Scalar glueball in a soft-wall model of AdS/QCD *

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Abstract: The scalar glueball is investigated in a soft-wall model of AdS/QCD. Constraints of the mass of the scalar glueball are given through an analysis of a relation between the bulk mass and the anomalous dimension. The mass of the ground scalar glueball is located at $0.96^{+0.04}_{-0.07}$ GeV $< m_{\rm G} <$ $1.36^{+0.05}_{-0.10}$ GeV. In terms of a background dilaton field $\Phi(z)=cz^2$, the two-point correlation function for the scalar gluon operator is obtained. The two-point correlation function at $\Delta=4$ gives a different behavior compared with the one in QCD.

Key words: AdS/QCD, soft-wall model, scalar glueball

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1 Introduction

The self-interaction among gluons is a distinctive feature in QCD. Bound gluon states, glueballs, may exist in QCD; they were first mentioned by Fritzsch et al. [1] Though the properties of glueballs cannot be obtained analytically through QCD, they are predicted in many models based on QCD or in lattice QCD. Therefore, the discovery of a glueball will be a direct test of QCD theory. Physicists have searched for the glueball for a long time; so far, there has been no conclusive evidence of the glueball, except for several observed candidates.

No matter whether glueballs exist or not, it is important to understand their spectroscopy. So far, there has not been much work concentrated on the production and decay of glueballs, but there have been many works on the mass of glueballs. Related references can be found in some reviews [2–4]. It is believed that the $J^{PC}=0^{++}$ scalar glueball has the lowest mass. The first assumption of the mass M of the scalar glueball is by Novikov et al., where M was assumed to be around 700 MeV [5]. QCD sum rules predict the mass of this kind of glueball to be around 1.5 GeV [6–8]; lattice theory gives the mass as being around 1.7 GeV [9].

Inspired by the gravity/gauge, or anti-de Sitter/conformal field theory (AdS/CFT) correspondence [10], QCD is in one side modeled on a deformation of the super Yang-Mills theory (the so called top-down [11] approach). On the other side, a five dimensional holographic QCD is constructed from the same correspondence (the so called bottom-up [12] approach). In the framework of holographic QCD, some unsolvable problems in QCD are solvable. In particular, the hadrons of QCD correspond to the normalizable modes of the five dimensional (5d) fields; therefore, many hadronic properties are predictive.

In AdS/QCD, two backgrounds have been introduced: the hard wall approach (an IR brane cutoff) [13] or the soft wall approach (a soft wall cutoff) [14]. For the soft wall cutoff, the dilaton soft wall and the metric soft wall are usually employed. The dilaton soft wall model with smooth cutoff seems suitable for the scalar glueball, which reproduces the expected Regge trajectory.

The mass spectrum of the scalar glueball has been computed in the holographic QCD. In Ref. [15], the mass square of the scalar glueball is predicted as being twice that of the mass square of the ρ meson: $m_{\rm G}^2 = 2m_{\rho}^2$. In Ref. [16], the scalar glueball mass is predicted at 1.34 GeV or 1.80 GeV, corresponding to Neumann or Dirichlet boundary conditions on the IR brane, respectively. Some theoretical predictions to the mass of scalar glueballs are listed in Table 1. Obviously, the mass pre-

Table 1. Mass predictions of the scalar glueball in some literature

literature	Ref. [5]	Ref. [6]	Ref. [7]	Ref. [17]	Ref. [8]	Ref. [9]	Ref. [18]	Ref. [16]
mass/GeV	0.7	1.5±0.2	1.7	1.4 and 1.0–1.25	1.25±0.2	1.71	0.65–0.75	1.34 or 1.80

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diction, which is waiting for an examination by experiment, may be largely different in different models.

The two-point correlation function of the scalar operator has been analyzed in Refs. [16, 19], where the behavior of the correlation function is given and compared with the result of the operator product expansion (OPE) in QCD.

In this paper, we focus on the properties of the scalar glueball in the framework of a dilaton soft-wall. We predict the possible mass of the scalar glueball in Section 2. In Section 3, we give the result of the two-point correlation function for the scalar glueball with $\Phi(z)=cz^2$. The conclusions are included in the final section.

2 Mass of scalar glueball in dilaton soft wall model

According to AdS/CFT correspondence, the correspondence between QCD local operators and fields in the AdS_5 bulk space can be constructed; the following relation exists [10, 15]:

$$m_5^2=(\Delta-P)(\Delta+P-4),$$

where m_5 is the AdS mass of the dual field in the bulk, Δ is the conformal dimension of a (p-form) operator.

It is noticed that the full conformal dimension Δ could be divided into a classical dimension $\Delta_{\rm class}$ and an anomalous dimension $\gamma(\mu)$ [20]:

$$\Delta=\Delta_{\rm class}+\gamma(\mu).$$

The operator ${\rm tr}\{G_{\mu\nu}G^{\mu\nu}\}$ defined on the boundary spacetime is related to the scalar glueball. Following Gubser's proposal [21], the full dimension of operator ${\rm tr}G^2$ (in correspondence to the scalar glueball) is

$$\Delta_{G^2}=4+\beta'(\alpha)-\frac{2\beta(\alpha)}{\alpha},$$

where the prime is denoted to the derivative with respect to α. This expression for scalar glueballs emerges from the trace anomaly of QCD energy momentum tensor T^μ_μ.

The $5d$ massive scalar bulk field X in the soft-wall model of AdS/QCD can be described by an action [20]

$$S_{\rm 5D}=\int {\rm d}^5x\sqrt{-g}{\rm e}^{-\Phi(z)}[g^{mn}\partial_m X\partial_n X+m_5^2X^2], \qquad (1)$$

where the background dilaton field $\Phi(z)=cz^2$ and g is the determinant of the metric tensor in AdS_5 space.

The metric is defined as follows

$${\rm d}s^2\equiv g_{mn}{\rm d}x^m{\rm d}x^n=\left(\frac{R}{z}\right)^2(\eta_{\mu\nu}{\rm d}x^\mu{\rm d}x^\nu+{\rm d}z^2),$$

where $\eta_{\mu,\nu}=(-,+,+,+)$ is the Minkowski metric. In the following, the AdS radius R is assumed to be the unit.

From the action of Eq. (1), the $1d$ Schödinger-type equation of motion is obtained [20]

$$-\tilde{Y}''[z]+\left(c^2z^2+\frac{15}{4z^2}+2c+\frac{{m_5}^2}{z^2}\right)\tilde{Y}[z]=-q^2\tilde{Y}[z], \quad (2)$$

where the Bogoliubov transformation

$$\tilde{X}(q,z)={\rm e}^{\frac{\left(cz^2+3\ln z\right)}{2}}\tilde{Y}[z]$$

is performed. Here, $\tilde{X}$ is the $4d$ Fourier transform of the field X. Through this $1d$ Schödinger-type equation of motion, the spectrum is given $m_{\rm G}^2=4cn+4c+2c\sqrt{4+m_5^2}$ [15].

In the classic case, $\Delta_{\rm class}=4$, ${m_5}^2=0$. Therefore, the mass of the scalar glueball is $m_{\rm G}^2=4c(n+2)$ [20], where n is identified with the radial quantum number. When the anomalous dimension of operator ${\rm tr}\{G_{\mu\nu}G^{\mu\nu}\}$ is taken into account, the conformal dimension is shifted. Such an effect was explored in Ref. [20], where the mass dependence of the Beta function in holographic QCD was also discussed.

The explicit expressions of the anomalous dimension and hence the bulk mass $m_5(z)$ in holographic QCD are usually difficult to obtain. When the anomalous dimension is taken into account, the bulk mass is shifted to

$$m_5^2(z)=\gamma(z)(\gamma(z)+4). \qquad (3)$$

Therefore, the corrected potential with this γ correction in the $1d$ schödinger-type equation becomes

$$\Delta V(z)=\frac{\gamma(z)(\gamma(z)+4)}{z^2}. \qquad (4)$$

It is straightforward to obtain an inequity

$$\Delta V(z)\geqslant\frac{-4}{z^2},$$

where the inequity is saturated for the constant dimension $\Delta=2$ or $\gamma=-2$. Similar analysis was performed to tetraquarks in Ref. [22].

Obviously, in such a case, the Breitenlohner-Freedman relation [23]

$$m_5^2>-\frac{(d-2P)^2}{4}, \qquad (5)$$

holds also, where the scalar and the vector correspond to $P=0$ and $P=1$, respectively [24]. As is well known, to keep the ground state wave-function integrable, the Breitnlohner-Freedman (BF) bound for the bulk mass $\left(\Delta=\frac{d}{2}+\sqrt{\frac{(d-2P)^2}{4}+m_5^2}\ [25]\right)$ must be satisfied.

Accordingly, the lowest limit for the ground ($n=0$) scalar glueball mass is achieved

$$m_{G,\Delta=2,n=0}^2=4c. \qquad (6)$$

On the other hand, the highest limit for the ground scalar glueball mass is

$$m_{G,\Delta=4,n=0}=2\sqrt{2c}.$$

Once c is fixed by the mass spectrum of the vector ρ mesons in AdS/QCD with $c = 0.2325\ \mathrm{GeV}^2 = (0.482\ \mathrm{GeV})^2$ [20], the possible mass of the ground scalar glueball becomes

$$0.96\ \mathrm{GeV} < m_{\mathrm{G}} < 1.36\ \mathrm{GeV}.$$

Of course, there is an uncertainty for the parameter c, which will result in an uncertainty for the mass. From a fitting result for the Regge trajectories of vector and axial-vector mesons ρ and a, c is found to be around 0.20–0.25 [26]. When $c = 0.2325\ \mathrm{GeV}^2 = (0.482\ \mathrm{GeV})^2$ is employed as the central value and the uncertainty of c is taken into account, the possible mass of the ground scalar glueball is

$$0.96^{+0.04}_{-0.07}\ \mathrm{GeV} < m_{\mathrm{G}} < 1.36^{+0.05}_{-0.10}\ \mathrm{GeV}.$$

3 Two-point correlation function for the scalar glueball

In the conjectured AdS/CFT correspondence, the $4d$ field is supposed to locate at the boundary of AdS_5. The bulk field coupling to an operator on the boundary could be detected through the interaction $\hat{O}X_0$ in the Lagrangian. One can compute the correlation function on the boundary according to the Gubser-Klebanov-Polyakov-Witten relation (GKPW) [10]

$$\left\langle \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4 x X_0 \hat{O}(x)} \right\rangle_{\mathrm{CFT}} = \mathrm{e}^{\mathrm{i}\int S_{5\mathrm{D}}[X]}, \tag{7}$$

where $S_{5\mathrm{D}}$ is defined in Eq. (1), and the index zero on the scalar field denotes the source on the boundary. The scalar field $X(x,z)$ is coupled to the lowest dimension operator $\hat{O} = \beta(\alpha)\mathrm{Tr}(G^2)$ for the scalar glueball.

Following the process in Refs. [16, 19], the two-point correlation function in AdS is obtained from the action as follows

$$\Pi_{\mathrm{AdS}}(q^2) = 2\frac{\mathrm{e}^{-cz^2}}{z^3}\tilde{K}(q,z)\partial_z \tilde{K}(q,z)\Big|_{z\to 0}, + 2m_5^2 \frac{\mathrm{e}^{-cz^2}}{z^5}\left(\tilde{K}(q,z)\right)^2\Big|_{z\to 0}, \tag{8}$$

where $\tilde{K}(q,z)$ is the momentum representation of the bulk-to-boundary propagator $K(x,z)$, which satisfies the same equation, Eq. (2), with the bulk field X. Here

$$X(x,z) = \int \mathrm{d}^4 x' K(x-x',z) X_0(x') \tag{9}$$

with $X_0(q) = X(x, z\to 0)$.

At $\Delta = 4$ ($m_5{}^2 = 0$), we have

$$\begin{aligned}\Pi_{\mathrm{AdS},\Delta=4}(q^2) = & -q^2 \upsilon^2 - \frac{1}{4}q^2(4c+q^2)(-2+\gamma_{\mathrm{E}}) \\ & -\frac{1}{4}q^2(4c+q^2) HN\left[1+\frac{q^2}{4c}\right] - \\ & \frac{1}{4}q^2(4c+q^2)\left(\log[c] + 2\log\left[\frac{1}{\upsilon}\right]\right),\end{aligned} \tag{10}$$

where z has been replaced by the renormalization scale υ^{-1}, and HN indicates the Harmonic number.

In the limit $q^2\to\infty$, the two-point correlation function in the short-distance regime transfers into

$$\begin{aligned}\Pi_{\mathrm{AdS},\Delta=4}(q^2) = & \frac{1}{4}\left(2 - 2\gamma_{\mathrm{E}} + \log[4] - \log\left[\frac{q^2}{\upsilon^2}\right]\right) q^4 \\ & + \frac{cq^2}{2} - \upsilon^2 q^2 + cq^2 \log[4] - 2cq^2\gamma_{\mathrm{E}} \\ & + cq^2 \log\left[\frac{\upsilon^2}{q^2}\right] - \frac{5c^2}{3} + \frac{4c^3}{3q^2} \\ & - \frac{8c^4}{15q^4} + O\left[\frac{1}{q^6}\right].\end{aligned} \tag{11}$$

A similar result was obtained in Ref. [19], where the authors discussed the important difference between the AdS expression and the one in QCD in detail. The behavior of the two-point correlation function is presented in Fig. 1, where the renormalization scale is chosen with $\nu = 1$ GeV and the scale parameter is chosen with $c = 0.233\ \mathrm{GeV}^2$. In the figure, the poles at the region $q^2 < 0$ give the mass spectrum $m_{\mathrm{G}}^2 = 4c(n+2)$ [15, 19] with n indicating the radial quantum number. The behavior of the two-point correlation function in QCD for

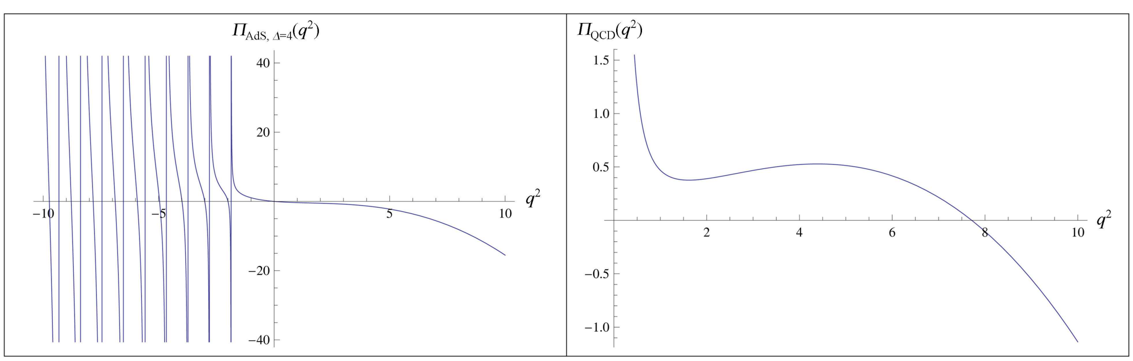

Fig. 1. Behavior of the two-point correlation functions for the scalar glueball in AdS and QCD.

the scalar glueball is also presented in the figure according to Eq. (2.9) in Ref. [19], where the parameters are those from Ref. [27].

In the limit $c \to 0$, namely in the pure anti-de Sitter bulk space, we have a massless boundary theory. In the case of a massive scalar bulk field background with a dilaton field, there appears a term of q^2 involving a dimension two condensate. Detailed discussions about this term have been made in Refs. [16, 19, 28]. As mentioned by Colangelo [19], such a term does not appear in the QCD short-distance expansion, and cannot be expressed as the vacuum expectation value of local operators.

4 Conclusion

The scalar glueball is investigated in the framework of a soft-wall model of AdS/QCD. Through the relation between the bulk mass and the anomalous dimension, we give the constraints on the mass of the scalar glueball. Our results indicate a light scalar glueball. The possible mass of the ground scalar glueball is located at the mass region: $0.96^{+0.04}_{-0.07}$ GeV $< m_{\rm G} < 1.36^{+0.05}_{-0.10}$ GeV. Obviously, the result depends on the choice of the parameter c in AdS/QCD. In order to predict the mass of the scalar glueball, the parameter c has to be fixed more precisely.

The dilaton field is chosen as $\Phi(z) = cz^2$, and the two-point correlation function for the scalar glueball is obtained. In particular, the exact expression of the correlation function is presented at $\Delta = 4$, which shows a different behavior compared with the one in QCD. Phenomenological results related to the two-point correlation for the scalar glueball deserve dedicated exploration.

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