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A scenario for high accuracy τ mass measurement at BEPC-Ⅱ *

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Abstract: Scenarios for the τ mass measurement at the upgraded Beijing Electron-Positron Collider (BEPC-Ⅱ) are studied. A nested minimization procedure is used to optimize the data taking plan. It is found that by using five energy points with the total integrated luminosity of 100 pb^{-1}, the τ mass can be determined with a statistical error of 50 keV.

Key words: τ mass, scan optimization uncertainty

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1 Introduction

The τ-lepton is a fundamental particle, and its mass is a Standard Model parameter, which requires that its mass be determined with high accuracy. According to the Particle Data Group [1], the current world average value of the τ−lepton mass is

$$m_\tau^{\rm PDG} = 1776.82 \pm 0.16 \text{ MeV}.$$

It is based mostly on four results:

$$m_\tau^{\rm BES} = 1776.96^{+0.18}_{-0.21}{}^{+0.25}_{-0.17} \text{ MeV [2]},$$

$$m_\tau^{\rm KEDR} = 1776.81^{+0.25}_{-0.23} \pm 0.15 \text{ MeV [3]},$$

$$m_\tau^{\rm BELLE} = 1776.61 \pm 0.13 \pm 0.35 \text{ MeV [4]},$$

$$m_\tau^{\rm BABAR} = 1776.68 \pm 0.12 \pm 0.41 \text{ MeV [5]}.$$

The latter two were obtained by using the pseudomass method. Employing the huge amount of data from B-factories, they have good statistical accuracy but have large systematic uncertainties related to the absolute calibration of the particle momentum measurements.

The former two experiments involve scans of the τ threshold region. The mass value was extracted from the dependence of the production cross section on the beam energy. In the BES [2] experiment, the beam energy was calibrated using the known masses of J/ψ and ψ′ by scans of these resonances, and the beam energy during the τ scan was calculated assuming the linearity of the accelerator energy scale. In the KEDR experiment, the beam energy was determined by the extrapolation of the resonant depolarization data [6, 7] and measured by using an infra-red Compton backscattering technique (CBS) [8]. The precise beam energy determination of the KEDR experiment reduced the systematic uncertainty, but the statistical uncertainty of the measurement was limited.

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A CBS beam energy measurement system with an accuracy of about 50 keV was recently put into operation at the BEPC-II collider [9]. This, together with the high luminosity of the collider (2×10^{32} cm$^{-2}\cdot$s^{-1} at τ threshold), will allow a τ mass measurement with improved accuracy. Below some aspects of the energy measurements using CBS are discussed, and possible experimental scenarios are considered.

2 Energy calibration of the τ threshold experiment

In the BEPC-II τ threshold scan, the accuracy of the beam energy determination should be verified in situ. This can be done by measuring the J/ψ and ψ′ meson masses $M^{\rm CBS}_{{\rm J}/\psi}$ and $M^{\rm CBS}_{\psi'}$ with statistical accuracies $\sigma_{M^{\rm CBS}_{\psi'}}$ and $\sigma_{M^{\rm CBS}_{{\rm J}/\psi}}$ of about 30 keV using the CBS method, and comparing them with the PDG values $M^{\rm PDG}_{\psi'}$ and $M^{\rm PDG}_{{\rm J}/\psi}$. The BEPC-II energy spread σ_W can also be determined from the scans.

From scans of the resonances, corrections to the CBS energy at the J/ψ and ψ′

$$\Delta'=\frac{M^{\rm PDG}_{\psi'}-M^{\rm CBS}_{\psi'}}{2}\ ,\quad \Delta=\frac{M^{\rm PDG}_{{\rm J}/\psi}-M^{\rm CBS}_{{\rm J}/\psi}}{2},\qquad (1)$$

are determined, and the energy correction at the τ-threshold

$$\Delta_{m_\tau}\approx\frac{\Delta'\cdot(2m^{\rm PDG}_\tau-M^{\rm PDG}_{{\rm J}/\psi})+\Delta\cdot(M^{\rm PDG}_{\psi'}-2m^{\rm PDG}_\tau)}{M^{\rm PDG}_{\psi'}-M^{\rm PDG}_{{\rm J}/\psi}}$$

may be estimated assuming that the energy correction has a linear dependence, as example. The systematical uncertainty of the energy determination (error of Δ_{m_τ} correction) may be estimated by using error propagation:

$$\sigma^{\rm CBS}_{m_\tau}\approx\frac{0.78}{2}\sqrt{\sigma^2_{M^{\rm CBS}_{\psi'}}+\sigma^2_{M^{\rm PDG}_{\psi'}}},$$

where $\sigma_{M^{\rm PDG}_{\psi'}}$ is the error of $M^{\rm PDG}_{\psi'}$. The error is dominated by the ψ′ mass measurement. The real analysis of systematic errors should be done after data acquisition.

3 Statistical error of the τ mass measurement

The cross section of the process $e^+e^-\to\tau^+\tau^-$ at a center-of-mass energy W is expressed as

$$\sigma(W)=\frac{1}{\sqrt{2\pi}\sigma_W}\int{\rm d}W'\exp\left\{-\frac{(W-W')^2}{2\sigma^2_W}\right\}$$
$$\times\int{\rm d}xF(x,W')\sigma_{\rm fs}(W'\sqrt{1-x}),$$

where the first integral takes into account the energy spread $\sigma_W=1.5$ MeV and the second one the initial state radiation (ISR) correction, where

$$\sigma_{\rm fs}(W)=\frac{4\pi\alpha^2}{3W^2}\frac{\beta(3-\beta^3)}{2}\frac{F_{\rm c}(\beta)F_{\rm r}(\beta)}{|1-\Pi(W)|^2}$$

includes the Coulomb interaction correction $F_{\rm c}$, the final state radiative correction $F_{\rm r}(\beta)$ [10], and the vacuum polarization $\Pi(W)$, and $\beta=(1-(2m_\tau/W)^2)^{1/2}$ is the τ velocity. The energy behavior of the τ production cross section is shown in Fig. 1.

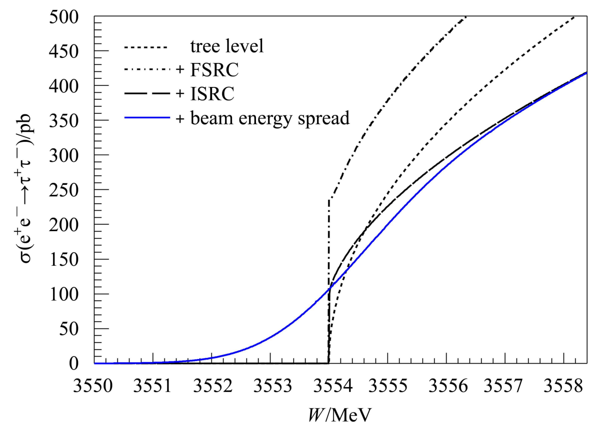

Fig. 1. The energy dependence of the $e^+e^-\to\tau^+\tau^-$ cross section near the production threshold.

The expected visible cross section is

$$\sigma^{\rm obs}(W,\ m_\tau,\ \varepsilon,\ \sigma_{\rm B})=\varepsilon\cdot\sigma(W)+\sigma_{\rm B},\qquad (2)$$

where ε and $\sigma_{\rm B}$ are the overall detection efficiency [11], and background cross section, respectively. For the τ-mass measurement, data are collected at several energy points around $\tau^+\tau^-$ production threshold. At each energy point $W_i=2E_i$, the integrated luminosity L_i and the number N_i of selected $e^+e^-\to\tau^+\tau^-$ candidate events are obtained. Here, E_i is the beam energy. The tau lepton mass can be found by fitting the expected number of events $N^{\rm exp}_i=\sigma^{\rm obs}(W_i)L_i$ to the number of selected events N_i. The free parameters of the fit are m_τ, $\sigma_{\rm B}$, and ε. Actually, ε is a general normalization factor, which implicitly includes the luminosity correction multiplier $\mathcal{R}_{\rm L}$. Unlike the background cross section, this normalization factor can be fixed reliably by using a Monte Carlo simulation. However, keeping it free substantially simplifies the data analysis.

In order to achieve the highest possible accuracy of the mass measurement, the optimization of luminosity and location of the energy points is necessary.

Optimization requires some constraints on the energies and luminosities to be imposed in order to avoid the merging of the points. It is necessary to have at least three energy points to obtain the three parameters of the fit. For the 3-point scenario, the only constraints required for optimization are the total integrated luminosity and the full energy range, which must be narrow enough to ensure a sufficient uniformity of the background and to minimize the efficiency variation.

The statistical uncertainty of the τ mass value is minimal in the 3-point scenario, and each extra point costs some additional luminosity. Nevertheless, extra points: (a) allow for a χ^2 check to confirm the threshold behavior of the observed cross section; (b) provide robustness to the fit against the change of the expected mass value and the background level variation during the selection criteria optimization; (c) give the possibility to check the systematic uncertainty due to background and efficiency uniformity; and (d) make the experiment more convincing.

Previously, a Monte Carlo simulation of the experiment was used with a sampling technique for the 3-point scenario [12, 13]. In this work, nested minimization procedures, which do not use Monte Carlo simulations, are used to optimize the data taking scenarios. We consider both 3-point and 5-point scenarios.

A scan scenario is parametrized by the point energy offsets $\Delta E_i = E_i - m_\tau^{\rm PDG}$ $(i=2,\ n-1)$ and the luminosity fractions L_i $(i=1,n)$. ΔE_1 and ΔE_n must be far enough below and above the threshold to determine the background level and detection efficiency in the fit but not so far that the efficiency would change. Thus $\Delta E_1 = -5$ MeV and $\Delta E_n = 15$ MeV are fixed in order to obtain ε and $\sigma_{\rm B}$ from the fit. The total integrated luminosity is taken as $L = 100\ {\rm pb}^{-1}$, assuming it can be collected during 10 days of data taking. The ratio $\sigma_{\rm B}/\varepsilon$ was varied from 0 to 16 pb in the fits.

To obtain the error of the τ-mass measurement for a particular scenario, the calculated number of events

$$N_i^{\rm calc} = L_i \sigma^{\rm obs}\left(2E_i,\ m_\tau^{\rm PDG},\ \varepsilon,\ \sigma_{\rm B}\right)$$

is fitted with the expected number of events

$$N_i^{\rm exp} = L_i \sigma^{\rm obs}\left(2E_i,\ m_\tau^*,\ \varepsilon^*,\ \sigma_{\rm B}^*\right)$$

through minimization of the likelihood function

$$-2\mathscr{L} = 2\sum_i \left[N_i^{\rm calc}\ln\left(\frac{N_i^{\rm calc}}{N_i^{\rm exp}}\right) + N_i^{\rm exp} - N_i^{\rm calc}\right].$$

The free parameters of the fit are m_τ^*, ε^*, $\sigma_{\rm B}^*$. The result of the fit is $N_i^{\rm calc} = N_i^{\rm exp}$ and $m_\tau^* = m_\tau^{\rm PDG}$, $\sigma_{\rm B}^* = \sigma_{\rm B}$, $\varepsilon^* = \varepsilon$. The τ-mass error σ_{m_τ} is the parabolic error of the parameter m_τ^*, which is obtained during the fit by the MINUIT [14] program.

Using a large set of scenarios, the error as a function of L_i and ΔE_i $(2n-2$ variables) was obtained using MINUIT, and the optimal scenario corresponds to the minimum of this function.

The dependences of the 3-point scenario optimal parameters on the $\sigma_{\rm B}/\varepsilon$ ratio are presented in Fig. 2.

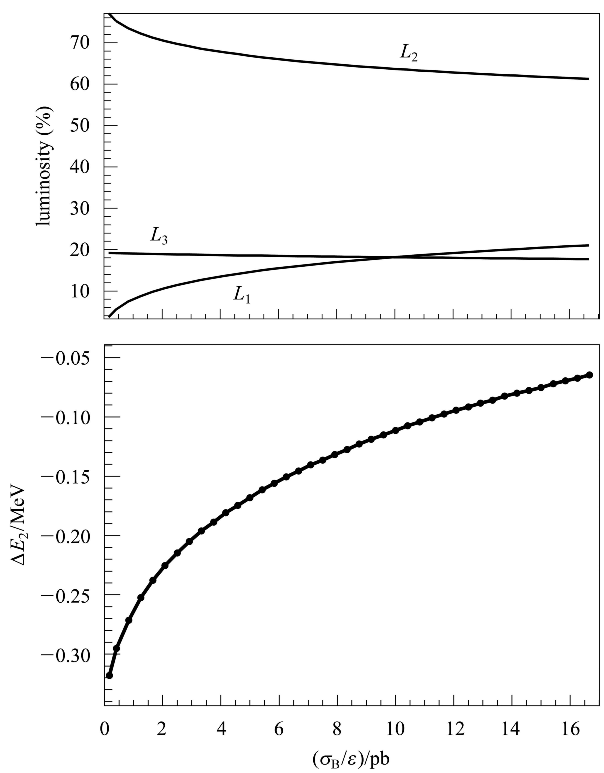

Fig. 2. Optimal parameters for the 3-point scan versus the ratio of background cross section to detection efficiency.

The 3-point scenario does not give information about possible instabilities during the scan (detector efficiency variation, beam (background) instabilities, energy measurement instabilities) and is sensitive to the uncertainty of the PDG tau mass value. Additional points must be added to check the threshold shape. For the 5-point scenario, additional constraints are needed to suppress the collapse to the 3-point scenario, which is an ideal case. The following are additional constraints with the 5-point scenario.

1) The distance between points 2 and 3 is fixed as $E_3 - E_2 = 2.5\,\sigma_{m_\tau}^{\rm PDG}$, where $\sigma_{m_\tau}^{\rm PDG}$ is the PDG error of the τ-mass. This constraint reduces the sensitivity to the assumed mass value.

2) A fourth energy point allows a chi-square value to be determined to see if the data fits the assumed

threshold shape. It should be close to the threshold to reduce the tau mass error but higher than the beam energy spread. The point is fixed at $\Delta E_4 = +3.5$ MeV.

3) The luminosity fractions of points 2 and 3 should be similar since they are close to the uncertain PDG tau mass value. We fix $L_2/L_3 = 1.5$ in order to break the symmetry of points 2 and 3.

4) The L_4 luminosity should not be higher than L_2 or L_3 because it is not an optimal energy point. The more integrated luminosity at the threshold, the higher measurement accuracy we will have. However, point 4 is needed to check the threshold shape. Therefore the L_4 luminosity value should be around that of L_5. The luminosity ratio of points 4 and 5 is fixed at $L_4/L_5 = 0.5$.

The dependence of the τ mass measurement error on the σ_B/ε ratio for the 5- and 3-point scenarios is shown in Fig. 3. The loss of precision in the 5-point scenario compared with the 3-point case is about 5%. Variations of E_4 and the ratio of L_4/L_5 increase the error of the τ mass σ_{m_τ} by less than 10%. The optimum 5-point τ threshold scan scenario is shown in Fig. 4 and Table 1.

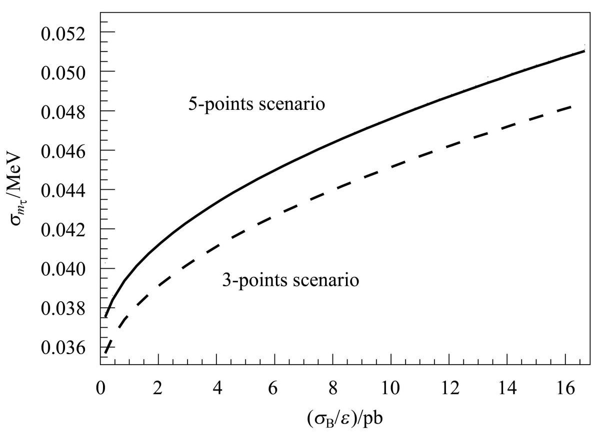

Fig. 3. The error of τ mass measurement versus σ_B/ε for $\sigma_W \simeq 1.35$ MeV. Solid and dashed lines correspond to the 5- and 3-point scenarios.

The expected statistical accuracy of the τ mass measurement can be parameterized as a function of total luminosity L, ε and σ_B:

$$\sigma_{m_\tau}^{\rm stat} \approx \left(\frac{\sigma_W}{\sigma_W^0}\right)^{0.75} \sqrt{\frac{L_0\varepsilon_0}{L\varepsilon}\left[a^2+b^2\left(\frac{\varepsilon_0}{\varepsilon}\frac{\sigma_B}{\sigma_B^0}\right)^{0.58}\right]}$$

where $L_0 = 100$ pb^{-1}, $\varepsilon_0 = 6\%$, $\sigma_B^0 = 0.3$ pb, $a = 41.4$ keV, $b = 28.9$ keV, $\sigma_W^0 = 1.49$ MeV. For $L = 10^2$ pb^{-1}, $\varepsilon = 6\%$ [11], $\sigma_B = 0.3$ pb and $\sigma_W = 1.5$ MeV the expected statistical accuracy is

$$\sigma_{m_\tau}^{\rm stat} \simeq 51 \text{ keV}.$$

Table 1. The optimum 5-point scenario.

points	ΔE_1	ΔE_2	ΔE_3	ΔE_4	ΔE_5
ΔE_i/MeV	−5	−0.325	+0.075	+3.5	+15
L_i%	14	39	26	7	14

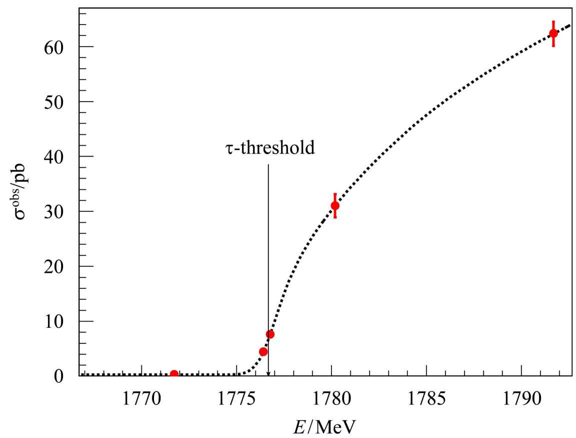

Fig. 4. The visible cross section as a function of the beam energy. The dots with error bars show the expected measurements in a 5-points tau threshold scan scenario.

4 The experimental procedure

An ideal experimental procedure should include the following steps:

1) choose selection criteria using simulated data;

2) take part of the background data to estimate the cross section;

3) take part of the data at maximal energy to fix the efficiency;

4) calculate the luminosity percentage for all points;

5) perform ψ(2S) and J/ψ scans;

6) take data at the first point near the threshold;

7) perform a ψ(2S) scan with reduced beam currents;

8) take data at the second point near the threshold;

9) perform ψ(2S) and J/ψ scans; and

10) take the remainder of the data.

In such an approach, both the statistical error and the systematic uncertainties are minimized. A ψ(2S) scan with reduced beam currents is necessary to study the beam energy measurement systematics due to the counting rate of the beam energy measurement system.

It might be difficult to change the collider energy from scans of the J/ψ and ψ(2S) to the τ threshold and vice versa. An alternative data taking scheme follows:

1) perform a J/ψ scan (24 hours to get sufficient CBS statistics);

2) take background data below the threshold;

3) take data at the first point near the threshold;

4) take data at the second point near the threshold;

5) take data at points 4 and 5 above the threshold; and

6) perform a ψ(2S) scan (24 hours to get sufficient CBS statistics).

In order to examine the stability of the beam energy measurements, this scanning should be repeated several times.

5 Conclusion

Scenarios of the τ-mass measurement based on τ threshold scans with the BES-Ⅲ detector at the BEPC-Ⅱ collider are studied. The beam energy is determined with the CBS method. A statistical accuracy of about 50 keV can be achieved during 10 days of data taking with a total integrated luminosity of 100 pb^{-1}. The systematic error is expected to be dominated by the uncertainty of the beam energy measurement and to be about 50 keV.

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