Determinate joint remote preparation of an arbitrary $\boldsymbol{W}$-class quantum state*

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Abstract: A novel determinate joint remote preparation scheme of an arbitrary W-class quantum state is proposed to improve the probability of successful preparation. The presented scheme is realized through orthogonal projective measurement of the Hadamard transferred basis, which converts a global measurement to several local measurements. Thus orthogonal projective measurement of the Hadamard transferred basis enables quantum information to be transmitted from different sources simultaneously, which is a breakthrough for quantum network node processing. Finally, analysis shows the feasibility and validity of the proposed method, with a 100% probability of successful preparation.

Key words: joint remote state preparation, Hadamard transformation, projective measurement, quantum network coding

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1 Introduction

Quantum information is the definition of quantum mechanical system based information dissemination. Two of its outstanding applications are quantum teleportation (QT, [1–5]) and remote state preparation (RSP, [6–10]). The former application proposed a novel single qubit state transfer protocol using one EPR pair and two classical bits (cbits), and the latter application is another embodiment of the former function under different conditions. In RSP, the sender completely knows the prepared state while the receiver has no knowledge of it. Furthermore, RSP makes a tradeoff between classical communication cost and quantum entanglement cost [11–13]. The success of QT and RSP laid a solid foundation for quantum communication networks, and thus joint remote state preparation (JRSP) was created as time passed [14–16]. JRSP deals with a message passing from several senders to several receivers who are located in different places. Similar to RSP, the senders in JRSP holds the full information while all receivers stay unknown. The implementation of JRSP marks the growth of quantum networks [17].

In this paper, a novel scheme for tripartite joint remote preparation of a W-class state is presented, including its preparation process and its application. The work is a further study of [16, 18, 19], which imports more complex quantum system and measurement usage. Information preparation in the proposed scheme is embodied by projective measurement of the Hadamard transferred basis. Hadamard transformation has been proved to be outstanding for its unitarity and resolvability, which leads to the 100% probability of successful preparation rather than probabilistic preparation [20]. At last, the presented scheme is applied as an embryo of simultaneous quantum network coding, which is greatly beneficial to quantum network design.

2 Preparation principals

Assume there are two senders (Alice and Bob) and only one receiver (Cliff), the proposed scheme

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involves Alice and Bob transmitting arbitrary W-class state [21–23] information to Cliff. The assigned W-class state can be described as: $W_{\rm C} = a_0|001\rangle + a_1|010\rangle + a_2|100\rangle + a_3|111\rangle$. The coefficient a_i is assumed to be real with normalization condition $a_1^2+a_2^2+a_3^2+a_4^2=1$. This state is fully known to Alice and Bob, and is fully unknown to Cliff. The difference between our scheme and the traditional ones [13, 14] lies in that the proposed method previously introduces shared information between Alice and Bob (the senders), which is carried by auxiliary qubits (A, B) with the state of $(a_0|00\rangle+a_1|01\rangle+a_2|10\rangle+a_3|11\rangle)_{\rm AB}$. In addition, the initial shared ground quantum state resource is under the basis: $|\psi\rangle_{12345}=|00000\rangle_{12345}$.

Suppose Alice has qubits (A, 1), Bob has qubits (B, 2), and Cliff has qubits (3, 4, 5). These three persons are located at three spatially separated places, respectively. The following Fig. 1 shows the detailed configuration.

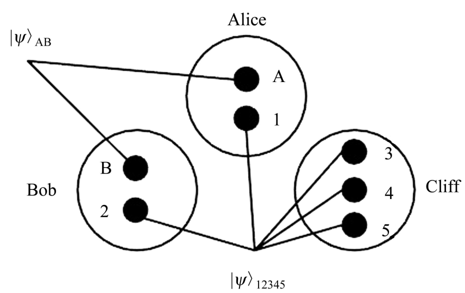

Fig. 1. The initial configuration.

To simplify the expression, $F(x)$ is defined as the function of converting (x mod 4) to a binary expression. For example, $|F(3)\rangle=|11\rangle$. So the whole system can be described in the following way:

$$\left(\sum_{i=0}^{3} a_i|F(i)\rangle\right)_{\rm AB} \otimes|00\rangle_{12}\otimes|000\rangle_{345}. \tag{1}$$

In the consideration of introducing a_i to qubits (1, 2), Alice should use CNOT operation on her local qubits (A, 1), with qubit (A) as control qubit and qubit (1) as operation qubit. Homologically, Bob operates qubits (B, 2), leading to the system of

$$\left(\sum_{i=0}^{3} a_i|F(i)\rangle|F(i)\rangle\right)_{\rm AB12} \otimes|000\rangle_{345}. \tag{2}$$

Then the senders should release the influence of qubits (A, B), so we can employ the idea in QT, applying a measurement basis transfer. The selected measurement basis transfer requires that transferred measurement bases be in orthogonality and normalization. Normalized quadravalence Hadamard unitary transformation is selected for the measurement basis transfer due to its simplicity:

$$\begin{bmatrix}|00\rangle\\|01\rangle\\|10\rangle\\|11\rangle\end{bmatrix}=H_4\begin{bmatrix}\delta_{00}\\\delta_{01}\\\delta_{10}\\\delta_{11}\end{bmatrix}=\frac{1}{2}\begin{bmatrix}\delta_{00}+\delta_{01}+\delta_{10}+\delta_{11}\\\delta_{00}-\delta_{01}+\delta_{10}-\delta_{11}\\\delta_{00}+\delta_{01}-\delta_{10}-\delta_{11}\\\delta_{00}-\delta_{01}-\delta_{10}+\delta_{11}\end{bmatrix}. \tag{3}$$

Note that the normalized Hadamard matrixes are in a good nature of

$$H_N^{\rm T} = H_N \quad \text{with} \quad H_N^{\rm T}\times H_N = I_N, \tag{4}$$

$$H_{2N} = \begin{bmatrix}H_N & H_N\\H_N & -H_N\end{bmatrix} \quad \text{with} \quad H_2=\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}. \tag{5}$$

The projective measurement basis chosen by Alice and Bob is a set of $\{\delta_{00},\delta_{01},\delta_{10},\delta_{11}\}$. The characteristics of normalized Hadamard transformation enable Alice and Bob to measure independently. In another word, Alice and Bob could separately use $\{\delta_0,\delta_1\}$ to measure qubit (A) and qubit (B). Because that

$$\begin{bmatrix}|\delta_{00}\rangle\\|\delta_{01}\rangle\\|\delta_{10}\rangle\\|\delta_{11}\rangle\end{bmatrix}=\frac{1}{2}\begin{bmatrix}|00\rangle+|01\rangle+|10\rangle+|11\rangle\\|00\rangle-|01\rangle+|10\rangle-|11\rangle\\|00\rangle+|01\rangle-|10\rangle-|11\rangle\\|00\rangle-|01\rangle-|10\rangle+|11\rangle\end{bmatrix}, \tag{6}$$

$$\begin{bmatrix}|\delta_{00}\rangle\\|\delta_{01}\rangle\\|\delta_{10}\rangle\\|\delta_{11}\rangle\end{bmatrix}=\frac{1}{2}\begin{bmatrix}(|0\rangle+|1\rangle)\times(|0\rangle+|1\rangle)\\(|0\rangle+|1\rangle)\times(|0\rangle-|1\rangle)\\(|0\rangle-|1\rangle)\times(|0\rangle+|1\rangle)\\(|0\rangle-|1\rangle)\times(|0\rangle-|1\rangle)\end{bmatrix}, \tag{7}$$

$$\begin{bmatrix}|\delta_{00}\rangle\\|\delta_{01}\rangle\\|\delta_{10}\rangle\\|\delta_{11}\rangle\end{bmatrix}=\begin{bmatrix}|\delta_0\rangle|\delta_0\rangle\\|\delta_0\rangle|\delta_1\rangle\\|\delta_1\rangle|\delta_0\rangle\\|\delta_1\rangle|\delta_1\rangle\end{bmatrix}, \tag{8}$$

$$\begin{bmatrix}|\delta_0\rangle\\|\delta_1\rangle\end{bmatrix}=\frac{1}{\sqrt{2}}\begin{bmatrix}|0\rangle+|0\rangle\\|0\rangle-|1\rangle\end{bmatrix}. \tag{9}$$

The above formula proves that the global measurement can be divided into several local measurements, and thus the idea of normalized Hadamard transformation based measurement basis transfer is an improvement of what has been done in [14, 15, 19]. After the measurements, Alice and Bob announce their outcomes (one cbit each) via a classical channel, leading to system (abbreviation for S) of:

IF $|\text{Result}\rangle=|\delta_{00}\rangle_{\rm AB}$

$$S_{00}=\begin{pmatrix}+a_0|00\rangle & +a_1|01\rangle\\+a_2|10\rangle & +a_3|11\rangle\end{pmatrix}_{12}\otimes|000\rangle_{345}, \tag{10}$$

IF $|\text{Result}\rangle=|\delta_{01}\rangle_{\rm AB}$

$$S_{01}=\begin{pmatrix}+a_0|00\rangle & -a_1|01\rangle\\+a_2|10\rangle & -a_3|11\rangle\end{pmatrix}_{12}\otimes|000\rangle_{345}, \tag{11}$$

IF $|\text{Result}\rangle = |\delta_{10}\rangle_{\text{AB}}$

$$S_{10} = \begin{pmatrix} +a_0|00\rangle & +a_1|01\rangle \\ -a_2|10\rangle & -a_3|11\rangle \end{pmatrix}_{12} \otimes |000\rangle_{345}, \quad (12)$$

IF $|\text{Result}\rangle = |\delta_{11}\rangle_{\text{AB}}$

$$S_{11} = \begin{pmatrix} +a_0|00\rangle & -a_1|01\rangle \\ -a_2|10\rangle & +a_3|11\rangle \end{pmatrix}_{12} \otimes |000\rangle_{345}. \quad (13)$$

We hope Cliff would receive a_i using qubits (3, 4, 5) rather than (1, 2). So CNOT operation is applied to qubits (1, 3) and (2, 4), with the former qubits as control qubits and the latter qubits as operation qubits, resulting in

$$\begin{aligned} S_{00} &= \begin{pmatrix} +a_0|00\rangle|00\rangle & +a_1|01\rangle|01\rangle \\ +a_2|10\rangle|10\rangle & +a_3|11\rangle|11\rangle \end{pmatrix}_{1234} \otimes |0\rangle_5 \\ S_{01} &= \begin{pmatrix} +a_0|00\rangle|00\rangle & -a_1|01\rangle|01\rangle \\ +a_2|10\rangle|10\rangle & -a_3|11\rangle|11\rangle \end{pmatrix}_{1234} \otimes |0\rangle_5 \\ S_{10} &= \begin{pmatrix} +a_0|00\rangle|00\rangle & +a_1|01\rangle|01\rangle \\ -a_2|10\rangle|10\rangle & -a_3|11\rangle|11\rangle \end{pmatrix}_{1234} \otimes |0\rangle_5 \\ S_{11} &= \begin{pmatrix} +a_0|00\rangle|00\rangle & -a_1|01\rangle|01\rangle \\ -a_2|10\rangle|10\rangle & +a_3|11\rangle|11\rangle \end{pmatrix}_{1234} \otimes |0\rangle_5 \end{aligned} . \quad (14)$$

To exchange the state of qubits (1, 2) and qubits (3, 4), Alice and Bob use CNOT operations again, which inversely employ qubits (3, 4) as control qubits and qubits (1, 2) as operation qubits. Then the system changes to be:

$$\begin{aligned} S_{00} &= |00\rangle_{12} \otimes \begin{pmatrix} +a_0|00\rangle & +a_1|01\rangle \\ +a_2|10\rangle & +a_3|11\rangle \end{pmatrix}_{34} \otimes |0\rangle_5 \\ S_{01} &= |00\rangle_{12} \otimes \begin{pmatrix} +a_0|00\rangle & -a_1|01\rangle \\ +a_2|10\rangle & -a_3|11\rangle \end{pmatrix}_{34} \otimes |0\rangle_5 \\ S_{10} &= |00\rangle_{12} \otimes \begin{pmatrix} +a_0|00\rangle & +a_1|01\rangle \\ -a_2|10\rangle & -a_3|11\rangle \end{pmatrix}_{34} \otimes |0\rangle_5 \\ S_{11} &= |00\rangle_{12} \otimes \begin{pmatrix} +a_0|00\rangle & -a_1|01\rangle \\ -a_2|10\rangle & +a_3|11\rangle \end{pmatrix}_{34} \otimes |0\rangle_5 \end{aligned} . \quad (15)$$

Currently qubits (1, 2) could be removed, and the remaining qubits (3, 4, 5) of Cliff are in state:

$$\begin{aligned} S_{00} &= (a_0|000\rangle + a_1|010\rangle + a_2|100\rangle + a_3|110\rangle)_{345} \\ S_{01} &= (a_0|000\rangle - a_1|010\rangle + a_2|100\rangle - a_3|110\rangle)_{345} \\ S_{10} &= (a_0|000\rangle + a_1|010\rangle - a_2|100\rangle - a_3|110\rangle)_{345} \\ S_{11} &= (a_0|000\rangle - a_1|010\rangle - a_2|100\rangle + a_3|110\rangle)_{345} \end{aligned} . \quad (16)$$

With former classical measurement results from Alice and Bob, Cliff is now able to reconstruct the W-class state using corresponding operation. The measurement result of $F(i)$ with $i \in \{0,1,2,3\}$ means that the system is in the state of $S_{F(i)}$, calling for transfer operation $U_{F(i)}$ to resume. Because the current state of Cliff fully contains information of a_i, $U_{F(i)}$ operations need only to transfer the present state to the W-class state, which are

$$\begin{aligned} U_{00} &= (|001\rangle\langle 000| + |010\rangle\langle 010| + |100\rangle\langle 100| + |111\rangle\langle 110|) + (|000\rangle\langle 001| + |011\rangle\langle 011| + |101\rangle\langle 101| + |110\rangle\langle 111|) \\ U_{01} &= (|001\rangle\langle 000| - |010\rangle\langle 010| + |100\rangle\langle 100| - |111\rangle\langle 110|) + (|000\rangle\langle 001| + |011\rangle\langle 011| + |101\rangle\langle 101| + |110\rangle\langle 111|) \\ U_{10} &= (|001\rangle\langle 000| + |010\rangle\langle 010| - |100\rangle\langle 100| - |111\rangle\langle 110|) + (|000\rangle\langle 001| + |011\rangle\langle 011| + |101\rangle\langle 101| + |110\rangle\langle 111|) \\ U_{11} &= (|001\rangle\langle 000| - |010\rangle\langle 010| - |100\rangle\langle 100| + |111\rangle\langle 110|) + (|000\rangle\langle 001| + |011\rangle\langle 011| + |101\rangle\langle 101| + |110\rangle\langle 111|) \end{aligned} . \quad (17)$$

These operations are similar in Refs. [14, 20]. While for each measurement result of qubits (A, B) Cliff is sure to resume the W-class state of $(a_0|001\rangle + a_1|010\rangle + a_2|100\rangle + a_3|111\rangle)_{345}$, Cliff is able to prepare the aimed state without the possibility of failure, whatever the measurement result of qubits (A, B) is 100% success means that the proposed method is reliable and steady.

3 Further discussion

It can be seen that the proposed scheme runs successfully. Since applications of entanglement in quantum information processing such as entanglement distribution [24–26] and controlled teleportation [27–30] are meritorious, then a problem arised in that this method seems useless for its real application value. However, a difference is made when the initial state of $(a_0|00\rangle + a_1|01\rangle + a_2|10\rangle + a_3|11\rangle)_{\text{AB}}$ can be written as $(A_0|0\rangle + A_1|1\rangle)_{\text{A}} \otimes (B_0|0\rangle + B_1|1\rangle)_{\text{B}}$, two independent inputs. This means the embodiment of simultaneous quantum network coding [31, 32] to some degree. A detailed description is presented in Fig. 2.

Network coding refers to network of nodes with information processing ability, and its typical example is a butterfly network [33, 34]. This theory points out that Shannon's maximal network capacity could be achieved by network coding, other than a store-forward routing plan. The nature of a quantum network endows quantum network coding with higher complexity, and our work could be viewed as quantum network node processing [35]. Due to the nature of the projective measurement of the Hadamard transferred basis which is analyzed before, the advanced

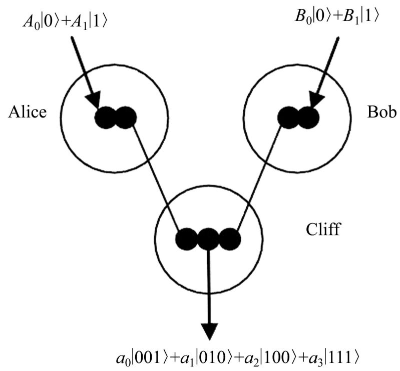

Fig. 2. Quantum network node processing.

node could process the information from Alice and Bob simultaneously. Quantum network nodes with this scheme are able to encode information from different sources without extra time consumption, which is of importance in quantum network coding.

4 Conclusion

In summary, a novel determinate joint remote preparation scheme of an arbitrary W-class quantum state has been proposed. The presented method is implemented by projective measurement of the Hadamard transferred basis. Compared with other works, the proposed scheme is advanced for the conversion of a global measurement to several local measurements, which means synchronous information processing. Furthermore, the proposed scheme prepares the quantum state successfully, without the probability of failure. The characteristics of the proposed scheme fit the requirements of updated quantum network nodes, which are beneficial to simultaneous quantum network coding.

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