Modeling and analysis of SLED

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Abstract: SLED (SLAC Energy Doubler) is a crucial component for the C-band microwave acceleration unit of a soft X-ray Free Electron Laser (SXFEL). To study the behavior of SLED, a mathematical model is commonly built and analyzed. In this paper, a new method is proposed to build the model of SLED at the Shanghai Institute of Applied Physics. With this method, the parameters of the two cavities can be analyzed separately. Also it is suitable to study parameter optimization of SLED and analyze the effect from the parameters variations. Simulation results of our method are also presented.

Key words: SLED, mathematical model, energy multiplication factor, coupling coefficient

PACS: 29.20.-C   **DOI:** 10.1088/1674-1137/39/4/047002

1 Introduction

A compact soft X-ray Free Electron Laser (SXFEL) facility is presently planned at the Shanghai Institute of Applied Physics, CAS [1], and some analytical modeling and simulation research is ongoing. The high power RF system for SXFEL comprises an RF power source, a constant gradient accelerating structure and waveguide components. In order to obtain a high constant gradient field in the accelerating structure, the existing klystron power source of 50 MW cannot meet the power requirement of the field target, and a pulse compressor is required to multiply the power from the klystron [2]. There are different types of pulse compressor which satisfy the requirements. In our case, a SLAC Energy Doubler- (SLED) type pulse compressor is proposed for the C-band RF system in SXFEL.

To study the performance of the pulse compressor and analyze the parameters, an effective way is to build a mathematic model, then the model based simulations can be implemented to verify the design. In this paper, a mathematic model of the SLED is presented, which is a powerful tool for control system development. With this model, the parameters of SLED are optimized and the effect of the parameter variations are analyzed correspondingly.

2 Modeling of SLED

2.1 Structure of SLED

SLED is an RF pulse compressor which was first invented by Farkers Z in 1974 [3]. The SLED is composed of two identical high Q-factor cavities attached to a 3 dB coupler. The structure of SLED is shown in Fig. 1.

The performance of SLED is determined by the structure of the storage cavities. The energy multiplication factor M can be expressed as:

$$M = \gamma e^{-\frac{T_a}{T_c}} \left[1 - (1+g)^{1+v}\right] \left[g(1+v)\right]^{-1} - (\alpha - 1), \quad (1)$$

where T_a is the filling time of the accelerating structure, T_c is the filling time of the cavity. $\alpha = 2\beta/(\beta+1)$, $v = T_a/(T_c \ln(1-g)) - 1$, g is the gradient of the group velocity along the accelerating structure. $\gamma = \beta(2 - e^{-T_1})$, β is the cavity coupling coefficient.
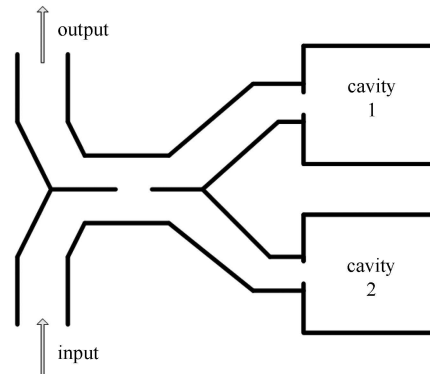

Fig. 1.   The structure of the SLED.

2.2 Modeling SLED using S11

Based on Refs. [3–5], a model of SLED can be constructed by the energy conservation. However the models based on these methods contain only the amplitude

information of the input and output signal, no phase information can be reflected, and is not suitable for the case of two asymmetry cavities.

In the paper the technology of the two-port terminal network is used to model the behavior of the cavity and input coupler. The cavity itself is equivalent to an RLC circuit, and the input coupler can be presented by an ideal transformer as shown in Fig. 2.
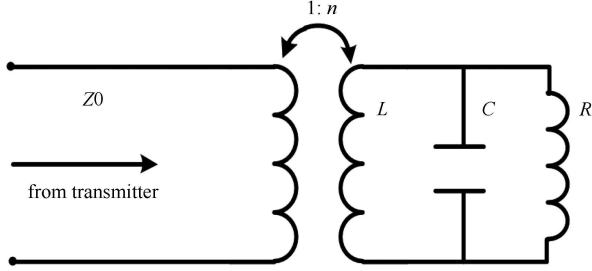

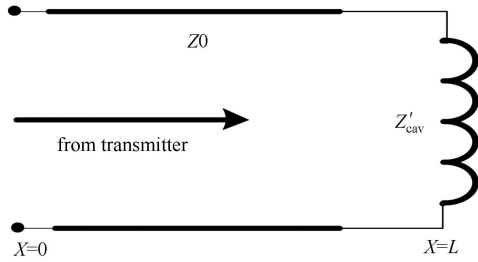Fig. 2.   The equivalent circuit of SLED.

Fig. 3.   Cavity model at the transmission line side.

For the systems engineer, it is important to know the relationship between the input and output voltage. The cavity model is transferred to the transmission line side shown in Fig. 3. Using the formula of the ideal transformer, the impedance at the transmission line side can be attained:

$$Z'_{\text{cav}} = \frac{R}{n^2} \frac{2\sigma s}{s^2 + 2\sigma s + \omega_0^2}, \qquad (2)$$

where $\omega_0 = \dfrac{1}{\sqrt{LC}}$ is the cavity resonant frequency, $Q_0 = \omega_0 RC$ is the cavity unload quality factor and $\sigma = \dfrac{\omega_0}{2Q_0}$. Applying the definition of coupling coefficient $\beta = \dfrac{Q_0}{Q_{\text{ext}}} = \dfrac{R}{n^2 Z_0}$, the equation can be presented as:

$$Z'_{\text{cav}} = \beta Z_0 \frac{2\sigma s}{s^2 + 2\sigma s + \omega_0^2}. \qquad (3)$$

Using the definition of the reflection coefficient for the two-port microwave network in Fig. 3, the formula of the reflection coefficient can be presented as [6]:

$$S_{11} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{Z'_{\text{cav}} - Z_0}{Z'_{\text{cav}} + Z_0}. \qquad (4)$$

Substituting the impedance Z'_{cav} (Eq. (3)) into Eq. (4), then Eq. (4) can be represented by the differential equation between the input and output voltage

$$\frac{\text{d}^2 V_{\text{out}}}{\text{d}t^2} + 2\sigma(\beta+1) \frac{\text{d}V_{\text{out}}}{\text{d}t} + \omega_0^2 V_{\text{out}}$$

$$= -\frac{\text{d}^2 V_{\text{in}}}{\text{d}t^2} + 2\sigma(\beta-1) \frac{\text{d}V_{\text{in}}}{\text{d}t} - \omega_0^2 V_{\text{in}}. \qquad (5)$$

The input voltage V_{in} and output voltage V_{out} are modulated sine wave with the frequency of ω, the phase and amplitude information is the most useful part that we should consider, so, they can be written into phasor as

$$V_{\text{in}} = \boldsymbol{V}_{\text{in}} e^{\text{j}\omega t}, \ V_{\text{out}} = \boldsymbol{V}_{\text{out}} e^{\text{i}\omega t}.$$

Where the $\boldsymbol{V}_{\text{in}}$ and $\boldsymbol{V}_{\text{out}}$ are the complex amplitude as vectors which contain the amplitude and phase information, which can be expressed by their real and imaginary parts. Put the phasor definition into the equation above and assume $\sigma(\beta-1) \ll \omega$, and define the detuning as $\Delta\omega = \omega_0 - \omega$. If the detuning is smaller than the working frequency, we will get the approximation of $\omega_0^2 - \omega^2 = 2\omega\Delta\omega$. Normally, the voltage changes slowly, so the item for the second derivation is always smaller than the others. So Eq. (5) can be simplified as

$$\frac{\text{d}\boldsymbol{V}_{\text{out}}}{\text{d}t} + [\sigma(\beta+1) + \text{j}\Delta\omega]\boldsymbol{V}_{\text{out}}$$

$$= -\frac{\text{d}\boldsymbol{V}_{\text{in}}}{\text{d}t} + [\sigma(\beta+1) + \text{j}\Delta\omega]\boldsymbol{V}_{\text{in}}. \qquad (6)$$

Eq. (3) can also be expressed by the transfer function as

$$\boldsymbol{V}_{\text{out}}(s) = \frac{-s + [\sigma(\beta-1) + \text{j}\Delta\omega]}{s + [\sigma(\beta+1) + \text{j}\Delta\omega]} \boldsymbol{V}_{\text{in}}(s). \qquad (7)$$

Due to the 3 dB power divider between the input and output ports, the relationship between the input and output of the SLED for each cavity can be presented respectively as

$$\boldsymbol{V}_{\text{sled\_out\_cavity1}}(s)$$
$$= \frac{1}{\sqrt{2}} \frac{-s + [\sigma(\beta-1) + \text{j}\Delta\omega]}{s + [\sigma(\beta+1) + \text{j}\Delta\omega]} e^{\text{j}\frac{\pi}{2}} \frac{1}{\sqrt{2}} \boldsymbol{V}_{\text{sled\_in}}(s), \quad (8)$$
$$\boldsymbol{V}_{\text{sled\_out\_cavity2}}(s)$$
$$= e^{\text{j}\frac{\pi}{2}} \frac{1}{\sqrt{2}} \frac{-s + [\sigma(\beta-1) + \text{j}\Delta\omega]}{s + [\sigma(\beta+1) + \text{j}\Delta\omega]} \frac{1}{\sqrt{2}} \boldsymbol{V}_{\text{sled\_in}}(s). \quad (9)$$

From the structure of the SLED as shown in Fig. 1, the output signal of the SLED is formed by the reflected signals of the two cavities. Based on Eqs. (8) and (9), the SLED model can be constructed as Fig. 4.
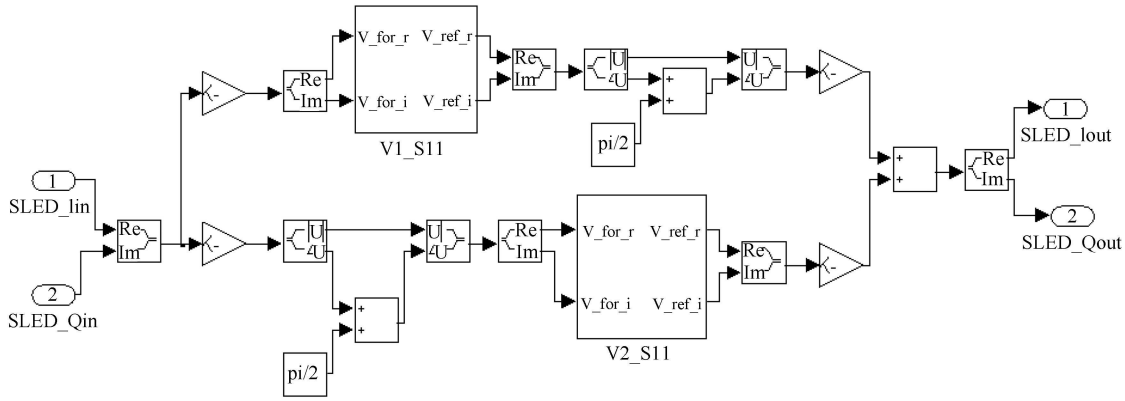
Fig. 4.   The SLED model.

3   Results of simulation by SLED model

Based on previous theoretical analysis, we have carried out some simulations using the SLED model to optimize parameters.

3.1   Study of the working point of SLED

The coupling coefficient β and quality factor Q_0 of the cavity dominate the performance of the RF pulse compressor, such as the energy multiplication factor and power efficiency. The parameters relevant to the calculation of the energy multiplication factor and RF power efficiency are listed in Table 1. Using the parameters in table 1 and by tuning the coupling coefficient β and quality factor Q_0, the tendency of the energy multiplication factor and RF power efficiency can be mapped, as shown in Fig. 5. The power efficiency and the energy multiplication increase with the quality factor and the input coupling coefficient.

Table 1.   Key parameters for energy gain factor calculation.

RF frequency	accelerating structure filling time	RF pulse length	reverse time
5712 MHz	372 ns	2.5 μs	2.0 μs

The optimal parameters are decided by the practical requirements. The point on the straight line in Fig. 5 shows the optimal operating point where both β and Q_0 are small when the energy multiplication factor reaches the maximum value. According to the simulation results, the operating point is selected when quality factor $Q_0 = 160000$ and the coupling coefficient $\beta = 7.0$, and the corresponding energy multiplication factor is 1.9029. Compared with the original working point ($Q_0=180000$, $\beta=8.5$), the fabrication is easier and the energy multiplication factor was reduced by about 0.1%.

3.2   Course tuning before operation

In practice, more attention is paid to the energy

multiplication factor. There are some frequency deviations from the operating frequency due to the temperature drift, some coupling coefficient deviations from the
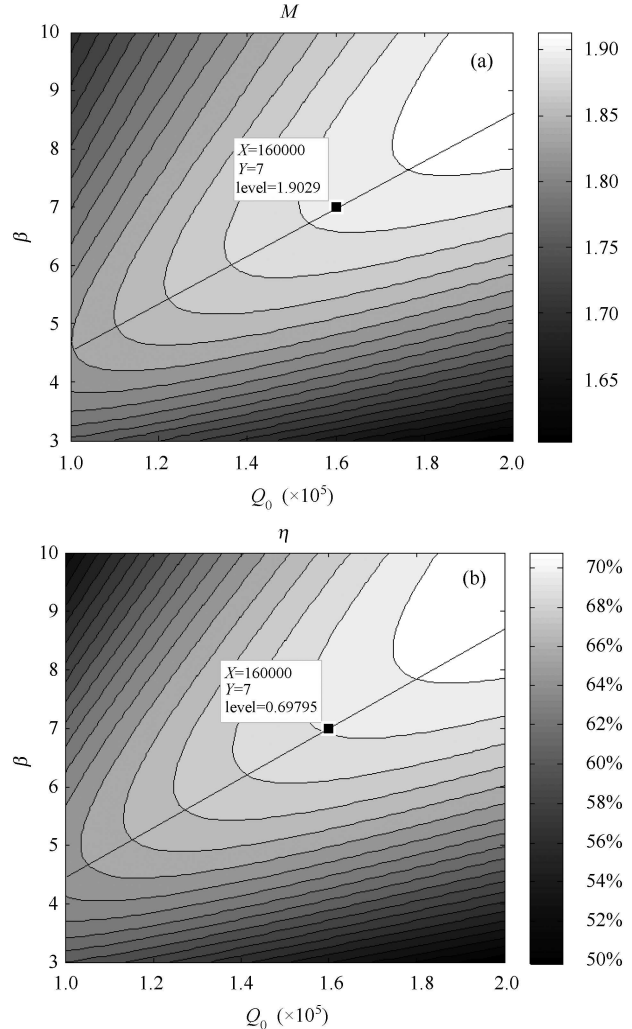

Fig. 5.   The energy multiplication factor M and RF power efficiency η mapped with the unloaded quality factor Q_0 and coupling coefficient.
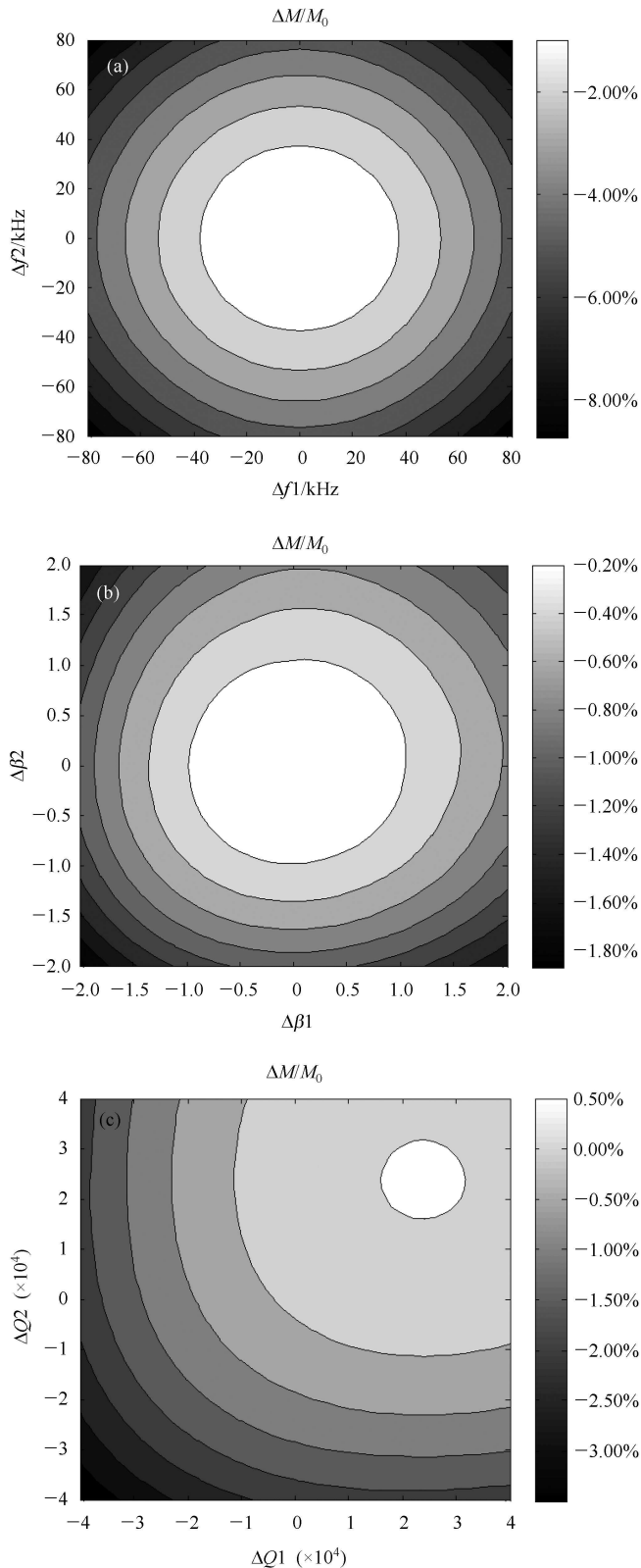
Fig. 6. The energy multiplication factor changes with the cavity frequency deviation (a), the input coupling coefficient deviation (b) and unloaded quality factor fluctuation (β=7.0, Q_0=160000).

desired value and the unloaded quality factor error of the cavity due to machining tolerance. Using the SLED model, we can get the Energy multiplication factor change caused by the cavity frequency deviation, the input coupling coefficient deviation and the unloaded quality factor fluctuation, as shown in Fig. 6. Fig. 6(a) and (b) show that the energy multiplication factor declines with the increase of frequency deviation and the coupling coefficient is far from the desired value. The effect caused by the unloaded quality factor error of the cavity is shown in Fig. 6(c). The design value is not the optimal value for getting the maximum energy multiplication factor which agrees well with the analysis in the above section.

Before the actual operation, the RF pulse compressor should be tuned to maintain the energy multiplication factor fluctuation less than 1%. According to Fig. 6, the frequency should be controlled in the range of ±30 kHz, the accuracy of the β in the range of ±2, which is identical with the result in Fig. 5(a) and the Q_0 in the range of $\pm2\times10^4$. Since the coupling coefficient β and Q_0 cannot be tuned during the operation, during the cold test they should be tuning in an optimal range in order to get a high precision, such as ±0.25 for the coupling constant and $\pm2\times10^3$ for Q_0 in order to attain the flatness of the energy gain factor 0.01%.

3.3   Fine tuning during operation

After the cold test, the coupling coefficient and unloaded quality factor are tuned at the proper values and during the operation of the RF pulse compressor, they are fixed and cannot be tuned. There is only one parameter, the frequency deviation, which can be changed and tuned by controlling the temperature of the cooling water. During the operation, as the requirement of the
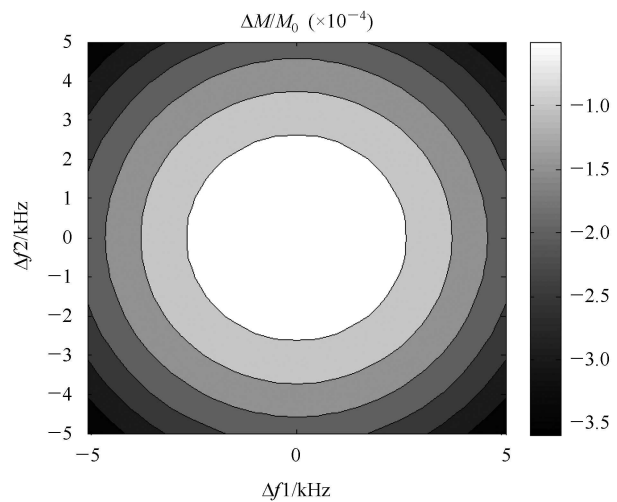

Fig. 7. The energy multiplication factor change with the cavity frequency deviation.

flatness of the energy gain factor is less than 0.01%, the frequency of the cavity should be controlled within ±0.2 kHz as shown in Fig. 7. As the temperature expansion coefficient of the SLED cavity is about 106 kHz/℃, the water temperature should be controlled within ±0.02 ℃.

According to the analysis above, the parameter requirements for getting energy multiplication factor flatness 0.01% are listed in Table 2.

Table 2.   Parameter control range.

parameters	coarse tuning	fine tuning
frequency deviation	±30 kHz	
temperature		±0.02 ℃(±2 kHz)
coupling efficient deviation	±0.25	
unloaded quality factor difference	$\pm2\times10^4$	

4   Conclusion

RF compressors as a key technology for particle accelerators have been widely studied in many accelerator laboratories, such as KEK, CERN, IHEP and SLAC. But there are only a few researchers using an equivalent circuit model to study the behavior of SLED and analyze the parameters. A detailed process for building a mathematic model is shown in this paper. The simulation results and the analysis of the parameter deviations of the cavities are also presented. During our modeling analysis, the flatness of the energy gain factor 0.01% can be achieved with our model when the temperature of the cooling water $|T_c|\leqslant0.02$ ℃, then the maximum frequency detuning will be controlled in 2 kHz.

The SLED model can be used to further study the RF pulse compressor. More specific measurements will be carried out in the future.

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