A unified model with a generalized gauge symmetry and its cosmological implications

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Abstract: A unified model is based on a generalized gauge symmetry with groups $[SU_{3c}]_{\rm color}\times(SU_2\times U_1)\times[U_{1b}\times U_{1l}]$. It implies that all interactions should preserve conservation laws of baryon number, lepton number, and electric charge, etc. The baryonic U_{1b}, leptonic U_{1l} and color SU_{3c} gauge transformations are generalized to involve non-integrable phase factors. One has gauge invariant fourth-order equations for massless gauge fields, which leads to linear potentials in the $[U_{1b}\times U_{1l}]$ and color $[SU_{3c}]$ sectors. We discuss possible cosmological implications of the new baryonic gauge field. It can produce a very small constant repulsive force between two baryon galaxies (or between two anti-baryon galaxies), where the baryon force can overcome the gravitational force at very large distances and leads to an accelerated cosmic expansion. Based on conservation laws in the unified model, we discuss a simple rotating dumbbell universe with equal amounts of matter and anti-matter, which may be pictured as two gigantic rotating clusters of galaxies. Within the gigantic baryonic cluster, a galaxy will have an approximately linearly accelerated expansion due to the effective force of constant density of all baryonic matter. The same expansion happens in the gigantic anti-baryonic cluster. Physical implications of the generalized gauge symmetry on charmonium confining potentials due to new SU_{3c} field equations, frequency shift of distant supernovae Ia and their experimental tests are discussed.

Key words: unified model, nonintegrable phase factors, conservation laws, cosmology, accelerated expansion

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1 Introduction

It appears that particle physics has developed to a stage that all established basic forces, such as the strong and the electroweak, can be understood on the basis of gauge symmetry within the framework of local fields [1, 2]. Each symmetry in flat space-time is associated with a conservation law, as implied by Noether's theorem [3]. All these fundamental laws of conservation appear to imply equal amounts of matter and anti-matter in the physical universe. In view of our limited knowledge of the physical universe, it is premature to conclude that all galaxies in the universe are made of baryons (and electrons).

Previous discussions of unified models are based on the conventional gauge symmetry, which involves the usual phase factors and has some unsatisfactory properties related to the confinement mechanism of quarks. Moreover, since there are no observable gauge fields associated with baryon and lepton numbers, people speculate that baryon and lepton numbers are only approximately conserved [4].

This paper is based on a unified model with the postulates:

(i) a generalized gauge symmetry, which is associated with the established conservation laws such as the baryon and lepton numbers and leads to new massless gauge fields satisfying fourth-order equations, and

(ii) the baryon-lepton (or quark-lepton) symmetry in the Lagrangian of the cosmic baryon–lepton dynamics.

We demonstrate that the coupling constants of these baryonic and leptonic gauge fields may be too small to be detected in the laboratory, but they can have interesting observable cosmological consequence such as the accelerated cosmic expansion. To bring out the observable implications of the universal principle of gauge symmetry, we discuss a unified model from the new viewpoint of generalized gauge symmetry with non-integrable phase factors. The unified model follows the idea of unification discussed by Glashow, Salam, Ward and Weinberg [4–7]. The model is based on gauge groups, $[SU_3]_{\rm color}\times(SU_2\times U_1)\times[U_{1b}\times U_{1l}]$. All internal gauge groups have the usual covariant derivatives of the form $(\partial_\mu+{\rm i}gH^a_\mu L^a)$, where $L^a{'}s$ are group generators. The

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generalized gauge symmetry is based on new gauge transformations, which involve (Lorentz) vector gauge functions $\omega_\mu(x)$ and non-integrable phase factor $\mathrm{i}g\int^x \omega_\mu(x')\mathrm{d}x'^\mu$ [8, 9]. For example, we have new baryonic $U_{1\mathrm{b}}$ gauge transformations, $B_\mu(x)\to B'_\mu(x)=B_\mu(x)+\omega_\mu(x)$, and $\psi(x)\to\psi'(x)=\exp[-\mathrm{i}g\int^x \omega_\mu(x')\mathrm{d}x'^\mu]\psi(x)$. These new $U_{1\mathrm{b}}$ gauge transformations with non-integrable phase factors can be generalized to SU_N groups. In our previous work, we used non-integrable phase factors to explore forms of gauge fields, wrapping numbers and quantization conditions in gauge field theories [10]. In this connection, it is interesting to note that T. T. Wu and C. N. Yang pointed out in 1975 that electromagnetism is the gauge invariant manifestation of the non-integrable phase factor $\exp(ie\oint A_\mu \mathrm{d}x^\mu)$[1].

The original motivation to generalize gauge symmetries was to search for a mechanism to confine quarks. One approach is to have a gauge field equation which can lead to a linear potential. We found a new type of gauge field which satisfies the fourth-order equation associated with a generalized color SU_3 gauge symmetry and can lead to a linear potential between two color charges [8]. We apply the same approach to U_1 gauge symmetry associated with the established conservation laws of the baryon and lepton numbers to explore cosmological implications such as the accelerated cosmic expansion.

In postulate (i), only those groups associated with exact conservation laws (for color charges, baryon and lepton numbers) have general vector gauge functions $\omega^a_\mu(x)$, which do not have the form $\partial_\mu\omega^a(x)$, where $\omega^a(x)$ is a usual function (i.e., not a path-dependent function). In this case, we have massless gauge bosons and new fourth-order gauge field equations. On the other hand, if the groups are associated with non-exact conservation laws (for weak isospins and hypercharges), then the vector gauge functions ω^a_μ take the special form $\omega^a_\mu(x)=\partial_\mu\omega^a(x)$. This happens in the $(SU_2\times U_1)$ sector of the model. These groups, SU_2 and U_1, are generated by the weak isospin and weak hypercharge respectively. Their non-integrable phase factors reduce to the usual phase factors because $\int^x \omega^a_\mu(x')\mathrm{d}x'^\mu=\omega^a(x)$. It is interesting to note that only the $(SU_2\times U_1)$ sector in the unified model suffers spontaneous symmetry breakdown. In this sector, we have the usual second-order field equations for massive and massless gauge bosons. The massless gauge boson is the photon, which is associated with the electromagnetic U_1 group and the exact conservation law of the electric charge. So far, we do not have in-depth understanding of this breakdown of the gauge symmetry in the electroweak $(SU_2\times U_1)$ sector in the unified model.

For almost half a century since physicists discussed higher-order field equations in quantum field theory, it has been widely thought that such field theories do not have positive energy. One might think that any field theory based on the fourth-order field equation is too speculative to be physically interesting. Nevertheless, it has been shown recently that this is not necessarily so for the fourth-order field equations, which will be discussed later. On the other hand, in the present unified model, we argue that the negative energy associated with, say, new color $SU_{3\mathrm{c}}$ gauge bosons does not contradict experiments because such gauge bosons are off-mass-shell and they can provide the confinement mechanism for quarks. Massless color gauge bosons are also confined in quark systems. These properties of the fourth-order field equations will be discussed in Section 4.

2 Cosmic baryon–lepton dynamics with a new gauge symmetry and linear potentials

Let us first consider the generalized $U_{1\mathrm{b}}$ gauge symmetry of a physical system with gauge field $B_\mu(x)$ and spin 1/2 quark field $q(x)$. The generalized gauge transformations for $B_\mu(x)$ and $q(x)$ are assumed to be

$$B'_\mu(x)=B_\mu(x)+\omega_\mu(x), \qquad \mu=0,1,2,3, \tag{1}$$

$$q'(x)=\mathrm{e}^{-iP_\omega(x)}q(x), \qquad \overline{q}'(x)=\overline{q}(x)\mathrm{e}^{iP_\omega(x)}, \tag{2}$$

$$P_\omega(x)=g_\mathrm{b}\int^x \omega_\lambda(x')\mathrm{d}x'^\lambda, \qquad \partial_\mu P_\omega(x)=g_\mathrm{b}\omega_\mu(x), \tag{3}$$

where $P_\omega(x)$ is a non-integrable (i.e., path-dependent) scalar phase [10], and g_b denotes a super weak coupling constant (or baryonic charge). The path in (3) could be arbitrary, as long as it ends at the point $x\equiv x^\nu$, and $\omega_\mu(x)$ may be considered as space-time dependent parameter of the Lie group $U_{1\mathrm{b}}$ and is required to satisfy

$$\partial^2\omega_\mu(x)-\partial_\mu\partial^\lambda\omega_\lambda(x)=0, \qquad \partial^2=\partial_\mu\partial_\nu\eta^{\mu\nu}, \tag{4}$$

where $\eta^{\mu\nu}=(1,-1,-1,-1)$. The solutions of (4) are four sets of infinitely many functions $\omega_\mu(x)$, $\mu=0,1,2,3$. To distinguish the new gauge symmetry from the usual gauge symmetry, let us call transformations (1)–(3) 'taiji gauge transformations', and the Lagrangian invariant under the transformations (1)–(3) taiji invariant Lagrangian. As usual, the taiji $U_{1\mathrm{b}}$ gauge covariant derivative $\Delta_{b\mu}$ is defined as

$$\Delta_{b\mu}=\partial_\mu+\mathrm{i}g_\mathrm{b}B_\mu, \qquad c=\hbar=1. \tag{5}$$

In this connection, we may remark that, strictly speaking, equations (1)–(4) in the generalized gauge symmetry are not exactly the same as the corresponding equations in the usual U_1 gauge symmetry. In particular, we have

1) Cf. Tai Tsun Wu and Chen Ning Yang, in Ref. [13], p. 504.

four functions, $\omega_\mu(x)$, rather than a scalar function, in the gauge transformations. However, they are not all completely arbitrary because they are assumed to satisfy four partial differential equations (4). Only the gauge covariant derivative (5) and the 'scalar' phase $P_\omega(x)$ in (2) are similar to those in the usual U_1 gauge symmetry. Interestingly, in the special case when the vector gauge function ω_μ can be expressed in terms of the space-time derivative of an arbitrary scalar function $\omega(x)$, i.e., $\omega_\mu(x)=\partial_\mu\omega(x)$, the equation (4) becomes a trivial identity. In this case, the expressions (1), (2), (3) and (5) are identical to those in the usual U_1 gauge transformations. Thus, it seems fitting to call this new gauge symmetry based on modified gauge transformations (1)–(3) the generalized or taiji U_1 gauge symmetry. To stress its basic nature related to fourth-order gauge field equations, quark confinement and cosmic accelerated expansion, we also call it taiji gauge symmetry to distinguish it from the usual gauge symmetry.

Similarly, the taiji gauge transformations for the lepton gauge field $L_\mu(x)$ and the electron field $\psi(x)$ are given by

$$L'_\mu(x)=L_\mu(x)+\Lambda_\mu(x), \tag{6}$$

$$\psi'(x)=\Omega(x)\psi(x), \qquad \overline{\psi}'(x)=\overline{\psi}(x)\Omega(x)^{-1}, \tag{7}$$

$$\Omega(x)=\mathrm{e}^{-\mathrm{i}P_\Lambda(x)}, \qquad P_\Lambda(x)=g_l\int^x \Lambda_\lambda(x')\mathrm{d}x'^\lambda. \tag{8}$$

The $U_{1\mathrm{b}}$ gauge curvature $B_{\mu\nu}$ is, as usual, given by the $U_{1\mathrm{b}}$ commutator of the gauge covariant derivative (5),

$$[\Delta_{\mathrm{b}\mu},\Delta_{\mathrm{b}\nu}]=\mathrm{i}g_\mathrm{b}B_{\mu\nu}, \tag{9}$$

$$B_{\mu\nu}=\partial_\mu B_\nu-\partial_\nu B_\mu. \tag{10}$$

It follows from equations (1)–(4), we have the following taiji gauge transformations for $B_{\mu\nu}(x)$, $\partial^\mu B_{\mu\nu}(x)$ and $\Delta_{b\mu}q(x)$:

$$B'_{\mu\nu}(x) = B_{\mu\nu}(x)+\partial_\mu\omega_\nu(x)-\partial_\nu\omega_\mu(x)\neq B_{\mu\nu} \tag{11}$$

$$\begin{aligned}\partial^\mu B'_{\mu\nu}(x) &= \partial^\mu B_{\mu\nu}(x)+\partial^2\omega_\nu(x)-\partial_\nu\partial^\lambda\omega_\lambda(x)\\ &= \partial^\mu B_{\mu\nu}(x),\end{aligned} \tag{12}$$

$$\begin{aligned}\Delta'_{\mathrm{b}\mu}q'(x) &= \mathrm{e}^{-\mathrm{i}P_\omega(x)}(\partial_\mu+\mathrm{i}g_\mathrm{b}B_\mu(x))q(x)\\ &= \mathrm{e}^{-\mathrm{i}P_\omega(x)}\Delta_{\mathrm{b}\mu}q(x),\end{aligned} \tag{13}$$

where we have used the relation

$$\partial_\mu\mathrm{e}^{-\mathrm{i}P_\omega(x)}=-\mathrm{i}g_\mathrm{b}\omega_\mu(x)\mathrm{e}^{-\mathrm{i}P_\omega(x)}, \tag{14}$$

and (4) to obtain (12) and (13). The result (11) shows that the $U_{1\mathrm{b}}$ gauge curvature $B_{\mu\nu}$ is no longer invariant under the taiji gauge transformations (1), in contrast to the usual U_1 gauge symmetry. However, the space-time derivative of the gauge curvature, i.e., $\partial^\mu B_{\mu\nu}$, is taiji gauge invariant under the new gauge transformations (1) with constraint (4).

Furthermore, based on equations (1), (2) and (13), one can show that $\overline{q}(x)\gamma^\mu\Delta_{\mathrm{b}\mu}q(x)$ is taiji gauge invariant, i.e.,

$$\overline{q}'(x)\gamma^\mu\Delta'_{\mathrm{b}\mu}q'(x)=\overline{q}(x)\gamma^\mu\Delta_{\mathrm{b}\mu}q(x). \tag{15}$$

Thus, one can construct baryon-leptonic dynamics (BL dynamics) based on the taiji gauge symmetry and the conservations of baryon and lepton numbers, similar to electrodynamics. The taiji gauge invariant Lagrangian L_BL for BL dynamics involving quarks $q(x)$, electrons ψ, baryonic gauge field $B_\mu(x)$ and leptonic gauge field $L_\mu(x)$, is given by

$$\begin{aligned}L_\mathrm{BL} = &\frac{L_\mathrm{s}^2}{2}(\partial^\mu B_{\mu\lambda}\partial_\nu B^{\nu\lambda}+\partial^\mu L_{\mu\lambda}\partial_\nu L^{\nu\lambda})\\ &+\mathrm{i}\overline{q}(x)\gamma^\mu\Delta_{\mathrm{b}\mu}q(x)-m_\mathrm{q}\overline{q}q\\ &+\mathrm{i}\overline{\psi}(x)\gamma^\mu(\partial_\mu+\mathrm{i}g_l L_\mu)\psi(x)-m\overline{\psi}\psi,\end{aligned} \tag{16}$$

where L_s is a universal constant scale with the dimension of length, so that all taiji gauge fields have the same dimension as the usual vector fields (which satisfy second-order field equations).

The field equations of the taiji gauge field B_μ and the quark $q(x)$, which carries baryon number (or baryon charge g_b), can be derived from (16). We have

$$\partial^2\partial^\lambda B_{\lambda\mu}-\frac{g_\mathrm{b}}{L_\mathrm{s}^2}\overline{q}\gamma_\mu q=0, \quad B_{\lambda\mu}=\partial_\lambda B_\mu-\partial_\mu B_\lambda, \tag{17}$$

$$\begin{aligned}(\mathrm{i}\gamma^\mu[\partial_\mu+\mathrm{i}g_\mathrm{b}B_\mu]-m_q)q &= 0,\\ \mathrm{i}\gamma^\mu[\partial_\mu-\mathrm{i}g_\mathrm{b}B_\mu]\overline{q}\gamma^\mu+m_q\overline{q} &= 0.\end{aligned} \tag{18}$$

The fourth-order equation (17) is invariant under the taiji gauge transformations (1) and (4), thus, it may be called the taiji gauge field equation.

Suppose we chose a gauge condition $\partial^\lambda B_\lambda=0$, (17) leads to the following field equations

$$\partial^2\partial^2 B_\mu=\frac{g_\mathrm{b}}{L_\mathrm{s}^2}\overline{q}\gamma_\mu q, \tag{19}$$

if one puts a point-like baryon charge at the origin, the zeroth component static gauge potential $B_0(\boldsymbol{r})$ satisfies the fourth-order equation,

$$\nabla^2\nabla^2 B_0(\boldsymbol{r})=\frac{g_\mathrm{b}}{L_\mathrm{s}^2}\delta^3(\boldsymbol{r}). \tag{20}$$

This equation leads to a linear gauge potential $B_0(\boldsymbol{r})$[1)], [9]

$$B_0(\boldsymbol{r})=-\frac{g_\mathrm{b}r}{8\pi L_\mathrm{s}^2}, \tag{21}$$

which is the potential generated by a quark carrying the baryon charge g_b. The potential energy $V_\mathrm{b}(\boldsymbol{r})$ between two quarks is given by

$$V_\mathrm{b}(\boldsymbol{r})=g_\mathrm{b}B_0(\boldsymbol{r})=-\frac{g_\mathrm{b}^2 r}{8\pi L_\mathrm{s}^2}. \tag{22}$$

1) We have used the Fourier transformation of the generalized functions.

It leads to a repulsive force between two baryon charges,

$$F_{\rm b}(\boldsymbol{r})=\frac{g_{\rm b}^2}{8\pi L_{\rm s}^2}\frac{\boldsymbol{r}}{r}, \tag{23}$$

which is independent of the distance between the two baryon charges.

Similarly, there is a repulsive force $F_\ell(\boldsymbol{r})$ between two lepton charges g_ℓ:

$$F_\ell(\boldsymbol{r})=\frac{g_\ell^2}{8\pi L_{\rm s}^2}\frac{\boldsymbol{r}}{r}. \tag{24}$$

The relation between g_ℓ and $g_{\rm b}$ will be discussed on the basis of the baryon–lepton symmetry in Section 4.

Since we are interested in the order of magnitude of physical quantities involved in cosmological phenomena, the constant acceleration related to the lepton gauge field will not be considered in the following discussions. According to results (22) and (23) in the cosmic BL dynamics, for a given value of the coupling strength $g_{\rm b}$, however small it may be, there is a large cosmic scale, beyond which the baryonic force overcomes the gravitational force and controls the physics and the evolution of the universe on large length scale.

3 Experimental tests, accelerated expansion and a rotating dumbbell universe

The linear potential energy (22) implies a constant repulsive force between two isolated baryonic charges, independent of their distances. The equation of motion of a freely moving test particle (a baryon) around a macroscopic object is, in general, given by [9] $\mathrm{d}^2\boldsymbol{r}/\mathrm{d}t^2\approx\boldsymbol{g}+\boldsymbol{g}_{\rm bm}$, where $\boldsymbol{g}$ is the gravitational acceleration produced by the macroscopic object, while the acceleration $\boldsymbol{g}_{\rm bm}$ is related to the baryonic charges contained in the object.

For experimental test of the model by measuring the acceleration of a supernova in the future, let us consider a more realistic model of a supernova with mass $m_{\rm s}$ located in a sphere of roughly 100 billion galaxies (as revealed by Hubble). We idealize baryonic galaxies as points uniformly distributed in a big sphere with a radius $R_{\rm o}$ and with a constant baryon density $\rho_{\rm b}$. Each nucleon carries three units of baryonic charge $g_{\rm b}$ and the total mass of the sphere is $M=\rho_{\rm b}4\pi R_{\rm o}^3/3$. The force between two baryon charges is given by (23). One can calculate the total force $F_{\rm tot}$ of the sphere that acts on a supernova at a distance $r<R_{\rm o}$ from the center of the sphere. It is approximately given by[1].

$$\boldsymbol{F}_{\rm tot}=\frac{9g_{\rm b}^2}{8\pi L_{\rm s}^2}\frac{m_{\rm s}M}{m_p^2}\left(\frac{\boldsymbol{r}}{R_{\rm o}}-\frac{r^2\boldsymbol{r}}{5R_{\rm o}^3}\right),$$

$$\frac{\mathrm{d}^2\boldsymbol{r}}{\mathrm{d}t^2}=\frac{\boldsymbol{F}_{\rm tot}}{m_{\rm s}}=\frac{9g_{\rm b}^2M}{8\pi L_{\rm s}^2m_p^2}\left(1-\frac{r^2}{5R_{\rm o}^2}\right)\frac{\boldsymbol{r}}{R_{\rm o}}, \tag{25}$$

where the gravitational acceleration produced by the distribution of matter in the sphere is not included. Note that the total baryonic force $F_{\rm tot}$ in (25) differs from the force in (23) because (23) is not an inverse-square force. If it were an inverse square force, similar to the gravitational force, then the total force on a supernova S for $r<R_{\rm o}$ would still be an inverse square force, calculated by assuming that the entire mass in the sphere interior to S is concentrated at the center, the mass exterior to this sphere not contributing at all to the field at S.

We note that the physical source of the cosmic acceleration in (25) is ubiquitous baryonic matter rather than some mysterious 'dark energy' in the universe. The gauge field theory based on the taiji gauge invariant Lagrangian (16) suggests that the accelerated cosmic expansion could be understood in terms of the cosmic force produced by the taiji $U_{1\rm b}$ gauge field obeying equation (17).

In contrast, if one were to assume a cosmological constant λ in Einstein's gravitational field equation, one would obtain a solution for a static potential involving the term $+\lambda r^2/6$ in the Newtonian limit [11]. Instead of (25), a cosmological constant leads to the following r-dependent acceleration for the cosmic expansion:

$$\mathrm{d}^2\boldsymbol{r}/\mathrm{d}t^2=C\boldsymbol{r}, \tag{26}$$

where C is a constant [11]. Thus, this difference between the predictions of the new baryon–lepton dynamics and the Einstein gravity with a cosmological constant could be tested experimentally. However, such a cosmological constant (in the Hilbert–Einstein Lagrangian for gravity) upset the original foundational principle of general coordinate invariance, which enables one to deduce the precise form of Einstein's law of gravitation "from general requirements of mathematical simplicity without any arbitrariness" [12]. In the Newtonian limit, the cosmological constant λ appears as a non-local constant source in the universe and produces a quadratic potential field, $+\lambda r^2/6$ for the accelerated cosmic expansion. In this sense, the cosmological constant appears to be some sort of 'new aether' with a constant density everywhere in the universe, resembling the old aether of 19th century electromagnetism.

Equation (25) is derived on the basis of local field theory with a generalized gauge symmetry associated with the established conservation law of baryon number. In contrast, (26) is obtained on an ad hoc basis and has little to do with field theory. We hope that the difference between (25) and (26) could be tested by experiments in the future, say, when the frequency shift of a distant supernova can be measured more accurately.

1) Hsu J P and Hsu L, 2015, A model of cosmic acceleration of a supernova and experimental test

Based on the unified model, it is worthwhile to consider a simple model of the universe [13], which is consistent with all the conservation laws of electric charge, baryon number, lepton number, etc. One simple picture of the universe is a 'dumbbell universe' with one end dominated by a gigantic cluster of baryon galaxies (or matter) and the other dominated by another gigantic cluster of anti-baryon galaxies (or anti-matter). It is reasonable to assume that, so far, we have only observed one lobe of the dumbbell universe. With limited knowledge about the physical universe, it seems premature to conclude the non-existence of anti-matter, in contradiction of the established conservation laws in particle physics. These two gigantic clusters could rotate around a center of mass, attracting each other with the force they produce, which is independent of the distance between them. This r-independence is in contrast to the picture presented earlier of the force (25) experienced by a nucleon immersed within a cluster of baryon galaxies.

The effectively constant force that is produced by the baryon and anti-baryon clusters (with a sufficient distance between them) is attractive between baryon matter and anti-baryon matter. This constant force is presumably too weak to be observed within our galaxy. Nevertheless, because the strengths of these basic forces are independent of distance, they would be larger than the gravitational force at extremely large distances similar to that between the two lobes of the dumbbell. This rotating dumbbell universe may also be pictured as the 'taiji yin-yang diagram[1]. One lobe of the dumbbell, in which our Milky Way galaxy is located, is dominated by baryon galaxies. Roughly speaking, there is an accelerated cosmic expansion due to the repulsive force among baryon galaxies in our observable portion of the universe. Similarly, the other lobe of the dumbbell is dominated by anti-baryon galaxies; there is also an accelerated cosmic expansion due to the repulsive force among anti-baryon galaxies. However, the whole universe with these two gigantic clusters will be permanently confined by the attractive force between the baryon cluster and the anti-baryon cluster. This rotating dumbbell model of the physical universe should be stable due to the presence of the attractive forces generated by the taiji gauge field.

If we include the existence of an anti-baryon-galaxy cluster far away (much, much larger than R_o) from the baryon-galaxy cluster, then the acceleration in (25) will be modified by an additional constant acceleration. That is, although the prediction (25) in the cosmic BL dynamics still holds in the sense that the r-dependence of the repulsive baryonic force acting on a supernova S within a sphere of baryonic galaxies is not affected, the additional constant acceleration due to the presence of an anti-baryon-galaxy cluster far away introduces a slight anisotropy in the observed cosmic accelerated expansion. This anisotropy provides a "signature" of the dumbbell universe model, which will eventually allow it to be distinguished from a modified conventional model. (See Fig. 1.)

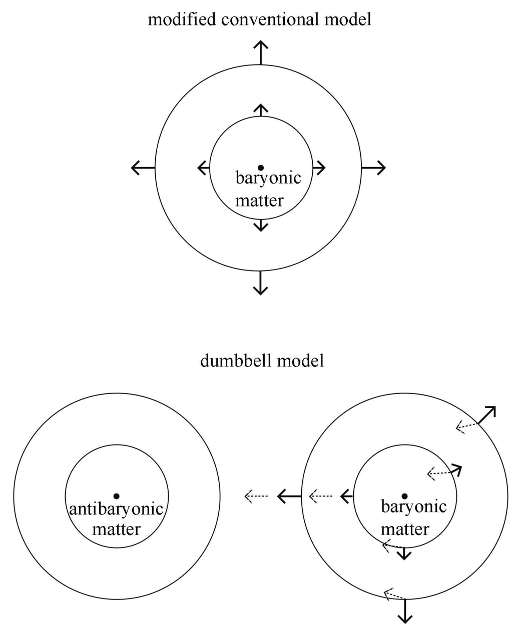

Fig. 1. How the strength and direction of the net force varies. Top: modified conventional model. Bottom: dumbbell model of the universe. In the modified conventional model, the force is roughly proportional to distance r from the center at very large distances (where gravity is negligible), and produces a radial repulsion. This is true for a universe made of baryonic or antibaryonic matter. Current observations indicate that the preponderance of matter is baryonic. For the dumbbell model shown above, the lobe to the right is baryonic and the lobe to the left is antibaryonic. Dotted vectors indicate the attractive force exerted by the antibaryonic lobe on the baryonic lobe. Solid vectors indicate the baryonic repulsive force. The net force at opposite ends of the dumbbell tends toward zero. The net force grows as one moves toward the central region of the dumbbell, for a given value of r (measured from the center of each lobe), when the effect of gravity is negligible. Force vectors are drawn on the baryonic lobe; corresponding vectors may be drawn on the antibaryonic lobe. The relative magnitudes and directions of the vectors are meant to be illustrative only, and are not exact.

1) The taiji circle is divided into two intertwined dark and light halves to represent yin and yang. This diagram was used by Bohr to illustrate his idea of complementarity in quantum mechanics. For taiji yin-yang diagram, see F. Louis, (http://www.biroco.com/yijing/louis_taiji_diagram.pdf).

In the present stage of the evolution of the universe, one may speculate that, based on the laws of physics in the unified model, the greatest energy source that one could observe is due to the annihilation of a baryonic galaxy and an anti-baryonic galaxy in the middle region between the two gigantic clusters. Experimentally, the baryon and anti-baryon annihilation radiation can be distinguished from other atomic radiations, as discussed by Vlasov [14]. The observation of such an enormous energy source (over a very long period of time) and the baryon-annihilation radiation (produced on the 'boundary' between regions occupied by baryon galaxies and anti-baryon galaxies) may also be considered as a test of the dumbbell universe.

4 Discussions of the unified model

4.1 Universal principle of taiji gauge symmetry

The U_{1l} and U_{1b} corresponding to the lepton and baryon numbers are usually considered not to be a local gauge symmetry, because they have been experimentally established to be exact symmetries. The reason is that if they were gauged, there must be massless gauge fields, which have not been observed in the laboratory. However, we believe that taiji gauge symmetry is a universal principle and can be more general than the conventional gauge symmetry. The universal principle of gauge symmetry states that there are massless gauge fields associated with each and every conserved internal quantum number, such as lepton and baryon numbers (or charges), and that the new gauge transformations involve vector gauge functions and non-integrable phase factors [8, 9]. The conventional gauge symmetry in the $SU_2\times U_1$ sector of the unified model is a special case when the vector gauge function $\omega^a_\mu(x)$ takes a special form $\partial_\mu\omega^a(x)$, where $\omega^a(x)$ is an arbitrary function.

It is intriguing that only the $SU_2\times U_1$ sector corresponds to weak isospin and hypercharge (which are not exactly conserved) and involve spontaneous symmetry breaking. Then, it leads to the electromagnetic U_1 gauge symmetry associated with the exactly conserved electric charge. In the unified model, only this electroweak sector involves second-order gauge field equations and massive gauge bosons. So far, it is fair to say that we do not have in-depth understanding of spontaneous symmetry breaking in the total-unified model.

In principle, there will be massless gauge fields associated with electron–lepton, muon–lepton and tauon–lepton numbers, if these lepton family numbers are exactly conserved. However, the observed neutrino oscillations indicate that these lepton family numbers are only approximately conserved.

4.2 Total-unified model and taiji gauge symmetry

In order to resolve the problems of quantization of gravitational field and others [13], we have formulated and discussed Yang–Mills gravity on the basis of external translational gauge symmetry in flat space-time, which corresponds to the exact conservation laws of energy and momentum. Such Yang–Mills gravity can be quantized to obtain Feynman–Dyson rules and to calculate the S-matrix, which satisfies gauge invariance and unitarity [15, 16]. Moreover, it is consistent with experiments and its equation of motion for classical objects (in the form of relativistic Hamilton–Jacobi equations) are formally the same as the corresponding equations in Einstein's gravity. The classical equations of motion in Yang–Mills gravity are obtained from the quantum wave equations with translational gauge symmetry by taking the geometric-optics (or classical) limit. They have the form of relativistic Hamilton–Jacobi equations with effective Riemanian metric tensors. In this sense, the apparent curvature of space-time appears to be simply a manifestation of the flat space-time translational gauge symmetry (T_4) for the motion of quantum particles in the classical limit [12]. These classical equations with the effective metric tensor $G^{\mu\nu}$,

$$G^{\mu\nu}(x)[\partial_\mu S(x)][\partial_\nu S(x)]-m^2=0, \qquad (27)$$

in Yang–Mills gravity are called Einstein–Grossmann equations of motion. They lead to observable results, which are consistent with the experiments of the perihelion shift of Mercury. For the bending of light (with optical frequency) by the sun, the total-unified model with Yang–Mills gravity leads to a deflection angle which is 12% smaller than that of Einstein gravity. So far, the experimental uncertainty is about 10%–20% for light rays with optical frequency (which corresponds to the geometric-optics limit). If the experimental errors can be reduced to a few percent in the future, then one can test this prediction of Yang–Mills gravity.

These properties and results suggest that one can include the external non-compact T_4 translational group in a total-unified model with the gauge groups

$$G_{\rm tot}=(T_4)_{\rm YMgravity}\times[SU_3]_{\rm color}\times(SU_2\times U_1)\times[U_{1b}\times U_{1l}].$$

We postulate to interpret that $G_{\rm tot}$ is the generalized taiji gauge symmetry with vector gauge functions. The gauge functions in Yang–Mills gravity for the transformation of, say, a scalar field is a vector function $\Lambda_\mu(x)$ [15],

$$\Phi(x)\to\Phi(x)-\Lambda^\lambda(x)\partial_\lambda\Phi(x).$$

Only the vector gauge functions ω^a_μ in the electroweak sector, $(SU_2\times U_1)$, can be expressed in the special form, $\omega^a_\mu=\partial_\mu\omega^a(x)$, so that it has the usual gauge symmetry in the total-unified model.

4.3 Cosmic baryon–lepton symmetry

A baryon–lepton symmetry in weak interactions was first discussed by Gamba, Marshak and Okubo in 1959 [17]. They observed a symmetry between baryon and lepton in the charged weak current under the interchange e↔n, μ↔Λ and ν↔p. After the advent of quark model for baryons, this observed symmetry became known as quark–lepton symmetry.

The interesting and basic idea of Gamba, Marshak and Okubo can be naturally generalized to the cosmic baryon–lepton dynamics with super weak interactions based on taiji gauge symmetry. Let us consider the baryon–lepton system involving quarks, leptons, B_μ and L_μ gauge fields. We postulate the complete BL Lagrangian to be invariant under the exchanges:

$$B_\mu \leftrightarrow L_\mu, \quad q \leftrightarrow \ell, \quad m_q \leftrightarrow m_\ell, \tag{28}$$
$$q=(u,d,s,c,b,t), \quad \ell=(e,\nu_e,\mu,\nu_\mu,\tau,\nu_\tau).$$

where all quarks in the set q are assigned a value of baryon number $+1/3$, anti-quarks $-1/3$ and non-baryons 0. All leptons in the set ℓ are assigned a value of lepton number $+1$, anti-lepton -1 and non-leptons 0. Baryon number may also be called baryon charge, g_b, which can be determined experimentally [9]. The complete BL Lagrangian takes the following general form

$$\begin{aligned} L_{CBL} = & \frac{L_b^2}{2}\partial^\mu B_{\mu\lambda}\partial_\nu B^{\nu\lambda}+\frac{L_\ell^2}{2}\partial^\mu L_{\mu\lambda}\partial_\nu L^{\nu\lambda} \\ & +i\overline{q}\gamma^\mu(\partial_\mu+ig_b B_\mu)q-m_q\overline{q}q \\ & +i\overline{\ell}\gamma^\mu(\partial_\mu+ig_l L_\mu)\ell-m_l\overline{\ell}\ell, \end{aligned} \tag{29}$$

where

$$L_b=L_\ell=L_s, \quad g_b=g_\ell. \tag{30}$$

Thus, we have a simple BL dynamics with a universal length scale L_s and coupling constant $g_b=g_\ell=g_e$.

4.4 New sources and confining dual potentials in the color SU_{3c} sector

In the unified model with generalized gauge symmetry, the point source for the static U_{1b} gauge field is the same as the usual field theory, as shown in (20). But the point sources for the color SU_{3c} sector could be more complicated due to the self-coupling of the SU_{3c} gauge field, which satisfies the fourth-order equations,

$$\begin{aligned} & L_s^2\left[\partial^2\partial_\nu H_a^{\nu\mu}-g_c f_{abc}H_{\lambda b}(\partial^2\partial^\mu H_c^\lambda-\partial^2\partial^\lambda H_c^\mu)\right] \\ & +g_c\overline{q}\gamma^\mu(\lambda_a/2)q+\cdots=0. \end{aligned} \tag{31}$$

The quark confining potential can be derived from the time-component of (31) in the static limit,

$$\begin{aligned} & L_s^2\nabla^2\nabla^2 H_a^0+L_s^2\nabla^2[g_c f_{abc}\partial_i(H_b^i H_c^0)]+g_c\overline{q}\gamma^0(\lambda_a/2)q+\cdots \\ & =0. \end{aligned} \tag{32}$$

Suppose we consider point color charge g_c as the source to generate the static gauge field $H_a^0(r)$, equation (32) suggests two types of singular point sources for the fourth-order equation,

$$L_s^2\nabla^2\nabla^2 H^0(\boldsymbol{r})=g_c\delta^3(\boldsymbol{r})-g_c L_s^2\nabla^2\delta^3(\boldsymbol{r}), \tag{33}$$

where the unusual second term is due to the self-coupling of the color gauge fields, as shown in the second term in (32). The fourth-order equation (32) leads to spherically symmetric static dual potentials,

$$H^0(r)=-\frac{g_c r}{8\pi L_s^2}+\frac{g_c}{4\pi r}. \tag{34}$$

For charmonium, the potential energy V_c for the charmed quark–antiquark system is given by $(-g_c)H_a^0(r)$. We note that the second term, a non-electromagnetic Coulomb type potential, is needed for a good fit of the charmonium energy spectra. Using the empirical formula of Cornell group for charmonium potential, we obtain the approximate strong coupling strength and the confining length scale L_s,

$$V_c(r)=\frac{g_c^2 r}{8\pi L_s^2}-\frac{g_c^2}{4\pi r}, \quad \frac{g_c^2}{4\pi}\approx 0.2, \quad L_s\approx 0.14 \text{ fm}, \tag{35}$$

where the charmed quark mass is taken to be $m_c \approx$ 1.6 GeV[1)] [18].

Note that the SU_{3c} gauge invariant Lagrangian with higher order derivatives appears to be renormalizable by power counting, which can be verified by counting the degree of divergences in Feynman diagrams. This renormalizability seems to be related to and also supported by the following properties: Suppose one compares the rules for Feynman diagrams obtained in the taiji gauge invariant Lagrangian L_{su3c} with those in the usual QCD. One sees that the quark vertices have the same structures in both cases. The three and four gauge-boson vertices in L_{su3c} involve two more momentum factors than the corresponding vertices in QCD, and they are compensated by the fact that the propagators of taiji gauge bosons have two more inverse momentum factors. The same thing occurs in the vertices and propagators involving ghost particles. This property suggests that all previous calculations in the usual QCD will not be upset in the present model with color SU_{3c} generalized gauge symmetry by the high order corrections [19–21]. The renormalizable property of the unified model in the color SU_{3c} sector will be discussed in a separate paper.

4.5 New gauge symmetry and fourth-order gauge field equations

In general, gauge fields with generalized gauge symmetry satisfy fourth-order differential equations. We

1) Recent experimental value for the charmed quark mass is about 1.27±0.08 GeV.

note that the fourth-order field equation is usually considered as unphysical because of the presence of indefinite energies (or non-positive norm) in the dynamical system [22–25]. However, it was shown recently that this is not necessarily so [26–28]. A no-ghost theorem was proved for the fourth-order derivative Pais–Uhlenbeck oscillator model [26]. The no-ghost theorem makes field theories with fourth-order field equations interesting. But the classical solutions of the original field equations may be altered. In this connection, we would argue from a different viewpoint because, say, the fourth-order color gauge-field equation (31) differs from the usual cases. To be specific, let us consider the problems of negative energies of the new color gauge bosons associated with the fourth-order field equations in the color SU_{3c} sector [29]. In this unified model, the new color gauge bosons will be permanently confined by the linear potential (35) in quark systems. We can consistently postulate to interpret the color gauge bosons to be off-mass-shell, in contrast to usual bosons which satisfy the second-order equation. Thus, the new gauge boson with negative energy will not appear in the external states of the S-matrix of the total unified model and, hence, does not contradict experiments.

4.6 Dark energy vs. 'dark' fields

If one uses Einstein's field equation with a cosmological constant (or dark energy), one will have the result that the acceleration of expansion depends linearly on the distance between two galaxies, as shown in (26). In contrast, the unified model provides a field-theoretic understanding of accelerated cosmic expansion, as shown in Sections 2 and 3. The coupling strengths of the new gauge fields associated with the conserved baryon and lepton numbers are extremely weak, so that their effects cannot be observed in laboratories even in our Milky Way galaxy. In this sense, we give a re-interpretation of the baryonic gauge field originally proposed by Lee and Yang, so that they will have observable effects on the cosmic scale [13, 30]. Thus, a crucial question is whether the acceleration in the expansion of the universe agrees with the predictions (25) or (26). Since we have accelerated Wu transformations of space-time, which gives a new Wu–Doppler shift for radiation sources with linear acceleration on a straight line [31], it is hoped that experimental tests of (25) and (26) can be carried out in the near future.

4.7 Big-Jets vs. Big Bang

Among the most important experimental findings in cosmology are Hubble's original discovery of the expansion of the universe in 1929, and the discovery of the cosmic microwave background by Penzias and Wilson in 1965. These findings provide evidence for the occurrence of a hot Big Bang [32, 33]. One might ask, is such a beginning consistent with the dumbbell model of the universe? If one hypothesizes that the dumbbell universe originated with the formation of Big-Jets, i.e., two diametrically opposed jets (similar to the type of phenomena one might encounter in a particle collision in a high energy physics laboratory) composed of baryons and anti-baryons in each jet, the processes of their annihilations eventually lead to a baryon dominated jet and an anti-baryon dominated jet. Then, from the vantage point of an observer in either jet, the evolution of that observer's universe would be similar to the general features of a Big Bang. A detailed description of specific phenomenon implied by a Big Bang origin of the universe, such as Big Bang nucleosynthesis or cosmic microwave background, awaits future study. The authors acknowledge the fact that a more thorough cosmological model, supported by detailed calculations needs to be formulated before a dumbbell model of the universe can gain widespread acceptance.

In summary, the total-unified model, including Yang–Mills gravity with flat space-time translational gauge symmetry, paves the way to unify all interactions. In this sense, the total-unified model is a satisfactory framework that could accommodate all fundamental laws of physics in a unified manner. An unexpected and interesting feature of the unified model is that both quark confinement on the small scale and the accelerated cosmic expansion on the large scale can be understood on the basis of the generalized taiji gauge symmetry.

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Additional Notes

One may conjecture the generalized gauge symmetry based on gauge transformations (1)–(3) involving a new path-dependent phase factor may correspond to a generalized fiber bundle, which deserves further study. (See Ref. [13], p. 106.)

For a detailed discussion of Lagrangians and Hamiltonians involving higher order derivatives, see also CHANG Tsung Sui, Proc. Roy. Soc. A **183** 316 (1945). For CHANG's related work, see online: HQ3,Yin and Zhu. We would like to thank ZHU Zhongyuan for informing us of CHANG's work.

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