Probing the mass degeneracy of particles with different spins *

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Abstract: The spin is an important property of a particle. Although it is unlikely, there is still a possibility that two particles with different spins share similar masses. In this paper, we propose a method to probe this kind of mass degeneracy of particles with different spins. We use the cascade decay $B^+ \to X(3872)K^+$, $X(3872) \to D^+D^-$ to illustrate our method. It can be seen that the possible mass degeneracy of X(3872) can lead to interesting behavior in the corresponding cascade decay.

Key words: X(3872), mass degeneracy, B meson

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1 Introduction

It is always important to determine the spin of a new particle once it is discovered. However, the statistics of events corresponding to the new particle are usually low. As a result, sometimes we find that there are several possibilities for the spin of the new particle. Consequently, it is possible that several particles with different spins share similar masses. In this paper, we propose a method to confirm or exclude this possibility.

We will use the cascade decay $B^\pm \to X(3872)K^\pm$, $X(3872) \to D^+D^-$ as an example to explain our method. X(3872) was first discovered by the Belle Collaboration in 2003 and was the first exotic hadron to be discovered [1]. At first, the analysis of the X(3872) angular distributions in the decay to $J/\psi\pi^+\pi^-$ favored $J^{PC}=1^{++}$ or 2^{-+} [2, 3], and only in the last year did the LHCb Collaboration find that the spin of X(3872) is 1 [4]. Before this discovery by the LHCb Collaboration, there was a possibility that there were two particles with spin 1 and 2 degeneracy around 3872 MeV. We will show that this kind of mass degeneracy leads to some interesting behavior.

2 Formalism

To probe the degeneracy of X(3872), we assume that there are two particles with spin 1 and 2, respectively, and with masses about 3872 MeV. We will denote these two particles as X_1 and X_2 respectively.

When the invariant mass of the D^+D^- pair lies around 3872 MeV, the decay amplitude $\mathcal{M}$ for the cascade decay can be expressed as [5]

$$\mathcal{M}(s_{12},s_{13})=a_1P_1(g_{s_{D\overline{D}}}(s_{DK}))+a_2P_2(g_{s_{D\overline{D}}}(s_{DK})), \quad (1)$$

where P_l $(l=1,2)$ is the $(l+1)^{\text{th}}$ Legendre polynomial, a_lP_l represents the decay amplitude with X_l being the intermediate resonance, and $s_{D\overline{D}}$ and s_{DK} are the invariant mass squared of the D^+D^- pair and the D^+ and K^+ respectively ($D(\overline{D})$ in the subscript represents $D^+(D^-)$). The function $g_{s_{D\overline{D}}}(s_{DK})$ is defined as

$$g_{s_{D\overline{D}}}(s_{DK})=(\hat{s}_{DK}-s_{DK})/\Delta_{DK}, \quad (2)$$

where $\hat{s}_{DK}=(s_{DK}^{\max}+s_{DK}^{\min})/2$, $\Delta_{DK}=(s_{DK}^{\max}-s_{DK}^{\min})/2$, with $s_{DK}^{\max(\min)}$ being the kinematically allowed maximum (minimum) value of s_{DK}. The subscript $s_{D\overline{D}}$ of the function $g_{s_{D\overline{D}}}(s_{DK})$ indicates its dependence on $s_{D\overline{D}}$ (through

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$s_{\rm DK}^{\rm max}$ and $s_{\rm DK}^{\rm min}$). The definition of $s_{\rm DK}^{\rm max}$ and $s_{\rm DK}^{\rm min}$ can be found in Ref. [5].

The Legendre polynomials $P_1(x)$ and $P_2(x)$ have one and two zero points respectively, which for $P_1(x)$ is $x_0=0$ and for $P_2(x)$ are $x_\pm=\pm1/\sqrt{3}$. Correspondingly, the zero point(s) for $P_1\left(g_{s_{DD}}(s_{\rm DK})\right)$ and $P_2\left(g_{s_{\rm DD}}(s_{\rm DK})\right)$ are $s_{\rm DK}^{(0)}=\hat{s}_{\rm DK}$ and $s_{\rm DK}^{(\pm)}=\hat{s}_{\rm DK}\pm\Delta_{\rm DK}/\sqrt{3}$, respectively. This allows us to divide the allowed region of $s_{\rm DK}$ ($s_{\rm DK}^{\rm min}$,$s_{\rm DK}^{\rm max}$) into four regions, which are ($s_{\rm DK}^{\rm min}$,$s_{\rm DK}^{(-)}$), ($s_{\rm DK}^{(-)}$,$s_{\rm DK}^{(0)}$), ($s_{\rm DK}^{(0)}$,$s_{\rm DK}^{(+)}$), and ($s_{\rm DK}^{(+)}$,$s_{\rm DK}^{\rm max}$), and will be denoted as $\Omega_{\rm a}$, $\Omega_{\rm b}$, $\bar{\Omega}_{\rm b}$, and $\bar{\Omega}_{\rm a}$, respectively. As was shown in Refs. [5] and [6], localized CP asymmetry can be used to probe the interference of two resonances with different spins. However, for the cascade decay that we are considering, the weak phase is $\arg(V_{\rm tb}V_{\rm ts}^*/V_{\rm cb}V_{\rm cs}^*)$, which is too small to have any detectable effects. Instead, we propose some other quantities to probe the degeneracy of X(3872).

Since the weak phase is small, we can safely neglect it. Then, we can redefine a_l (l=1, 2) according to

$$a_l\rightarrow a_l{\rm e}^{{\rm i}\delta_l}, \tag{3}$$

so that the new defined a_ls are real, and the δ_ls are strong phases. The absolute value squared of $\mathcal{M}$ can be expressed as

$$|\mathcal{M}|^2=\mathcal{S}+\mathcal{A}, \tag{4}$$

where

$$\begin{aligned}\mathcal{S} &= \frac{a_1^2}{\Delta_{\rm DK}^2}\left(s_{\rm DK}^{(0)}-s_{\rm DK}\right)^2\\ &\quad+\frac{9a_2^2}{4\Delta_{\rm DK}^4}\left(s_{\rm DK}^{(-)}-s_{\rm DK}\right)^2\left(s_{\rm DK}^{(+)}-s_{\rm DK}\right)^2,\end{aligned} \tag{5}$$

$$\mathcal{A} = \frac{3a_1a_2\cos\delta}{\Delta_{\rm DK}^3}\left(s_{\rm DK}^{(0)}-s_{\rm DK}\right)\left(s_{\rm DK}^{(-)}-s_{\rm DK}\right)\left(s_{\rm DK}^{(+)}-s_{\rm DK}\right), \tag{6}$$

with $\delta=\delta_2-\delta_1$. $\mathcal{S}$ and $\mathcal{A}$ have a very interesting property. One can see from the above two equations that with the substitution

$$s_{\rm DK}\rightarrow2s_{\rm DK}^{(0)}-s_{\rm DK}, \tag{7}$$

$\mathcal{A}$ changes its sign, while $\mathcal{S}$ does not:

$$\mathcal{S}(s_{\rm DK}) = \mathcal{S}(2s_{\rm DK}^{(0)}-s_{\rm DK}), \tag{8}$$

$$\mathcal{A}(s_{\rm DK}) = -\mathcal{A}(2s_{\rm DK}^{(0)}-s_{\rm DK}). \tag{9}$$

In other words, as a function of $x=(s_{\rm DK}-\hat{s}_{\rm DK})/\Delta_{\rm DK}$, $\mathcal{S}$ is even, while $\mathcal{A}$ is odd. In Fig. 1, we depict $\mathcal{S}$, $\mathcal{A}$ and their sum as a function of x for $a_1=a_2=\cos\delta=1$. The four regions in phase space for fixed $s_{\rm D\bar{D}}$ that were mentioned above now correspond to $x\in(-1,\ 1/\sqrt{3})$, $(-1/\sqrt{3},\ 0)$, $(0,\ 1/\sqrt{3})$, and $(1/\sqrt{3},\ 1)$, respectively.

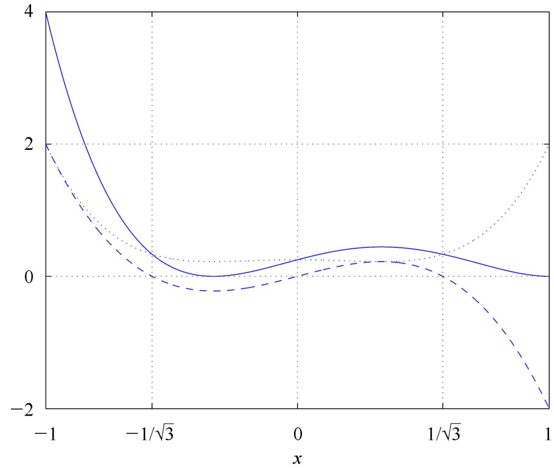

Fig. 1. $\mathcal{S}$ (dotted line), $\mathcal{A}$ (dashed line), and $|\mathcal{M}|^2$ (solid line) as a function of x, for the situation $a_1=a_2=\cos\delta=1$. $(-1,\ -1/\sqrt{3})$, $(-1/\sqrt{3},\ 0)$, $(0,\ 1/\sqrt{3})$, and $(1/\sqrt{3},\ 1)$ correspond to $\Omega_{\rm a}$, $\Omega_{\rm b}$, $\bar{\Omega}_{\rm b}$, and $\bar{\Omega}_{\rm a}$, respectively.

Inspired by the above property, we define two quantities $R_{\rm a}$ and $R_{\rm b}$ as

$$R_{\rm r}=\frac{N_{\Omega_{\rm r}}-N_{\bar{\Omega}_{\rm r}}}{N_{\Omega_{\rm r}}+N_{\bar{\Omega}_{\rm r}}}, \tag{10}$$

where $r=a$ or b, and N_ω ($\omega=\Omega_{\rm a}$, $\Omega_{\rm b}$, $\bar{\Omega}_{\rm b}$, $\bar{\Omega}_{\rm a}$) represents the event number of the cascade decay in the region ω. It can be seen for the situation we are considering that

$$R_{\rm r}=\frac{\mathcal{A}_{\Omega_{\rm r}}}{\mathcal{S}_{\Omega_{\rm r}}}, \tag{11}$$

where

$$\mathcal{S}_{\Omega_{\rm r}}=\int_{\Omega_{\rm r}}{\rm d}s_{\rm DK}\mathcal{S},\quad \mathcal{A}_{\Omega_{\rm r}}=\int_{\Omega_{\rm r}}{\rm d}s_{\rm DK}\mathcal{A}. \tag{12}$$

After doing the above integral, one has

$$\mathcal{S}_{\Omega_{\rm a}} = \left[\left(1-\frac{1}{3\sqrt{3}}\right)\frac{a_1^2}{3}+\left(1-\frac{2}{3\sqrt{3}}\right)\frac{a_2^2}{5}\right]\Delta_{\rm DK}, \tag{13}$$

$$\mathcal{S}_{\Omega_{\rm b}} = \left(\frac{a_1^2}{3}+\frac{2a_2^2}{5}\right)\frac{\Delta_{\rm DK}}{3\sqrt{3}}, \tag{14}$$

$$\mathcal{A}_{\Omega_{\rm a}} = \frac{1}{3}\Delta_{\rm DK}a_1a_2\cos\delta, \tag{15}$$

$$\mathcal{A}_{\Omega_{\rm b}} = -\frac{1}{12}\Delta_{\rm DK}a_1a_2\cos\delta. \tag{16}$$

With the above equations, one can further derive the relations between $R_{\rm a}$ and $R_{\rm b}$:

$$\frac{R_{\rm b}}{R_{\rm a}}=-\frac{1}{8}\left[3\sqrt{3}-2+\frac{3\sqrt{3}}{1+\dfrac{6a_2^2}{5a_1^2}}\right], \tag{17}$$

and

$$-\frac{1}{4}(3\sqrt{3}-1)<\frac{R_{\rm b}}{R_{\rm a}}<-\frac{1}{4}\left(\frac{3\sqrt{3}}{2}-1\right). \tag{18}$$

It can be seen that if there are two resonances lying around 3872 MeV, we should observe nonzero R_{a} and R_{b} with opposite signs. If there is only one resonance lying around 3872 MeV, this means a_1 or a_2 equals zero. By setting a_1 or a_2 equal to zero, one can immediately see that $\mathcal{A}=0$. As a result, both R_{a} and R_{b} are equal to zero. Thus, by measuring R_{a} and R_{b}, one can confirm or exclude the degeneracy of X(3872).

Note that all the above discussions are for fixed $\sqrt{s_{\mathrm{D\overline{D}}}}$, which in this case is around 3872 MeV. Practically, $\sqrt{s_{\mathrm{D\overline{D}}}}$ takes values which are in a small interval around 3872 MeV. The above discussion then should be modified by taking into account a proper integral over $s_{\mathrm{D\overline{D}}}$.

3 Conclusion

The upper limit for the branching ratio of the cascade decay is (at 90% confidence level) [7, 8]

$$\mathcal{B}(\mathrm{B}^+\to\mathrm{X(3872)K}^+)\times\mathcal{B}(\mathrm{X}\to\mathrm{D}^+\mathrm{D}^-)<4.0\times10^{-5}. \quad (19)$$

Thus the upper limit for the cascade decay event number N_{cas} is $N_{\mathrm{B+}}\times4.0\times10^{-5}$, where $N_{\mathrm{B+}}$ is the total number of events for the B^+ signal. To double the statistics, one can also include the CP conjugate channel. The method that we proposed above fails when the relative strong phase δ is close to $\pi/2$.

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