Hyperbolic Cardassian universe

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Abstract: We propose a hyperbolic function form of the Cardassian component in the Cardassian model. Using the repartition of this Cardassian component, we can obtain a non-zero gravitational interaction between the time derivative of Ricci scalar curvature and the baryon/lepton number current in the radiation-dominated universe. Furthermore, the other term that acts like a non-zero cosmological constant would give an accelerated expansion of current universe and the features of this model do not violate our desired requirements.

Key words: Cardassian, hyperbolic function, baryo/leptogenesis

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1 Introduction

The observational evidence for the spatially flat but accelerating universe[1] and the baryon/lepton number asymmetry in the universe today remain a mystery in modern cosmology and particle physics. For the former, three types of models are available: (1) a new form of particle; (2) some form of modified gravity; and, (3) a departure from the cosmological principle. As is well known, the role of dark energy models has been much exploited to achieve the cosmic acceleration[2–6]. Cardassian universe models that incarnates the hope of late-time acceleration due to the modified Friedmann equation alone has been of concern [7–9]. For the latter, the three conditions that Sakharov suggested [10] are: (1) the interactions that violate the baryon-number conservation; (2) C and CP violation; and, (3) a departure from thermal equilibrium seems to be necessary from the contemporary view; however, when the CPT-violation is present in the early radiation-dominated universe, the asymmetry can be generated while maintaining thermal equilibrium. One silent mechanism is the so-called gravitational baryogenesis [11]. Similar to dark energy, there have been many theoretical possibilities on the early gravitational baryo/leptogenesis [11–20]. From the point of view of a unified description, there may be unknown mechanisms for the origin of the observational universe. One of the possibilities of the generation of gravitational baryo/leptonesis in the radiation-dominated era and the accelerating expansion of current universe is to introduce a Cardassian component [21] while maintaining the thermal history and the formation of large scale structure in standard cosmology. Moreover, satisfying (1) at late times after the redshift $z\sim\mathcal{O}(1)$ the different form of the energy density function $g(\rho_{\rm M})$ drives the universe towards an observationally accelerated expansion; (2) the sound speed $c_{\rm s}^2$ of the cosmological fluid

$$c_{\rm s}^2=\left(\frac{\partial p}{\partial \rho}\right)_{\rm s}=\rho_{\rm M}\left(\frac{\partial^2 \rho}{\partial \rho_{\rm M}^2}\right)_{\rm s}\Big/\left(\frac{\partial \rho}{\partial \rho_{\rm M}}\right)_{\rm s}$$

is guaranteed to be positive ($c_{\rm s}^2>0$) to keep the classical solution of the expansion stable [7] (where p and ρ denote the total pressure and energy density of all the species present in the universe, $\rho_{\rm M}$ is the energy density and $p_{\rm M}$ is the pressure of ordinary matter). Any suitable Cardassian model also requires that it can be used on all scales, such as the cosmological scales as well as the galactic scales [8]. That is, in the Newtonian limit when the Euler's equation

$$\frac{\partial \vec{v}}{\partial t}+\left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}=-\frac{\vec{\nabla}p}{\rho+p}-\vec{\nabla}\phi$$

can give a result that closes to

$$\frac{{\rm d}\vec{v}}{{\rm d}t}=-\frac{\vec{\nabla}p_{\rm M}}{\rho_{\rm M}}+\vec{g}$$

in standard cosmology. (Where $\vec{g}=-\vec{\nabla}\phi$, ϕ is the Newtonian potential for the gravitational field and $\vec{v}$ is the 3-dimensional fluid velocity.) Note that the Cardassian universe contains only ordinary matter and radiation, which does not require new energy sources [9].

Conventionally, when we attribute the early gravitational baryo/leptogenesis to the Cardassian component, one can obtain some phenomenological constraints on the Cardassian cases from the concept of gravitational CPT-violation at a given epoch [21]. Nevertheless, the

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scenario of requirements for theoretical works given by the parametric method provides only unconnected distinct parts. We do not have a comparatively complete picture of the gravitational baryo/leptogenesis in the universe. In order to propose a Cardassian model including the early gravitational baryo/leptogenesis, as well as the current cosmic acceleration, it seems that the presentation of time function of the Cardassian component has to be a requisite.

In this paper, we try to propose a Cardassian component $f(\rho)\equiv g(\rho)-\rho$ that takes a hyperbolic tangent function form

$$f(t)\sim\tanh^2(t) \tag{1}$$

from the general solution of the CPT-conserving equation, where the energy density ρ contains only ordinary matter and radiation, and $g(\rho)$ is as a function of ρ, which returns to ρ at early stage but gives rise to an accelerated expansion in the recent past of the universe in the classical Cardassian models. Then, the possibility of gravitational baryo/leptogenesis using the time function but negative energy contribution within the hyperbolic energy density is under study, where the non-vanishing Ricci scalar curvature is

$$\mathcal{R}\sim\text{sech}^2(t), \tag{2}$$

and $\dot{\mathcal{R}}(t)\neq 0$ in the early radiation-dominated universe. By analogy, there exits another Cardassian component that seems to exactly drive the current accelerated expansion, just as a rather tiny but non-zero cosmological constant Λ. We show that the hyperbolic component is a potential Cardassian model that is able to deal with the two puzzles of the universe.

2 Hyperbolic Cardassian component

In this section we concentrate on the early radiation-dominated universe that follows inflation when $\omega\approx 1/3$ [11]. The contemporary view is that at this time all the energy components follow from the CPT-conserving equation [21]:

$$3x^{-1/2}\ddot{x}+12\dot{x}-\frac{3}{2}x^{-3/2}\dot{x}^2=0, \tag{3}$$

where $x=(\dot{a}/a)^2=H^2$, $H(t)$ is the Hubble parameter, $a(t)$ is the scale factor, and a dot represents the time derivative. We know that, besides the radiation component ρ_γ, there is another solution of the hyperbolic tangent function form:

$$x'=\frac{C_1}{4}\tanh^2\left[\sqrt{C_1}(t+C_2)\right], \tag{4}$$

or

$$H'=\frac{\sqrt{C_1}}{2}\tanh\left[\sqrt{C_1}(t+C_2)\right], \tag{5}$$

where $x'=H'^2$ and C_1, C_2 are constants. That is

$$\mathcal{R}'(t) = 12H'^2+6\dot{H}'=3C_1\left\{\text{sech}^2\left[\sqrt{C_1}(t+C_2)\right] + \tanh^2\left[\sqrt{C_1}(t+C_2)\right]\right\}=C. \tag{6}$$

If $C_1=0$, then we would reach back to the standard cosmology. However, the apparent cosmological constant [22, 23] may help us think otherwise: $C\sim\Lambda$, that C_1 may be a remnant small but non-zero constant. So that Eq. (6) gives

$$H'^2=\frac{C}{12}-\frac{1}{2}\dot{H}', \tag{7}$$

which means that Eq. (4) can be written as:

$$\rho'=\rho_{\rm C}+\rho_{\rm X}, \tag{8}$$

where $\rho'=3H'^2/(8\pi G)$, $\rho_{\rm C}={\rm C}/(4\cdot 8\pi G)$ and $\rho_{\rm X}=-3\dot{H}'/(2\cdot 8\pi G)$. Therefore,

$$\sqrt{3}(1-3\omega_{\rm C})(1+\omega_{\rm C})(8\pi G\rho_{\rm C})^{3/2}=0$$

shows that the state parameter of the energy component $\rho_{\rm C}$ is $\omega_{\rm C}=-1$; thereby, it has no gravitational interaction with the B–L number current. On the contrary, the term $\rho_{\rm X}=-3\dot{H}'/(2\cdot 8\pi G)$ would lead to a non-vanishing gravitational interaction because

$$H_{\rm X}^2=-\frac{\dot{H}'}{2} \tag{9}$$

and the time derivative of $H_{\rm X}(t)$ is then

$$\dot{H}_{\rm X}=\frac{{\rm i}\ddot{H}'}{4}\sqrt{\frac{2}{\dot{H}'}}, \tag{10}$$

where ${\rm i}^2=-1$ and the imaginary $\dot{H}_{\rm X}$ is thought to be an untruthful description in an expanding universe thus, the term $H_{\rm X}^2=\rho_{\rm X}=-3\dot{H}'/(2\cdot 8\pi G)$ remains variable in the universe. Note that the opposite sign energy contribution $\rho_{\rm X}(<0)$ satisfies the mechanism of gravitational baryo/leptogenesis [11] in which the interaction

$$\frac{1}{M_*^2}\int{\rm d}^4x\sqrt{-g}(\partial_\mu\mathcal{R})J^\mu$$

(where $\mathcal{R}$ is the Ricci scalar curvature, M_* is the cut-off scale of the effective theory, J^μ can be any current that yields a net baryon and lepton number charge in thermal equilibrium) gives opposite sign energy contributions that differ for particle versus antiparticle and dynamically violates CPT exactly. Henceforth, the "left" Ricci scalar curvature takes

$$\mathcal{R}(t) = 12H^2+6\dot{H}=12H'^2=4\cdot 8\pi G(\rho_{\rm C}+\rho_{\rm X}), \tag{11}$$

where the above energy component $\rho_{\rm C}+\rho_{\rm X}$ springs from the radiation contribution in the early era, which does not go beyond the Cardassian framework. We can see that the term x' comes from the CPT-conserving Eq. (3) but the opposite sign energy contribution $\rho_{\rm X}$ needs no

$\dot{H}_X$ in our observational universe; therefore, we have a statement of

$$x' = \frac{C}{12} - \frac{1}{2}\dot{H}'. \tag{12}$$

Furthermore, for the purpose of gravitational baryo/leptogenesis, it would be well that the expected gravitational interaction $\dot{\mathcal{R}}$ has to be of a function, which has a peak value ($\sim 10^{-10}$) in the radiation-dominated era and then slides down rapidly during later times to satisfy the present experiments. So that for any Cardassian model, including the early gravitational baryo/leptogenesis, there exist at least six requirements that should be satisfied. Besides the three conditions for any suitable Cardassian model for the present time, as mentioned earlier, a qualified Cardassian model facing the matter asymmetry should also follow the two constraints [21]: (1) in the early universe the ratio of the Cardassian term and the cosmic time is closely related to the required CPT-violation; and, (2) the current accelerated expansion of the universe is driven by a Cardassian component that belongs to a quintessence-like model. Meanwhile, the gravitational interaction $\dot{\mathcal{R}}$ function would take a desired value in the history of the cosmic time.

3 *CPT*-violating Cardassian universe

Now that the gravitational baryo/leptogenesis requires CPT-violation in the radiation-dominated epoch as the universe expands, the related Cardassian universe needs to cover the gravitational CPT-violating effect because the current tiny value of its remnant means that the experimental impact becomes insignificant. With the help of Eqs. (8, 11) we begin to investigate the following hyperbolic Cardassian model:

$$\begin{aligned} H^2 &= \frac{8\pi G}{3} g(\rho_{\text{ord}}) = \frac{8\pi G}{3}\rho_{\text{ord}} + x' \\ &= \frac{8\pi G}{3}\rho_{\text{ord}} + \frac{C_1}{4}\tanh^2\left[\sqrt{C_1}(t+C_2)\right], \end{aligned} \tag{13}$$

where ρ_{ord} is the ordinary energy density of the universe. The Cardassian component is

$$\begin{aligned} f(\rho_{\text{ord}}) &= \frac{3}{8\pi G}\cdot\frac{C_1}{4}\tanh^2\left[\sqrt{C_1}(t+C_2)\right] \\ &= \rho_C + \rho_X. \end{aligned} \tag{14}$$

First, in the very early universe the last equal sign of the right-hand side of Eq. (14) gives the required Cardassian component

$$f \approx \rho_X = -3\dot{H}'/(2\cdot 8\pi G), \tag{15}$$

where ρ_C has been neglected in the radiation-dominated era, where the ρ_X component brings the non-vanishing CPT-violating gravitational interaction

$$\begin{aligned} \dot{\mathcal{R}} &= 4\cdot 8\pi G\dot{\rho_X} = -6\ddot{H}' = 6C_1^{3/2}\text{sech}^2\left[\sqrt{C_1}(t+C_2)\right] \\ &\quad\cdot\tanh\left[\sqrt{C_1}(t+C_2)\right]. \end{aligned} \tag{16}$$

If taking $6C_1^{3/2} \sim 1$ (10^{-10}) then the evolution of function $U = \text{sech}^2\left[\sqrt{C_1}(t+C_2)\right]\cdot\tanh\left[\sqrt{C_1}(t+C_2)\right]$ as $T = \sqrt{C_1}(t+C_2)$ develops is shown in Fig. 1.

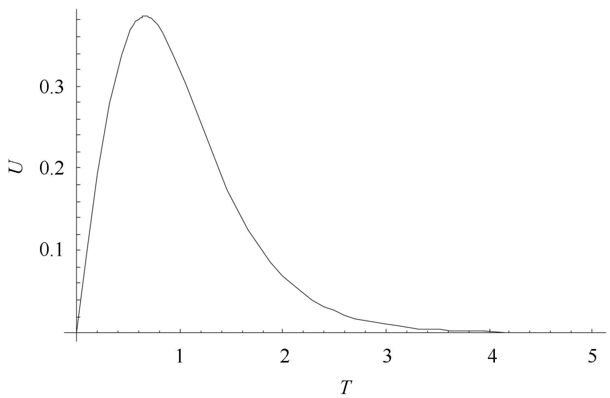

Fig. 1. The evolution of $U = \text{sech}^2(T)\cdot\tanh(T)$ as a function of $T = \sqrt{C_1}(t+C_2)$.

Obviously, the increase of the gravitational interaction $U = \dot{\mathcal{R}}$ has a maximum in the early stage, and then the non-thermal energy component $\rho_X (< 0)$ decreases rapidly, as does the gravitational interaction $\dot{\mathcal{R}}$. In other words, the effect of matter asymmetry due to the negative ρ_X contribution only takes place around the radiation-dominated era and has a faint impact on the present observational universe, as well as in the experimental tests. In this way, the constraint of the early CPT-violation is included in the Cardassian term, which can be found. Moreover, the gravitational interaction $\dot{\mathcal{R}}$ has a desired value and the CPT-violating effect would be available.

Second, for the present time, the ρ_X component is decreasing approximatively zero and the constant ρ_C ($\sim\Lambda$) becomes important on a very large scale, and then drives the universe towards an acceleration since [21]:

$$g(\rho_M) = \rho_{\text{ord}} + f(\rho_{\text{ord}}) = \rho_M + \rho_C, \tag{17}$$

and

$$\begin{aligned} &\dot{\rho} = [\dot{\rho_{\text{ord}}} + f(\dot{\rho_{\text{ord}}})] \sim -2(t-t_0)^{-3} < 0, \\ &\ddot{\rho} \sim 6(t-t_0)^{-4} > 0, \end{aligned} \tag{18}$$

where t_0 is constant and $\rho = g(\rho_M)$. Therefore,

$$\frac{\partial\rho}{\partial\rho_M} = \frac{\dot{\rho}}{\dot{\rho_M}} > 0 \tag{19}$$

and

$$\frac{\partial^2\rho}{\partial\rho_M^2} = \ddot{\rho}\cdot\dot{\rho_M}^{-2} > 0. \tag{20}$$

That is, the sound speed

$$c_s^2 > 0. \tag{21}$$

Meanwhile, $\omega_C = -1$ means that on galactic scales the hyperbolic model gives

$$\frac{d\vec{v}}{dt} = -\frac{\vec{\nabla} p_M}{\rho_M} + \vec{g}, \tag{22}$$

where $\vec{\nabla} p_C = 0$, $p_C + \rho_C = 0$ and $p_M \sim 0$. Which is the Euler' equation in the Newtonian limit and in standard cosmology [8]. Simultaneously,

$$\omega = \frac{p_M - \rho_C}{\rho_M + \rho_C} > -1, \tag{23}$$

thus, the second constraint which is on the quintessence-like Cardassian model [21] can also be found. Hence, the hyperbolic Cardassian model has met the six requirements that we have summed up above.

4 Conclusion

In summary, we propose in this paper a mechanism for the gravitational baryo/leptogenesis in which the Cardassian component takes a hyperbolic tangent function form. By using Eq. (14) properly, we can arrive at a non-zero $\dot{\mathcal{R}}$ in the early radiation-dominated universe. The time history of the gravitational interaction $\dot{\mathcal{R}}$ as the universe expands presents acceptable features for us. In addition, the constant ρ_C, which acts like a cosmological constant can give an accelerated expansion of current universe, is included in the framework of Cardassian expansion. The x' component of Eq. (4) is used as the Cardassian component directly. The key approach lies in Eq. (7) or Eq. (8), where the opposite sign energy contribution ρ_X differs for particle versus antiparticle and the ρ_C contribution currently exhibits a repulsive effect on a very large scale.

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