

# Effects of relativistic kinematics in heavy ion elastic scattering\*

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**Abstract:** Relativistic corrections to the reaction kinematic parameters were made for elastic scattering of  ${}^6\text{Li}$ ,  ${}^{12}\text{C}$  and  ${}^{40}\text{Ar}$  from  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  targets at incident energies between 20 and 100 MeV/nucleon. The results of optical model calculations show that the effects of such corrections are important when describing the angular distributions of elastic scattering cross sections for heavy ion scattering at incident energies as low as around 40 MeV/nucleon. The effects on the total reaction cross sections on the other hand, were found to be small within the energy range studied when the optical model potential is fixed.

**Key words:** heavy ion elastic scattering, optical model, relativistic kinematics

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## 1 Introduction

It is well-known that at sufficiently high incident energies, relativistic effects have to be taken into account when describing heavy ion elastic scattering [1, 2]. There are several aspects in treating the relativity, for example, the use of the Schrödinger equation is not relativistically correct and, at least, some re-interpolation of the nuclear optical potential should be made [3]; the parameters of reaction kinematics, namely, the atomic masses and incident energies, have to be modified. The latter aspect is rather simple, but it has been found to be important for some cases. For example, Farid and Satchler studied the latter aspect and their effects on the optical model potentials extracted by fitting experimental data. Changes in potential strength by larger than 20% were found for heavy ion elastic scattering at an incident energy as low as 44 MeV/nucleon, at which the relativistic effects were usually thought to be negligible [2]. The effects on the shapes of angular distributions of elastic scattering cross sections, however, have not been reported yet.

Recently, in our study of systematic heavy ion potentials, we also found the necessity of introducing the relativistic corrections to the reaction kinematic parameters when the incident energy of a projectile nucleus is above 40 MeV/nucleon [4]. One example is shown in Fig. 1, where optical model calculations with and without relativistic corrections were compared with the experimental data. Clearly one can see that the inclusion of relativistic corrections helps to improve the description of ex-

perimental data. In this paper, we examine the effect of relativistic corrections to reaction kinematic parameters on heavy ion elastic scattering and total reaction cross sections, and study their dependence on the projectile and target masses and incident energies. For such purposes,  ${}^6\text{Li}$ ,  ${}^{12}\text{C}$  and  ${}^{40}\text{Ar}$  elastic scattering from  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  are studied at incident energies between 10 and 100 MeV/nucleon, which is the range where the relativistic effects are usually thought to be negligible.

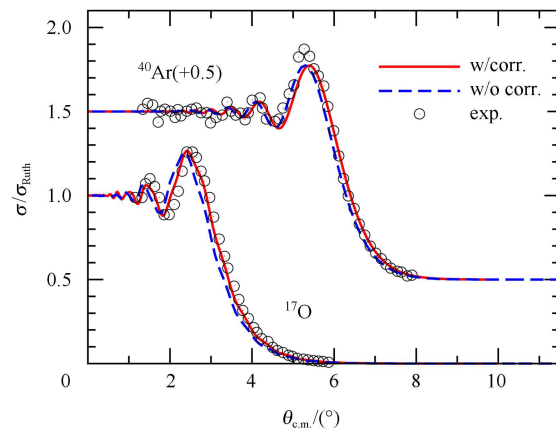


Fig. 1. (color online) Optical model calculations of  ${}^{17}\text{O}$  and  ${}^{40}\text{Ar}$  elastic scattering from  ${}^{208}\text{Pb}$  at 84 and 44 MeV/nucleon, respectively, with and without taking into account the relativistic corrections to the reaction kinematic parameters. The experimental data are from Refs. [5, 6].

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This paper is organized as follows: the relativistic corrections to the parameters of reaction kinematics are introduced in Section 2; the results of optical model calculations with and without taking into account such corrections are shown in Section 3; and the summary of this paper is given in Section 4.

## 2 Effects of relativistic corrections to reaction kinematic parameters

In Ref. [7], Satchler demonstrates that the Klein-Gordon equation describing the elastic scattering of a pion from an atomic nucleus can be transformed into a Schrödinger equation by modifying the mass number (in atomic mass unit) and incident energy of the pion projectile. Since the mass of a pion is much smaller than that of the target nucleus, correction to the target mass is neglected. Such a prescription can be generalized to cases of nucleus-nucleus scattering by taking into account the projectile-target symmetry. For such cases, the masses of the projectile and the target nuclei,  $m_p$  and  $m_t$ , respectively, and the effective bombarding energy,  $E_L$  should be translated as [7]:

$$M_p = m_p \gamma_p, \quad (1)$$

$$M_t = m_t \gamma_t, \quad (2)$$

$$E_L = E_{c.m.} M_p / \mu, \quad (3)$$

where  $\mu$  is the reduced mass

$$\mu = \frac{M_p M_t}{M_p + M_t}. \quad (4)$$

The relativistically correct center-of-mass energy  $E_{c.m.}$  is

$$E_{c.m.} = \frac{(k\hbar c)^2}{2\mu}, \quad (5)$$

in which  $c$  is the speed of light,  $k$  is the wave number:

$$k = \frac{m_p c^2}{\hbar c} (\gamma_p^2 - 1)^{1/2} \quad (6)$$

and

$$\gamma_p = \frac{x_p + \gamma_L}{\sqrt{1 + x_p^2 + 2x_p \gamma_L}}, \quad (7)$$

$$\gamma_t = \frac{x_t + \gamma_L}{\sqrt{1 + x_t^2 + 2x_t \gamma_L}}, \quad (8)$$

with

$$x_p = \frac{m_p}{m_t}, x_t = \frac{m_t}{m_p}, \text{ and } \gamma_L = 1 + \frac{E_{lab}}{m_p c^2}. \quad (9)$$

This prescription has been extensively used in, e.g., Refs. [8–11].

## 3 Results of optical model calculations

Recently a systematic nucleus-nucleus potential was proposed with a single-folding model approach,

with which a heavy ion potential is calculated by folding the nucleon-nucleus potential with the nucleon density distributions of the projectile [12, 13]. The Bruyères Jeukenne-Lejeune-Mahaux (JLMB) model nucleon-nucleus potential [14, 15] was used and the systematics of potential parameters was found for heavy targets ( $A \gtrsim 40$ ) with incident energies from 5 to 40 MeV/nucleon. This potential was found to give a reasonable account of the heavy ion potential, even up to 100 MeV/nucleon [4]. With this systematic potential, we are now able to study the effects of relativistic corrections to the reaction kinematic parameters and their dependence on projectile, target masses and incident energies realistically. Figs. 2 and 3 show the angular distributions of elastic scattering cross sections as the ratio to the Rutherford cross sections for  ${}^6\text{Li}$  and  ${}^{40}\text{Ar}$  projectiles, respectively, from  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  at 80 MeV/nucleon, with and without taking into account the relativistic corrections to the reaction kinematic parameters. Clearly, one can see that, for a fixed incident energy, the importance of the above relativistic corrections increases with the increase of the mass of the target nucleus.

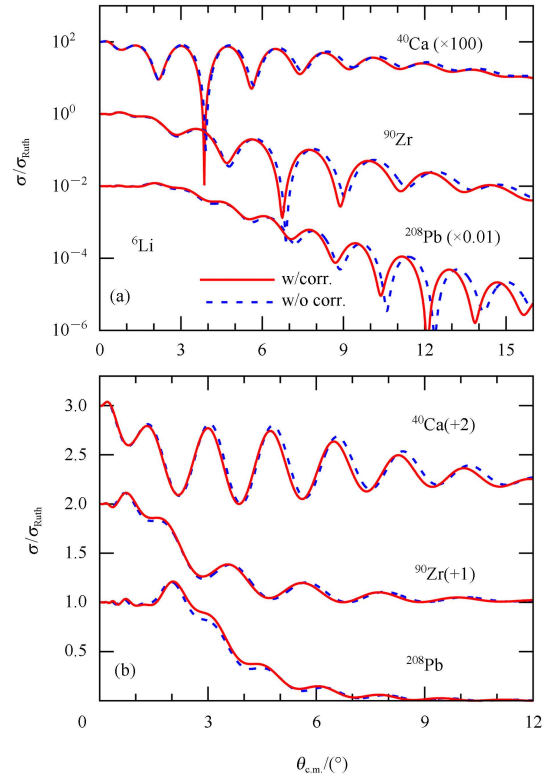


Fig. 2. (color online) Angular distributions of  ${}^6\text{Li}$  elastic scattering from  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  at 80 MeV/nucleon shown in (a) logarithmic scale and (b) linear scale, from optical model calculations with and without taking into account the relativistic corrections to the reaction kinematic parameters.

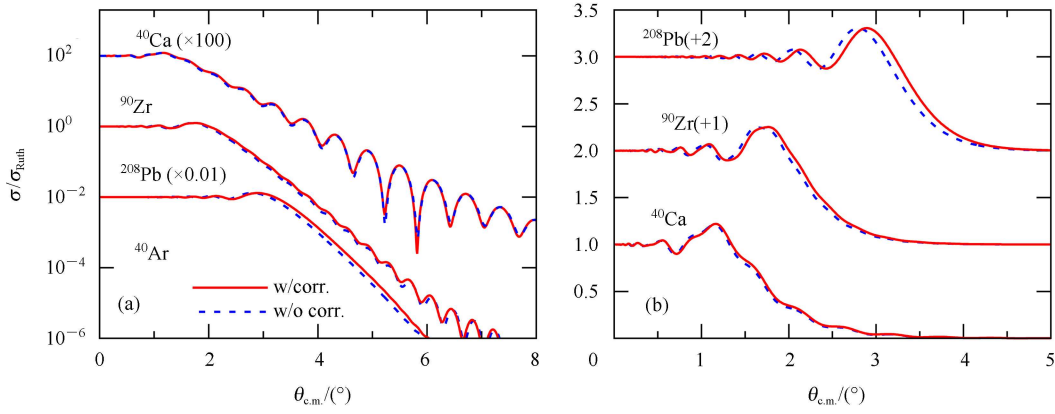


Fig. 3. (color online) The same as Fig. 2 but for the  $^{40}\text{Ar}$  projectile.

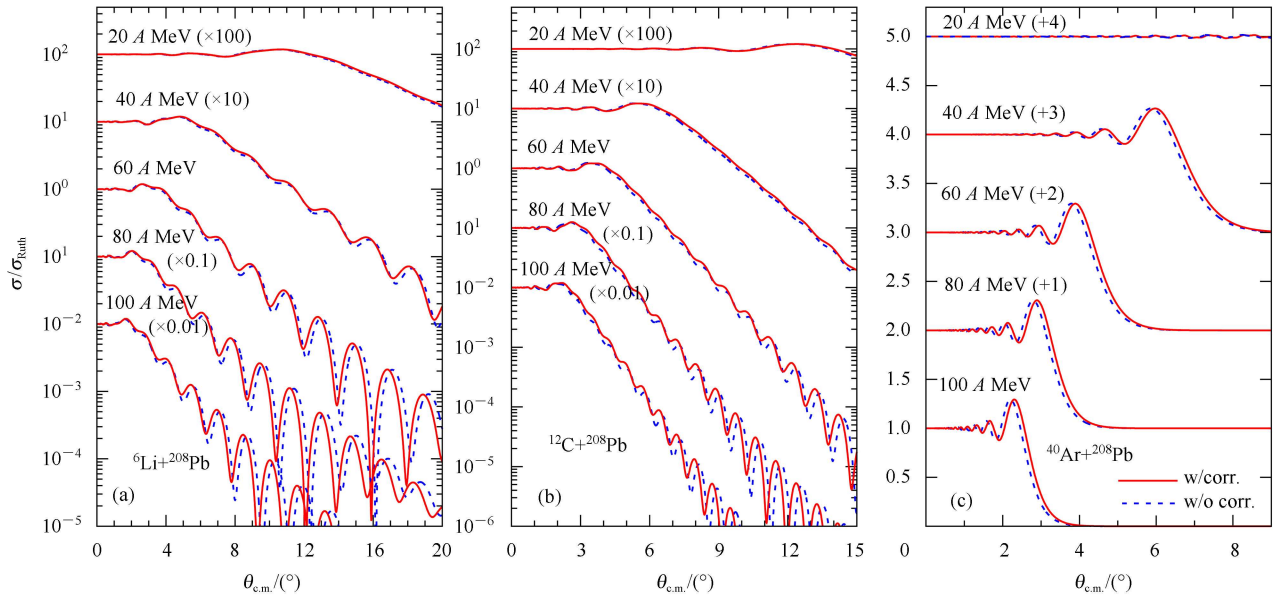


Fig. 4. (color online) Angular distributions of (a)  $^6\text{Li}$ , (b)  $^{12}\text{C}$  and (c)  $^{40}\text{Ar}$  elastic scattering from  $^{208}\text{Pb}$  at 20, 40, 60, 80 and 100 MeV/nucleon from optical model calculations with and without taking into account the relativistic corrections to the reaction kinematic parameters.

In order to study how the importance of the relativistic corrections evolves with the incident energy of the projectile we study  $^6\text{Li}$ ,  $^{12}\text{C}$  and  $^{40}\text{Ar}$  elastic scattering from  $^{208}\text{Pb}$  at 20, 40, 60, 80 and 100 MeV/nucleon. The results are shown in Fig. 4. Two types of angular distributions are clearly shown: (i) the Fraunhofer-like scattering, which shows oscillations at large angles when the Coulomb potential is not so strong compared with the nuclear potential (i.e.,  $\eta \lesssim 1$ , where  $\eta = Z_p Z_t e^2 \mu / \hbar^2 k$  is the Sommerfeld parameter with  $Z_p$  and  $Z_t$  being the charge numbers of the projectile and target nuclei, and  $\mu$  being the reduced mass) and (ii) the Fresnel-like scattering, which does not show oscillations at large angles when the Coulomb potential is strong ( $\eta \gg 1$ ) [16].

It is well known that the separation of successive maxima or minima of the Fraunhofer-like scattering,  $\Delta\theta$ , relates to the grazing angular momentum  $\lambda_g$  and the wave number  $k$  by:

$$\Delta\theta \approx \frac{\pi}{\lambda_g} \approx \frac{\pi}{k R_g}, \quad (10)$$

where  $R_g$  is the critical or grazing radius at which the projectile and target nuclei begin to experience the strong nuclear interaction acting between them [16]. We can thus quantify the effect of relativistic corrections to Fraunhofer-like scattering by the change of wave numbers  $k$ . The result is shown in Fig. 5, where the ratio  $k/k^{\text{rel}}$  is plotted as a function of the incident energy for

${}^6\text{Li}$ ,  ${}^{12}\text{C}$  and  ${}^{40}\text{Ar}$  projectiles with  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  targets with  $k^{\text{rel}}$  and  $k$  being the wave numbers calculated with and without taking into account the relativistic corrections. One can see that the change in  $k$  decreases with the increase of projectile mass and increases with the increase of the target mass, which is consistent with the observation in angular distributions shown in Figs. 2, 3 and 4.

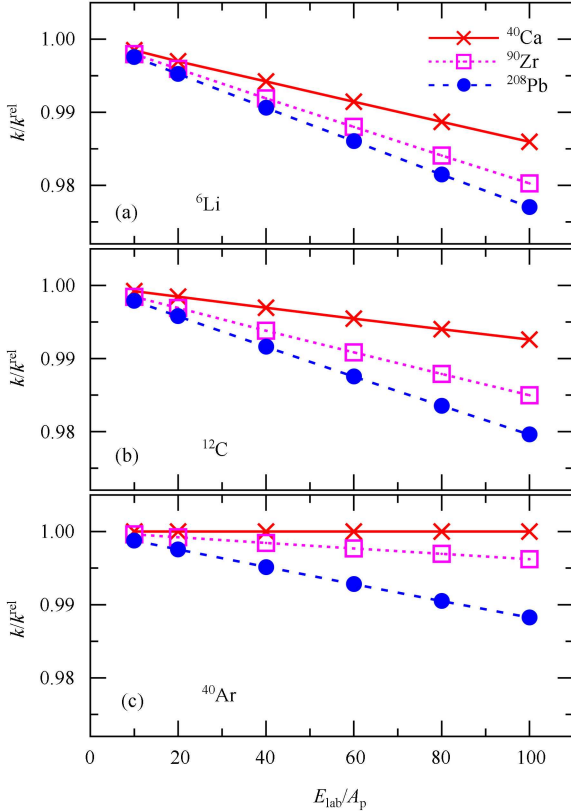


Fig. 5. (color online) Effect of the relativistic corrections on wave number  $k$  as a function of incident energies with target nuclei  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  for (a)  ${}^6\text{Li}$  and (b)  ${}^{12}\text{C}$  and (c)  ${}^{40}\text{Ar}$ .

A Fresnel-like scattering, on the other hand, does not show many oscillations in the angular distributions. It is characterized with the quarter-point angle  $\theta_{\frac{1}{4}}$ , which relates the wave number  $k$  by [16]:

$$kR_{\frac{1}{4}} = \eta \left[ 1 + \text{cosec} \left( \frac{1}{2} \theta_{\frac{1}{4}} \right) \right], \quad (11)$$

where  $R_{\frac{1}{4}}$  is the interaction radius, which is taken to be the closest approach for the orbit  $L_{\frac{1}{4}}$  that satisfies

$$L_{\frac{1}{4}} + \frac{1}{2} = \eta \cot \left( \frac{1}{2} \theta_{\frac{1}{4}} \right). \quad (12)$$

For such Fresnel-like scattering, we choose to quantify the effect of relativistic corrections by changes in  $\theta_{\frac{1}{4}}$ . The results of optical model calculations show that for

${}^{40}\text{Ar}$  elastic scattering from  ${}^{208}\text{Pb}$  at 20, 40, 60, 80 and 100 MeV/nucleon, the changes in  $\theta_{\frac{1}{4}}$  are 1%, 1.5%, 2.2%, 2.9% and 3.6%, respectively, which can be seen in Fig. 4(c). Note that such amount of changes are distinguishable with experimental data, as can be seen from Fig. 1.

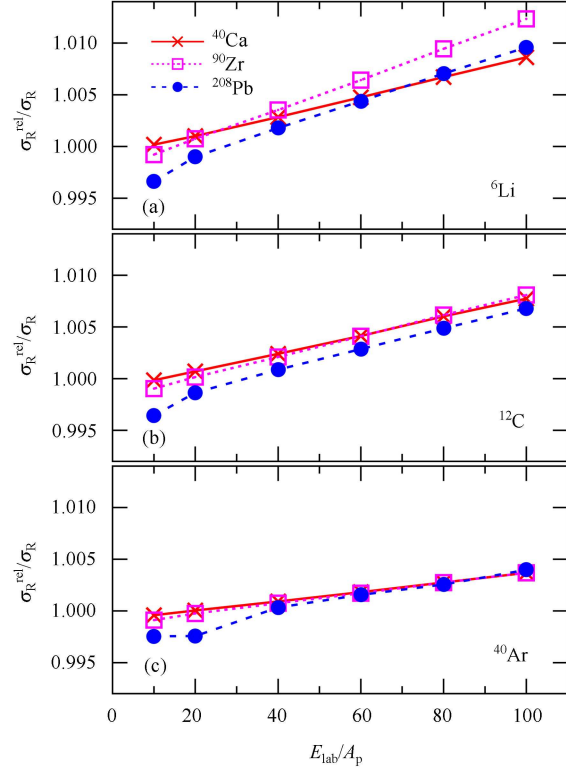


Fig. 6. (color online) Effect of relativistic corrections on total reaction cross sections from optical model calculations as a function of incident energies with target nuclei  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  for (a)  ${}^6\text{Li}$ , (b)  ${}^{12}\text{C}$  and (c)  ${}^{40}\text{Ar}$ .

We also evaluate the effect of the above relativistic corrections on the total reaction cross sections  $\sigma_{\text{R}}$  for projectiles  ${}^6\text{Li}$ ,  ${}^{12}\text{C}$  and  ${}^{40}\text{Ar}$  with  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  targets. The results are shown in Fig. 6 as ratios  $\sigma_{\text{R}}^{\text{rel}}/\sigma_{\text{R}}$ , where  $\sigma_{\text{R}}^{\text{rel}}$  and  $\sigma_{\text{R}}$  are the total reaction cross sections obtained with and without taking into account the relativistic corrections in the optical model calculations. For most of the cases we studied, the changes in  $\sigma_{\text{R}}$  are within 0.5%, which is negligible compared with the typical experimental uncertainties in a total reaction cross section measurement. Note that the authors of Ref.[2] found that the effect of relativistic corrections to the total reaction cross section of  ${}^{40}\text{Ar}$  and  ${}^{60}\text{Ni}$ ,  ${}^{120}\text{Sn}$  and  ${}^{208}\text{Pb}$  targets at 44 MeV/nucleon were 0.2%, 1.2% and 2.2%, respectively, which are much larger than the results shown in Fig. 6. There is no conflict between the

results in this paper and those in Ref. [2] because in this work, the optical model potentials were fixed. This is the reason why the changes in the total reaction cross sections are small. In Ref. [2], the reported changes in  $\sigma_R$  were due to the use of two different sets of optical model potentials obtained by fitting the experimental data with and without applying the relativistic corrections to the reaction kinematics.

## 4 Summary

In summary,  ${}^6\text{Li}$ ,  ${}^{12}\text{C}$  and  ${}^{40}\text{Ar}$  elastic scattering from  ${}^{40}\text{Ca}$ ,  ${}^{90}\text{Zr}$  and  ${}^{208}\text{Pb}$  with incident energies between 20

and 100 MeV/nucleon were studied with optical model calculations. The effects of relativistic corrections to the reaction kinematic parameters were studied for their angular distributions and the total reaction cross sections. The results of the calculations show that the relativistic corrections are important for describing the angular distributions of heavy ion elastic scattering cross sections at incident energies as low as around 40 MeV/nucleon. The total reaction cross sections are found to be little affected by such corrections. Besides the scattering problems studied in this paper, it might be interesting to study how important these relativistic effects are in decay problems, such as the  $\alpha$  decay studied in Ref. [17].

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