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Supersymmetric extension of the minimal dark matter model*

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Abstract: The minimal dark matter model is given a supersymmetric extension. A super $SU(2)_{\rm L}$ quintuplet is introduced with its fermionic neutral component still being the dark matter, and the dark matter mass is about 19.7 TeV. Mass splitting among the quintplet due to supersymmetry particles is found to be negligibly small compared to the electroweak corrections. Other properties of this supersymmetry model are studied, it has the solutions to the PAMELA and Fermi-LAT anomaly, and the predictions in higher energies need further experimental data to verify them.

Key words: dark matter, supersymmetry, mass splitting, coannihilation

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1 Introduction

It is known from many astrophysical measurements that the universe contains an enormous amount of invisible, non-baryonic dark matter (DM) which is not included in the Standard Model (SM). Among various hypotheses for the nature of the DM, that of weakly interacting massive particles (WIMPs) is very attractive. Focussing on the DM problem, one can explore a simple WIMP model, that is the minimal dark matter model (MDM) [1, 2]: adding to the SM a single matter X without introducing any additional discrete symmetry, and X is in a high dimensional representation of the usual SM $SU(2)_{\rm L}\times U(1)_{\rm Y}$ electroweak (EW) interactions. The stability of the DM candidate is guaranteed by the SM gauge symmetry and by the renormalizability. The minimality of the model lies in the fact that the new physics is determined by only one parameter, namely the mass M of the X multiplet. Therefore, the MDM is remarkably predictive. There are some extensions to the MDM [3].

In this work, we make SUSY extension to the MDM. Note that in the so-called minimal SUSY extension of the SM (MSSM), the DM candidate, that is the lightest SUSY particle, is there only after introducing an extra discrete symmetry by hand, which is the R-parity. Instead, in our SUSY MDM (SMDM), we still follow the logic of MDM; the existence of the DM lies in the fact that the DM is in a high dimensional representation of the SM gauge group without using discrete symmetries.

In Section 2, the SMDM is constructed. In Section 3, mass splitting of the X multiplet, the DM relic density, direct and indirect detection signatures of the SMDM are calculated. In Section 4, the conclusion is made. In the Appendix, we give basic facts about the representation of the $SU(2)$ group.

2 SMDM

The SMDM is simply constructed by supersym-

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metric extension to the MDM. The particle content is, in addition to that of the MSSM, the fermionic $SU(2)_{\rm L}$ 5-plet X of the MDM and its superpartner which is a complex scalar 5-plet $\tilde{\rm X}$. The charged components $\rm X^Q$ are slightly heavier than the neutral one $\rm X^0$ due to quantum corrections [1, 2]; and the superpartner $\tilde{\rm X}$ because of its soft mass $M_{\rm soft}$, is also heavier than X. Both $\rm X^Q$ and $\tilde{\rm X}$ will decay into $\rm X^0$. The relic particle in the SMDM is still $\rm X^0$ as in the MDM. The new parameters are M and $M_{\rm soft}$, the model is still predictive. As for the Lagrangian, in addition to that of the MSSM, we have

$$\begin{aligned}\mathcal{L}_{\rm SMDM} =\ & \frac{\rm i}{2}(X^{\dagger i}\bar{\sigma}^{\mu}\partial_{\mu}X_i+\bar{X}^{\dagger i}\bar{\sigma}^{\mu}\partial_{\mu}\bar{X}_i)-\frac{\rm i}{2}g_2A^{\mu}_a(X^{\dagger i}\bar{\sigma}_{\mu}(T^a)_i{}^jX_j-\bar{X}^{\dagger}{}_i\bar{\sigma}_{\mu}(T^a)_j{}^i\bar{X}_j)\\ &-\frac{\sqrt{2}}{2}g_2(\tilde{X}^{*i}(T^a)_i{}^jX_j\lambda^a+\lambda^{\dagger a}X^{\dagger i}(T^a)_i{}^j\tilde{X}_j-\tilde{\bar{X}}^{*i}(T^a)_j{}^i\bar{X}_j\lambda^a-\lambda^{\dagger a}\bar{X}^{\dagger i}(T^a)_j{}^i\tilde{\bar{X}}_j)\\ &+\frac{1}{2}(D^{\mu}\tilde{X}^*D_{\mu}\tilde{X}-M^2|\tilde{X}|^2)-\frac{1}{2}M(\bar{X}_iX_i+X^{\dagger i}\bar{X}^{\dagger i})+\frac{1}{2}g_2D_a\tilde{X}^i(T^a)_i{}^j\tilde{X}_j-\frac{1}{2}M^2_{\rm soft}|\tilde{X}|^2. \qquad (1)\end{aligned}$$

The component field notation has been used. In Eq. (1), T^a's are generators of the $SU(2)_{\rm L}$ in n representation. $\rm X_{\it i}$ and $\bar{\rm X}^i$ consist of the left-hand pairs of $\rm X_{\it i}$, transforming in $SU(2)_{\rm L}$ 5 representation with the generator $(T^a)_i{}^j$ and the complex conjugate representation with the generator $(T^a)^{*j}_i=(T^a)_j{}^i$, respectively. They are not independent. Actually they are dual to each other under the $SU(2)_{\rm L}$. We write the Lagrangian in the form of Eq. (1) just for convenience. Their superpartners compose a bosonic $SU(2)_{\rm L}$ 5-plets $\tilde{\rm X}^i$. Both $\rm X_{\it i}$ and $\tilde{\rm X}^i$ have trivial $SU(3)_{\rm c}\times U(1)_{\rm Y}$ quantum numbers (1,0).

In terms of 4-component notation, we can define following spinors,

$$\Psi^{+2}\equiv\begin{pmatrix}X^{+2}\\(\bar{X}^{+2})^{\dagger}\end{pmatrix},\Psi^{-2}\equiv\begin{pmatrix}X^{-2}\\(\bar{X}^{-2})^{\dagger}\end{pmatrix}=(\Psi^{+2})^{\rm C}, \quad (2)$$

$$\Psi^{+1}\equiv\begin{pmatrix}X^{+1}\\(\bar{X}^{+1})^{\dagger}\end{pmatrix},\Psi^{-1}\equiv\begin{pmatrix}X^{-1}\\(\bar{X}^{-1})^{\dagger}\end{pmatrix}=(\Psi^{+1})^{\rm C}, \quad (3)$$

$$\Psi^{0}\equiv\begin{pmatrix}X^{0}\\(\bar{X}^{0})^{\dagger}\end{pmatrix}=(\Psi^{0})^{\rm C}. \quad (4)$$

The neutral component Ψ^0 is a Majorana field.

The superpotential takes the simple from:

$$W=W_{\rm MSSM}+\frac{1}{2}M\tilde{X}^2, \quad (5)$$

which gives X and $\tilde{\rm X}$ the same unbroken supersymmetry mass M. $\tilde{\rm X}$ gets a soft mass after supersymmetry breaking. The D-term contribution to the scalar potential is:

$$V_D=\frac{1}{2}g_2^2\left[\sum\phi^*t^a\phi+\frac{1}{2}\tilde{X}^*T^a\tilde{X}\right]^2. \quad (6)$$

where ϕ denotes the $SU(2)_{\rm L}$ scalars in the MSSM. Compared with the MSSM, the extra term is:

$$\frac{1}{2}g_2^2\left[\sum(\phi^*t^a\phi)(\tilde{X}^*T^a\tilde{X})+\left(\frac{1}{2}\tilde{X}^*T^a\tilde{X}\right)^2\right]. \quad (7)$$

These couplings do not cause $\tilde{\rm X}$ to decay but to annihilate into MSSM $SU(2)_{\rm L}$ scalars, which give an extra negligible mass splitting between $\tilde{\rm X}^i$.

Considering non-renormalizable terms of the Lagrangian, there are dimension 5 operators $\tilde{X}^{ijkl}\phi_i\phi_j\phi_k\phi_l/\Lambda$ for the complex scalar 5-plet $\tilde{\rm X}$, and dimension 6 operators $X^{ijkl}\psi_i\phi_j\phi_k\phi_l/\Lambda^2$ for the fermionic 5-plet X allowed by the $SU(2)_{\rm L}\times U(1)_{\rm Y}$ gauge symmetry, where ψ_i is the left-hand leptons or the higgsinos in the MSSM, e.g. $\tilde{X}H_{\rm u}H_{\rm d}H_{\rm u}H^*_{\rm u}/\Lambda$, $\tilde{X}H_{\rm u}H_{\rm d}H_{\rm d}H^*_{\rm d}/\Lambda$, $X\tilde{H}_{\rm u}H_{\rm d}H_{\rm d}H^*_{\rm d}/\Lambda^2$, $XLH_{\rm u}H_{\rm d}H_{\rm u}/\Lambda^2$, etc.

We can generate these couplings by adding the corresponding higher dimension superpotential, eg.

$$W_{\rm non-ren}=\frac{\tilde{X}H_{\rm u}H_{\rm d}H_{\rm u}H_{\rm d}}{\Lambda^2}+\frac{\tilde{X}\tilde{L}H_{\rm u}H_{\rm d}H_{\rm u}}{\Lambda^2}+\cdots, \quad (8)$$

the equations of motion for the auxiliary fields are:

$$\begin{aligned}F_{H_{\rm d}} &= -\left(\frac{\partial W}{\partial H_{\rm d}}\right)^*\\ &= -\left(\mu H_{\rm u}+\frac{\tilde{X}H_{\rm u}H_{\rm u}H_{\rm d}}{\Lambda^2}+\frac{\tilde{X}\tilde{L}H_{\rm u}H_{\rm u}}{\Lambda^2}+\cdots\right)^*,\\ F_{H_{\rm u}} &= -\left(\frac{\partial W}{\partial H_{\rm u}}\right)^*\\ &= -\left(\mu H_{\rm d}+\frac{\tilde{X}H_{\rm u}H_{\rm d}H_{\rm d}}{\Lambda^2}+\frac{\tilde{X}\tilde{L}H_{\rm u}H_{\rm d}}{\Lambda^2}+\cdots\right)^*.\end{aligned} \quad (9)$$

This generates dim6 couplings for the fermionic 5-plet : $X\tilde{H}_{\rm u}H_{\rm d}H_{\rm d}H^*_{\rm d}/\Lambda^2$, $XLH_{\rm u}H_{\rm d}H_{\rm u}/\Lambda^2$ where $\Lambda\approx$

10^{15} GeV. These operators can induce 4-body decays with a typical life-time $\tau \sim \Lambda^4$ TeV$^{-5} \sim 10^{19}$ s which is longer than the age of the universe ($\sim 10^{17}$ s). So these couplings have no influence on the observed stability of the DM candidates. The F-term also generates the scalar dim.5 operators which are not suppressed by one power of Λ but two powers: $\mu\tilde{X}H_uH_dH_dH_d^*/\Lambda^2$, $\mu\tilde{X}\tilde{L}H_dH_dH_d^*/\Lambda^2$. The typical life-time of these operators is: $\tau \sim \Lambda^3$ TeV$^{-3}\mu^{-1} \sim 10^{20}$ s which is long enough. Of course there are even higher dimension couplings of the complex scalar $\tilde{\mathrm{X}}$, eg. $\tilde{X}\tilde{X}^*H_uH_u^*H_dH_d^*H_dH_d^*/\Lambda^4$, $\tilde{X}\tilde{X}^*\tilde{L}H_uH_dH_u^*H_d^*H_d^*/\Lambda^4$, etc. These higher dimension operators can be neglected in considering the decay.

Therefore the new introduced particles $(\tilde{\mathrm{X}}^{\pm2},\tilde{\mathrm{X}}^{\pm1},\tilde{\mathrm{X}}^0)$ only decay into $(\mathrm{X}^{\pm2},\mathrm{X}^{\pm1},\mathrm{X}^0)$ via gauge interactions. $(\mathrm{X}^{\pm2},\mathrm{X}^{\pm1},\mathrm{X}^0)$ are quite stable. We will further study the mass splitting among them in the next section, and see that the DM candidate is still X^0.

3 Properties of the SMDM

The DM candidate in the SMDM model is still X^0. The mass splitting due to SUSY particles is small because of SUSY breaking.

3.1 Mass splitting

Mass splitting should be studied in detail, like that in the MDM, because it can be calculated with little uncertainty in this simple model. Because of EW symmetry breaking, the gauge kinetic terms gives the fermonic 5-plet X a mass splitting through loop corrections [1, 2], $\Delta M^{\mathrm{Q}}_{\mathrm{EW}} \equiv M^{\mathrm{Q}} - M^0 \approx Q^2 \times 166$ MeV, where M^{Q} and M^0 are the pole masses of X^{Q} and X^0, respectively.

The scalar particles $\tilde{\mathrm{X}}$'s also contribute to the mass splitting of X. They are heavier than the fermions X's by a soft mass M_{soft} which is generally expected to be about 100 GeV–1 TeV. This further mass splitting is calculated by using the supersymmetric kinetic term, the second line of Eq. (1), at the loop level which involves $\tilde{\mathrm{X}}$'s and the gauginos. By using the two-component notation for fermions, the one-loop pole mass is written as [4]

$$M^{\mathrm{Q}}_{\mathrm{SUSY}} = M\left(1+\frac{1}{2}\Sigma^{\mathrm{Q}}_{\mathrm{L}}+\frac{1}{2}\Sigma^{\mathrm{Q}}_{\mathrm{R}}\right) \tag{10}$$

where $\Sigma^{\mathrm{Q}}_{\mathrm{L}}$, $\Sigma^{\mathrm{Q}}_{\mathrm{R}}$ are the 1PI self-energy functions as shown in Fig. 1 in $Q=0,+1$ cases.

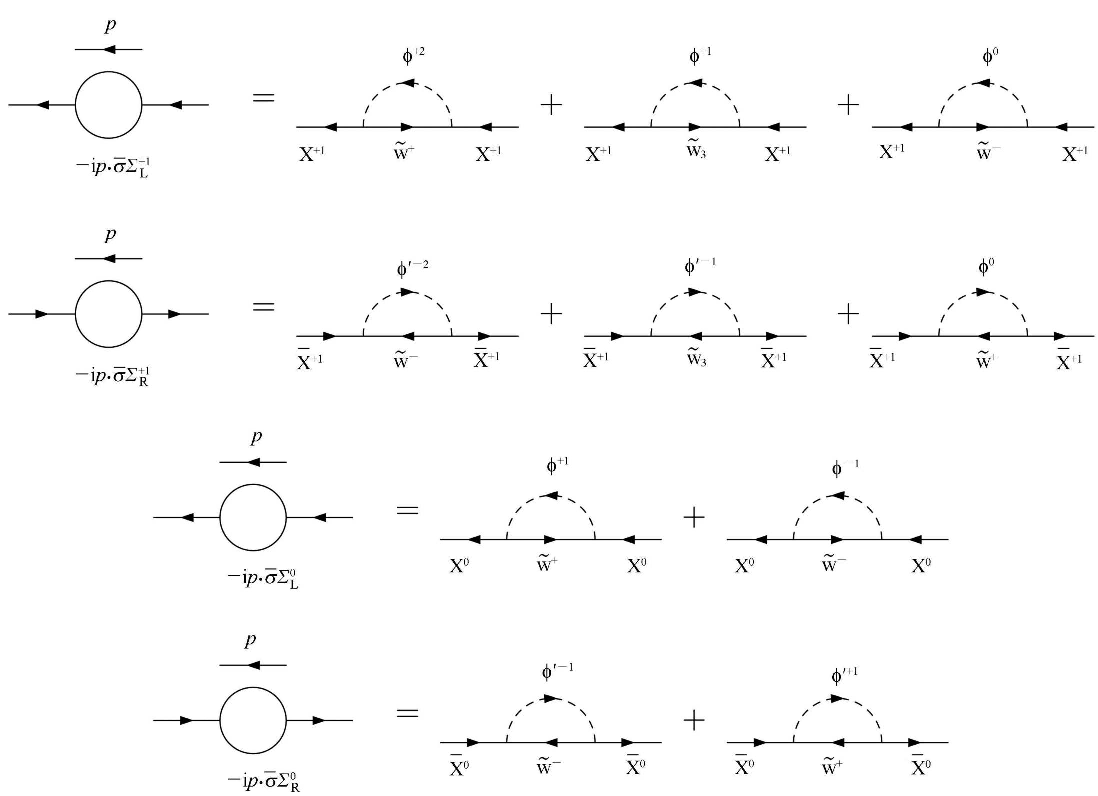

Fig. 1. One-loop corrections to the 1PI self-energy functions to the $Q=0,+1$ components of the SMDM.

In the diagrams of Fig. 1, we denote the correspondent superpartners:

$$\phi^{+2}\equiv\tilde{X}^{+2},\ \phi^{'-2}\equiv\tilde{\tilde{X}}^{+2},\ \phi^{+1}\equiv\tilde{X}^{+1},$$
$$\phi^{'-1}\equiv\tilde{\tilde{X}}^{+1},\ \phi^{0}\equiv\tilde{X}^{+1}. \tag{11}$$

$\tilde{w}^{\pm}$, $\tilde{w}_3$ are the superpartners of the $SU(2)_{\rm L}$ gauge bosons and $\tilde{w}^{+}=V_{11}^{-1}\tilde{C}_1^{+}+V_{12}^{-1}\tilde{C}_2^{+}$, $\tilde{w}^{-}=U_{11}^{-1}\tilde{C}_1^{-}+U_{12}^{-1}\tilde{C}_2^{-}$, $\tilde{w}_3=N_{2i}^{-1}\tilde{N}_i$, i=1–4. $\tilde{C}_1^{\pm}$, $\tilde{C}_2^{\pm}$ and $\tilde{N}_i$ are the charginos and neutralinos of the MSSM. U, V, N are the unitary matrices diagonalizing the mass matrices of charginos and neutralinos [5].

It is worth noting that the $\Sigma_D^{\rm Q}$ term which is related to the B-term $B^{ij}\tilde{X}_i\tilde{X}_j$, may also appear in the pole mass formula, and it does not cause divergences. But in our calculation we do not consider it for simplicity; it is enough for us to break the supersymmetry only through the soft mass $M_{\rm soft}$.

Using the superpartner notation mentioned in Section 1 we get:

$$\begin{aligned}
\Sigma_{\rm L}^{+1} &= \frac{g_2^2}{16\pi^2}\Bigg[V_{11}^*V_{11}B_1(\tilde{C}_1,\phi^{+2})+V_{21}^*V_{21}B_1(\tilde{C}_2,\phi^{+2})+\frac{3}{2}(U_{11}^*U_{11}B_1(\tilde{C}_1,\phi^{0})+U_{21}^*U_{21}B_1(\tilde{C}_2,\phi^{0}))\\
&\quad+\frac{1}{2}(N_{12}^*N_{12}B_1(\tilde{N}_1,\phi^{+1})+N_{22}^*N_{22}B_1(\tilde{N}_2,\phi^{+1})+N_{32}^*N_{32}B_1(\tilde{N}_3,\phi^{+1})+N_{42}^*N_{42}B_1(\tilde{N}_4,\phi^{+1})\Bigg],\\
\Sigma_{\rm R}^{+1} &= \frac{g_2^2}{16\pi^2}\Bigg[U_{11}^*U_{11}B_1(\tilde{C}_1,\phi^{'-2})+U_{21}^*U_{21}B_1(\tilde{C}_2,\phi^{'-2})+\frac{3}{2}(V_{11}^*V_{11}B_1(\tilde{C}_1,\phi^{0})+V_{21}^*V_{21}B_1(\tilde{C}_2,\phi^{0}))\\
&\quad+\frac{1}{2}(N_{12}^*N_{12}B_1(\tilde{N}_1,\phi^{'-1})+N_{22}^*N_{22}B_1(\tilde{N}_2,\phi^{'-1})+N_{32}^*N_{32}B_1(\tilde{N}_3,\phi^{'-1})+N_{42}^*N_{42}B_1(\tilde{N}_4,\phi^{'-1})\Bigg],\\
\Sigma_{\rm L}^{0} &= \frac{g_2^2}{16\pi^2}\left[\frac{3}{2}(V_{11}^*V_{11}B_1(\tilde{C}_1,\phi^{+1})+V_{21}^*V_{21}B_1(\tilde{C}_2,\phi^{+1})+U_{11}^*U_{11}B_1(\tilde{C}_1,\phi^{-1})+U_{21}^*U_{21}B_1(\tilde{C}_2,\phi^{-1}))\right],\\
\Sigma_{\rm R}^{0} &= \frac{g_2^2}{16\pi^2}\left[\frac{3}{2}(U_{11}^*U_{11}B_1(\tilde{C}_1,\phi^{'-1})+U_{21}^*U_{21}B_1(\tilde{C}_2,\phi^{'-1})+V_{11}^*V_{11}B_1(\tilde{C}_1,\phi^{'+1})+V_{21}^*V_{21}B_1(\tilde{C}_2,\phi^{'+1}))\right],
\end{aligned}\tag{12}$$

where B_1 is the one rank two point integral

$$B_1(p^2,m_1,m_2)=-\frac{1}{2\varepsilon}+\frac{A_0(m_1)-A_0(m_2)+(m_2^2-m_1^2-p^2)B_0(p^2,m_1,m_2)}{2p^2},\tag{13}$$

with A_0 and B_0 being the Passarino-Veltman functions.

Because all the superpartners ϕ^i have the same mass $M+M_{\rm soft}$, we can simplify the above four equations to get the final result of the mass splitting due to SUSY particles:

$$\begin{aligned}
\Delta M_{\rm SUSY}^{\rm Q} &= \frac{Q^2}{2}(\Sigma_{\rm L}^{\rm Q}+\Sigma_{\rm R}^{\rm Q}-\Sigma_{\rm L}^{0}-\Sigma_{\rm R}^{0})\\
&= \frac{g_2^2Q^2}{16\pi^2}\Big[-(V_{11}^*V_{11}+U_{11}^*U_{11})B_1(\tilde{C}_1,\phi)\\
&\quad-(V_{21}^*V_{21}+U_{21}^*U_{21})B_1(\tilde{C}_2,\phi)\\
&\quad+\frac{1}{2}N_{i2}^*N_{i2}B_1(\tilde{N}_i,\phi)\Big].
\end{aligned}\tag{14}$$

The poles in the B1 function are cancelled as expected using the unitarity of the U, V and N. The mass splitting is a function of M_1, M_2, $\tan\beta$, μ, $M_{\rm soft}$ and M. In the correct EW breaking parameter space, our numerical result for the mass splitting due to SUSY particles is that

$$\Delta M_{\rm SUSY}^{\rm Q}\sim 0.01Q^2\ {\rm MeV},\tag{15}$$

which is negligibly small compared with the pure EW corrections.

3.2 The thermal relic density

The thermal relic density fixes the WIMP mass. In the MDM, the relic species are $\{{\rm X}^{\pm2},\ {\rm X}^{\pm1},\ {\rm X}^{0}\}$ and the coannihilation channels are:

$${\rm X}^i{\rm X}^j\to {\rm AA},{\rm f\bar{f}}.\tag{16}$$

where A and f denote an EW gauge boson and the SM fermion, respectively. The mass splitting among them are very small compared with their masses. In the density calculation, such mass splitting are negligible. The relic particle thermal average cross section is [1]:

$$\langle\sigma_{\rm A}v\rangle({\rm X}^i{\rm X}^j\rightarrow {\rm AA},{\rm f\bar{f}})\approx\frac{\pi\alpha_2^2}{8M^2}\times 166. \tag{17}$$

Matching to the relic abundance, the DM particle mass is determined to be $M=4.4$ TeV without considering Sommerfeld corrections.

Once SUSY is introduced, a whole bunch of superpartners of those in MDM will present. SUSY breaking gives soft masses to the scalars and gauginos. Looking at $\tilde{\rm X}$'s, they have a mass $M+M_{\rm soft}$. In general when $M_{\rm soft}\leqslant 0.1M$, e.g. $M_{\rm soft}\leqslant 440$ GeV for $M = 4.4$ TeV, they have sizable effects on the relic abundance and must be included in the the relic species [6, 7]. So now the coannihilation relic species are $\{(\tilde{\rm X}^{\pm2},\tilde{\rm X}^{\pm1},\ \tilde{\rm X}^0),({\rm X}^{\pm2},{\rm X}^{\pm1},{\rm X}^0)\}$ and the coannihilation channels are:

$$\begin{aligned}&\tilde{\rm X}^i\tilde{\rm X}^j\rightarrow {\rm AA},{\rm f\bar{f}},\tilde{\rm G}\tilde{\rm G},\\ &\tilde{\rm X}^i{\rm X}^j\rightarrow {\rm f}\bar{\tilde{\rm f}},\tilde{\rm G}{\rm A},\\ &{\rm X}^i{\rm X}^j\rightarrow {\rm AA},{\rm f\bar{f}},\tilde{\rm G}\tilde{\rm G},\end{aligned} \tag{18}$$

where $\tilde{\rm G}$ denotes MSSM gauginos and $\tilde{\rm f}$ the superpartner of f.

Furthermore, because M is also much larger than the electroweak scale, the physics of the above-mentioned coannihilation is basically supersymmetric and EW gauge symmetric, and we can make unbroken EW symmetry and unbroken supersymmetry approximation when calculating the thermal average cross section. Nevertheless, it is still a hard work. In terms of two-component fields, there are 24 gauge kinetic vertices and another 24 vertices involving superpartners. But actually we can have a useful and proper estimate for the relations between cross sections for the three kinds of processes in Eq. (21). It is found that introducing SUSY has nearly 4 times influence on the thermal average cross section. In Figs. 2–7 a series of explicit examples by using two-component spinor techniques [4] will be shown and the results read:

$$v\sigma(\bar{\rm X}^{+2}{\rm X}^{+2}\rightarrow {\rm W}_3{\rm W}_3)=16\times\frac{8\pi\alpha^2}{3M^2}, \tag{19}$$

$$v\sigma(\phi^{*+2}\phi^{+2}\rightarrow {\rm W}_3{\rm W}_3)=32\times\frac{8\pi\alpha^2}{3M^2}, \tag{20}$$

$$v\sigma(\bar{\rm X}^{+2}{\rm X}^{+2}\rightarrow \tilde{\omega}_3\tilde{\omega}_3)\sim 0, \tag{21}$$

$$v\sigma(\phi^{*+2}\phi^{+2}\rightarrow \tilde{\omega}_3\tilde{\omega}_3)\sim 0, \tag{22}$$

$$v\sigma(\phi^{+2}\phi^{'-2}\rightarrow \tilde{\omega}_3\tilde{\omega}_3)\simeq 16\times\frac{3\pi\alpha^2}{2M^2}, \tag{23}$$

$$v\sigma(\bar{\rm X}^{+2}\phi^{+2}\rightarrow {\rm W}_3\tilde{\omega}_3)\simeq 16\times\frac{3\pi\alpha^2}{8M^2}, \tag{24}$$

$$v\sigma({\rm X}^{+2}\phi^{'-2}\rightarrow {\rm W}_3\tilde{\omega}_3)\simeq 16\times\frac{3\pi\alpha^2}{8M^2}, \tag{25}$$

where v is the relative velocity in the lab frame, Eqs. (24) and (25) are the results of the p-wave suppression.

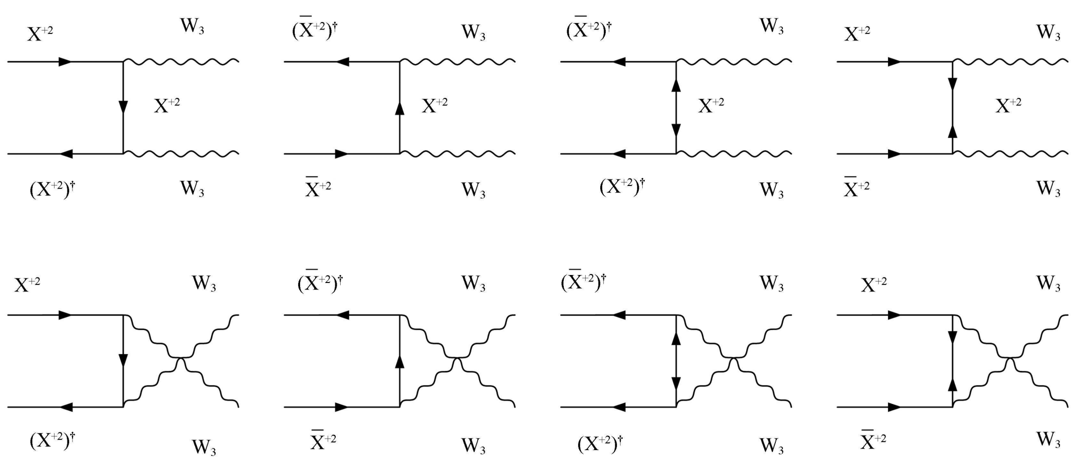

Fig. 2. The eight Feynman diagrams for $\bar{\rm X}^{+2}{\rm X}^{+2}\rightarrow {\rm W}_3{\rm W}_3$.

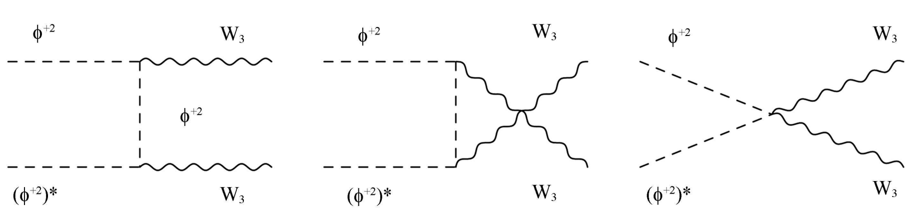

Fig. 3. The three Feynman diagrams for $\phi^{*+2}\phi^{+2}\rightarrow {\rm W}_3{\rm W}_3$.

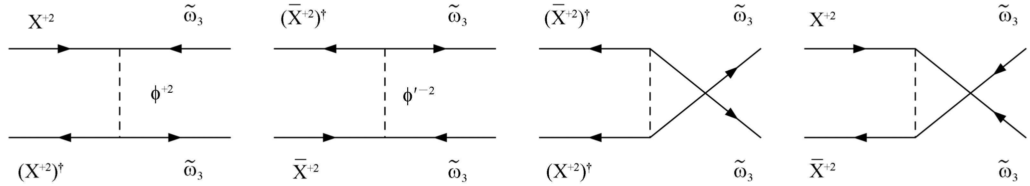

Fig. 4. The four Feynman diagrams for $\bar{\mathrm{X}}^{+2}\mathrm{X}^{+2} \to \tilde{\omega_3}\tilde{\omega_3}$.

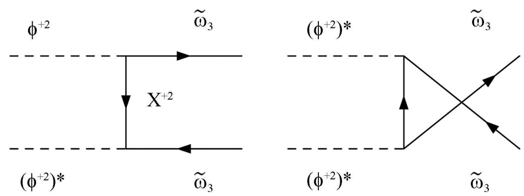

Fig. 5. The two Feynman diagrams for $\phi^{*+2}\phi^{+2} \to \tilde{\omega_3}\tilde{\omega_3}$.

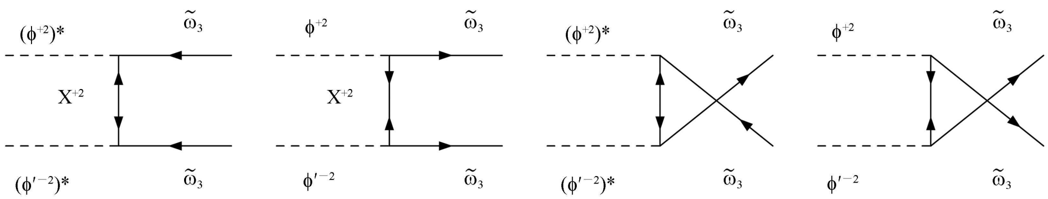

Fig. 6. The four Feynman diagrams for $\phi^{+2}\phi^{'-2} \to \tilde{\omega_3}\tilde{\omega_3}$.

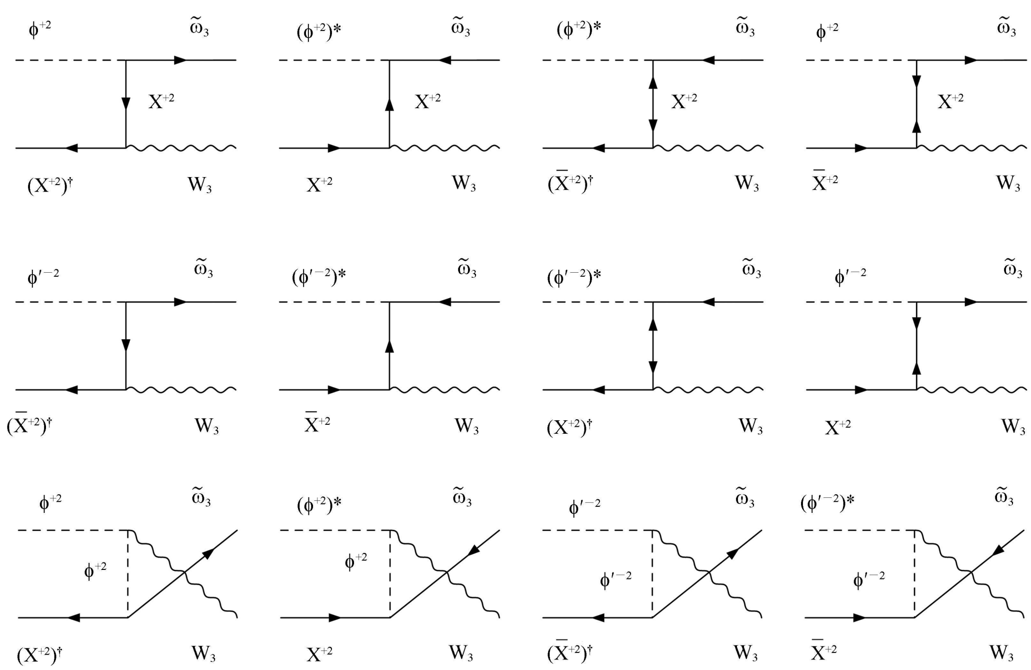

Fig. 7. The twelve Feynman diagrams for $\bar{\mathrm{X}}^{+2}\phi^{+2} \to \mathrm{W}_3\tilde{\omega_3}$ and $\mathrm{X}^{+2}\phi^{'-2} \to \mathrm{W}_3\tilde{\omega_3}$.

We have already calculated a complete series of thermal cross sections and we end up with:

$$v\sigma_{\rm eff}=\sum v\sigma\approx 4v\sigma(\bar{\rm X}^{+2}{\rm X}^{+2}\rightarrow {\rm W}_3{\rm W}_3) \qquad (26)$$

considering the freedom for X^{+2}, $\bar{X}^{+2}$ and ϕ^{+2}, ϕ'^{-2} are all $g=2$. This is very different from the dramatic influence from the higher representation such as $SU(2)$-5 in our problem. Despite this conclusion which we have made from some specific examples, we think it is a common result.

Once the DM particle is as heavy as a few TeV, the Sommerfeld effect due to hundreds GeV particles should be taken into consideration. In the MDM, the Sommerfeld enhancement effect due to SM particles, especially due to W and Z bosons, has been calculated and the factor is about 5 [8]. We expect approximately the same result in our case. While our case is $N=1$ SUSY, all the SUSY particles should be included in the ladder diagram calculation in determining the potential which tells us the enhancement factor. However including the SUSY particles does not essentially change the Sommerfeld effect. In the extreme case of $N=4$ SUSY, the extra symmetries just keep the gauge coupling from running. The potential itself has the same form as in the non-SUSY Yang-Mills case. In our $N=1$ SUSY case with soft breaking, the logarithmic running of the gauge coupling of the MDM is expected to be only mildly reduced, the Sommerfeld effect is then approximately the same. So it is reasonable to say that the Sommerfeld enhancement factor is about the same as that calculated in the MDM, $M\simeq\sqrt{5}\times2\times4.4\simeq19.7$ TeV.

3.3 Direct and indirect signatures

As for the direct DM detection rate, this SMDM is of the same order as the MDM. The DM particle interacts with quarks via loops. Although more heavy particles appear in the loops, the total cross section has the same order, $\sigma_{\rm SI}\propto10^{-44}$ cm^2, which is within the sensitivity of the current experiments, such as Super-CDMS and Xenon 1-ton [9, 10].

The DM annihilation in the galaxy may have observable signatures. The estimation of the cross section is also like that in the MDM [11, 12], the result has no order change. Note that SUSY does not change the Sommerfeld effect essentially, the predominant annihilation channel is still into EW W bosons, $\langle\sigma v\rangle_{\rm WW}\sim10^{-23}$ cm$^3\cdot$s^{-1}. Because the DM mass $M\simeq19.7$ TeV, we expect this model predicts: (1) continuous rise e^+/e^++e^- spectrum up to about 20 TeV; (2) flat e^++e^- spectrum up to M; (3) $\overline{p}/p$ flux has excess above the energy probed by PAMELA. SMDM has the solutions to the PAMELA and Fermi-LAT anomaly [13, 14], the predictions in higher energies need further experimental data to verify them.

4 Summary and discussion

We have made SUSY extension to the MDM model by introducing a complex scalar quintuplet as the superpartner of the fermion quintuplet. The neutral component of the fermionic 5-plet is still the DM particle as in the MDM model. Mass splittings among the fermionic 5-plets due to SUSY have been calculated in detail and they are found to be small. By considering new relic species and new coannihilation channels in the MSSM final states, the DM mass is estimated to be 19.7 TeV.

The direct and indirect signals are basically the same as those in the MDM. Numerically the DM elastic scattering cross section with a nucleus is about 10^{-44} cm$^3\cdot$s^{-1} and the cross section of the predominant annihilation channel into W bosons is about 10^{-23} cm$^3\cdot$s^{-1}. SMDM predicts e^+, e^++e^-, $\overline{p}$ spectrum in agreement with the previous PAMELA, Fermi-LAT data, $\overline{p}$ flux has excess above the energy probed by PAMELA which need further experimental tests.

In the near future, suppose SUSY is discovered, say at LHC, it will still be a question if MSSM itself provides a DM particle, because R-parity as a discrete symmetry is still an assumption which is not as solid as gauge invariance and SUSY. It is plausible that R-parity is violated. In that case, it is still simple and interesting to have the DM via introducing $SU(2)_{\rm L}$ high dimensional representations.

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Appendix A

The $SU(2)$-n representation

The $SU(2)$-n representation U^j (n=2j+1) is self-conjugate , when j is integer (n-odd), U^j is real , when j is half-integer (n-even), U^j is self-conjugate also but not real:

$$\mathrm{X}\mathrm{U}^j\mathrm{X}^{-1}=\mathrm{U}^{j*}\Rightarrow\begin{cases}\mathrm{U}^j=U^{j*} & j=0,1,2,\cdots\ ,\ \mathrm{X}\text{ is symmetric}\\ \mathrm{U}^j\text{ is not real} & j=\dfrac{1}{2},\dfrac{3}{2},\cdots\ ,\ \ \mathrm{X}\text{ is antisymmetric}\end{cases}, \tag{A1}$$

it is important in proving some identities including the $SU(2)$ generators.

The generators of the $SU(2)$-n representation is:

$$\begin{aligned}(T_1^j)_{\nu\mu}&=\frac{1}{2}[\delta_{\nu(\mu+1)}\Gamma_\nu^j+\delta_{\nu(\mu-1)}\Gamma_{-\nu}^j],\\ (T_2^j)_{\nu\mu}&=-\frac{\mathrm{i}}{2}[\delta_{\nu(\mu+1)}\Gamma_\nu^j-\delta_{\nu(\mu-1)}\Gamma_{-\nu}^j],\\ (T_3^j)_{\nu\mu}&=\mu\delta_{\nu\mu},\end{aligned} \tag{A2}$$

where

$$\Gamma_\nu^j=\Gamma_{-\nu+1}^j=(j+\nu)(j-\nu+1)^{1/2}, \tag{A3}$$

for example

$$T_3^1=\frac{\sqrt{2}}{2}\begin{pmatrix}0&1&0\\1&0&1\\0&1&0\end{pmatrix},\quad T_3^2=\frac{\sqrt{2}}{2}\begin{pmatrix}0&-\mathrm{i}&0\\ \mathrm{i}&0&-\mathrm{i}\\0&\mathrm{i}&0\end{pmatrix},\quad T_3^3=\begin{pmatrix}1&0&0\\0&0&0\\0&0&-1\end{pmatrix}, \tag{A4}$$

$$T_5^1=\begin{pmatrix}0&1&0&0&0\\1&0&\frac{\sqrt{6}}{2}&0&0\\0&\frac{\sqrt{6}}{2}&0&\frac{\sqrt{6}}{2}&0\\0&0&\frac{\sqrt{6}}{2}&0&1\\0&0&0&1&0\end{pmatrix},\quad T_5^2=\begin{pmatrix}0&-\mathrm{i}&0&0&0\\ \mathrm{i}&0&-\mathrm{i}\frac{\sqrt{6}}{2}&0&0\\0&\mathrm{i}\frac{\sqrt{6}}{2}&0&-\mathrm{i}\frac{\sqrt{6}}{2}&0\\0&0&\mathrm{i}\frac{\sqrt{6}}{2}&0&-\mathrm{i}\\0&0&0&\mathrm{i}&0\end{pmatrix},\quad T_5^3=\begin{pmatrix}2&0&0&0&0\\0&1&0&0&0\\0&0&0&0&0\\0&0&0&-1&0\\0&0&0&0&-2\end{pmatrix}, \tag{A5}$$

and have

$$\bar{T^a}T^2=-T^2T^a. \tag{A6}$$

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