Structure of A=11 double-Λ hypernuclei studied with three-body forces

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Abstract: The energy levels and ΛΛ bond energy of the double-Λ hypernucleus ^{11}Be are calculated considering two- and three-nucleon forces. The interactions between the constituent particles are contact interactions that reproduce the low-energy binding energy of the nuclei. Effective action is constructed to involve the three-body forces. In this paper, we compare the binding energy result that is obtained with the experimental and other modern nucleon-nucleon potentials. The results of all the schemes agree very well, showing the high accuracy of our present ability to calculate the many-nucleon bound state with three-body forces. The experimental value of $B_{\Lambda\Lambda}(^{11}_{\Lambda\Lambda}\mathrm{Be})$=20.83 MeV seems to be more compatible with our calculated value of $B_{\Lambda\Lambda}(^{11}_{\Lambda\Lambda}\mathrm{Be})$=19.31 MeV in comparison with the calculated result of 18.23 MeV by Hiyama et al.

Key words: five-body cluster, A=11 double-Λ hypernuclei, binding energy, three-body forces

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1 Introduction

In nuclear physics, a fundamental problem is to describe the different facets of the interactions among the nucleons in two- and many-body hyperonic nuclei. Recent advances in computational facilities, together with the development of new methods and the refinement of older ones, allow very precise calculations of few-body systems. These advances are especially remarkable in nuclear physics, considering the complexity of the nuclear interaction. Three-nucleon interactions are a frontier, and a coherent theoretical and experimental effort to constrain them in few- to many-nucleon systems is necessary. Moreover, three-nucleon forces with independent T=3/2 components have been developed and implemented by the Urbana-Argonne-Los Alamos group. When combined with the modern nucleon-nucleon Argonne v18 potential, these forces describe the spectra of light nuclei and low-energy scattering in the five-body system [1, 2]. Theoretically, hyperon-nucleon interactions have many similarities, but also many differences with nucleon-nucleon interactions: no one-pion-exchange in Lambda-N, but one-kaon-exchange; important two-pion-exchange with the excitation of an intermediate sigma; and hyperon-nucleon-nucleon forces that may be relatively more important than three-nucleon forces, depending on whether the sigmas are explicitly kept. Better hyperon-nucleon forces would help us understand the role of hyperonic degrees of freedom in high-density matter, e.g. compact stars.

So far, several cluster models have appeared to estimate the ground-state binding energies of double-Λ species [3–9]. Recently, ΛN interaction models have been constructed by utilizing various Λ hypernuclear data to complement the limited ΛN scattering data. A recent finding of the double-Λ hypernucleus in the KEK-E373 experiment has had a great impact not only on the study of baryon-baryon interactions in the strangeness $S=-2$ sector, but also on the study of the dynamics of many-body systems with multi-strangeness [10]. Hiyama et al. reported on a pioneering five-body ααnΛΛ cluster-model calculation of $^{11}_{\Lambda\Lambda}$Be in order to confront a possible interpretation of the KEK-E373 HIDA event [11]. Gal et al. also reported on a six-body ααnnΛΛ calculation of $^{12}_{\Lambda\Lambda}$Be to confront another possible interpretation that is beyond reach at present [12]. They obtained binding-energy shell-model estimates for both $^{11,12}_{\Lambda\Lambda}$Be, using experimental B_Λ values with small corrections based on recently determined ΛN spin-dependent interaction parameters. The results of their calculation conclude that neither $^{11}_{\Lambda\Lambda}$Be nor $^{12}_{\Lambda\Lambda}$Be provide satisfactory interpretation of the HIDA event.

On the other hand, for the baryon-baryon interactions with $S=-2$ sectors that are of concern presently, experimental information has been highly limited due to the extreme difficulties of two-body scattering experiments. Therefore, the observed ΛΛ bond energies of double-Λ hypernuclei should be the most reliable source

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for the $S=-2$ interaction, and such data play a decisive role in determining the strength of underlying $\Lambda\Lambda$ interactions.

In this paper, we present the results of three-body Faddeev-type calculations for the systems of three clusters interacting through short-range nuclear as well as long-range Coulomb interaction. Here we apply the three-cluster Faddeev formalism to the five-body $\alpha\alpha n\Lambda\Lambda$ model for ${}^{11}_{\Lambda\Lambda}$Be. From the early years of hypernuclear study, ${}^{11}_{\Lambda\Lambda}$Be was considered to be a prototype of a five-cluster structure, in which the two α clusters form a loosely bound subsystem, nucleon and the effect of the two Λ hyperon. We have succeeded in performing a five-body calculation of ${}^{11}_{\Lambda\Lambda}$Be using an $\alpha\alpha n\Lambda\Lambda$ cluster model in comparison with the recent observation of the Hida event for a new double Λ hypernucleus. The calculated $\Lambda\Lambda$ binding energy also shows good agreement with other theoretical methods.

The paper is organized as follows. In Section 2 we derive the Faddeev equations for the scattering amplitude. The Faddeev equations will be solved by iteration yielding a multiple scattering series. We tabulate the calculated binding energy, discuss the theoretical errors, and compare our results with the corresponding experimental and theoretical values in Section 3. The summary and conclusions follow in Section 4.

2 Two-cluster $\Lambda\Lambda$, Λn, $\Lambda\alpha$, αn and $\alpha\alpha$ interactions

2.1 αx interactions

As for the potentials $V_{\alpha x}$ between the clusters α and x, we employ those which have often been used in the OCM-based cluster-model study of light nuclei. Namely, they are the $V_{\alpha N}$ potential introduced in Ref. [13], the $V_{\alpha d}$ and $V_{\alpha t}$ potentials given in Ref. [14] and the $V_{\alpha\alpha}$ potential used in Ref. [15], which reproduce the low-lying states and low-energy scattering phase shifts of the αx systems reasonably well. The potentials are described in the following parity-dependent form with the central and spin-orbit terms:

$$V_{\alpha x}(r) = \sum_{i=1}^{i_{\max}} V_i \mathrm{e}^{-\beta_i r^2} + \sum_{i=1}^{i'_{\max}} (-)^l V_i^{\mathrm{p}} \mathrm{e}^{-\beta_i^{\mathrm{p}} r^2} + \left[\sum_{i=1}^{i''_{\max}} V_i^{ls} \mathrm{e}^{-\gamma_i r^2} + \sum_{i=1}^{i'''_{\max}} (-)^l V_i^{ls,\mathrm{p}} \mathrm{e}^{-\gamma_i^{\mathrm{p}} r^2} \right] \boldsymbol{l}\cdot\boldsymbol{s}_x, \quad (1)$$

where V_i^{p} shows the parity-dependent form of the potentials and $\boldsymbol{l}$ is the relative angular momentum between α and x, and $\boldsymbol{s}_{\mathrm{x}}$ is the spin of x. In the $\alpha\alpha$ system, the spin-orbit term is missing and the odd wave is forbidden by the Pauli principle. The additional Coulomb potentials are constructed by folding the proton-proton Coulomb force into the proton densities of the α and x clusters.

2.2 Λx interactions

The interaction between the Λ particle and the x cluster can be described by folding the G-matrix-type YN interaction into the density of the x cluster. The obtained interactions are given in Ref. [16] as

$$v_{\Lambda N}(r;k_{\mathrm{F}}) = \sum_{i=1}^{3} \left[(v_{0,\mathrm{even}}^{(i)} + v_{\sigma\sigma,\mathrm{even}}^{(i)} \boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_{\mathrm{N}}) \frac{1+P_r}{2} + (v_{0,\mathrm{odd}}^{(i)} + v_{\sigma\sigma,\mathrm{odd}}^{(i)} \boldsymbol{\sigma}_\Lambda\cdot\boldsymbol{\sigma}_{\mathrm{N}}) \frac{1-P_r}{2} \right] \mathrm{e}^{-\mu_i r^2}, \quad (2)$$

where P_r is the space exchange (Majorana) operator. The strengths $v^{(i)}$ are represented as quadratic functions of k_{F}; see Eq. (2.7) of Ref. [16] and Table V of Ref. [8] for various original YN interactions. The Λx interaction is derived by folding the above $v_{\Lambda N}(r;k_{\mathrm{F}})$ interaction into the x-cluster wave function. The k_{F} depends on the mass number of the cluster x. Because of the operator P_r in Eq. (3.3), the resultant Λx potential becomes nonlocal [8]. We summarize the functional form of the local and nonlocal parts of the Λx potentials as

$$V_{\Lambda x}(\boldsymbol{r},\boldsymbol{r}') = \sum_{i=1}^{3} (V_i + V_i^s \boldsymbol{s}_\Lambda\cdot\boldsymbol{s}_{\mathrm{x}}) \mathrm{e}^{-\beta_i r^2} \delta(\boldsymbol{r}-\boldsymbol{r}') + \sum_{i=1}^{3} (U_i + U_i^s \boldsymbol{s}_\Lambda\cdot\boldsymbol{s}_{\mathrm{x}}) \mathrm{e}^{-\gamma_i(\boldsymbol{r}+\boldsymbol{r}')^2 - \delta_i(\boldsymbol{r}-\boldsymbol{r}')^2}, \quad (3)$$

where $\boldsymbol{s}_\Lambda = \boldsymbol{\sigma}_\Lambda/2$.

2.3 The Coulomb interaction

The T-matrix of the full Coulomb force is divergent at the diagonal part and the strong oscillation in the momentum representation. The Faddeev calculation with the cut-off Coulomb force is a simple approximation to the unsolvable Faddeev equation with the complete Coulomb force. The Coulomb potential W_{C}, due to its long range, does not satisfy the mathematical properties required for the formulation of standard scattering theory as given in the previous subsection for short-range interaction V_α. The comparison between the data from the nuclear physics experiments and the theoretical predictions with full Coulomb is meaningful only if the full and screened Coulomb becomes physically indistinguishable. We base our treatment of the Coulomb interaction as a Ref. [17], and choose the screened Coulomb potential in configuration-space representation as

$$W_R(r) = W_{\mathrm{C}}(r) \mathrm{e}^{-(r/R)^n}, \quad (4)$$

and then transform it to momentum-space. Here, R is the screening radius and n controls the smoothness of

the screening. The standard scattering theory is formally applicable to the screened Coulomb potential W_R, i.e., the Lippmann-Schwinger equation yields the two-particle transition matrix

$$T_R = W_R + W_R g_0 t_R, \tag{5}$$

where g_0 is the two-particle free resolvent.

2.4 Three-body nuclear reactions

The three-body system is the only nuclear three-particle system that may be considered realistic in the sense that the interactions are given by high precision potentials valid over a broad energy range. The description of three-body nuclear reactions involves a number of approximate methods that have been developed. The methods are the distorted-wave Born approximation, various adiabatic approaches [18], and the continuum-discretized coupled-channels method [19]. The present method based on Faddeev is more technically and numerically involved and has some disadvantages. The Faddeev method may be more flexible with respect to dynamic input, and therefore allows us to test novel aspects of the nuclear interaction that are not accessible with the traditional approaches. The comparison between traditional nuclear reaction approaches and the momentum-space Faddeev method for various systems is summarized here.

The standard nucleon-nucleus optical potentials employed in three-body calculations have central and, eventually, spin-orbit parts that are local. This local approximation yields a tremendous simplification in the practical realization of distorted-wave Born approximation, continuum-discretized coupled-channels and other traditional approaches that are based on configuration-space representations where the use of nonlocal optical potentials has never been attempted. There are very few nonlocal parametrizations of the optical potentials available. We take the one from Refs. [20, 21] defined in the configuration space as

$$v_\gamma(\boldsymbol{r}',\boldsymbol{r}) = H_{\rm c}(x)[V_{\rm c}(y)+{\rm i}W_{\rm c}(y)]+2\boldsymbol{S}_\gamma\cdot\boldsymbol{L}_\gamma H_{\rm s}(x)V_{\rm s}(y), \tag{6}$$

with $x = |\boldsymbol{r}'-\boldsymbol{r}|$ and $y = |\boldsymbol{r}'+\boldsymbol{r}|/2$. The central part has real volume and imaginary surface parts, whereas the spin-orbit part is real; all of them are expressed in the standard way by Woods-Saxon functions. Some of their strength parameters were readjusted in Ref. [22] to improve the description of the experimental nucleon-nucleus scattering data. The range of the nonlocality is determined by the functions $H_i(x) = (\pi\beta_i^2)^{-3/2}\exp(-x^2/\beta_i^2)$ with the parameters β_i being of the order of 1 fm. The description of the (α,Λ,n) three-particle system with real potentials is quite successful at low energies, but becomes less reliable with increasing energy where the inner structure of the α particle cannot be neglected anymore [23]. The methods based on the Faddeev equations can also be applied in this case, however, the potentials within the pairs that are bound in the initial or final channel must remain real.

3 The Faddeev integral equation for the $\alpha\alpha$n$\Lambda\Lambda$ system

We describe the scattering process in a system of three-clusters interacting with short-range, strong interactions v_α, α=1, 2, 3; where, v_1 is the potential between clusters 2($\alpha\alpha$) and 3($\Lambda\Lambda$) [24, 25]. In the framework of non-relativistic quantum mechanics, the center-of-mass (c.m.) and the internal motion can be separated by introducing Jacobi momenta

$$\boldsymbol{p}_\alpha = \frac{m_\gamma\boldsymbol{k}_\beta - m_\beta\boldsymbol{k}_\gamma}{m_\beta+m_\gamma}, \tag{7}$$

$$\boldsymbol{q}_\alpha = \frac{m_\alpha(\boldsymbol{k}_\beta+\boldsymbol{k}_\gamma)-(m_\beta+m_\gamma)\boldsymbol{k}_\alpha}{m_\alpha+m_\beta+m_\gamma}, \tag{8}$$

where $(\alpha\beta\gamma)$, $\boldsymbol{k}_\alpha$ and m_α are the cyclic permutations of (123), the individual cluster momenta and the masses, respectively. We start from the triad of the Lippmann-Schwinger (LS) equations [25] acting on a three-cluster initial state given by

$$\Phi_\beta^{(c)} = |\boldsymbol{p}_\beta\rangle^{(c)}|\boldsymbol{q}_\beta\rangle, \tag{9}$$

where $|\boldsymbol{p}_\beta\rangle^{(c)}$ is a two-cluster state, and the index β=1, 2, 3 indicates the three choices of pairs characterized by the third particle or cluster. Furthermore, $U^\beta = \sum_{\gamma\neq\beta}U_\gamma$, where U_γ (γ=1, 2, 3) are the pair forces. Three-body forces can also be incorporated in a straightforward fashion. The LS equations are given by:

$$\Psi_0^{(c)} = \Phi_\beta^{(c)} + G_\beta U^\beta \Psi_0^{(c)}, \tag{10}$$

where $G_\beta^{-1} = (E+i\varepsilon-H_0-U_\beta)^{-1}$ is Green's function. By using standard Jacobi momenta $\boldsymbol{p}_\alpha$ and $\boldsymbol{q}_\alpha$, Eq. (7) and suitable multiplication of the three equations in the triad from the left by V_γ, one obtains the transition operators $V_{\beta 0} \equiv (U_\gamma+U_\theta)\Psi_0^{(c)}$, with $\gamma\neq\beta$, $\beta\neq\theta$, which fulfill the set of equations

$$V_{\beta 0} = \sum_{\gamma\neq\beta} t_\gamma\Phi_0 + \sum_{\gamma\neq\beta} t_\gamma G_0 U_{\gamma 0}, \tag{11}$$

where $\Phi_0 = |\boldsymbol{p}\rangle|\boldsymbol{q}\rangle$ is the three-cluster state. We consider the system of three-clusters with charges z_β of equal sign interacting via pairwise strong short-range and screened Coulomb potentials $V_\beta+W_{\beta R}$ with β being 1, 2, or 3. The corresponding two-particle transition matrices are calculated with the full channel interaction, see [22]

$$T_\beta^{(R)} = (V_\beta+W_{\beta R}) + (V_\beta+W_{\beta R})G_0 T_\beta^{(R)}, \tag{12}$$

For the three-body break-up operator, one can generate the multiple scattering series directly by decomposing

V_{00} as

$$V_{00}\equiv\sum_{\theta}V_{\theta}, \tag{13}$$

where V_θ is the coupled set of Faddeev equations

$$V_{\theta}=T_{\theta}+T_{\theta}G_0\sum_{\alpha\neq\theta}V_{\beta}. \tag{14}$$

By iterating Eq. (14) and inserting the result into Eq. (13), this leads exactly to the multiple scattering series. So, we have a set of three coupled equations [26]

$$\begin{aligned} V_1 &= T_1+T_1G_0(V_2+V_3),\\ V_2 &= T_2+T_2G_0(V_3+V_1),\\ V_3 &= T_3+T_3G_0(V_1+V_2). \end{aligned} \tag{15}$$

For the calculations, the ααnΛΛ five-body cluster is quite challenging because of: (i) three species of particles (α, Λ, and neutron), (ii) five different kinds of interactions (Λ-Λ, neutron-Λ, α-Λ, α-neutron and α-α) involved, and (iii) the Pauli principle between the two Λ particles and between the Λ and neutron. There are interactions that are determined so as to reproduce the observed binding energies of the two (ΛΛ, Λn, Λα, αn and αα) and three-body (ΛΛα, Λαα, ΛΛn and αnΛ) sub-clusters.

Fixing clusters arbitrarily, like the neutron as the spectator and labelling it as "1", and the two others (αα and ΛΛ two-body clusters) as particles "2" and "3", the scattering wave function $\Psi_0^{(c)}$ must be antisymmetric under the exchange of particles "2" and "3". Thus, defining the exchange operator P_{23}, the scattering wave function must fulfill $P_{23}\Psi_0^{(+)}=-\Psi_0^{(+)}$. By applying the driving terms to the free state, we obtain [26]:

$$\begin{aligned} V_1\Phi_{0,a}&=T_1\Phi_{0,a}+T_1G_0(1-P_{23})V_2\Phi_{0,a}\\ V_2\Phi_{0,a}&=T_2\Phi_{0,a}+T_2G_0(-P_{23}V_2\Phi_{0,a}+V_1\Phi_{0,a}). \end{aligned} \tag{16}$$

where, $\Phi_{0,a}\equiv(1-P_{23})|\boldsymbol{p}_1\boldsymbol{q}_1\rangle|0m_2m_3\rangle\left|0\frac{1}{2}\frac{1}{2}\right\rangle$, which is antisymmetric under the exchange of the two clusters.

The matrix element for the n+αα+ΛΛ$\rightarrow{}^{11}_{\Lambda\Lambda}$Be process is simply related to the time-reversed photo disintegration process of ${}^{11}_{\Lambda\Lambda}$Be into three free clusters. It is necessary that one can formulate the photodisintegration of ${}^{11}_{\Lambda\Lambda}$Be based on a three-particle picture. Let O be the photon absorption operator and $|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle$ the ${}^{11}_{\Lambda\Lambda}$Be ground state. The break-up amplitude into n+αα+ΛΛ can then be written as an infinite series of processes

$$\begin{aligned} &\langle\Phi_{0,a}|V_0|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle\\ &=\langle\Phi_{0,a}|O|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle+\sum_i\langle\Phi_{0,a}|U_iG_0O|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle\\ &\quad+\sum_{ij}\langle\Phi_{0,a}|U_iG_0U_jG_0O|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle+\cdots. \end{aligned} \tag{17}$$

Here, U_i are the pair forces among the ΛΛ, Λn,Λα particles. This infinite series in terms of pair forces represents the final state interactions. The first term is the direct break-up process generated by O. By defining

$$\begin{aligned} \langle\Phi_{0,a}|V_0|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle &= \langle\Phi_{0,a}|O|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle\\ &\quad+\sum_i\langle\Phi_{0,a}|V_{0i}|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle, \end{aligned} \tag{18}$$

one can obtain the T-matrices T_i, which leads to three coupled Faddeev equations (i=1, 2, 3) [26],

$$V_{0i}|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle=T_iG_0O|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle+T_iG_0\sum_{j\neq i}V_{0j}|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle, \tag{19}$$

and the complete break-up amplitude

$$\begin{aligned} \langle\Phi_{0,a}|U_0|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle &= \langle\Phi_{0,a}|O|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle+\langle\Phi_{0,a}|U_{01}|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle\\ &\quad+\langle\Phi_{0,a}|(1-P_{23})U_{02}|\Psi^{11}_{\Lambda\Lambda\,\mathrm{Be}}\rangle. \end{aligned} \tag{20}$$

We employ the two-, three-, and four-body subsystem interactions and single-particle and two-body currents, and these coupled equations can be solved using standard techniques [27].

4 Results and discussion

We derived two coupled Faddeev equations for the three-cluster scattering amplitudes, and the results of three-body Faddeev-type calculations for systems with three particles interacting through short-range nuclear plus long-range Coulomb potentials. Realistic applications of the three-body theory to three-cluster nuclear reactions have only become possible in recent years since a reliable and practical momentum-space treatment of the Coulomb interaction was developed [22]. In the present ααnΛΛ five-body system for ${}^{11}_{\Lambda\Lambda}$Be, it is absolutely necessary that any sub-cluster systems composed of the two, three, or four constituent particles are reasonably described by taking the interactions among these systems as discussed in Section 2. We provided Faddeev equations for the n+αα+ΛΛ capture process to the ${}^{11}_{\Lambda\Lambda}$Be ground state.

In the present five-body calculation, we employ the interactions of Ref. [10] so that those severe constraints are also successfully met in our two-, three-, and four-body subsystems. By using other work on double-Λ hypernuclei, cluster model interactions were determined so as to reproduce the following observed quantities well: (a) the energies of the low-lying states and scattering phase shifts in the αn and αα systems; (b) the Λ-binding energies B_Λ in ${}^{5}_{\Lambda}$He (= αΛ), ${}^{6}_{\Lambda}$He (= αnΛ) and ${}^{9}_{\Lambda}$Be (=ααΛ); (c) the double-Λ binding energies $B_{\Lambda\Lambda}$ in ${}^{6}_{\Lambda\Lambda}$He (=αΛΛ), the NAGARA event [10].

Tables 1 and 2 show the RMS distances $\bar{r}_\alpha$, $\bar{r}_\Lambda$ and $\bar{r}_{\rm n}$ and of Λ, n and α, and the RMS. The distances are $\bar{r}_{\alpha-\alpha}$ between two α particles, $\bar{r}_{\alpha-{\rm n}}$ between α and n and $\bar{r}_{(\alpha\alpha)-{\rm n}}$ between (αα) and n in ${}^{11}_{\Lambda\Lambda}$Be, respectively. When the three-body force is added to the calculation, the reduction in these distances is about 6%; this is similar to the case of the double-Λ hypernuclei studied with the ααnΛΛ five-body model and in comparison with its results [10].

Table 1 also shows that the shrinkage effect is seen in ${}^{11}_{\Lambda\Lambda}$Be due to the addition of two Λ particles.

Table 1. The calculated RMS distances $\bar{r}_\alpha$, $\bar{r}_\Lambda$ and $\bar{r}_{\rm n}$ and of Λ, n and α, respectively, measured from the c.m. of ${}^{11}_{\Lambda\Lambda}$Be.

literature	$\bar{r}_\alpha$/fm	$\bar{r}_\Lambda$/fm	$\bar{r}_{\rm n}$/fm
Hiyama et al. [10]	1.62	2.18	2.65
This calculation	1.56	2.05	2.51

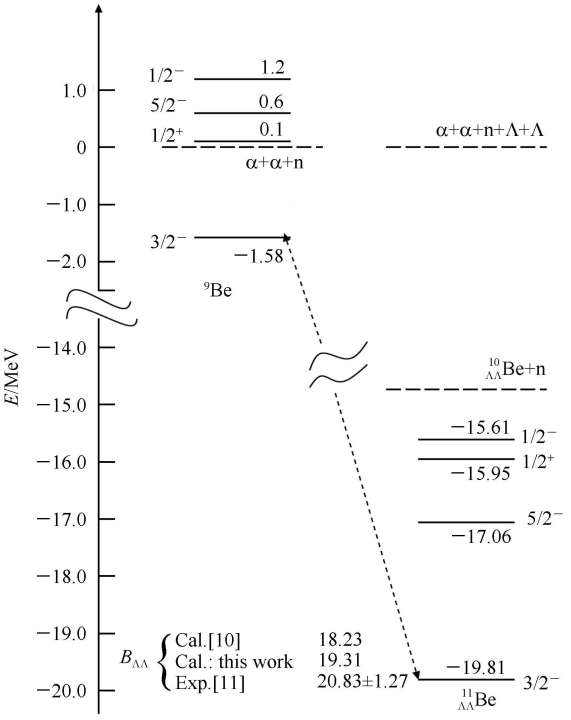

Fig. 1. The calculated energy spectra of the low-lying states of ${}^{11}_{\Lambda\Lambda}$Be together with the results by Hiyama et al. [10] and the lowest threshold, ${}^{10}_{\Lambda\Lambda}$Be+n threshold. The value of the ground state is also found to be −19.81 MeV by including all other types of configurations such as ${}^{10}_{\Lambda\Lambda}{\rm Be}^*$+n, ${}^{10}_{\Lambda}{\rm Be}^*+\Lambda$, ${}^{7}_{\Lambda\Lambda}{\rm He}^*+\alpha$ and ${}^{6}_{\Lambda}{\rm He}^*+{}^{5}_{\Lambda}{\rm He}^*$ [10]. The experimental result is taken from Ref. [11].

Using the same framework of Ref. [10], we calculated the energies and wave functions of the $5/2^-_1$, $1/2^+_1$ and $1/2^-_1$ states of ${}^{11}_{\Lambda\Lambda}$Be. As seen in Fig. 1, the calculated value of $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}{\rm Be})$ is 19.31 MeV in comparison with the calculated result of 18.23 MeV by Hiyama et al. [10] and the experimental value of 20.83 MeV [11] for the $3/2^-$ ground state. There is a correction of about 7%, which is much smaller than the result by Hiyama et al. [10].

The $V^{\rm bond}_{\Lambda\Lambda}$ is defined, which is useful for estimating the strength of the ΛΛ interaction [10]

$$V^{\rm bond}_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)\equiv B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)-B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z{:}V_{\Lambda\Lambda}=0)\ , \qquad (21)$$

where $B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z:V_{\Lambda\Lambda}=0)$ denotes the $B_{\Lambda\Lambda}$ value calculated by putting $V_{\Lambda\Lambda}=0$. The calculated $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}{\rm Be})$ is 19.67 MeV in ${}^{11}_{\Lambda\Lambda}$Be and the calculated $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}{\rm Be};V_{\Lambda\Lambda}=0)$ is 19.13 MeV. Then, $V^{\rm bond}_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)=0.54$ MeV, which is in comparison with the calculated value of $V^{\rm bond}_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z)$=0.56 MeV by Hiyama et al. [10].

Table 2. The calculated RMS distances $\bar{r}_{\alpha-\alpha}$, $\bar{r}_{\alpha-{\rm n}}$ and $\bar{r}_{(\alpha\alpha)-{\rm n}}$ in ${}^{11}_{\Lambda\Lambda}$Be.

literature	$\bar{r}_{\alpha-\alpha}$/fm	$\bar{r}_{\alpha-{\rm n}}$/fm	$\bar{r}_{(\alpha\alpha)-{\rm n}}$/fm
Hiyama et al. [10]	3.10	3.33	2.94
This calculation	3.02	3.21	2.05

5 Summary and conclusions

We presented the results of three-body Faddeev-type calculations for systems with three clusters interacting through short-range nuclear as well as long-range Coulomb interaction. Realistic applications of three-body theory to three-cluster nuclear reactions have only become possible in recent years, when a reliable and practical momentum-space treatment of the Coulomb interaction was developed. For recent observation of the Hida event for a new double-Λ hypernucleus, we succeeded in performing a five-body calculation of ${}^{11}_{\Lambda\Lambda}$Be using an ααnΛΛ cluster model. The calculated ΛΛ binding energy is in good agreement with other theoretical methods. More precise data are needed in order to test our present result, together with the three-body force, quantitatively. Many data for double-Λ hypernuclei are expected to be found in the new emulsion experiment E07 at J-PARC in the near future. Then, our predictions will be clearly tested.

The experimental value of $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}{\rm Be})$=20.83 MeV seems to be more compatible with our calculated value of $B_{\Lambda\Lambda}({}^{11}_{\Lambda\Lambda}{\rm Be})$=19.31 MeV in comparison with the calculated 18.23 MeV result by Hiyama et al.

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