi^{J^{PC}}_{\mathcal{C}_1\mathcal{C}_3\lambda_1\lambda_3}(\boldsymbol{k})b^\dagger_{\mathcal{C}_1\lambda_1}(\boldsymbol{k})d^\dagger_{\mathcal{C}_3\lambda_3}(-\boldsymbol{k})|0\rangle, \tag{19}$$

with convention 1 for a quark having momentum $\boldsymbol{k}$ and 3 for an anti-quark having momentum $-\boldsymbol{k}$. Color and spin are represented by $\mathcal{C}_i(i=1, 3)$ and $\lambda_i(i=1, 3)$, respectively. The wavefunction $\Psi^{J^{PC}}_{\mathcal{C}_1\mathcal{C}_3\lambda_1\lambda_3}(\boldsymbol{k})$ has the form

$$\Psi^{J^{PC}}_{\mathcal{C}_i\lambda_i}(\boldsymbol{k}) = \delta_{\mathcal{C}_1\mathcal{C}_3}f(k)\sum_{m_Lm_S}\left\langle\frac{1}{2}\frac{1}{2}\lambda_1\lambda_3|Sm_S\right\rangle \times\langle LSm_Lm_S|Jm_J\rangle(-1)^{\frac{1}{2}-\lambda_3}Y_L^{m_L}(\boldsymbol{k}), \tag{20}$$

with spin and angular momentum coupling $\hat{S}_1+\hat{S}_3=\hat{S}$, where $\hat{L}+\hat{S}=\hat{J}$, $Y_L^m(\boldsymbol{k})$ is the spherical harmonic function and $f(k)$ is the radial wavefunction with variational parameter α:

$$f(k)=k^{2L}\mathrm{e}^{-\frac{k^2}{\alpha}}. \tag{21}$$

The meson mass is given by

$$\begin{aligned}M &= \frac{\langle\Psi^{J^{PC}}|H_\mathrm{t}|\Psi^{J^{PC}}\rangle}{\langle\Psi^{J^{PC}}|\Psi^{J^{PC}}\rangle}\\ &= \frac{\langle\Psi^{J^{PC}}|H_\mathrm{kine}|\Psi^{J^{PC}}\rangle}{\langle\Psi^{J^{PC}}|\Psi^{J^{PC}}\rangle}+\frac{\langle\Psi^{J^{PC}}|H_{I0}|\Psi^{J^{PC}}\rangle}{\langle\Psi^{J^{PC}}|\Psi^{J^{PC}}\rangle}\\ &\quad+\frac{\langle\Psi^{J^{PC}}|H_{I1}|\Psi^{J^{PC}}\rangle}{\langle\Psi^{J^{PC}}|\Psi^{J^{PC}}\rangle}+\frac{\langle\Psi^{J^{PC}}|H_\mathrm{ch}|\Psi^{J^{PC}}\rangle}{\langle\Psi^{J^{PC}}|\Psi^{J^{PC}}\rangle}\\ &= M_\mathrm{kine}+M_0+M_1+M_\mathrm{ch},\end{aligned} \tag{22}$$

where

$$\begin{aligned}M_\mathrm{kine} &= \int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}(\sqrt{m_1^2+\boldsymbol{k}^2}+\sqrt{m_3^2+\boldsymbol{k}^2})f^2(k)\\ &\quad\times\left[\left\langle\frac{1}{2}\frac{1}{2}\lambda_1\lambda_3|Sm_S\right\rangle\langle LSm_Lm_S|Jm_J\rangle\right]^2\\ &\quad\times Y_L^{*m_L}(\boldsymbol{k})Y_L^{m_L}(\boldsymbol{k}),\end{aligned} \tag{23}$$

$$\begin{aligned}M_0 &= \frac{4}{3}\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3}\hat{V}_0(|\boldsymbol{k}-\boldsymbol{k}'|)f(k)f(k')\\ &\quad\times\left\langle\frac{1}{2}\frac{1}{2}\lambda'_1\lambda'_3|S'm'_S\right\rangle\langle L'S'm'_Lm'_S|J'm'_J\rangle\\ &\quad\times\left\langle\frac{1}{2}\frac{1}{2}\lambda_1\lambda_3|Sm_S\right\rangle\langle LSm_Lm_S|Jm_J\rangle\\ &\quad\times(-1)^{\frac{1}{2}-\lambda_3+\frac{1}{2}-\lambda'_3}Y_{L'}^{*m'_L}(\boldsymbol{k}')Y_L^{m_L}(\boldsymbol{k})\\ &\quad\times\left[\bar{u}_{\lambda'_1}(\boldsymbol{k}')\gamma^0u_{\lambda_1}(\boldsymbol{k})\right]\left[\bar{v}_{\lambda_3}(-\boldsymbol{k})\gamma^0v_{\lambda'_3}(-\boldsymbol{k}')\right],\end{aligned} \tag{24}$$

$$\begin{aligned}M_1 &= \frac{4}{3}\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3}\hat{V}_1(|\boldsymbol{k}-\boldsymbol{k}'|)f(k)f(k')\\ &\quad\times\left\langle\frac{1}{2}\frac{1}{2}\lambda'_1\lambda'_3|S'm'_S\right\rangle\langle L'S'm'_Lm'_S|J'm'_J\rangle\\ &\quad\times\left\langle\frac{1}{2}\frac{1}{2}\lambda_1\lambda_3|Sm_S\right\rangle\langle LSm_Lm_S|Jm_J\rangle\\ &\quad\times(-1)^{\frac{1}{2}-\lambda_3+\frac{1}{2}-\lambda'_3}Y_{L'}^{*m'_L}(\boldsymbol{k}')Y_L^{m_L}(\boldsymbol{k})\\ &\quad\times\left[\bar{u}_{\lambda'_1}(\boldsymbol{k}')\Gamma u_{\lambda_1}(\boldsymbol{k})\right]\left[\bar{v}_{\lambda_3}(-\boldsymbol{k})\Gamma v_{\lambda'_3}(-\boldsymbol{k}')\right].\end{aligned} \tag{25}$$

The above expressions omit the summations on λ_i, λ'_i, m_L, m'_Lm_S, m'_S. Similarly, the contribution from the meson-exchange interaction is

$$\begin{aligned}M_\mathrm{ch} &= -\int\frac{\mathrm{d}\boldsymbol{k}}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}'}{(2\pi)^3}\hat{V}_\mathrm{ch}(|\boldsymbol{k}-\boldsymbol{k}'|)f(k)f(k')\\ &\quad\times\left\langle\frac{1}{2}\frac{1}{2}\lambda'_1\lambda'_3|S'm'_S\right\rangle\langle L'S'm'_Lm'_S|J'm'_J\rangle\\ &\quad\times\left\langle\frac{1}{2}\frac{1}{2}\lambda_1\lambda_3|Sm_S\right\rangle\langle LSm_Lm_S|Jm_J\rangle\\ &\quad\times(-1)^{\frac{1}{2}-\lambda_3+\frac{1}{2}-\lambda'_3}Y_{L'}^{*m'_L}(\boldsymbol{k}')Y_L^{m_L}(\boldsymbol{k})\\ &\quad\times\left[\bar{u}_{\lambda'_1}(\boldsymbol{k}')\gamma_5u_{\lambda_1}(\boldsymbol{k})\right]\left[\bar{v}_{\lambda_3}(-\boldsymbol{k})\gamma_5v_{\lambda'_3}(-\boldsymbol{k}')\right].\end{aligned} \tag{26}$$

4 Baryons

The baryon state can be constructed using quark creation operators acting on the vacuum state:

$$|qqq,J^P\rangle = \sum_{\mathcal{C}_i\lambda_i f_i}\int\frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}\Psi^{J^P}_{\mathcal{C}_i\lambda_i f_i}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times b^\dagger_{\mathcal{C}_1\lambda_1 f_1}(\boldsymbol{k}_1)b^\dagger_{\mathcal{C}_2\lambda_2 f_2}(\boldsymbol{k}_2)b^\dagger_{\mathcal{C}_3\lambda_3 f_3}(\boldsymbol{k}_3)|0\rangle, \quad (27)$$

here $\mathcal{C}_i$, λ_i, $f_i(i=1,\ 2,\ 3)$ represent the color, spin and flavor of the three quarks, respectively. We work in the rest frame where $\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3=0$. The baryon wave function, $\Psi^{J^P}_{\mathcal{C}_i\lambda_i f_i}(\boldsymbol{k}_1,\ \boldsymbol{k}_2,\ \boldsymbol{k}_3)$, can be written as the product of momentum, flavor-spin and color wave functions

$$\Psi^{J^P}_{\mathcal{C}_i\lambda_i f_i}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) = f(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times\psi_{\mathrm{fs}}(\lambda_1,\lambda_2,\lambda_3,f_1,f_2,f_3) \times\psi_{\mathrm{color}}(\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3), \quad (28)$$

where ψ_{fs} is the flavor-spin wave function and ψ_{color} is the color wave function. We only study the ground octet and decuplet states and no angular momentum part appears. Fermi-Dirac statistics require that the total baryon wave function must be antisymmetric under the exchange of quarks. The baryon color state is a singlet and is antisymmetric

$$\psi_{\mathrm{color}}(\mathcal{C}_1,\mathcal{C}_2,\mathcal{C}_3)=\varepsilon_{\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3}. \quad (29)$$

Hence the remaining wave function must be symmetric. Since the ground state momentum wave function $f(\boldsymbol{k}_1,\ \boldsymbol{k}_2,\ \boldsymbol{k}_3)$ is symmetric, the flavor-spin wave function ψ_{fs} must also be symmetric. In the following calculation, we will take the proton and Δ^+ as examples.

To construct a completely symmetric momentum space wave function the momentum Jacobi coordinates are utilized:

$$\boldsymbol{\rho}_{12}=\frac{1}{\sqrt{2}}(\boldsymbol{k}_1-\boldsymbol{k}_2),\boldsymbol{\lambda}_{12}=\frac{1}{\sqrt{6}}(\boldsymbol{k}_1+\boldsymbol{k}_2-2\boldsymbol{k}_3), \quad (30)$$

$$\boldsymbol{\rho}_{23}=\frac{1}{\sqrt{2}}(\boldsymbol{k}_2-\boldsymbol{k}_3),\boldsymbol{\lambda}_{23}=\frac{1}{\sqrt{6}}(\boldsymbol{k}_2+\boldsymbol{k}_3-2\boldsymbol{k}_1), \quad (31)$$

$$\boldsymbol{\rho}_{31}=\frac{1}{\sqrt{2}}(\boldsymbol{k}_3-\boldsymbol{k}_1),\boldsymbol{\lambda}_{31}=\frac{1}{\sqrt{6}}(\boldsymbol{k}_3+\boldsymbol{k}_1-2\boldsymbol{k}_2). \quad (32)$$

The proper symmetric variational wave function can then be written as

$$f(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)=\mathrm{e}^{-\frac{\rho_{12}^2}{\alpha_1^2}-\frac{\lambda_{12}^2}{\alpha_2^2}}+\mathrm{e}^{-\frac{\rho_{23}^2}{\alpha_1^2}-\frac{\lambda_{23}^2}{\alpha_2^2}}+\mathrm{e}^{-\frac{\rho_{31}^2}{\alpha_1^2}-\frac{\lambda_{31}^2}{\alpha_2^2}}, \quad (33)$$

where α_1 and α_2 are determined by the variational method.

The proton is taken as an example for calculating the baryon octet mass. According to the discussion above, the proton state can be expressed as

$$|\mathrm{proton},\frac{1}{2}\rangle = \sum_{\mathcal{C}_i\lambda_i f_i}\int\frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}f(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times\psi_{\mathrm{fs}}\left(\mathrm{proton},\frac{1}{2}\right)\times\varepsilon_{\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3} \times b^\dagger_{\mathcal{C}_1\lambda_1 u}(\boldsymbol{k}_1)b^\dagger_{\mathcal{C}_2\lambda_2 u}(\boldsymbol{k}_2)b^\dagger_{\mathcal{C}_3\lambda_3 d}(\boldsymbol{k}_3)|0\rangle,$$

where

$$\psi_{\mathrm{fs}}\left(\mathrm{proton},\frac{1}{2}\right) = \frac{2}{3\sqrt{2}}[u(\uparrow)u(\uparrow)d(\downarrow)-u(\uparrow)u(\downarrow)d(\uparrow) -u(\downarrow)u(\uparrow)d(\uparrow)+u(\uparrow)d(\downarrow)u(\uparrow) -u(\downarrow)d(\uparrow)u(\uparrow)-u(\uparrow)d(\uparrow)u(\downarrow) +(\downarrow)u(\uparrow)u(\uparrow)-d(\uparrow)u(\downarrow)u(\uparrow) -d(\uparrow)u(\uparrow)u(\downarrow)]. \quad (34)$$

Contributions to the Hamiltonian expectation value are summarized in Fig. 1. The expectation value for the proton mass is

$$\left\langle\mathrm{proton},\frac{1}{2}|H_{\mathrm{t}}|\mathrm{proton},\frac{1}{2}\right\rangle=M_{\mathrm{kine}}+M_{12}+M_{23}+M_{31},$$

where the kinetic energy has the form

$$M_{\mathrm{kine}} = \int\frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}f^2(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)(\sqrt{m_{\mathrm{u}}^2+\boldsymbol{k}_1{}^2} +\sqrt{m_{\mathrm{u}}^2+\boldsymbol{k}_2{}^2}+\sqrt{m_{\mathrm{u}}^2+\boldsymbol{k}_3{}^2}). \quad (35)$$

The matrix elements M_{12}, M_{23}, M_{31} have many terms due to the complexity of proton flavor-spin wave function ψ_{fs}. For example M_{12} is

$$M_{12} = -F_{\mathrm{C}}\int\frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}f(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times f(\boldsymbol{k}'_1,\boldsymbol{k}'_2,\boldsymbol{k}'_3)V(|\boldsymbol{q}|)\left(\frac{4}{18}E_1+\frac{2}{18}E_2+\frac{2}{18}E_3 +\frac{8}{18}E_4+\frac{2}{18}E_5+\frac{2}{18}E_6-\frac{4}{18}E_7-\frac{4}{18}E_8\right), \quad (36)$$

where F_{C} is the color factor. It is $-2/3$ and 1 for color-octet and coler-singlet interactions, respectively. The initial and final momentum have the following relationship: $\boldsymbol{k}'_1=\boldsymbol{k}_1+\boldsymbol{q}$, $\boldsymbol{k}'_2=\boldsymbol{k}_2-\boldsymbol{q}$, $\boldsymbol{k}'_3=\boldsymbol{k}_3$. The eight contributions E_i are given in Table 1. They are classified by different spin configurations. The above Eq. is the compressed form for all kinds of interactions. The expressions for M_{13} (M_{23}) are similar to M_{12} with the replacement of k_2 and $k_2^{'}$ by k_3 and $k_3^{'}$ (k_1 and $k_1^{'}$ by k_3 and $k_3^{'}$). Due to the symmetry of the wave function, the numerical values of M_{12}, M_{13} and M_{23} are the same.

Table 1. Expressions E_i and coefficients M_{12} for the proton.

spin	contributing terms	coefficient	matrix element
$\uparrow\uparrow\to\uparrow\uparrow$	$u(\uparrow)u(\uparrow)\to u(\uparrow)u(\uparrow)$	$\frac{4}{18}$	$E_1=[\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_2)]$
	$u(\uparrow)d(\uparrow)\to u(\uparrow)d(\uparrow)$	$\frac{2}{18}$	$E_2=[\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{d}_{\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma d_{\frac{1}{2}}(\boldsymbol{k}_2)]$
$\uparrow\downarrow\to\uparrow\downarrow$	$u(\uparrow)u(\downarrow)\to u(\uparrow)u(\downarrow)$	$\frac{2}{18}$	$E_3=[\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{u}_{-\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma u_{-\frac{1}{2}}(\boldsymbol{k}_2)]$
	$u(\uparrow)d(\downarrow)\to u(\uparrow)d(\downarrow)$	$\frac{8}{18}$	$E_4=[\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{d}_{-\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma d_{-\frac{1}{2}}(\boldsymbol{k}_2)]$
	$d(\uparrow)u(\downarrow)\to d(\uparrow)u(\downarrow)$	$\frac{2}{18}$	$E_5=[\bar{d}_{\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma d_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{u}_{-\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma u_{-\frac{1}{2}}(\boldsymbol{k}_2)]$
$\uparrow\downarrow\to\downarrow\uparrow$	$u(\uparrow)u(\downarrow)\to u(\downarrow)u(\uparrow)$	$\frac{2}{18}$	$E_6=[\bar{u}_{-\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma u_{-\frac{1}{2}}(\boldsymbol{k}_2)]$
	$u(\uparrow)d(\downarrow)\to u(\downarrow)d(\uparrow)$	$-\frac{4}{18}$	$E_7=[\bar{u}_{-\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{d}_{\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma d_{-\frac{1}{2}}(\boldsymbol{k}_2)]$
	$d(\uparrow)u(\downarrow)\to d(\downarrow)u(\uparrow)$	$-\frac{4}{18}$	$E_8=[\bar{d}_{-\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma d_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma u_{-\frac{1}{2}}(\boldsymbol{k}_2)]$

Table 2. Expressions $E_1^{'}$, $E_2^{'}$ and coefficients $M_{12}^{'}$ for the Δ^+.

spin	contributing terms	coefficient	matrix element
$\uparrow\uparrow\to\uparrow\uparrow$	$u(\uparrow)u(\uparrow)\to u(\uparrow)u(\uparrow)$	$\frac{1}{3}$	$E_1^{'}=[\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_2)]$
	$u(\uparrow)d(\uparrow)\to u(\uparrow)d(\uparrow)$	$\frac{2}{3}$	$E_2^{'}=[\bar{u}_{\frac{1}{2}}(\boldsymbol{k}_1{}')\Gamma u_{\frac{1}{2}}(\boldsymbol{k}_1)][\bar{d}_{\frac{1}{2}}(\boldsymbol{k}_2{}')\Gamma d_{\frac{1}{2}}(\boldsymbol{k}_2)]$

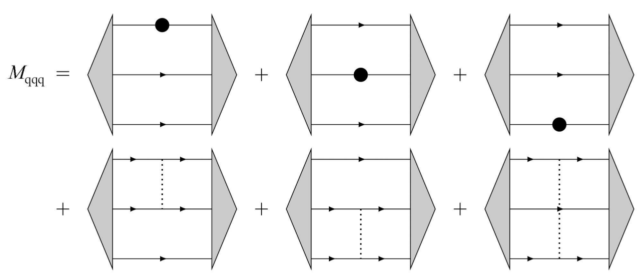

Fig. 1. Baryon diagrams for $\langle\psi_{\rm qqq}|H_{\rm t}|\psi_{\rm qqq}\rangle$. The solid dots are for the kinetic energy and the dashed lines are for the quark-quark interaction.

For the decuplet states the Δ^+ is used as a representative example and has a wavefunction given by

$$\left|\Delta^+,\frac{3}{2}\right\rangle = \sum_{\mathcal{C}_i\lambda_i f_i}\int\frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}f(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times\psi_{\rm fs}\left(\Delta^+,\frac{3}{2}\right)\times\varepsilon_{\mathcal{C}_1\mathcal{C}_2\mathcal{C}_3} \times b^{\dagger}_{\mathcal{C}_1\lambda_1 u}(\boldsymbol{k}_1)b^{\dagger}_{\mathcal{C}_2\lambda_2 u}(\boldsymbol{k}_2)b^{\dagger}_{\mathcal{C}_3\lambda_3 d}(\boldsymbol{k}_3)|0\rangle, \tag{37}$$

in which

$$\psi_{\rm fs}\left(\Delta^+,\frac{3}{2}\right) = \frac{1}{\sqrt{3}}[u(\uparrow)u(\uparrow)d(\uparrow)+u(\uparrow)d(\uparrow)u(\uparrow) +d(\uparrow)u(\uparrow)u(\uparrow)], \tag{38}$$

and the mass is given by

$$\left\langle\Delta^+,\frac{3}{2}\middle|H_{\rm t}\middle|\Delta^+,\frac{3}{2}\right\rangle=M_{\rm kine}^{'}+M_{12}^{'}+M_{23}^{'}+M_{31}^{'}, \tag{39}$$

where

$$M_{\rm kine}^{'} = \int\frac{\mathrm{d}\boldsymbol{k}_1}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k}_2}{(2\pi)^3}f^2(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3)\left(\sqrt{m_1^2+\boldsymbol{k}_1{}^2} +\sqrt{m_2^2+\boldsymbol{k}_2{}^2}+\sqrt{m_3^2+\boldsymbol{k}_3{}^2}\right), \tag{40}$$

$$M_{12}^{'} = -F_{\rm C}\int\frac{\mathrm{d}\boldsymbol{k_1}}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{k_2}}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}f(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3) \times f(\boldsymbol{k}_1',\boldsymbol{k}_2',\boldsymbol{k}_3')V(|\boldsymbol{q}|)[E_1^{'}+E_2^{'}]. \tag{41}$$

The expressions for $E_1^{'}$ and $E_2^{'}$ are given in Table 2. Again, due to the symmetry of the wave function, the numerical values for $M_{13}^{'}$ and $M_{23}^{'}$ are the same as $M_{12}^{'}$.

5 Numerical results

The new spin splitting interaction Hamiltonian H_{I1} was investigated by calculating the meson and baryon masses for all possible Γ matrices. The interaction with matrices 1 and γ_5 invert the baryon octet and decuplet spectra, i.e. produce larger octet masses than decuplet masses. The matrices $\vec{\gamma}\gamma_0,\gamma_5\gamma_0$, and $\vec{\gamma}\gamma_5$ produce P wave meson masses several hundred MeV lower than the experimental values. The calculated pseudoscalar meson mass is about 600 MeV, which is only half of the experimental data. For the matrix γ_5, though meson mass can be reproduced well, the baryon masses are far away from the experimental data. For example, the calculated nucleon mass is around 1200 MeV, while the Δ mass is 900–1000 MeV, which is even smaller than the nucleon mass. Only the interaction with the $\Gamma=\vec{\gamma}$ could produce reasonable baryon and meson masses simultaneously. This is the same Lorenz structure as in Ref. [30] using an effective one gluon exchange hyperfine interaction.

Table 3. Unified model parameters. The different meson [23] and baryon [24] parameters from Isgur et al. are also listed.

parameters	ξ=2.1	ξ=1.0	Ref. [23]	Ref. [24]
(m_u/m_d)/MeV	313	50	220	220
m_s/MeV	660	640	419	419
α_s	0.40	0.40	0.60	0.60
σ/GeV^2	0.18	0.18	0.18	0.15
C/MeV	−195	−198	−253	−615
α_1	0.762	0.490		
σ_1/GeV^2	0.207	0.0625		

For the potential V_0 in Hamiltonian H_{I0}, we use the predetermined values $\sigma=0.18$ GeV^2 and $\alpha_s=0.4$. Then the remaining free model parameters, the u/d, s quark masses, the potential strengths σ_1 and α_1 in H_{I1} and constant C in H_{I0}, were determined by reproducing the light quark meson spectrum. These parameters are listed in Table 3 along with Godfrey and Isgur's quark model values for comparison. The inclusion of the meson-exchange interaction does not add a free parameter. We should mention that our potential is different from the one-gluon exchange potential though the interacting Γ matrix is the same. The momentum dependence of our additional potential is the combination of linear and Coulomb potential which is determined by the numerical calculation.

In this model the relativistic four-component spinor u_λ is related to the free quark propagator by

$$\sum_{\lambda=1,2} u_\lambda(\boldsymbol{k})\bar{u}_\lambda(\boldsymbol{k})=\not{p}+m. \tag{42}$$

However for confined quarks lattice results obtain a different propagator, so modified spinors of the form

$$u_\lambda(\boldsymbol{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{\frac{\omega+\xi m}{\omega}}\chi_\lambda \\ \sqrt{\frac{\omega-\xi m}{\omega}}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}\chi_\lambda \end{pmatrix}, \tag{43}$$

$$v_\lambda(\boldsymbol{k}) = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{\frac{\omega-\xi m}{\omega}}\boldsymbol{\sigma}\cdot\hat{\boldsymbol{k}}\chi_\lambda \\ \sqrt{\frac{\omega+\xi m}{\omega}}\chi_\lambda \end{pmatrix}, \tag{44}$$

have also been investigated. Here $\omega=\sqrt{k^2+\xi^2m^2}$ with parameter ξ. We should mention that here ω does not correspond to the energy of the particle with mass m. The parameter ξ will only modify the wave function. The free quark spinor is obtained for $\xi=1$ while $\xi\to\infty$ produces the non-relativistic two component spinor χ_λ.

The hadron spectra were studied for different values of ξ and quark masses and the results are listed in Table 4. While different sets of values produce comparable hadron spectra, the value $\xi=2.1$ yields quark masses similar to the constiuent quark model while for $\xi=1$ a much lighter u quark mass of 50 MeV is required. Note the quark masses are purely parameters which should not necessarily be identified as constituent quark masses.

Table 4. Calculated hadron spectrum in MeV. The experimental values from PDG are listed in the last column.

J^{PC}	meson	this work(ξ=2.1)	this work(ξ=1.0)	PDG
	π	141	137	135
0^{-+}	K	494	498	493
	ρ	778	779	776
1^{--}	K^*	891	888	894
	ϕ	1029	995	1020
	b_1	1195	1043	1235
1^{+-}	K_{1B}	1346	1277	
	h_1	1512	1485	1380
	a_0	1460	1352	1450
0^{++}	K_0^*	1519	1473	1430
	f_0	1623	1609	1710
$\frac{1}{2}^+$	N	934	941	938
	Λ	1158	1180	1116
	Σ	1204	1196	1189
	Ξ	1360	1340	1314
$\frac{3}{2}^+$	Δ	1233	1254	1232
	Σ^*	1385	1400	1385
	Ξ^*	1544	1543	1533
	Ω	1704	1681	1672

The hadron masses are obtained by variationally using the Monte Carlo method and are compared to the experimental results in Table 4. Only meson states suggested as $q\bar{q}$ in the PDG review table (Table 14.2) [42] are addressed. For mesons the Goldstone exchange interaction is small, less than 20 MeV. However for baryons it is larger, reducing the decuplet masses by about 30 MeV and for octets between 60 and 100 MeV which is now sufficient to reproduce the observed 300 MeV NΔ mass splitting. This is gratifying because this splitting without Goldstone exchange is only 250 MeV. It appears Goldstone exchange interactions play an important role in the baryon spectrum [43].

The calculated meson masses agree quite well with the PDG data, especially the 0^{-+} and 1^{--} states. The πρ splitting is close to 640 MeV and the KK^* splitting is about 400 MeV. In traditional quark models these splittings are produced by the color hyperfine interaction. In this work it is predominantly obtained from the Hamiltonian H_{I1} which lowers the 0^{-+} masses by about 500 MeV while reducing 1^{--} masses less than 100 MeV.

The model parameters were mainly determined by the π, ρ/ω, K and K* masses and then the remaining hadron masses were predicted. With the exception of three states (h_1, K_0^* and f_0) the overall meson and baryon spectra are in very good agreement with previous observations. This model calculation also predicts that the lightest scalar mesons have mass well above 1 GeV. This would indicate that the $a_0(980)$, $f_0(980)$ and $f_0(500)$ mesons are not pure $q\bar{q}$ states and possibly have a tetraquark structure.

6 Summary

A new, unified Hamiltonian model has been developed which combines the attractive features of phenomenologically based quark models with many of the theoretical ingredients common to QCD. A new spin interaction has also been investigated for a variety of Lorentz structures with a clear preference for $\Gamma=\vec{\gamma}$. A Goldstone exchange interaction was also included and, along with the spin interaction, found necessary to accurately reproduce the NΔ mass splitting.

The parameters are mainly determined by fitting the π, ρ/ω, K and K* masses. The remaining meson and baryon masses were then predicted and found to be in good agreement with previous observations. All scalar mesons are predicted to have mass well above 1 GeV suggesting the $a_0(980)$, $f_0(980)$ and $f_0(500)$ are not simple $q\bar{q}$ states but perhaps tetraquarks. Most significantly, a good Hamiltonian description for the meson, baryon (octet and decuplet) spectra has been obtained with a common set of parameters which has previously not been achieved.

Future work will address heavy quark systems to further test this model. If robust results are obtained, applications to exotic systems will be performed.

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References

1 Allton C R et al. (UKQCD collaboration). Phys. Rev. D, 1999, **60**: 034507
2 Aoki S et al. (CP-PACS collaboration). Phys. Rev. Lett., 2000, **84**: 238
3 Leinweber D B, Thomas A W, Tsushima K et al. Phys. Rev. D, 2000, **61**: 074502
4 Sasaki S, Blum T, Ohta S. Phys. Rev. D, 2002, **65**: 074503
5 Edwards R G, Dudek J J, Richards D G et al. Phys. Rev. D, 2011, **84**: 074508
6 Melnitchouk W, Thompson S O, Bonnet F D et al. Phys. Rev. D, 2003, **67**: 114506
7 Shifman M A, Vainshtein A I, Zakharov V I. Nucl. Phys. B, 1979, **147**: 385
8 Shifman M A, Vainshtein A I, Zakharov V I. Nucl. Phys. B, 1979, **147**: 448
9 Ioffe, B L. Nucl. Phys. B, 1981, **188**: 317
10 DAI Y B, HUANG C S, LIU C et al. Phys. Rev. D, 2003, **68**: 114011
11 WANG Z G, WAN S L. Phys. Rev. D, 2006, **74**: 014017
12 Brambilla N, Pineda A, Soto J et al. Nucl. Phys. B, 2000, **566**: 275
13 Kniehl B A, Penin A A, Smirnov V A et al. Nucl. Phys. B, 2002, **635**: 357
14 Pineda A. Phys. Rev. D, 2002, **65**: 074007
15 Jenkins E E. Nucl. Phys. B, 1992, **368**: 190
16 GUO F K, Hanhart C, Meissner U G. Eur. Phys. J. A, 2009, **40**: 171
17 WANG P, WANG X G. Phys. Rev. D, 2012, **86**: 014030
18 WANG P, WANG X G. Phys. Rev. Lett., 2013, **111**: 042002
19 GUO F K, LIU L, Meissner U G, Wang P. Phys. Rev. D, 2013, **88**: 074506
20 Godfrey S, Napolitano J. Rev. Mod. Phys., 1999, **71**: 1411
21 Eichten E, Godfrey S, Mahlke H, Rosner J L. Rev. Mod. Phys., 2008, **80**: 1161
22 Rujula A, Georgi H, Glashow S L. Phys. Rev. D, 1975, **12**: 147
23 Godfrey S, Isgur N. Phys. Rev. D, 1985, **32**: 189
24 Capstick S, Isgur N. Phys. Rev. D, 1986, **34**: 2809
25 Glozman L Y, Riska D O. Phys. Rept., 1996, **268**: 263
26 ZHANG Z Y, Faessler A, Straub U, Glozman L Y. Nucl. Phys. A, 1994, **578**: 573
27 ZHANG Z Y, YU Y W, SHEN P N et al. Nucl. Phys. A, 1997, **625**: 59
28 DAI L R, ZHANG Z Y, YU Y W, WANG P. Nucl. Phys. A, 2003, **727**: 321
29 Llanes-Estrada F J, Cotanch S R. Phys. Rev. Lett., 2000, **84**: 1102
30 Llanes-Estrada F J, Cotanch S R, Szczepaniak A P, Swanson E S. Phys. Rev. C, 2004, **70**: 035202
31 Llanes-Estrada F J, Cotanch S R. Nucl. Phys. A, 2002, **697**: 303
32 Szczepaniak A P, Swanson E S, JI C R, Cotanch S R. Phys. Rev. Lett., 1996, **76**: 2011
33 Cotanch S R, Szczepaniak A P, Swanson E S, JI C R. Nucl. Phys. A, 1998, **631**: 640C.
34 Cotanch S R, Llanes-Estrada F J. Nucl. Phys. A, 2001, **689**: 481
35 Llanes-Estrada F J, Cotanch S R. Phys. Lett. B, 2001, **504**: 15
36 General I J, Cotanch S R, Llanes-Estrada F J. Eur. Phys. J. C, 2007, **51**: 347
37 General I J, Wang P, Cotanch S R, Llanes-Estrada F J. Phys. Lett. B, 2007, **653**: 216
38 WANG P, Cotanch S R, General I J. Eur. Phys. J. C, 2008, **55**: 409
39 XIE W, MO L Q, WANG P, Cotanch S R. Phys. Lett. B, 2013 **725**: 148
40 Gross F, Milana J. Phys. Rev. D, 1991, **43**: 2401
41 Bugg D V. Eur. Phys. J. C, 2004, **33**: 505
42 Beringer J et al. Phys. Rev. D, 2012, **86**: 010001
43 Theberge S, Thomas A W, Miller G A. Phys. Rev. D, 1980, **22**: 2838