

Rearrangements of interacting Fermi liquids^{*}

YANG Rong-Yao(杨荣瑶) JIANG Wei-Zhou(蒋维洲)¹⁾

Department of Physics, Southeast University, Nanjing 210000, China

Abstract: The stability condition of the Landau Fermi liquid theory may be broken when the interaction between particles is strong enough. In this case, the ground state is reconstructed to have a particle distribution different from the Fermi-step function. For specific instances, one case with the vector boson exchange and another with the relativistic heavy-ion collision are taken into consideration. With the vector boson exchange, we find that the relative weak interaction strength can lead to the ground-state rearrangement as long as the fermion mass is large enough. It is found that the relativistic heavy-ion collision may also cause the ground-state rearrangement, affecting the statistics of the collision system.

Key words: Landau Fermi liquid theory, ground-state instability, relativistic heavy-ion collisions, vector bosons

PACS: 71.10.Hf, 24.10.Jv, 26.60.+c **DOI:** 10.1088/1674-1137/38/10/104104

1 Introduction

Since the high T_c superconductor and quantum Hall effect were discovered in the 1980s, more and more systems which disobey the Landau Fermi liquid theory have been springing up. It is known that the Landau Fermi liquid theory is not valid for one-dimensional (1D) systems and strongly correlated 2D liquids [1, 2]. Khodel et al. [3, 4] demonstrated that a state with the fermion condensation or multi-connected momentum distribution arises as the stability condition is broken and accordingly the ground state is rearranged. When the ground-state rearrangement occurs, systems can possess novel properties that are characteristic of a non-Fermi-liquid behavior. Such a rearrangement can also occur for nuclear ground states at very high densities [5, 6], and can have possible implications in astrophysical issues. For instance, the direct Urca process may be remodelled when the rearrangement occurs, so that the fast cooling of neutron stars is probable [7]. It is interesting to notice that the rearrangement discussed in the literature is driven by the enhancement of repulsion in a strongly correlated system [4–6].

In the present work, we consider two extreme examples that may undergo a ground-state rearrangement due to repulsive interactions. One is that the Fermi liquid system with the weak repulsion provided by the vector boson exchanges. The building block of the very Fermi

liquid system can be the weakly interacting massive particles that may possibly aggregate in the core of compact stars. It was pointed out [8] that it could exist in the fifth force whose strength is certainly very weak. At least, this possibility has not been excluded completely. Recently, considering this in terms of the exchange of the U-boson, a few groups found that the U-boson that couples with nucleons weakly can have a significant effect on properties of neutron stars [9–11]. It is thus interesting to examine whether and how the weak coupling can also cause the ground-state rearrangement in specific systems.

Another example regards the furious deceleration in the heavy-ion collisions in the time scale of the typical strong interaction [12, 13]. According to the equivalence principle of the general relativity, the deceleration or acceleration is identical to a local gravitational field. As the usual particle statistics neglects the space-time character, its consideration is of special interest and significance [14]. The present investigation serves the purpose to mimic fermion rearrangements in strongly curved space that exist during spectacular supernova explosions and in the horizon of a black hole. We will focus on determining the conditions of the occurrence of the ground-state rearrangement in these two scenarios.

The paper is arranged as follows. In Section 2, the stability condition of ground states together with the ground-state rearrangement is demonstrated for Fermi liquids. The numerical results are presented in Section 3. Finally, the summary is given.

Received 30 December 2013, Revised 25 March 2014

^{*} Supported by National Natural Science Foundation of China (10975033, 11275048) and China Jiangsu Provincial Natural Science Foundation (BK20131286)

1) E-mail: wzjiang@seu.edu.cn

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2 Stability condition of ground states

2.1 Necessary stability condition for the Fermi liquid

The ground-state stability condition can be established on the fact that the variation of the ground state energy is positive for any admissible variations of distribution. Namely, the excitation states have a higher energy than the ground state, which means that

$$\delta E_0 = \int [\epsilon(k) - \mu] \delta n(\mathbf{k}) \frac{d^3 \mathbf{k}}{(2\pi)^3}, \quad (1)$$

is positive when the conservative condition for the particle number is satisfied: $\int \delta n(\mathbf{k}) \frac{d^3 \mathbf{k}}{(2\pi)^3} = 0$ [15], where ϵ is the single-particle energy and μ is the chemical potential. The Landau Fermi liquid theory tells us that the ground-state distribution of a homogeneous Fermi liquid is a Fermi-step function $n_{\text{F}}(k) = \theta(k_{\text{F}} - k)$ with k_{F} being the Fermi momentum. Namely, the states are filled below the Fermi momentum and are empty above it. The stability condition is satisfied if the single-particle energy above the Fermi surface is larger than that below it. Otherwise, it is violated. That is to say, $s(k) = \frac{\epsilon(k) - \epsilon(k_{\text{F}})}{k - k_{\text{F}}}$, needs to be positive for the whole momentum range. Indeed, $\epsilon(k_{\text{F}})$ is the chemical potential μ when the temperature $T \approx 0$. As $\lim_{k \rightarrow k_{\text{F}}} s(k) = \left. \frac{d\epsilon(k)}{dk} \right|_{k=k_{\text{F}}}$, the ground state of the Landau Fermi liquid theory becomes unstable provided that $d\epsilon/dk$ is negative or zero in the vicinity of the Fermi momentum. In this case, the mass of quasi-particles $M_{\text{Q}} = k_{\text{F}}/v_{\text{F}}$ is consistently negative or infinity once the value of the group velocity

$$v_{\text{F}} \leq 0 \left(v_{\text{F}} = \left. \frac{d\epsilon(k)}{dk} \right|_{k=k_{\text{F}}} \right).$$

In a word, we obtain a necessary stability condition

$$\begin{aligned} \epsilon(k) = & \sqrt{k^2 + M^{*2}} + \left[\frac{\gamma g_{\omega}^2}{2\pi^2 m_{\omega}^2} \int_0^{\infty} p^2 n(p) dp + \sum_{i=\omega, \rho} \frac{g_i^2 c_i}{8\pi^2 k} \int_0^{\infty} p n(p) \ln \frac{(k+p)^2 + m_i^2}{(k-p)^2 + m_i^2} dp \right] \\ & + \frac{\gamma g_{\nu}^2}{2\pi^2 m_{\nu}^2} \int_0^{\infty} p^2 n(p) dp + \frac{g_{\nu}^2}{8\pi^2 k} \int_0^{\infty} p n(p) \ln \frac{(k+p)^2 + m_{\nu}^2}{(k-p)^2 + m_{\nu}^2} dp, \end{aligned} \quad (2)$$

where M^* is the fermion effective mass which is density dependent and moderately model-dependent, m_{ν} is the mass of the vector boson of interest (e.g. the U boson [11]), γ is the degree of degeneracy which is 4 with the spin and isospin considered, g_{ν} is the boson-fermion coupling constant, and $c_i = 1, 3$ for the ω and ρ meson, respectively. The terms in square brackets represent the contribution of the vector mesons ω and ρ in symmetric nuclear matter. In Eq. (2), besides the usual vector

for the Landau Fermi liquid theory: $d\epsilon/dk$ has to be positive near k_{F} (in this paper we define the interval $0.95k_{\text{F}} \sim 1.05k_{\text{F}}$ as the nearby k_{F} to examine the stability condition).

With the breaking of the stability condition, the ground state can only be reconstructed by means of the rearrangement. In Fig. 1, we show as an example the reconstructed ground-state distribution after the rearrangement.

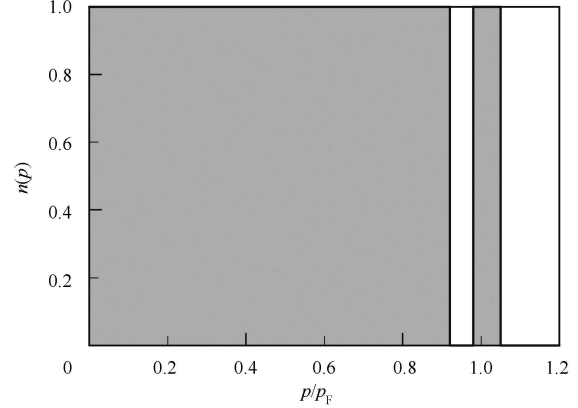


Fig. 1. Reconstructed ground-state distribution after the rearrangement. In this case, the ground state is multi-connected.

2.2 Stability condition for systems with vector boson exchanges

The first example that concerns the ground-state rearrangement is realized by the addition of repulsion with the exchange of the vector boson. The addition of repulsion may be supposed in strongly interacting nuclear matter or in weakly interacting dark matter. We can examine the system stability by analyzing its energy spectrum as above. In the relativistic Hartree-Fock (RHF) approximation, the energy momentum distribution is given as

mesons ω and ρ in the relativistic nuclear models, we have included the additional vector boson (denoted by the subscript ν) in the original model to provide the additional repulsion that may result in a limited modification to the nucleon effective mass.

Substituting the Fermi-step momentum distribution into Eq. (2), we can obtain the explicit expression of the derivative

$$\begin{aligned} \frac{d\epsilon(k)}{dk} = & \frac{k}{\sqrt{k^2+M^{*2}}} - \sum_{i=\omega,\rho} \left(\frac{g_i^2 c_i}{8\pi^2 k} \int_0^{k_F} p \left[\frac{1}{k} \ln \frac{(k+p)^2+m_i^2}{(k-p)^2+m_i^2} - \frac{2(k+p)}{(k+p)^2+m_i^2} + \frac{2(k-p)}{(k-p)^2+m_i^2} \right] dp \right) \\ & - \frac{g_v^2}{8\pi^2 k} \int_0^{k_F} p \left[\frac{1}{k} \ln \frac{(k+p)^2+m_v^2}{(k-p)^2+m_v^2} - \frac{2(k+p)}{(k+p)^2+m_v^2} + \frac{2(k-p)}{(k-p)^2+m_v^2} \right] dp. \end{aligned} \quad (3)$$

It must be noted that the direct Hartree terms in Eq. (2) have no contribution to the derivative as they are independent of momentum k . As discussed before, the momentum distribution of the ground state deviates from the step function once $d\epsilon/dk \leq 0$ in the vicinity of the Fermi surface. When the coupling constant g_v is larger than some critical value g_{vc} , the zero point of $d\epsilon/dk$ will appear near k_F . By numerical integration, we can search the zero point for $d\epsilon/dk$ in the interval $0.95k_F \leq k \leq 1.05k_F$, and the minimum g_v for possessing at least one zero point is the critical coupling constant g_{vc} .

2.3 Necessary stability condition for collision systems

It is well-known that the local effect of gravity on a physical system is identical to the effect of linear acceleration on query1"of" changed to "on" please check that this retains the authors intended meaning the system according to Einstein's equivalence principle, and vice versa. In accelerated systems, the Hamiltonian comprises of a term provided by the non-inertial effect

$$H_{\text{nin}} = \frac{A^2}{c^2} \sum_i M_i z_i^2, \quad (4)$$

where A is the acceleration of the system, c is the velocity of light (equal to 1 in the natural unit), M is the mass of the particle (usually the effective mass M^* is used instead), z is the particle's coordinate along the direction of acceleration of the reference frame (in the case of collision, z is equal to half the distance between colliding particles, i.e. $z=r/2$ in the center of the mass reference frame) [16].

The non-inertial contribution in Eq. (4) has a form of the harmonic oscillator. In the Green function method or Feynman diagrammatic approach, an explicit expression of the propagator of the harmonic oscillator is mathematically complicated, and the numerics seem to be not straightforward [17, 18]. In order to simplify this tedious issue, we suppose that the repulsive force decelerating the particles during collision is provided by a postulated vector meson exchange through a well-known Yukawa coupling. At the same time, we assume that the step function distributions are applied to systems prior to the collision. The non-inertial effect can then be described by the vector meson exchange between nucleons which is sophisticatedly applied in modern physics. Instead of the given non-inertial effect term, we can use the Yukawa po-

tential $g^2 e^{-rm_v}/4\pi r$, i.e., we let $MA^2 z^2 = g^2 e^{-rm_v}/4\pi r$, where g is a dimensionless coupling constant, m_v is the mass of the postulated meson (generally in the same order of magnitude with nucleon mass), r is the distance between nucleons. The propagator of the Yukawa potential is very simple: $F(k) = g^2/(k^2+m_v^2)$. The single-particle energy for colliding systems can be expressed as

$$\begin{aligned} \epsilon(\mathbf{k}) = & \sqrt{\mathbf{k}^2+M^{*2}} + \sum_{i=\omega,\rho} \left[\frac{g_i^2 c_i}{8\pi^2 k} \int_0^\infty pn(p) \ln \frac{(k+p)^2+m_i^2}{(k-p)^2+m_i^2} dp \right] \\ & + \int \frac{g^2}{(\mathbf{k}-\mathbf{p})^2+m_v^2} n(\mathbf{p}) \frac{d^3\mathbf{p}}{(2\pi)^3}, \end{aligned} \quad (5)$$

where the direct Hartree terms are neglected because they do not contribute to the momentum derivative that is used to determine the critical coupling constant. Here, we quote the same coefficient c_i as in Eq. (2) by assuming the symmetric matter after the collision. This is a reasonable assumption to neglect the effect of isospin asymmetry because the coupling strength of the ρ meson is much less than that of the ω meson and the isospin asymmetry is usually not large for the colliding system. After some straightforward calculations, one can find that the derivative $d\epsilon/dk$ is exactly the same as Eq. (3), provided g_v and g_{vc} are replaced by g and g_c , respectively. With the given quantities M^* and m_v at the density ρ , we adjust the coupling constant g to exceed some critical value g_c where $d\epsilon/dk$ has the zero point in the vicinity of the Fermi surface. Then we can find out the critical coupling constant for the ground-state rearrangement.

3 Numerical results and discussions

To obtain the fermion distribution function, one needs first to solve the single-particle energy $\epsilon(k)$. Once the rearrangement occurs, the fermion distribution function becomes multi-connected and the numerical realization is quite tricky due to the existence of sharp edges. We do not elaborate the numerical details, and readers can be referred to Ref. [4] for details. In the following, we discuss in turn the occurrence of ground-state rearrangement in cases of the vector boson exchanges and relativistic heavy-ion collisions.

3.1 Rearrangement with vector boson exchanges

Here we first work on the relativistic model SLC [19]. The additional vector boson is added to this model to provide additional repulsion, and the nucleon effective

mass is obtained in the RHF approximation [20]. With the density-dependent nucleon effective mass, we are able to investigate the rearrangement self-consistently at various densities. In what follows, we will discuss the rearrangement in a simple system that consists of dark matter in which the fermion mass is taken as a free parameter by assuming that the dark matter is just weakly interacting. In this case, we intend to see whether the rearrangement can occur with the weak repulsion.

After obtaining the nucleon effective mass in the RHF approximation, we can calculate the critical coupling constant for the ground-state rearrangement. In Fig. 2, the critical coupling constant is plotted as a function of density for various choices of the vector boson mass: $m_v=0.001, 0.010$ and 0.100 GeV. As one can see from Fig. 2, the critical coupling constant g_{vc} ascends monotonically with increasing density when m_v is relative small ($m_v=0.001, 0.010$ GeV), while for the large mass ($m_v=0.100$ GeV), g_{vc} increases fast in the subsaturation density region, then goes down slowly with the successive increase of the density. In all cases, the critical coupling constant saturates at high densities.

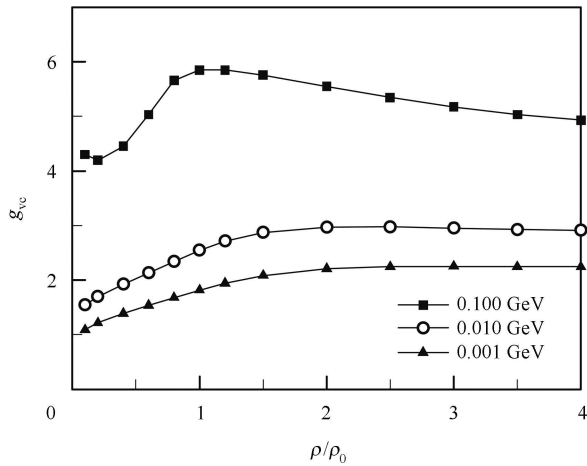


Fig. 2. The critical coupling constant as a function of density. It is solved in the RHF approximation with the parametrization of the model SLC with the addition of the additional vector boson exchange.

Recently, the vector boson beyond the standard model, dubbed the U-boson that may account for the modification to the inverse square law of the Newtonian gravity, has been introduced to compensate the excessively small pressure suggested by the super-soft symmetry energy [10]. The super-soft equation of state (EOS)

can be successfully remedied by the repulsion provided by the U-boson with the ratio of the coupling constant to the mass of vector boson ranging from 7.07 GeV^{-1} to 12.25 GeV^{-1} [10]. Later on, successive work indicates that the U-boson with the ratio around 10 GeV^{-1} can even remedy largely the deviation between the soft and stiff EOS's and the difference in the mass-radius relation of neutron stars as well [11]. We are intrigued to see whether the ground-state rearrangement can occur with these ratio parameters of the U-boson. Unfortunately, as displayed in the Fig. 2, various g_{vc}/m_v are all larger than 40 GeV^{-1} . Thus, it is usually impossible for nucleon systems to rearrange its ground state with the regular ratio g_v/m_v that is around 10 GeV^{-1} , and we are not able to anticipate the constraint from whether or not the ground-state rearrangement occurs on the U-boson parameters. On the other hand, this also reveals the fact that using the Landau Fermi liquid theory is appropriate for the usual nuclear Fermi liquid.

As seen in Eq. (3), the positive kinetic term drops with the increase of the fermion mass so that smaller repulsion is needed for the rearrangement of the ground state. This intrigues us to consider dark matter that is invisible by the standard-model strong, electro-weak but gravitational interaction, because no constraint on the mass of dark matter candidates can be used to exclude the mass region from the magnitude of eV to several hundreds of GeV [21]. Supposing that dense dark matter can be formed, we now examine the ground-state rearrangement of dark matter by taking the mass of fermionic dark matter candidates as a free parameter. In this way, we still use Eq. (3) by simply neglecting the terms concerning the meson exchanges that characterize the strong interactions. Specifically, we calculate the g_{vc}/m_v at $m_v=0.010$ GeV for various M at the number density that is equal to the nuclear saturation density. The results are listed in Table 1. It is seen that the g_{vc}/m_v drops clearly with increasing the mass of the dark matter candidate. For $M=300.00$ GeV, the critical coupling constant becomes as small as 0.11 which gives an interaction strength comparable to the electromagnetic interaction. This reveals a fact that the matter consisting of superheavy fermions can not be regarded as a simple Fermi liquid as long as a weak repulsion exists. Supposing that the heavy dark matter candidates stack up in the core of dense stars, the rearrangement may easily occur therein provided the repulsion comes up with the exchange of weakly interacting bosons.

Table 1. Ratios of the critical coupling constant to the vector boson mass at various M for $m_v=0.010$ GeV and $\rho=0.16 \text{ fm}^{-3}$.

M/GeV	0.10	0.50	1.00	5.00	10.00	50.00	100.00	200.00	300.00
$(g_{vc}/m_v)/\text{GeV}^{-1}$	352.44	248.15	183.29	83.27	58.91	26.35	18.63	13.18	10.76

Table 2. Critical coupling constants and accelerations for various masses of a postulated vector meson. Here, we take $\rho=0.16 \text{ fm}^{-3}$ and $R_0=10 \text{ fm}$.

m_ν/GeV	0.1350	0.2626	0.4976	0.5478	0.6424	0.7800	0.8900	0.9578
g_c	7.19	12.89	27.81	31.91	40.57	55.44	69.29	78.71
$(g_c/m_\nu)/\text{GeV}^{-1}$	53.27	49.09	55.89	58.25	63.16	71.08	77.86	82.17
A_c/fm^{-1}	0.0441	0.0538	0.0855	0.0946	0.114	0.147	0.179	0.200

3.2 The case of collision

For the heavy-ion collision [12, 13], one needs in principle to determine the density of crashed matter that depends on the collision energy. However, it is complicated and beyond the scope of the present work. Considering the fact that the critical coupling constant for the ground-state rearrangement does not change much as the density rises up around the saturation density, see Fig. 2, we demonstrate the rearrangement at saturation density. For numerical trials at other densities, the results are qualitatively similar, and we will not specify the numerical details. Again, we adopt the basic SLC parametrization to include the non-inertial effect and work out the nucleon effective mass self-consistently in the RHF.

Though the non-inertial effect is hypothetically replaced by the exchange of the vector meson, we do not have the constraint on its mass. In practical calculations, we thus change the postulated meson mass from the lowest pion mass to the one close to 1 GeV. At the same density, we can calculate different critical coupling constants for different postulated meson masses. For the instance of $m_\nu=0.5478 \text{ GeV}$, we obtain that at $k \approx 0.95k_F$ the derivative $d\epsilon/dk$ is negative for $g > g_c = 31.91$. The results are tabulated in Table 2. We see that the critical coupling constant of the postulated meson increases clearly with the mass, while the ratio of the coupling constant to the mass changes much slower.

As one can anticipate, to make sure the occurrence of rearrangement, the inequality $|A| \geq A_c$ should hold, while A_c is the minimum acceleration to rearrange the ground state when the distance between the colliding particles is given. Supposing the full stopping of the colliding system leads eventually to a fireball with the radius of a few femtometres, we estimate the A_c by averaging the Yukawa-type coupling and the coupling of the harmonic oscillator within such a sphere, namely,

$$A_c^2 = \left(\int_0^{R_0} d^3r g_c^2 \frac{e^{-rm}}{r} \right) / \int_0^{R_0} d^3r \pi M^* r^2. \quad (6)$$

In this way, different critical decelerations for different postulated meson masses can be estimated. Here, we use $\rho = 0.16 \text{ fm}^{-3}$ and $R_0=10 \text{ fm}$ as an example, and results are tabulated in Table 2. As one can see, for $m=0.5478 \text{ GeV}$, with $g_c = 31.91$, we get

$|A| \geq 0.0946 \text{ fm}^{-1}$ for the occurrence of rearrangement ($|A_c| \approx 8.50 \times 10^{30} \text{ m/s}^2$). This critical deceleration is really tremendous. However, it is not too difficult to reach such an enormous deceleration in relativistic heavy-ion collisions. For instance, two nuclei, being accelerated almost up to the light velocity, crash against each other to be in full stopping in the time scale of a few femtometres, e.g., 5–10 fm, which is comparable to the size of stopped matter. Then, the deceleration is 0.1–0.2 fm^{-1} which gives a reasonable and consistent magnitude of deceleration as compared with those in Table 2. The present estimate indicates that the non-inertial effect because of the huge deceleration can lead to the ground-state rearrangement. Note that we do not consider the effect of thermalization at the late stage of the collision. The estimate also implies that as long as the gravitational field is sufficiently strong, the ground-state rearrangement may lead to a deviation from the quasi-particle Fermi liquid. For the case of a larger density formed by the collision, we can get moderately smaller critical decelerations. However, the conclusion does not alter much for various densities of matter.

4 Summary

We have investigated the applicability of the Landau Fermi liquid theory and found that the necessary stability condition for this theory may be broken in some circumstances. Once the necessary stability condition is broken, the ground state is rearranged to be different from the step function. In our relativistic model, the nuclear Fermi liquid is stable with the usual strong interaction strength and no rearrangement occurs. This is nevertheless in sharp contrast with the system consisting of very heavy fermions that may be regarded as dark matter candidates. We have found that it is possible for heavy-fermion matter to rearrange the ground state even by exchanging the weakly coupling vector boson. In the case of the relativistic heavy-ion collisions, the ground state of the colliding systems can be rearranged due to the strong non-inertial effect which changes the ways of the quasi-particle occupation. This finding is potentially instructive in investigating the fermion system during the supernova explosions and in the horizon of black holes.

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