

# Constraining parameter space of the little Higgs model using data from tera-Z factory and ILC\*

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**Abstract:** The Standard Model (SM) prediction on the forward-backward asymmetry for  $b\bar{b}$  production ( $A_{FB}^b$ ) is well consistent with the data of LEP I at the Z-pole, but deviates from the data at  $\sqrt{s}=89.55$  and  $92.95$  GeV which are slightly away from the pole. This deviation implies that there is still room for new physics. We calculate the  $A_{FB}^b$  at the vicinity of the Z-pole in the little Higgs model as well as other measurable parameters such as  $R_b$  and  $R_c$ , by which we may constrain the parameter space of the little Higgs model. This can be further tested in the newly proposed tera-Z factory. With the fitted parameters we further make predictions on  $A_{FB}^b$  and  $A_{FB}^t$  for  $t\bar{t}$  production at the International Linear Collider (ILC).

**Key words:** tera-Z factory, ILC, little Higgs model, forward-backward asymmetry

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## 1 Introduction

As is well recognized, the hadron colliders are machines for discovery. With regards to other aspects, the electron-positron collider, muon-collider and even the proposed photon collider would provide detailed information about the discovered new physics candidates. When some peculiar phenomena are observed at the hadron colliders such as Tevatron or LHC beyond the expectation of the Standard Model (SM), one is tempted to associate them with new physics. Generally, making confirmation is difficult, especially as there are too many new physics models available and most of them can offer a plausible interpretation towards the new observation. One of the reasons is that from the data obtained at hadron colliders, it is difficult to study the details which are crucial for identifying the new interaction and/or new particles observed in the physical process accompanied by an enormous background. That is why people will turn to high-energy lepton colliders after successful operation of hadron colliders.

More precisely speaking, to discover new physics, one is looking for phenomena beyond the SM expecta-

tion through experimental measurements carried out at hadron colliders, whereas confirming the existence of new physics needs the measurement of several characteristic quantities at electron-positron colliders.

The forward-backward asymmetry ( $A_{FB}^t$ ) in top-antitop production at the Tevatron is one such measurement. In  $t\bar{t}$  (here we write as  $Q\bar{Q}$  which can apply to the case of  $b\bar{b}$  production) the rest frame is defined as

$$A_{FB}^Q \equiv \frac{N_Q(y_Q - y_{\bar{Q}} > 0) - N_Q(y_Q - y_{\bar{Q}} < 0)}{N_Q(y_Q - y_{\bar{Q}} > 0) + N_Q(y_Q - y_{\bar{Q}} < 0)}, \quad (1)$$

where  $N_Q$  is the number of heavy quarks (t or b) and  $y_Q - y_{\bar{Q}}$  is the difference of the rapidities of the Q and  $\bar{Q}$  which is the Lorentz invariant and defined as

$$y_Q - y_{\bar{Q}} = 2 \operatorname{arctanh} \left( \sqrt{1 - \frac{4m_Q^2}{s}} \cos\theta \right), \quad (2)$$

with  $s = (p_1 + p_2)^2$  and  $p_1, p_2$  being the momenta of Q and  $\bar{Q}$ .  $A_{FB}$  can be further rewritten as

$$A_{FB} = \frac{N_Q(\cos\theta > 0) - N_Q(\cos\theta < 0)}{N_Q(\cos\theta > 0) + N_Q(\cos\theta < 0)}, \quad (3)$$

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where  $\theta$  is the angle between the outgoing top quark and the injecting proton beam. Obviously, the sign of  $y_Q - y_{\bar{Q}}$  is the same as  $\cos\theta$ .

The data of the Tevatron at the Fermilab of  $A_{\text{FB}}^t$  [1] are as follows: the measurements of the CDF and D0 collaborations yield  $A_{\text{FB}}^t = 0.158 \pm 0.075$  [2],  $A_{\text{FB}}^t = 0.162 \pm 0.047$  [3] and  $A_{\text{FB}}^t = 0.196 \pm 0.065$  [4], which are significantly larger than the SM prediction  $A_{\text{FB}}^{\text{SM}} = 0.089$  [5] for top pair production. This discrepancy would suggest a hint of existence of new physics beyond SM. Numerous models beyond SM have been proposed to explain the deviation from the SM prediction, and we list a few of them in our reference list as examples [6–18], but definitely still many important works should also be included.

We showed in our previous work [19] that the deviation of the theoretical prediction from the data can be mended in the little Higgs model (LHM). It is known that the LHM is one of the promising models which is an extension of the SM. Definitely, a natural tendency is to check the validity of this model at the lepton collider and furthermore to constrain the model parameters. An ideal place for this job was the LEP experiments, especially the forward-backward asymmetry of  $b\bar{b}$  pair production at the Z-pole which is a more sensitive quantity for checking the model than the cross section. One notices that at the Z-pole the SM prediction on the forward-backward asymmetry  $A_{\text{FB}}^b$  for  $b\bar{b}$  production which is similar to the definition for the  $t\bar{t}$  pair production at the Tevatron, is well consistent with the LEP data [20], but deviates from the data at the Z-pole vicinity energies at 89.55 GeV and 92.95 GeV. Even though the absolute deviations are not extremely large, they are indeed beyond a few  $\sigma$ 's.  $A_{\text{FB}}^b$  was systematically calculated with the SM in [20], and the results show that the gap between the theoretical value and the experimental data is about  $1 \sim 2\sigma$  at 89.55 and 92.95 GeV. It is also noted that the errors at 89.55 and 92.95 GeV are larger than that at the Z-pole, so there  $2\sigma$  implies larger deviations. Of course the distinction might be due to the measurement errors, but one cannot exclude the possibility that it comes from the contributions of new physics. Taking the difference as a signal of new physics beyond SM (BSM) we hope that the  $1 \sim 2\sigma$  deviations can be explained. Indeed, in this work we introduce the LHM and see if we can reach the goal by adjusting the model parameters which do not conflict with other experimental results. Indeed, we need more accurate measurements at the vicinity of the Z-pole; fortunately, the recently proposed tera-Z factory may play an important role in providing us with more information.

By a direct observation, the  $A_{\text{FB}}$  is induced by the odd power of  $\cos\theta$  in the amplitude square. Obviously, such terms imply that the parity in the process is violated. In the SM, the parity violation in the process

$e^+e^- \rightarrow b\bar{b}$  is due to the Z boson exchange, whose interaction with fermions has both vector and axial vector components. For next-to-leading order (NLO), the box diagrams also generate an asymmetry, because it results in odd powers of  $\cos\theta$ ; meanwhile their interference with the photon can also enlarge the asymmetry. In the framework of the LHM, we notice that there are two extra bosons, the heavy  $Z_H$  and heavy photon  $A_H$  whose interactions with fermions possess both vector and axial vector components. Therefore, they contribute to the asymmetry directly via the axial part of their interaction with fermions and interference with the SM Z-boson and photon. In fact,  $Z_H$  may be too heavy to make a substantial contribution to the asymmetry  $A_{\text{FB}}^b$  at the tera-Z factory energy, so that the new physics contribution to the asymmetry is almost totally caused by the heavy photon.

The strategy of this work is to investigate the contributions of both SM and BSM to the asymmetries in  $e^+e^- \rightarrow b\bar{b}$  and  $e^+e^- \rightarrow t\bar{t}$  with a special BSM, i.e. the LHM which we used to explain  $A_{\text{FB}}^t$  observed at the Tevatron [19]. The energies we set are that of the tera-Z factory and International Linear Collider (ILC) or Compact Linear Collider (CLIC) respectively. A comparison of the asymmetries obtained for  $t\bar{t}$  at ILC and  $b\bar{b}$  at the tera-Z factory may help gain better understanding of the model. Even though we employ a special model BSM, the obtained results can make sense of the role of BSM for the asymmetries, and moreover we can use the data to constrain the model parameters which might be applied to other physical processes and be further tested.

This paper is organized as follows. After this introduction, in Section 2, we formulate the total scattering cross section,  $A_{\text{FB}}^Q$  to NLO within the frameworks of SM+LHM and as well as the measurable  $R_b$  and  $R_c$ . The numerical results along with all the input parameters are presented in Section 3. The obtained results are shown explicitly in several figures and tables. The last section is devoted to a simple discussion and conclusion.

## 2 The contributions of SM and LHM to the asymmetry up to NLO

In this section we formulate the contributions to the  $A_{\text{FB}}^Q$  and the total cross sections for the processes of  $e^+e^- \rightarrow Q\bar{Q}$  in the framework of SM+LHM up to NLO. The derivation in the SM at one-loop level was done a long time ago [20, 21]. Here we just repeat the derivation and confirm their numerical results for  $\sqrt{s}$  near the Z-pole. Then we focus on the contribution of new physics, namely the LHM [22].

### 2.1 SM contribution

For completeness, we first briefly review the calcula-

tion in SM. Since we first discuss the processes at energies near the Z-pole, Q refers only to the b quark.

The amplitude of the process  $e^+e^- \rightarrow b\bar{b}$  at the leading order of SM is formulated as

$$\begin{aligned} \mathcal{M}_1 = & \bar{u}(p_4)\gamma^\mu \frac{-ie}{4\sin\theta_W \cos\theta_W} \left( -\left(1 - \frac{4}{3}\sin^2\theta_W\right) + \gamma^5 \right) \\ & \times v(p_3) \frac{-i}{s-m_Z^2} \bar{v}(p_2)\gamma_\mu \frac{-ie}{4\sin\theta_W \cos\theta_W} \\ & \times (-(1-4\sin^2\theta_W) + \gamma^5) u(p_1) + \bar{u}(p_4) \\ & \times \left( -ie\frac{1}{3}\gamma^\mu \right) v(p_3) \frac{-i}{s} \bar{v}(p_2) (-ie\gamma_\mu) u(p_1), \end{aligned} \quad (4)$$

where  $\theta_W$  is the Weinberg angle,  $p_1$  and  $p_2$  respectively stand for the four-momenta of the initial electron and positron, and  $p_3, p_4$  denote the four-momenta of the final  $\bar{b}$  and b. The NLO in SM contribution comes from the renormalized propagator, vertex correction, box diagrams and QED corrections. The first two corrections are included in the effective vector and axial vector coupling constants of the Z-fermions ( $I_3^f$  is the weak isospin of the fermion f) as follows:

$$\begin{aligned} v_f \rightarrow & \left( \frac{e^2}{4\sin^2\theta_W \cos^2\theta_W} \rho_f \right)^{\frac{1}{2}} (I_3^f - 2Q_f \kappa_f \sin^2\theta_W) \\ a_f \rightarrow & \left( \frac{e^2}{4\sin^2\theta_W \sin^2\theta_W} \rho_f \right)^{\frac{1}{2}} I_3^f, \end{aligned} \quad (5)$$

where  $\rho_f$  and  $\kappa_f$  are

$$\begin{aligned} \rho_f = & 1 + \frac{3e^2}{64\pi^2 \sin^2\theta_W} \left( \frac{m_t^2}{m_W^2} - \frac{\sin^2\theta_W}{\cos^2\theta_W} \left( \ln \frac{m_H^2}{m_W^2} \right) - \frac{5}{6} \right) \\ & + \Delta\rho_f \\ \kappa_f = & 1 + \frac{3e^2}{64\pi^2 \sin^2\theta_W} \left( \frac{m_t^2 \cos^2\theta_W}{m_W^2 \sin^2\theta_W} - \frac{10}{9} \left( \ln \frac{m_H^2}{m_W^2} \right) - \frac{5}{6} \right) \\ & + \Delta\kappa_f. \end{aligned} \quad (6)$$

For the b quark,  $\Delta\rho_f$  and  $\Delta\kappa_f$  are not negligible and can be written as

$$\begin{aligned} \Delta\rho_b = & -2\Delta\kappa_b \\ \Delta\kappa_b = & \frac{e^2}{64\pi^2 \sin^2\theta_W} \left( 2\frac{m_t^2}{m_W^2} \right). \end{aligned} \quad (7)$$

The box diagram contribution was estimated [21] to be very small, so that can be safely neglected. As for the QED corrections, only the initial state radiation (ISR) is substantial [21] which is expressed in terms of a convolution integral for the integrated cross section.

$$\sigma(s) = \int_{z_0}^1 dz H_{\text{QED}}(z, s) \sigma_{\text{ew}}(z, s), z_0 \geq \frac{4m_f^2}{s}. \quad (8)$$

where

$$\begin{aligned} H_{\text{QED}}(s) = & \frac{2\alpha}{\pi} (L_e - 1) (1-z)^{\frac{2\alpha}{\pi}(L_e - 1) - 1} \\ & \times \left( 1 + \frac{\alpha}{\pi} \left( \frac{3}{2}(L_e - 1) + \frac{\pi^2}{3} - \frac{1}{2} \right) \right) \\ & + \frac{\alpha}{\pi} \left( \left( \frac{4z}{(1+z)^2} \frac{1+z^2}{1-z} - \frac{2}{1-z} \right) (L_e - 1) \right. \\ & \left. - \frac{4z}{(1+z)^2} \ln \frac{4z}{(1+z)^2} \right), \end{aligned} \quad (9)$$

$$\alpha = \frac{e^2}{4\pi}, \quad L_e = \ln \frac{s}{m_e^2}.$$

With these corrections, we can obtain the complete SM amplitude for the process  $e^+e^- \rightarrow b\bar{b}$  at NLO.

## 2.2 LHM contribution

In the LHM [22], there are four neutral bosons, two are an SM photon and a Z-boson and the two extra bosons are a heavy Z-boson and a heavy photon pertaining to LHM. In our previous study [19], by fitting the data of the Tevatron, we determined that the mass of the heavy Z-boson is much heavier than that of the SM Z-boson, thus at the LEP energy scale its contribution can be neglected. Meanwhile the mass of the heavy photon is around the LEP energy scale, so would modify the values of  $R_b$ ,  $R_c$  and  $A_{\text{FB}}^b$  near the Z-pole predicted by the SM. The coupling of  $A_H$  to fermion is written as

$$\mathcal{L}_{A_H} = A_H \bar{q} \gamma^\mu (g_v^q + g_a^q \gamma^5) q + A_H \bar{e} \gamma^\mu (g_v^e + g_a^e \gamma^5) e, \quad (10)$$

and the relevant parameters are listed in Table 1 in the next section.

Similar to the SM correction, the LHM vertex corrections are depicted in the Feynman diagrams of Fig. 1. To explicitly demonstrate the procedure used for deriving the contribution, let us present the amplitude determined by the first diagram of Fig. 1 and only the heavy photon exchange is provided as an example, that is:

$$\begin{aligned} \mathcal{M}_2 = & \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2 - m_W^2} \bar{u}(p_4) \frac{-ie}{\sin\theta_W 2\sqrt{2}} V_{bt} \gamma^\mu (1 - \gamma^5) \\ & \times \frac{i(\not{p}_4 - \not{k} + m_4)}{(p_4 - k)^2 - m_4^2} (i(g_v^q + g_a^q \gamma^5) \gamma_\nu) \frac{i(-\not{p}_3 - \not{k} + m_4)}{(p_3 + k)^2 - m_4^2} \\ & \times \frac{-ie}{\sin\theta_W 2\sqrt{2}} V_{bt} \gamma^\mu (1 - \gamma^5) v(p_3) \\ & \times \bar{v}(p_2) (i(g_v^e + g_a^e \gamma^5) \gamma_\nu) \frac{-i}{s} u(p_1), \end{aligned} \quad (11)$$

where  $V_{bt}$  is the CKM matrix element. For the rest of the diagrams, the corresponding amplitudes can be obtained in a similar way with different coupling constants and masses of the intermediate fermions and bosons which are exchanged at s or t-channels.

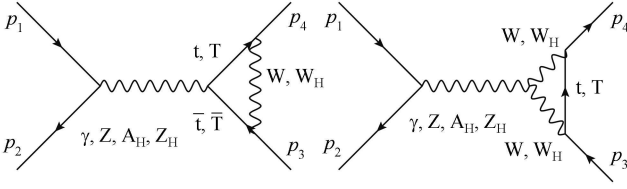


Fig. 1. The Feynman diagrams of vertex corrections for  $e^+e^- \rightarrow b\bar{b}$  where  $A_H$  and  $Z_H$  are the heavy photon and heavy Z-boson in LHM

Averaging spin projections of initial electron-positron and summing over the spins and colors of the produced quarks, the differential cross section with respect to the production angle  $\theta$  is:

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} &= 3 \times \frac{2\pi\sqrt{1-\frac{4m_Q^2}{s}}}{64\pi^2s} \frac{1}{4} \sum |\mathcal{M}_1 + \mathcal{M}_2|^2 \\ &\approx 3 \times \frac{2\pi\sqrt{1-\frac{4m_Q^2}{s}}}{64\pi^2s} \frac{1}{4} (|\mathcal{M}_1|^2 + 2\text{Re}(\mathcal{M}_1^* \mathcal{M}_2)), \end{aligned} \quad (12)$$

and then we integrate over the positive and negative ranges of  $\cos\theta$  respectively. The asymmetry which is expressed in terms of the Lorentz invariant rapidity difference  $y_Q - y_{\bar{Q}}$  defined in Eq. (2) and (1) is eventually derived. Moreover, we have also derived the relevant  $R_b$  and  $R_c$  [23] which are commonly defined in the literature, in the SM+LHM framework. The numerical results will be presented in the next section.

### 3 Numerical results

Here we list all the inputs which are needed in our numerical computation. The masses of charm, bottom and top quarks are taken as 1.27, 4.18 and 173.5 GeV and the masses of light quarks (u, d, s) are neglected. In the center of the mass frame, the kinematics are determined

as

$$\begin{aligned} p_1 \cdot p_2 &= \frac{s}{2}, \quad p_3 \cdot p_4 = \frac{s}{2} - m_Q^2, \\ p_1 \cdot p_3 &= p_2 \cdot p_4 = \frac{s}{4} \left( 1 + \sqrt{1 - \frac{4m_Q^2}{s}} \cos\theta \right), \\ p_1 \cdot p_4 &= p_2 \cdot p_3 = \frac{s}{4} \left( 1 - \sqrt{1 - \frac{4m_Q^2}{s}} \cos\theta \right). \end{aligned} \quad (13)$$

For the energy of the LEP I experiment, we set  $\sqrt{s} = 92.95$  GeV and  $m_Z = 91.2$  GeV,  $m_W = 80.4$  GeV,  $m_H = 125$  GeV [24–27]. The electromagnetic coupling constant and weak mixing angle are running with energy, and at different energy scales we take  $\alpha_e = 1/128.878$ ,  $\sin^2\theta_W = 0.2316$  for  $\sqrt{s} = 91.2$  GeV;  $\alpha_e = 1/128.516$ ,  $\sin^2\theta_W = 0.2398$  for  $\sqrt{s} = 500$  GeV;  $\alpha_e = 1/128.369$ ,  $\sin^2\theta_W = 0.2444$  for  $\sqrt{s} = 1$  TeV [28–30]. At the proposed tera-Z factory the center-of-mass (CM) energy will be around the vicinity of the Z mass, so the on-mass-shell resonance effect would be dominant and the Breit-Winger formulation is an appropriate approach.

The coupling constants between the heavy photon and various fermions are listed in Table 1. The mass of the heavy photon is  $m_{A_H} = 0.08138 \left( \frac{1}{a} + a \right) f$  GeV, and  $f$  is a vacuum expectation value of LHM [22].

It is noted that for the heavy photon, all its couplings to fermions uniquely depend on parameters  $a$  and  $b$ , which are not determined in the model, so that here we treat them as free parameters. The only way to determine them so far, before a more fundamental principle appears, is by fitting available experimental data.

Even though the SM prediction on the asymmetry  $A_{FB}^b$  is generally consistent with the LEP data, as indicated in the introduction, there are still deviations between data and theoretical prediction as  $\sqrt{s}$  being away from the pole mass of the Z boson. Thus we may expect that when the contribution of LHM is included, the

Table 1. The coupling constants between the heavy photon and fermion. In the table  $a = \tan\theta'$  [22] and  $b = \frac{\lambda_1^2}{\lambda_1^2 + \lambda_2^2}$

with  $\lambda_1$  and  $\lambda_2$  satisfy  $\frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2} \approx \left( \frac{v}{m_t} \right)^2 \approx 2$  [22],  $v$  is the VEV of SM.

	$g_v^q$	$g_a^q$
$A_H \bar{u}u$	$-0.0292 \left( \frac{3}{a} - 2a \right)$	$-0.0175 \left( \frac{3}{a} - 2a \right)$
$A_H \bar{d}d$	$0.2742 \frac{1}{a} + 0.245a$	$-0.0175 \left( \frac{3}{a} - 2a \right)$
$A_H \bar{t}t$	$-0.0292 \left( \frac{3}{a} - 2a \right) - 0.035 \left( \frac{1}{a} + a \right) b$	$-0.0175 \left( \frac{3}{a} - 2a \right) - 0.035 \left( \frac{1}{a} + a \right) b$
	$g_v^l$	$g_a^l$
$A_H \bar{e}e$	$0.0525 \left( \frac{3}{a} - 2a \right)$	$0.0175 \left( \frac{3}{a} - 2a \right)$

theoretical prediction can be in better agreement with experimental data. Our numerical results are shown in Fig. 2 and Fig. 3 and an improvement is noted. It is worth pointing out, for the  $b\bar{b}$  production, the contribution of the SM box diagrams is small, but not negligible; in comparison, the box contributions induced by the LHM are too small to be involved. Thus, we have the contributions from five sources: the heavy photon of LHM, the SM box diagrams, the  $\gamma$ , Z boson of SM and the interferences among them.

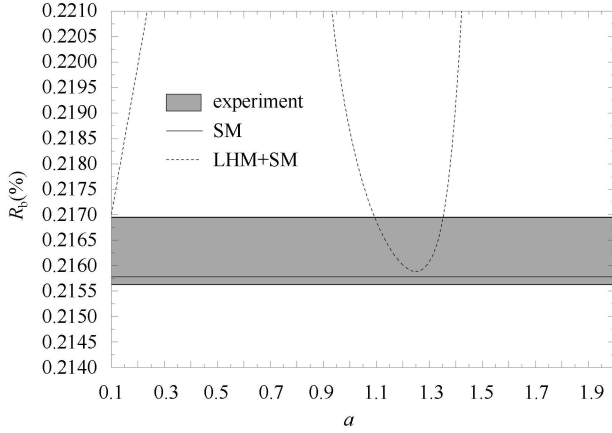


Fig. 2. Dependence of  $R_b$ , which is theoretically evaluated by SM-only and SM+LHM on parameter  $a$  at  $\sqrt{s}=91.2$  GeV and the experimental data [30].

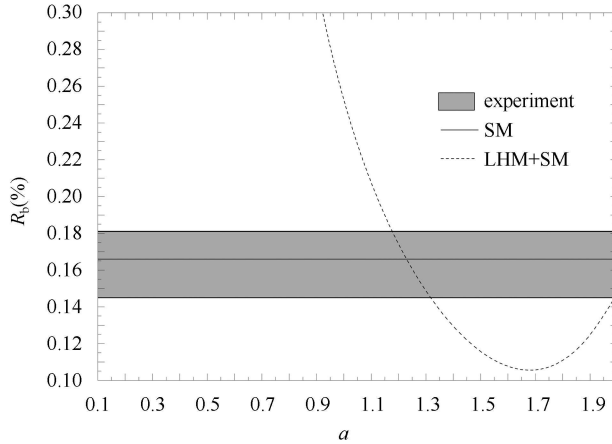


Fig. 3. Dependence of  $R_b$ , which is theoretically evaluated by SM-only and SM+LHM on parameter  $a$  at  $\sqrt{s}=189$  GeV and the experimental data [23].

Figure 2 and Fig. 3 show the dependence of  $R_b$ , which was measured at LEP I and II on the parameter  $a$  in the scenario of LHM+SM where the ratio  $R_b$  is defined as [23]:

$$R_b = \frac{\sigma(e^+e^- \rightarrow b\bar{b})}{\sigma(e^+e^- \rightarrow q\bar{q})}. \quad (14)$$

The results indicate that parameter  $a$  must fall into a narrow range from 1.1 to 1.3 to fit the LEP I and II data.

We also show  $R_c$  at the Z-pole versus parameter  $a$  predicted by LHM+SM in Fig. 4. From the results one can see that by fitting the LEP I data, two windows exist for parameter  $a$ : 0.1–0.46 and 0.77–2.

Combining the constraints from the measured values of  $R_b$  and  $R_c$ , parameter  $a$  should be in a range of 1.1–1.3.

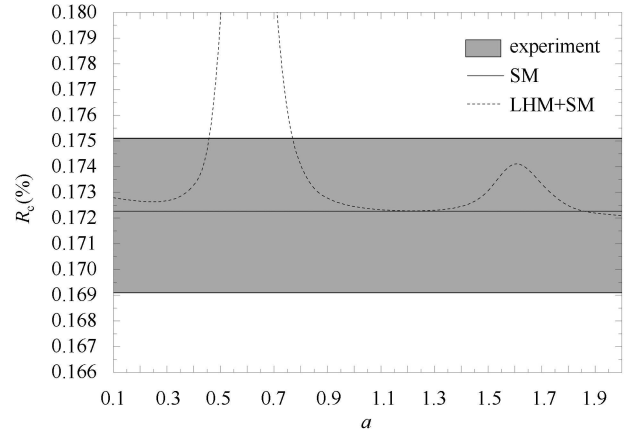


Fig. 4. The dependence of the ratio  $R_c$  theoretically evaluated by SM-only and SM+LHM on parameter  $a$  at  $\sqrt{s}=91.2$  GeV and the experimental data [30].

In Fig. 5, we present the dependence of  $A_{FB}^b$  on  $\sqrt{s}$  with  $a$  being 1.22 and 1.23 respectively, where we choose the CM energy  $\sqrt{s}$  close to the Z boson mass which is the energy range of the proposed tera-Z factory. To be more explicit, let us show the theoretical prediction on the asymmetry  $A_{FB}^b$  at different CM energies  $\sqrt{s}=89.55$  GeV and  $\sqrt{s}=92.95$  GeV in Fig. 6. It is shown that with the LHM, agreement between the theoretical prediction in the scenario of the SM+LHM on the asymmetry  $A_{FB}^b$  and the experimental data is improved compared with that in SM only as long as the model parameter  $a$  exists in a narrow window. However, we observe that at  $\sqrt{s}=M_Z$  the predicted  $A_{FB}^b$  coincides well with the data, but for the CM energy at  $\sqrt{s}=89.55$  GeV and  $\sqrt{s}=92.95$  GeV, neither SM nor SM+LHM predictions can be perfectly consistent with the data. Moreover, the theoretical estimate sensitively depends on the value of  $a$ . In other words, a common value does not exist for  $a$  which can simultaneously satisfy the measured data at  $\sqrt{s}=89.55$  GeV and  $\sqrt{s}=92.95$  GeV. We will discuss this point in the last section.

To confirm the validity of our results, let us compute the total cross sections of  $e^+e^- \rightarrow Q\bar{Q}$  at the Z-pole. The computation of the total cross sections of  $e^+e^- \rightarrow Q\bar{Q}$  is

a bit tricky. The total energy  $\sqrt{s}$  for the production is chosen to be close to the Z-pole, but due to the effect of ISR where an unobservable photon is radiated from either an electron or positron, the real available collision energy  $\sqrt{s'}$  would be slightly lower than  $\sqrt{s}$ . Due to the Breit-Wigner structure of the Z-propagator, the small deviation of colliding energy from the Z-pole would obviously decrease the total cross section.

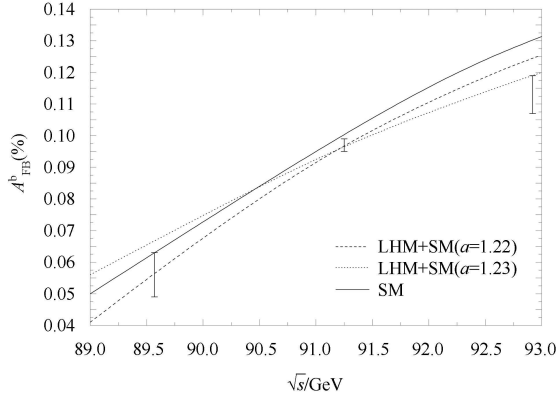


Fig. 5. Dependence of  $A_{\text{FB}}^b$  evaluated with SM+LHM on the CM energy  $\sqrt{s}$  of the proposed tera-Z factory and the experimental data of LEP I [20]. The solid line is the SM result. On the graph three experimental error bars at 89.55 GeV, 91.26 GeV and 92.95 GeV respectively are explicitly shown.

Thus we give two sets of the cross sections in the new version, one (set 1) is that directly calculated provided the collision energy is exactly  $\sqrt{s} = \sqrt{s'} = Z$  mass, whereas for the other set, we adopt the scheme provided in [21] (Eqs. (8) and (10)), i.e. properly take into account the effects of ISR (set 2). Surely, the values in the two sets are quite different; the second set corresponds to the value which is suppressed by the deviation of  $\sqrt{s'}$  from the Z-pole. Obviously, when  $\sqrt{s}$  deviates from the Z-mass the ISR effect would be less important, so that the two sets of the computed cross sections are closer to each other at  $\sqrt{s} \neq M_Z$ .

Namely, we have:

For set 1:

Without taking into account the ISR effects, we have the cross section of  $e^+e^- \rightarrow b\bar{b}$  at the Z-pole in SM it is 8623.156 pb, and in LHM is 8623.136–8623.288 pb with parameter  $a$  varying from 1.22 to 1.23. Those results are consistent with that given in [21]. The corresponding values for process  $e^+e^- \rightarrow c\bar{c}$  are 6835.286 pb in SM and 6835.287–6835.288 pb in LMH.

For set 2:

Here we consider the ISR effects, and have the cross section of  $e^+e^- \rightarrow b\bar{b}$  at the Z-pole which in SM is 6232.146 pb, and in LHM is 6232.126~6232.278 pb with

parameter  $a$  varying from 1.22 to 1.23. Those results also coincide with the values given in [21]. The corresponding values for process  $e^+e^- \rightarrow c\bar{c}$  are 4940.436 pb in SM and 4940.438~4940.438 pb in LMH.

In fact, the ISR is absolutely present in our measurements, therefore only the values given in set 2 correspond to the measured data.

Now let us turn to the ILC case. For that energy range, not only the heavy photon, but also the new heavy vector boson  $Z_H$  all contribute to the asymmetry  $A_{\text{FB}}^b$ ; moreover, since  $t\bar{t}$  pairs are produced, the asymmetry  $A_{\text{FB}}^t$  can also be measured.

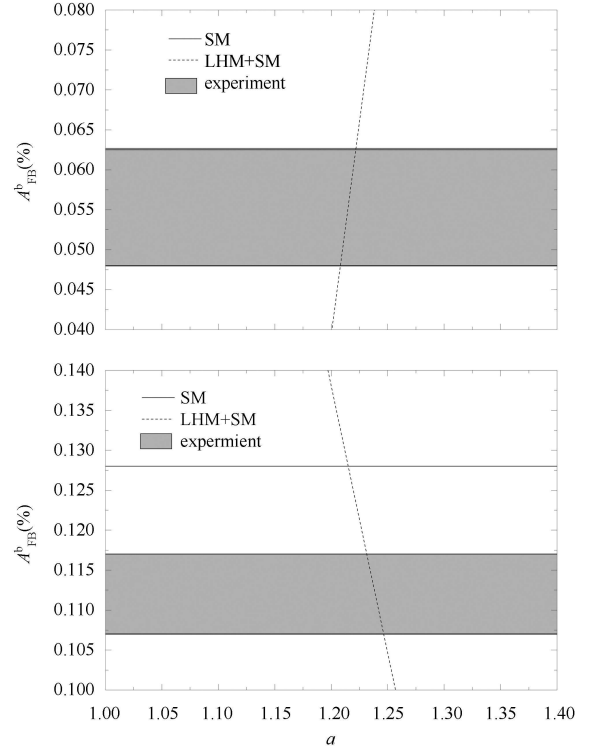


Fig. 6. The dependence of the asymmetry  $A_{\text{FB}}^b$  on the parameter  $a$  at  $\sqrt{s} = 89.55$  GeV (up) and  $\sqrt{s} = 92.95$  GeV (down).

Figure 7 and Fig. 8 respectively demonstrate the dependence of  $A_{\text{FB}}^b$  and  $A_{\text{FB}}^t$  on the CM energies at the proposed ILC.

Figure 7 shows that as the mass of heavy  $Z_H$  being set at 450 GeV, the  $A_{\text{FB}}^b$  evaluated with LHM+SM has a minimum near  $\sqrt{s} = 410$  GeV, and a maximum at  $\sqrt{s} = 450$  GeV, this is understood as the effects of interference between the heavy  $Z_H$ , the heavy photon of LHM and the SM Z boson.

We depict the dependence of the evaluated asymmetry for top pair production in Fig. 8 which shows that  $A_{\text{FB}}^t$  behaves quite differently for the SM-only and LHM+SM predictions. The behavior of  $A_{\text{FB}}^t$  evaluated with LHM+SM has a bump peaked at  $\sqrt{s} = 430$  GeV.

This is also caused by an interference between  $Z_H$  and SM particles while the contribution from heavy photons can be safely ignored. As  $\sqrt{s}$  is above 500 GeV and below 400 GeV, the theoretically predicted value of  $A_{FB}^t$  tends gradually to be dominated by SM.

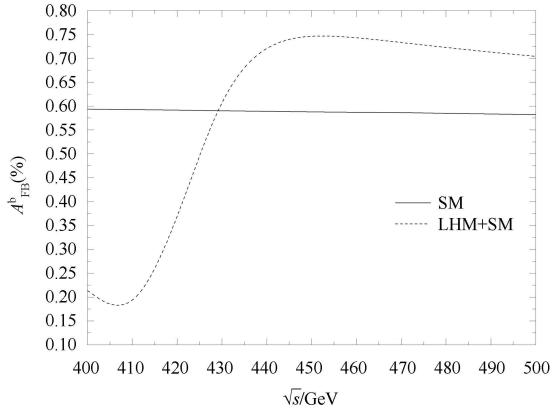


Fig. 7.  $A_{FB}^b$  evaluated with SM-only and LHM+SM versus the CM energy  $\sqrt{s}$  of the proposed ILC

Table 2 presents the ratios for the production rates of top quark  $R_t$  and bottom quark  $R_b$  which are defined

Table 2. The evaluated  $R_t$  and  $R_b$ . (The values predicted by LHM correspond to  $a$  varying from 1.22 to 1.23).

	$R_t^{\text{theor}}$		$R_t^{\text{exp}}$
	SM	LHM+SM	
ILC(500 GeV)	0.165956	0.142362	
ILC(1 TeV)	0.217092	0.205868	
	$R_b^{\text{theor}}$		$R_b^{\text{exp}}$
	SM	LHM+SM	
ILC(500 GeV)	0.118978	0.149088	
ILC(1 TeV)	0.115201	0.124723	
tera-Z(92.95 GeV)	0.21406	0.21495–0.21493	
LEPI(91.2 GeV)	0.21576	0.21580–0.21579	0.21629±0.00066 [30]
LEPII(189 GeV)	0.16035	0.15758–0.15757	0.163±0.013(stat)±0.005(syst) [23]

Table 3. Theoretical predicted  $A_{FB}^Q$  ( $Q=t, b$ ), in LHM+SM  $a$  is in the narrow range of 1.22–1.23.

	$A_{FB}^{t(\text{theor})}(\%)$		$A_{FB}^{t(\text{exp})}(\%)$
	SM	LHM+SM	
ILC(500 GeV)	46.68	51.86	
ILC(1 TeV)	56.63	54.81	
	$A_{FB}^{b(\text{theor})}(\%)$		$A_{FB}^{b(\text{exp})}(\%)$
	SM	LHM+SM	
ILC(500 GeV)	58.83	70.64	
ILC(1 TeV)	55.06	61.27	
tera-Z(92.95 GeV)	12.84	12.41–11.71	
LEPI (91.2 GeV)	10.07	9.86–9.71	9.89±0.27±0.13 [31]
LEPII (189 GeV)	66.57	55.29–54.21	61±18(stat)±9(syst) [23]

## 4 Discussion and conclusion

The observation of the asymmetry of top pair production  $A_{FB}^t$  at the Tevatron [2–4], which is obviously larger

in Eq. (14) at various energies, and experimental data if they are available.

In Table 3, we list the  $A_{FB}^t$  and  $A_{FB}^b$  evaluated with SM and LHM+SM, and available experimental data.

One other point is noted: that to fit the data of the asymmetry, the parameter  $a$  is required to fall into a rather narrow window—an aspect which needs to be fine-tuned. So far, we are unable to avoid it.

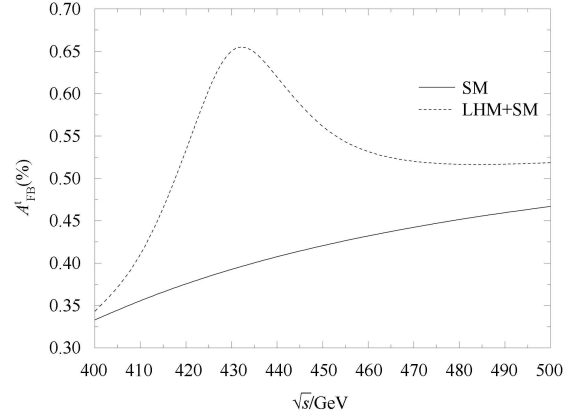


Fig. 8.  $A_{FB}^t$  evaluated with SM-only and LHM+SM vs the CM energy  $\sqrt{s}$  of ILC.

than the SM prediction, implies the possible existence of new physics BSM. Many authors [6–18] have tried to explain the discrepancy between theoretical predictions and data in terms of various models BSM, and LHM is one of them. The LHM was first proposed to cancel the quadratic divergence induced by the SM top quark at the self-energy loop of Higgs to solve the hierarchy problem for the Higgs boson. This model, besides the cancellation, has more phenomenological applications to various processes. For example, the authors of [32] studied its effects on  $\rho$  which is defined as  $\rho = \frac{1}{\cos^2 \theta_W} \frac{M_W^2}{M_Z^2}$  and is 1 in SM at tree level [33]. [33] also presents the high order corrections to the  $\rho$  parameter.

Introducing the effect of LHM, the authors of [32] set a constraint on the parameter space of the LHM by

fitting the measured  $\rho$  and further  $\Delta\rho$  whose definition is given in [33]. Moreover, the authors of [34–38] have studied the processes at high energy leptonic colliders, whereas some processes at LHC are discussed in [39–42]. Their conclusions show that the predictions on the relevant quantities made in the framework of SM+LHM can coincide with the experimental data as long as the VEV varies from hundreds of GeV to several TeV, and the parameter space of LHM gained in their works does not contradict the data we obtained in this work.

The D0 collaboration recently announced their new data as  $A_{\text{FB}}^t = 0.106 \pm 0.030$  [43, 44]; that is lower than their earlier result and closer to SM prediction than before. Even though the data are closer to the SM prediction, there is still room for new physics.

At the energy scale of Z mass, it is noted that the SM prediction of the forward-backward asymmetry of bottom quark pair production  $A_{\text{FB}}^b$  coincides with the LEP I data well at Z-pole[20], but deviates from the data at the Z-pole vicinity energy 89.55 GeV and 92.95 GeV about  $1 \sim 2\sigma$ .

The reason is that at the Z-pole the contribution of the Z boson resonance is overwhelmingly dominant, and

the other contributions from interference among SM particles and new physics BSM are relatively small and almost do not manifest themselves. However, when the CM energies deviate from the Z-pole, the effect of those interactions becomes more significant. Then the effects of new physics would show up at the vicinity of the Z-pole mass. Incorporating the LHM, we find that the consistency between theoretical predictions of  $A_{\text{FB}}^b$  at 89.55 GeV and 92.95 GeV can be improved as the model parameter  $a$  takes a value of 1.22–1.23. By contrast, by fitting the data of  $R_b$  and  $R_c$ , the value of  $a$  can take a wider range of 1.1–1.3. This fine-tuning of 1.22–1.23 makes us slightly uncomfortable, even though, by incorporating LHM, the theoretical prediction is closer to the LEP I data.

Therefore, our conclusion is that more precise measurements are badly needed. Fortunately, the recently proposed tera-Z factory may do the job and provide valuable information. Even the SM+LHM scenario cannot provide a satisfactory solution; one can conjecture other possible models. To confirm or negate the LHM by comparing its prediction with the available data would definitely be interesting and important.

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