Quantum information splitting of a two-qubit Bell state using a four-qubit entangled state*

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Abstract: A scheme is proposed for quantum information splitting of a two-qubit Bell state by using a four-qubit entangled state as a quantum channel. In the scenario, it is supposed that there are three legitimate parties, say Alice, Bob and Charlie. Alice is the sender of quantum information. Bob and Charlie are two agents. Alice first performs GHZ state measurement and tells Bob and Charlie the measurement results via a classical channel. It is impossible for Bob to reconstruct the original state with local operations unless help is obtained from Charlie. If Charlie allows Bob to reconstruct the original state information, he needs to perform a single-qubit measurement and tell Bob the measurement result. Using the measurement results from Alice and Charlie, Bob can reconstruct the original state. We also consider the problem of security attacks. This protocol is considered to be secure.

Key words: quantum information, quantum information splitting, two-qubit Bell state, GHZ state measurement

PACS: 03.65.Ud, 03.67.Hk, 03.67.Ac **DOI:** 10.1088/1674-1137/39/4/043103

1 Introduction

Traditional cryptography with symmetric keys has some well-known defects. Based on mathematical cryptosystems which generally use a difficult-to-solve math problem, its security is limited by the computing power available. It is difficult to find examples in classical cryptography of keys which cannot be stolen or changed in the transfer process. On the basis of the principles of quantum mechanics, quantum cryptography can solve these problems in traditional cryptography with a more reliable secure key system. Quantum information is the term used for microscopic qubit state information. The basic principles of quantum mechanics, in particular the Heisenberg Uncertainty Principle and the quantum cloning theorem, are the most important physical bases for the description of quantum information behavior. At present, research in quantum cryptography communication mainly includes quantum key distribution (QKD), quantum secure direct communication (QSDC), quantum cryptography shared, quantum digital signatures, etc. QKD aims to establish a private key between two long-range authorized users, and is one of the most pressing current applications of quantum mechanics. Since Bennett and Brassard proposed the first QKD protocol (BB84) in 1984 [1], more and more quantum information schemes have been proposed, such as quantum teleportation and quantum secret sharing [2–5]. Unlike QKD, the purpose of QSDC is to transmit the secret message directly, without first establishing a key to encrypt it [6–10]. There are two main types of research on QSDC: a communication protocol based on a two-step approach using entangled photon pairs, and a communication protocol based on high dimensional quantum states. Long and Liu first proposed the QSDC scheme realization based on EPR pairs in 2000, with a quantum data block proposed to test eavesdropping efficiently [11]. Deng et al. first proposed a two-step QSDC scheme with entangled qubit pairs in 2003 [12]. Thereafter, Deng and Long proposed a QSDC scheme based on single photons, which acted as a quantum one-time pad cryptosystem [13]. Wang et al. proposed high-dimensional quantum superdense coding and a muti-step QSDC scheme in 2005 [14, 15]. Zhu et al. proposed a QSDC scheme with a secret transmitting order for the particles in 2006 [16]. Gao et al. proposed a new inspection strategy for each Bell state by using two photons incorporated randomly into the sending sequence in 2009, thus improving the security of the ping-pong protocol [17]. Wang et al. proposed a quantum superdense coding protocol in 2011 [18]. Liu

Received 13 May 2014, Revised 23 September 2014

* Supported by Fundamental Research Funds for the Central Universi-ties (ZYGX2011J064), National Nature Science Foundation of China (60903157, 61133016), and National High Technology Joint Re-search Program of China (863 Program, 2011AA010706)

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et al. proposed a high-capacity QSDC scheme with single photons in 2012 [19].

Since 1993, when Bennett suggested the classical communication and EPR entangled channels transmitted to an unknown single qubit [20], quantum transmission has become a hotspot in the field of quantum information content and experiments [21]. In 1999, a quantum physics method was proposed for the first time by Hillery to realize the concept of quantum secret sharing and quantum information separation, and an entanglement correlation design based on the GHZ state, which is the first quantum secret sharing protocol (the HBB protocol) [22]. This protocol controls the transmission of the unknown quantum state through a controller. However, a system with more than three quantum-entangled states is complex, and has particular characteristics. H.J Ridgel and R. Raussendorf put forward a special quantum state when the qubit number is greater than three, called a cluster state [23–27]. The cluster state has the nature of GHZ classes and class W entangled states, but is more difficult than GHZ state entanglement because it has been damaged by the local operation, so clusters are a very important type of state. After Hillery et al. proposed that GHZ states can be used for quantum information splitting (QIS), QIS has been investigated extensively [28–33]. In recent years, Nie et al. proposed using a four-qubit entangled state as a quantum channel which can be a control contact transmission scheme for a single qubit [34–41]. Quantum states containing Bell states [42], four-qubit cluster states, five-qubit cluster states, six-qubit cluster states, GHZ states [43] and so on have all been considered as quantum channels. Zhan et al. have proposed a scheme for teleportation via high-dimensional entangled states [44]. Yin et al. have proposed the quantum teleportation of a four-qubit cluster state [45]. Nie et al. have proposed a scheme for quantum teleportation by a GHZ state [46]. Nie at al. have also proposed using four-particle entangled states as a quantum channel via single particle QIS [47–51]. Li et al. have proposed a scheme for QIS for an arbitrary three-qubit state using a four-qubit cluster state and GHZ-state [52].

In this paper, we use a four-qubit entangled state as the quantum channel via a new two-qubit Bell state division scheme for quantum information splitting. In the scenario, Alice, Bob and Charlie safely share a four-qubit entanglement state as a quantum channel. First of all, Alice performs a GHZ state measurement on her qubits. Alice can obtain one of 8 measurement results with equal probability, then the remaining qubits may collapse into one of 8 states $|\Psi^i\rangle_{234}(i{=}1,2,\cdots\cdots,8)$ after the measurement. Alice sends the measurement results to Bob and Charlie via a classical channel. Whether or not Bob can reconstruct the original state with local operations to the state$|\Psi^i\rangle_{234}$ depends on the controller Charlie. If Charlie allows Bob to reconstruct the original unknown state, he needs to perform a single qubit measurement using the basis $\{|0\rangle,|1\rangle\}$ and tell Bob the result. By combining the information from Alice and Charlie, Bob can reconstruct the original state on his qubits.

This article mainly includes the following three points:

(1) Alice performs GHZ state measurement and tells Bob and Charlie the measurement results via a classical channel.

(2) Charlie performs Z-basis of a single qubit measurement.

(3) According to the measurement results, Bob makes appropriate unitary transformation on a two-qubit, and can then rebuild the two-qubit Bell states.

2 Quantum information splitting of four-qubit states

In this section, we will briefly describe quantum information splitting of four-qubit states using a two-qubit Bell state. The main steps are as follows:

Step 1: Alice needs to prepare a four-qubit entangled state as a quantum channel, which can be described as follows:

$$\begin{aligned}|\Phi\rangle_{1234} &= \frac{1}{2}[|00\rangle(|00\rangle+|11\rangle)+|11\rangle(|01\rangle+|10\rangle)]_{1234}\\ &= \frac{1}{2}(|0000\rangle+|0011\rangle+|1101\rangle+|1110\rangle)_{1234}. \end{aligned} \quad (1)$$

Alice's own two-qubit Bell state can be written as

$$\begin{aligned}|\Phi^+\rangle_{x1x2}&=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)\\ |\Phi^-\rangle_{x1x2}&=\frac{1}{\sqrt{2}}(|00\rangle-|11\rangle)\\ |\Psi^+\rangle_{x1x2}&=\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle)\\ |\Psi^-\rangle_{x1x2}&=\frac{1}{\sqrt{2}}(|01\rangle-|10\rangle).\end{aligned} \quad (2)$$

It is assumed that Alice owns qubit 1 and the two-qubit Bell state, Bob owns qubits 2 and 3, and Charlie owns qubit 4. The state of the whole system is

$$\begin{aligned}|\Pi\rangle_{x1x21234} = |\Phi^\pm\rangle_{x1x2}\otimes|\Phi\rangle_{1234}=&\frac{1}{2\sqrt{2}}(|000000\rangle+|000011\rangle\\ &+|001101\rangle+|001110\rangle\pm|110000\rangle\pm|110011\rangle\\ &\pm|111101\rangle\pm|111110\rangle)_{x1x21234}\end{aligned} \quad (3)$$

or

$$\begin{aligned}|\Pi\rangle_{x1x21234} = |\Phi^\pm\rangle_{x1x2}\otimes|\Phi\rangle_{1234}=&\frac{1}{2\sqrt{2}}(|010000\rangle+|010011\rangle\\ &+|011101\rangle+|011110\rangle\pm|100000\rangle\pm|100011\rangle\\ &\pm|101101\rangle\pm|101110\rangle)_{x1x21234}.\end{aligned} \quad (4)$$

Step 2: In order to achieve the QIS of the two-qubit Bell state simply, Alice performs GHZ-state measurements on her qubits. The GHZ state is

$$|GHZ\rangle^{\pm} = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle), \tag{5}$$

$$|H\rangle^{\pm} = \frac{1}{\sqrt{2}}(|011\rangle \pm |100\rangle), \tag{6}$$

$$|G\rangle^{\pm} = \frac{1}{\sqrt{2}}(|010\rangle \pm |101\rangle), \tag{7}$$

$$|Z\rangle^{\pm} = \frac{1}{\sqrt{2}}(|001\rangle \pm |110\rangle). \tag{8}$$

After the measurements, the remaining qubits may collapse into one of the following states:

$$\begin{aligned} |\Psi^{1,2}\rangle_{234} &= \frac{1}{4}(|000\rangle + |011\rangle \pm |101\rangle \pm |110\rangle)_{234} \\ |\Psi^{3,4}\rangle_{234} &= \frac{1}{4}(|101\rangle + |110\rangle \pm |000\rangle \pm |011\rangle)_{234} \\ |\Psi^{5,6}\rangle_{234} &= \frac{1}{4}(|000\rangle + |011\rangle \mp |101\rangle \mp |110\rangle)_{234} \\ |\Psi^{7,8}\rangle_{234} &= \frac{1}{4}(|101\rangle + |110\rangle \mp |000\rangle \mp |011\rangle)_{234}. \end{aligned} \tag{9}$$

Step 3: As a result of the action for entanglement exchange, the quantum information sent by Alice has been passed to Bob and Charlie's quantum states; the quantum information distribution process has been completed at this time. Any one of Bob and Charlie, if not through interaction and exchange of information, can rely on local measurements of their qubits to get Alice's unknown quantum state. According to the no-cloning theorem, however, there can be only one receiver for initial state information. Generally, assume Alice's GHZ measurement result is $|\Psi^2\rangle_{234}$. According to (9), Bob and Charlie's corresponding qubits state can be written as

$$\begin{aligned} |\Psi^2\rangle_{234} &= \frac{1}{4}(|000\rangle + |011\rangle - |101\rangle - |110\rangle) \\ &= \frac{1}{\sqrt{2}}[|0\rangle_2(|00\rangle + |11\rangle)_{34} - |1\rangle_2(|01\rangle + |10\rangle)_{34}]. \end{aligned} \tag{10}$$

For one of the eight states in (9) , which two of the four unitary operators is required depends on the states obtained by Bob.

The four unitary operators can be written as

$$\begin{aligned} I &= I = U_0 = |0\rangle\langle 0| + |1\rangle\langle 1| \\ Z &= \sigma_z = U_1 = |0\rangle\langle 0| - |1\rangle\langle 1| \\ X &= \sigma_x = U_2 = |0\rangle\langle 1| + |1\rangle\langle 0| \\ Y &= -\mathrm{i}\sigma_y = U_3 = |0\rangle\langle 1| - |1\rangle\langle 0|. \end{aligned} \tag{11}$$

Suppose Alice can license Bob to split quantum information. If Charlie agrees to help Bob rebuild the original information, Charlie needs to perform a single-qubit measurement and tell Bob the measurement results.

If Charlie's measurement result is $|0\rangle_2$, then Bob's qubit 3 and qubit 4 will collapse into the state $(|00\rangle + |11\rangle)_{34}$. This is the initial quantum state information which was sent by Alice, and Bob does not need to do anything for qubit 3 and qubit 4 to obtain the initial quantum information.

If Charlie's measurement result is $|1\rangle_2$, then Bob's qubit 3 and qubit 4 will collapse into the state $(|01\rangle + |10\rangle)_{34}$. Bob then needs to apply the U_2 operator to qubit 3 and qubit 4 to obtain the initial quantum information.

Step 4: If Charlie permits Bob to reconstruct the original state, he needs to perform a single-qubit measurement of qubit 4 based on $\{|0\rangle, |1\rangle\}$. After measurement, the state of the remaining qubits collapses into the state

$$\begin{aligned} &|\xi^{1,2}\rangle = (|00\rangle \pm |11\rangle)_{23} \quad &&|\xi^{3,4}\rangle = (|01\rangle \pm |10\rangle)_{23} \\ &|\xi^{5,6}\rangle = (|11\rangle \pm |00\rangle)_{23} \quad &&|\xi^{7,8}\rangle = (|10\rangle \pm |01\rangle)_{23} \\ &|\xi^{9,10}\rangle = (|00\rangle \mp |11\rangle)_{23} \quad &&|\xi^{11,12}\rangle = (|01\rangle \mp |10\rangle)_{23} \\ &|\xi^{13,14}\rangle = (|11\rangle \mp |00\rangle)_{23} \quad &&|\xi^{15,16}\rangle = (|10\rangle \mp |01\rangle)_{23}. \end{aligned} \tag{12}$$

After doing those operations, Bob can successfully reconstruct the original two-qubit Bell state $|\Phi^{\pm}\rangle$ and $|\Psi^{\pm}\rangle$.

The process is illustrated in Fig. 1.

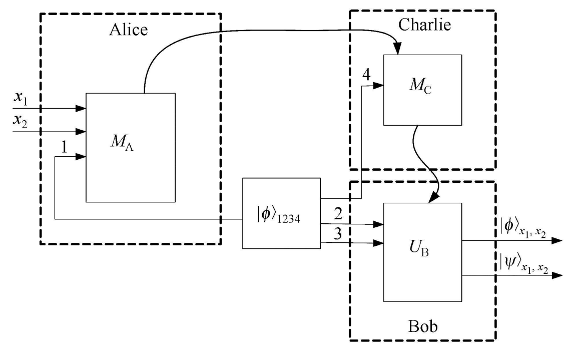

Fig. 1. QIS for the two-qubit Bell state. The curved lines represent classical channels. Qubits x_1, x_2 and 1 are owned by Alice, qubit 4 is owned by Charlie, and qubits 2, 3 are owned by Bob. M_A is the GHZ state collective measurement on qubits x_1, x_2 and 1. U_B is the unitary operation on qubits 2 and 3. M_C is the single qubit measurement on qubit 4. Details are explained in the paper.

In order to achieve QIS, a GHZ state measurement on qubits 1, x_1,x_2 is made by Alice, which will cause qubits 2,3 and 4 to collapse. The state obtained by Bob and the set of unitary operators needed after Charlie performs the single-qubit measurement are shown in Table 1.

3 Security analysis

In this section, we discuss the security of this scheme against certain eavesdropping attacks. Using the same

Table 1. Measurement results performed by Alice and the collapsed states obtained after measuring, Charlie's measurement, the collapsed state obtained and Bob's unitary operator.

Alice's measurement	the state obtained by Bob and Charlie	Charlie's measurement	states obtained by Bob	unitary operators (U_i)
$\|GHZ\rangle^+$	$\|\Psi^1\rangle_{234}=\frac{1}{4}(\|000\rangle+\|011\rangle+\|101\rangle+\|110\rangle)_{234}$	$\|0\rangle$	$\|\xi^1\rangle=(\|00\rangle+\|11\rangle)_{23}$	U_0
		$\|1\rangle$	$\|\xi^2\rangle=(\|00\rangle-\|11\rangle)_{23}$	U_1
$\|GHZ\rangle^-$	$\|\Psi^2\rangle_{234}=\frac{1}{4}(\|000\rangle+\|011\rangle-\|101\rangle-\|110\rangle)_{234}$	$\|0\rangle$	$\|\xi^3\rangle=(\|01\rangle+\|10\rangle)_{23}$	U_2
		$\|1\rangle$	$\|\xi^4\rangle=(\|01\rangle-\|10\rangle)_{23}$	U_3
$\|G\rangle^+$	$\|\Psi^3\rangle_{234}=\frac{1}{4}(\|101\rangle+\|110\rangle+\|000\rangle+\|011\rangle)_{234}$	$\|0\rangle$	$\|\xi^5\rangle=(\|11\rangle+\|00\rangle)_{23}$	U_0
		$\|1\rangle$	$\|\xi^6\rangle=(\|11\rangle-\|00\rangle)_{23}$	$-U_1$
$\|G\rangle^-$	$\|\Psi^4\rangle_{234}=\frac{1}{4}(\|101\rangle+\|110\rangle-\|000\rangle-\|011\rangle)_{234}$	$\|0\rangle$	$\|\xi^7\rangle=(\|10\rangle+\|01\rangle)_{23}$	U_2
		$\|1\rangle$	$\|\xi^8\rangle=(\|10\rangle-\|01\rangle)_{23}$	$-U_3$
$\|H\rangle^+$	$\|\Psi^5\rangle_{234}=\frac{1}{4}(\|000\rangle+\|011\rangle-\|101\rangle-\|110\rangle)_{234}$	$\|0\rangle$	$\|\xi^9\rangle=(\|00\rangle-\|11\rangle)_{23}$	U_1
		$\|1\rangle$	$\|\xi^{10}\rangle=(\|00\rangle+\|11\rangle)_{23}$	U_0
$\|H\rangle^-$	$\|\Psi^6\rangle_{234}=\frac{1}{4}(\|000\rangle+\|011\rangle+\|101\rangle+\|110\rangle)_{234}$	$\|0\rangle$	$\|\xi^{11}\rangle=(\|01\rangle-\|10\rangle)_{23}$	U_3
		$\|1\rangle$	$\|\xi^{12}\rangle=(\|01\rangle+\|10\rangle)_{23}$	U_2
$\|Z\rangle^+$	$\|\Psi^7\rangle_{234}=\frac{1}{4}(\|101\rangle+\|110\rangle-\|000\rangle-\|011\rangle)_{234}$	$\|0\rangle$	$\|\xi^{13}\rangle=(\|11\rangle-\|00\rangle)_{23}$	$-U_1$
		$\|1\rangle$	$\|\xi^{14}\rangle=(\|11\rangle+\|00\rangle)_{23}$	U_0
$\|Z\rangle^-$	$\|\Psi^8\rangle_{234}=\frac{1}{4}(\|101\rangle+\|110\rangle+\|000\rangle+\|011\rangle)_{234}$	$\|0\rangle$	$\|\xi^{15}\rangle=(\|10\rangle-\|01\rangle)_{23}$	$-U_3$
		$\|1\rangle$	$\|\xi^{16}\rangle=(\|10\rangle+\|01\rangle)_{23}$	U_2

method, we assume that an eavesdropper (called Eve) has managed to entangle an accessorial qubit to a qubit owned by Charlie, so that she can measure the accessorial qubit to obtain information about the unknown qubit state. Suppose that all the three legitimate partners are unaware of the attacker Eve. Then, after Alice performs the three-qubit GHZ state measurement, the combined state of Charlie, Bob and Eve collapses into a four-qubit entangled state. However, after Charlie makes his single-qubit measurement, the Bob-Eve system collapses into a product state, leaving Eve with no information about the unknown qubit. To view this scenario more explicitly, we assume that the accessorial qubit entangled with qubit 2 of the entangled channel, possessed by Charlie, to be $\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_{\mathrm{E}}$. If Alice gets the measurement outcomes $|\Psi^i\rangle_{234\mathrm{E}}(i=1,2,\cdots\cdots,8)$, then the combined state of Charlie, Bob and Eve would be

$$
\begin{aligned}
|\Omega^{1,2}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|0000\rangle+|0110\rangle\pm|1010\rangle\pm|1100\rangle)_{234\mathrm{E}}\\
|\Omega^{3,4}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|0001\rangle+|0111\rangle\pm|1011\rangle\pm|1101\rangle)_{234\mathrm{E}}\\
|\Omega^{5,6}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|1010\rangle+|1100\rangle\pm|0000\rangle\pm|0110\rangle)_{234\mathrm{E}}\\
|\Omega^{7,8}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|1011\rangle+|1101\rangle\pm|0001\rangle\pm|0111\rangle)_{234\mathrm{E}}\\
|\Omega^{9,10}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|0000\rangle+|0110\rangle\mp|1010\rangle\mp|1100\rangle)_{234\mathrm{E}}\\
|\Omega^{11,12}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|0001\rangle+|0111\rangle\mp|1011\rangle\mp|1101\rangle)_{234\mathrm{E}}\\
|\Omega^{13,14}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|1010\rangle+|1100\rangle\mp|0000\rangle\mp|0110\rangle)_{234\mathrm{E}}\\
|\Omega^{15,16}\rangle_{234\mathrm{E}} &= \frac{1}{4}(|1011\rangle+|1101\rangle\mp|0001\rangle\mp|0111\rangle)_{234\mathrm{E}}.
\end{aligned} \tag{13}
$$

When Charlie performs the single-qubit measurement of his qubit 4, the Bob-Eve system will then collapse into a product state

$$
\begin{aligned}
|\Xi^{1,2}\rangle &= \frac{1}{4}(|000\rangle+|010\rangle\pm|100\rangle\pm|110\rangle)_{23\mathrm{E}}\\
|\Xi^{3,4}\rangle &= \frac{1}{4}(|001\rangle+|011\rangle\pm|101\rangle\pm|111\rangle)_{23\mathrm{E}}\\
|\Xi^{5,6}\rangle &= \frac{1}{4}(|100\rangle+|110\rangle\pm|000\rangle\pm|010\rangle)_{23\mathrm{E}}\\
|\Xi^{7,8}\rangle &= \frac{1}{4}(|101\rangle+|111\rangle\pm|001\rangle\pm|011\rangle)_{23\mathrm{E}}\\
|\Xi^{9,10}\rangle &= \frac{1}{4}(|000\rangle+|010\rangle\mp|100\rangle\mp|110\rangle)_{23\mathrm{E}}\\
|\Xi^{11,12}\rangle &= \frac{1}{4}(|001\rangle+|011\rangle\mp|101\rangle\mp|111\rangle)_{23\mathrm{E}}\\
|\Xi^{13,14}\rangle &= \frac{1}{4}(|100\rangle+|110\rangle\mp|000\rangle\mp|010\rangle)_{23\mathrm{E}}\\
|\Xi^{15,16}\rangle &= \frac{1}{4}(|101\rangle+|111\rangle\mp|001\rangle\mp|011\rangle)_{23\mathrm{E}}.
\end{aligned} \tag{14}
$$

For example, if Charlie gets the $|0\rangle_1$ state, then the Bob-Eve system collapses into the product state

$$|\beth^1\rangle=(|00\rangle+|01\rangle+|10\rangle+|11\rangle)_{\mathrm{BE}}. \tag{15}$$

It is clear that Eve has no chance of obtaining any information about the unknown original state, so this scheme is secure.

4 Conclusions

In this paper, we proposed a QIS scheme for two-qubit Bell-states by using a four-qubit entangled state as a quantum channel. The sender Alice, the receiver Bob and the controller Charlie share the quantum channel. Alice sends unknown information to Bob; it is impossible for him to reconstruct the original state with local operations, so he must cooperate with Charlie. It is impossible for any single party to get total initial information, as Alice owns one qubit, Bob owns two qubits, and Charlie owns one qubit. First, Alice performs a GHZ state measurement on the three qubits in her hand, then tells Charlie and Bob her measurement results. Charlie performs a single-qubit measurement based on $\{|0\rangle,|1\rangle\}$, and tells his measurement results to Bob. Using the measurement results from Alice and Charlie, Bob performs the appropriate unitary transformation operation on his qubits, and thus can reconstruct the two-qubit Bell state sent from Alice.

We have also shown that this scheme is secure under certain eavesdropper attacks. By collaboration, it is possible that the original state can be restricted in a probabilistic way by combined accessorial qubits. So the scheme we propose in the paper has sufficiently high security.

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