

# Mixing of two-quasineutron and two-quasiproton $K^\pi=6^+$ configurations in the vicinity of $^{174}\text{Yb}^*$

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**Abstract:** In the framework of the projected shell model, we investigate the competition between the two-quasineutron and two-quasiproton  $K^\pi=6^+$  states in the ytterbium isotopes and  $N=104$  isotones adjacent to  $^{174}\text{Yb}$ . The  $^{174}\text{Yb}$  results are compared with the experimental data. The  $K^\pi=6^+$  isomer observed in  $^{174}\text{Yb}$  is assigned as an admixture of the  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  and  $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$  intrinsic structure, which explains the experimental  $|g_K - g_R|$  value. Similar mixing would appear in  $^{172}\text{Yb}$ ,  $^{176}\text{Hf}$ , and  $^{178}\text{W}$ . The low-lying  $K^\pi=6^+$  states are also predicted in  $^{170-178}\text{Yb}$ .

**Key words:** K isomers, projected shell model, rotational band,  $g$  factor

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## 1 Introduction

Research on multi-quasiparticle (multi-qp) metastable states provides a unique opportunity to understand the interplay between collective and quasiparticle excitation degrees of freedom in nuclear systems. In particular, an abundance of long-lived high- $K$  isomers (where  $K$  is the total angular momentum projection onto the symmetry axis) was found in the  $A \sim 170-180$  mass region of prolate-deformed nuclei, which has attracted intensive interest both experimentally and theoretically [1–5].  $K$  isomers are of great importance for enhancing the stability of unstable nuclei [6] and novel energy-storage applications [1]. More fundamentally, they offer an insight into the underlying single-particle structure and their pairing strength around the Fermi surface.

In order to study the underlying structural information, a confirmation of the intrinsic structures for  $K$  isomers is needed. The gyromagnetic ratio (or  $g$  factor) is essential for determining quasiparticle configurations. Recently, a rotational band built on the  $830 \mu\text{s}$   $6^+$  isomer in  $^{174}\text{Yb}$  has been observed, which makes the  $g$  factor measurement accessible for this  $K^\pi=6^+$  intrinsic state [7]. Although the  $K^\pi=6^+$  state in  $^{174}\text{Yb}$  is commonly associated with the  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  configuration [8], the newly measured  $g$  factor shows deviation from the expected value for the two-quasineutron state [7]. This implies that the  $K^\pi=6^+$  isomer observed in  $^{174}\text{Yb}$  is unlikely to be a “pure” two-quasineutron state.

In addition, the existence of both two-quasineutron [9] and two-quasiproton  $K^\pi=6^+$  isomers [10] in neighboring nuclei indicates the complexity of the configuration assignment for the  $6^+$  isomer in this mass region.

The present work aims at a thorough analysis of the two-quasineutron and two-quasiproton  $K^\pi=6^+$  states in the  $Z=70$  isotopes and  $N=104$  isotones adjacent to  $^{174}\text{Yb}$  by using the projected shell model (PSM) [11]. As a shell-model-type calculation truncated in the angular-momentum-projected multi-qp basis, the PSM is ideal for studying high- $K$  isomers and the associated rotational bands [5]. Such calculations may shed light on the microscopic origin of high- $K$  multi-qp states by analyzing the wave functions generated in the PSM. Moreover, the well-defined wave functions allow us to compute the  $g$  factor, which provides direct indication of dominant multi-qp configurations.

## 2 The model

In the present calculations, single-particle energies are given by the axially symmetric Nilsson model [12], in which the  $\kappa$  and  $\mu$  parameters are empirically fitted [13]. The Lipkin-Nogami (LN) pairing [14] has been used in the present PSM calculation. In the PSM, the many-body wave function can be written as a superposition of projected multi-qp states [11, 15]

$$|\Psi_{IM}\rangle = \sum_{K\kappa} f_{IK\kappa} \hat{P}_{MK}^I |\Phi_\kappa\rangle, \quad (1)$$

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where  $\hat{P}_{MK}^I$  is the angular-momentum-projection operator,  $|\Phi_\kappa\rangle$  denotes the multi-qp states as a basis set, and  $f_{IK_\kappa}$  is their weight factor. For even-even nuclei,  $|\Phi_\kappa\rangle$  consists of the multi-qp states (up to four-qp states) associated with the Nilsson-LN vacuum  $|\phi\rangle$ :

$$\{|\phi\rangle, a_{\nu_1}^\dagger a_{\nu_2}^\dagger |\phi\rangle, a_{\pi_1}^\dagger a_{\pi_2}^\dagger |\phi\rangle, a_{\nu_1}^\dagger a_{\nu_2}^\dagger a_{\pi_1}^\dagger a_{\pi_2}^\dagger |\phi\rangle\}, \quad (2)$$

where  $a_\nu^\dagger$  and  $a_\pi^\dagger$  are the creation operators for neutrons and protons, respectively.

The deformed Nilsson-LN states are created with axial symmetry in the present work. Thus each multi-qp state  $|\Phi_\kappa\rangle$  has a definite  $K$  as a good quantum number. The state of Eq. (1) is a linear combination of various  $K$  states, which represents the mixing among different  $K$  components. Hence the calculation takes  $K$ -mixing effects into account. The energies and wave functions (expressed in terms of  $f_{IK_\kappa}$ ) are determined by a variational procedure, i.e., solving the following variational equation [11]

$$\sum_{K'\kappa'} \{H_{K_\kappa K'\kappa'}^I - E_I N_{K_\kappa K'\kappa'}^I\} f_{IK'\kappa'} = 0, \quad (3)$$

where  $H_{K_\kappa K'\kappa'}^I$  and  $N_{K_\kappa K'\kappa'}^I$  are the Hamiltonian and norm matrix elements, respectively.

We employ the total Hamiltonian, which is defined as [11, 16]

$$\hat{H} = \hat{H}_0 - \frac{\chi}{2} \sum_{\mu} \hat{Q}_{2\mu}^\dagger \hat{Q}_{2\mu} - G_M \hat{P}^\dagger \hat{P} - G_Q \sum_{\mu} \hat{P}_{2\mu}^\dagger \hat{P}_{2\mu}, \quad (4)$$

where  $\hat{H}_0$  is the spherical Nilsson Hamiltonian. The other terms are standard, i.e., for the quadrupole-quadrupole, monopole-pairing, and quadrupole-pairing interactions, respectively. The strength  $\chi$  of the quadrupole-quadrupole force is determined by comparison with the deformed Nilsson potential [11]. The monopole-pairing strength  $G_M$  is determined by the average gap method [17]. The quadrupole-pairing strength  $G_Q$  is taken to be proportional to  $G_M$  with a constant of 0.20, which is consistent with the previous PSM calculations [11].

The  $g$  factor is ideal for identifying the multi-qp contribution to the wave function. It is defined by

$$g(I) = g_\nu(I) + g_\pi(I), \quad (5)$$

with  $g_\tau(I)$  ( $\pi$  for protons and  $\nu$  for neutrons) given by

$$g_\tau(I) = \frac{1}{\mu_N \sqrt{I(I+1)}} [g_1^\tau \langle \Psi_{IM} | \hat{j}^\tau | \Psi_{IM} \rangle + (g_s^\tau - g_1^\tau) \langle \Psi_{IM} | \hat{s}^\tau | \Psi_{IM} \rangle], \quad (6)$$

where  $|\Psi_{IM}\rangle$  is the wave function of Eq. (1). In the present work, we use the standard values for  $g_1$  and  $g_s$  (i.e.,  $g_1^\pi = 1$ ,  $g_1^\nu = 0$ ,  $g_s^\pi = 5.586 \times 0.75$ , and  $g_s^\nu = -3.826 \times 0.75$ ).  $g_s^\pi$  and  $g_s^\nu$  are both reduced by a quenching factor of 0.75 from the free-nucleon values to account for

the core-polarization and meson-exchange current corrections [18].

### 3 Calculations and discussions

To determine the deformation first, which is needed in the PSM calculations, we calculate the ground-state (g.s.) potential energy surfaces (PESs) by using the Strutinsky method [19], in which the energy is decomposed into macroscopic, quantal shell-correction, and residual pairing energies. The macroscopic energy can be approximated by the standard liquid-drop model [20]. The g.s. deformation can be determined by minimizing the PES. Table 1 lists the calculated quadrupole ( $\varepsilon_2$ ) and hexadecapole ( $\varepsilon_4$ ) deformations for the  $Z=70$  isotopes and  $N=104$  isotones in the vicinity of  $^{174}\text{Yb}$ . The calculated quadrupole deformations  $\varepsilon_2$  are in good agreement with the experimentally adopted values [21]. In the following PSM calculations, we thus construct the multi-qp basis at the deformations that are listed in Table 1.

Table 1. The quadrupole and hexadecapole deformation parameters for  $Z=70$  isotopes and  $N=104$  isotones.

$Z=70$	$^{170}\text{Yb}$	$^{172}\text{Yb}$	$^{174}\text{Yb}$	$^{176}\text{Yb}$	$^{178}\text{Yb}$
$\varepsilon_2$	0.284	0.287	0.286	0.279	0.274
$\varepsilon_4$	0.034	0.043	0.055	0.064	0.074
$N=104$	$^{172}\text{Er}$	$^{174}\text{Yb}$	$^{176}\text{Hf}$	$^{178}\text{W}$	$^{180}\text{Os}$
$\varepsilon_2$	0.289	0.286	0.271	0.248	0.233
$\varepsilon_4$	0.049	0.055	0.049	0.039	0.040

Figure 1 shows the calculated level spectra for  $^{174}\text{Yb}$ , including rotational bands built on different intrinsic structures. It can be seen that the calculated ground-state band,  $K^\pi = 6^+$  band, and  $K^\pi = 14^+$  band reproduce the experimental data well. Ref. [7] associates the observed  $K^\pi = 6^+$  isomer with a two-quasineutron ( $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$ ) configuration. However, our calculation gives two  $K^\pi = 6^+$  bands with nearly degenerate energies. One is based on the  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  configuration, and the other is built on a two-quasiproton ( $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$ ) configuration which lies only 26 keV higher than the two-quasineutron one. Both bands have equivalent rotational behaviors, which are in accord with the experimental  $K^\pi = 6^+$  band. Nearly degenerate energies would reinforce the mixing between these two bands. Indeed, mixing of the two-quasiproton and two-quasineutron  $K^\pi = 6^+$  bands has been found in neighboring nuclei,  $^{176}\text{Hf}$  [22] and  $^{174}\text{Hf}$  [10]. However, the extent of the mixing between high- $K$  two-quasineutron and two-quasiproton states is very small in our calculations as well as in previous PSM calculations [23]. This may be related to the model space and the QQ interaction employed in the Hamiltonian of the PSM [23].

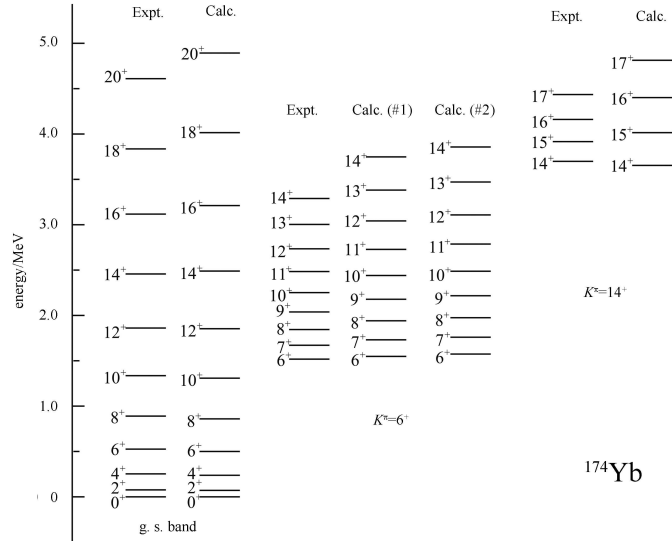


Fig. 1. The calculated level spectra sorted by bands built on different intrinsic structures for  $^{174}\text{Yb}$ . The dominant configurations of the prolate high- $K$  bands are  $K^\pi=6^+$  (#1):  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$ ;  $K^\pi=6^+$  (#2):  $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$ ;  $K^\pi=14^+$ :  $\nu 7/2^- [514] \otimes \nu 5/2^- [512] \otimes \pi 9/2^- [514] \otimes \pi 7/2^- [523]$ . The experimental data are taken from Ref. [7].

Table 2 lists the calculated  $g$  factors for the  $K^\pi=6^+$  states. Our calculation gives  $g$  factor values of 0.07 for the two-quasineutron  $K^\pi=6^+$  state and 0.89 for the two-quasiproton  $K^\pi=6^+$  state, respectively. We use  $g = g_R + (g_K - g_R) \cdot \frac{K^2}{I(I+1)}$  and assume  $g_R = 0.35$  [7] to obtain  $|g_K - g_R| = 0.33(0.63)$  for the two-quasineutron (two-quasiproton) configuration. Experimentally, the observed branching ratios for the  $K^\pi=6^+$  band in  $^{174}\text{Yb}$  give a weighted mean of  $|g_K - g_R| = 0.437(6)$  [7], which is higher than the calculated value for the two-quasineutron configuration and is lower than that for the two-quasiproton configuration. The intermediate value of the measured  $|g_K - g_R|$  indicates the mixing of the two  $K^\pi=6^+$  bands. As a consequence of this mixing, one may expect to observe interband transitions between members of the two mixed bands. Further experimental data are needed to clarify this.

Table 2. The calculated  $g$  factors for the two-quasineutron and two-quasiproton  $K^\pi=6^+$  states in  $^{174}\text{Yb}$ .  $g_R$  is assumed to be 0.35 [7]. The experimental data are from Ref. [7].

$K^\pi$	configuration	$g$ factor (Calc.)	$ g_K - g_R $ (Calc.)	$ g_K - g_R $ (Expt.)
$6^+$	two-quasineutron or	0.07	0.33	0.437(6)
	two-quasiproton	0.89	0.63	

Systematic observations of the  $K^\pi=6^+$  isomers can be found in  $^{172-178}\text{Hf}$  [10, 22]. Most of them are dominated by the  $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$  configuration, except for  $^{176}\text{Hf}$  ( $N=104$ ), in which the  $K^\pi=6^+$  isomer is an admixture of two-quasiproton and two-quasineutron

states [22]. On the contrary, in the  $N=104$  isotones (except  $^{176}\text{Hf}$ ), the  $K^\pi=6^+$  isomers which have been observed from  $^{172}\text{Er}$  to  $^{180}\text{Os}$  are associated with the  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  intrinsic structure [9]. This implies that the two-quasineutron and two-quasiproton structures would compete with each other in this mass region. The competition between these different multi-qp excitations provides useful information about the shell structure of nucleon orbits.

We therefore make a thorough investigation on both two-quasineutron and two-quasiproton  $K^\pi=6^+$  states in the  $N=104$  isotones and  $Z=70$  isotopes. Fig. 2 displays the comparison between the calculated and experimental energies in the  $N=104$  isotones close to  $^{174}\text{Yb}$ . Good agreement between the calculations and experiments is obtained for the known  $K^\pi=6^+$  states. Since the  $\nu 7/2^- [514]$  and  $\nu 5/2^- [512]$  orbits are located around the  $N=104$  neutron Fermi surface, the calculated energies for the two-quasineutron configurations are low and nearly constant. The calculated energies for the two-quasiproton configurations decrease rapidly from  $Z=68$  to  $72$  and increase remarkably from  $Z=72$  to  $76$ . In the range of  $Z=70-74$ , the two-quasiproton  $K^\pi=6^+$  state is predicted to be at an energy similar to the two-quasineutron one. This is because of the proton Fermi surfaces of  $70 \leq Z \leq 74$  near the  $\pi 7/2^+ [404]$  and  $\pi 5/2^+ [402]$  orbits. Small energy differences may lead to the mixing of the two  $K^\pi=6^+$  states. Accordingly, the  $K^\pi=6^+$  isomers in  $^{174}\text{Yb}$ ,  $^{176}\text{Hf}$ , and  $^{178}\text{W}$  would be based on an admixture of the two-quasineutron  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  and two-quasiproton  $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$  intrinsic states.

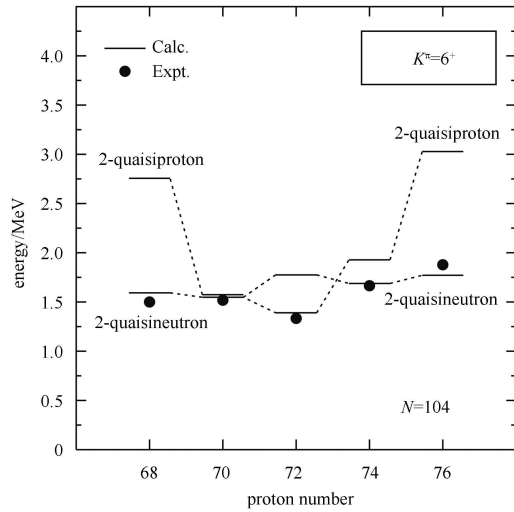


Fig. 2. The energies for the  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  ( $K = 6^+$ ) and  $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$  ( $K = 6^+$ ) states in the  $N = 104$  isotones adjacent to  $^{174}\text{Yb}$ . The experimental data are taken from Refs. [7, 9, 24].

The experimental information concerning the  $K^\pi = 6^+$  isomers in ytterbium isotopes is relatively scarce. Our calculations predict the systematic existence of the low-lying  $K^\pi = 6^+$  states in  $^{170-178}\text{Yb}$ . As can be seen in Fig. 3, the calculated energies for the  $K^\pi = 6^+$ ,  $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$  states lie at similar energies in  $^{170-178}\text{Yb}$ , whereas the energies for the  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  states change significantly along with the increasing neutron number. In  $^{172}\text{Yb}$  and  $^{174}\text{Yb}$ , the two-quasineutron configuration is at a comparable energy to the two-quasiproton one. This may result in serious mixing of these two  $K^\pi = 6^+$  states, and a corresponding experimental search is needed.

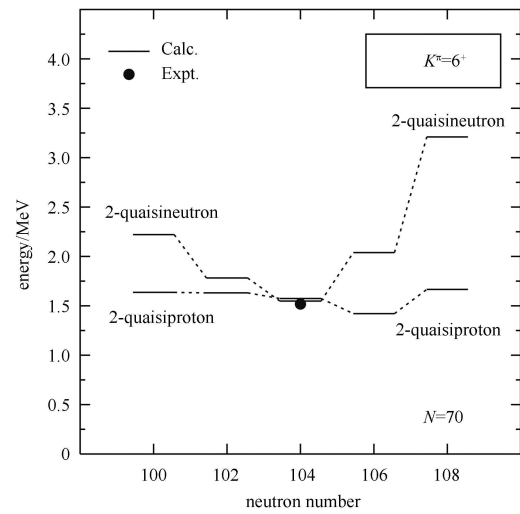


Fig. 3. Similar to Fig. 3, but for the  $K = 6^+$  states in the ytterbium isotopes adjacent to  $^{174}\text{Yb}$ . The experimental data are taken from Ref. [7].

## 4 Summary

In summary, we investigated the competition between the two-quasineutron and two-quasiproton  $K^\pi = 6^+$  states in the  $Z = 72$  isotopes and  $N = 104$  isotones in the vicinity of  $^{174}\text{Yb}$  by using the projected shell model. The present calculations reproduce the experimental observations well. Our calculation associated the  $K^\pi = 6^+$  isomer observed in  $^{174}\text{Yb}$  with an admixture of the  $\nu 7/2^- [514] \otimes \nu 5/2^- [512]$  and  $\pi 7/2^+ [404] \otimes \pi 5/2^+ [402]$  intrinsic structure, which is supported by the experimental  $|g_K - g_R|$  value. Similar mixing would also occur in  $^{172}\text{Yb}$ ,  $^{176}\text{Hf}$ , and  $^{178}\text{W}$ . In addition, the systematic existence of the low-lying  $K^\pi = 6^+$  states is predicted for  $^{170-178}\text{Yb}$ .

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