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T2K indication of relatively large θ_{13} and a natural perturbation to the democratic neutrino mixing pattern*

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Abstract: The T2K Collaboration has recently reported a remarkable indication of the $\nu_\mu \to \nu_e$ oscillation which is consistent with a relatively large value of θ_{13} in the three-flavor neutrino mixing scheme. We show that it is possible to account for such a result of θ_{13} by introducing a natural perturbation to the democratic neutrino mixing pattern, without or with CP violation. A testable correlation between θ_{13} and θ_{23} is predicted in this ansatz. We also discuss the Wolfenstein-like parametrization of neutrino mixing, and comment on other possibilities of generating sufficiently large θ_{13} at the electroweak scale.

Key words: T2K experiment, θ_{13}, democratic neutrino mixing

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1 Introduction

Current solar, atmospheric, reactor and accelerator neutrino oscillation experiments have provided us with very convincing evidence that neutrinos are massive and lepton flavors are mixed [1]. In the basis where the flavor eigenstates of three charged leptons are identified with their mass eigenstates, the mixing of neutrino flavors is effectively described by a 3×3 unitary matrix $\boldsymbol{V}$ whose nine elements can be parametrized in terms of three rotation angles and three CP-violating phases:

$$\boldsymbol{V} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23}-s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23}-s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix} P_\nu, \quad (1)$$

where $c_{ij}\equiv\cos\theta_{ij}$, $s_{ij}\equiv\sin\theta_{ij}$ (for $ij=12,13,23$), and $P_\nu=\mathrm{Diag}\{e^{i\rho},e^{i\sigma},1\}$ is a diagonal phase matrix which is physically relevant if three neutrinos are the Majorana particles. The latest global analysis of current neutrino oscillation data, done by Schwetz et al [2], yields $s^2_{12}=0.312^{+0.017}_{-0.015}$, $s^2_{13}=0.010^{+0.009}_{-0.006}$ (NH) or $0.013^{+0.009}_{-0.007}$ (IH) and $s^2_{23}=0.51\pm0.06$ (NH) or 0.52 ± 0.06 (IH) at the 1σ level, where "NH" and "IH" correspond respectively to the normal and inverted neutrino mass hierarchies. The central values of three mixing angles are approximately $\theta_{12}\approx34^\circ$, $\theta_{13}\approx6^\circ$ and $\theta_{23}\approx46^\circ$. Unfortunately, three CP-violating phases of $\boldsymbol{V}$ remain entirely unconstrained. The ongoing and forthcoming neutrino oscillation experiments will measure θ_{13} and δ, and the neutrinoless double-beta decay experiments will hopefully help to probe or constrain ρ and σ.

The magnitude of θ_{13} is one of the central concerns in today's neutrino phenomenology. The most stringent upper bound on this angle is $\theta_{13}<11.4^\circ$ at the 90% confidence level, as set by the CHOOZ [3] and MINOS [4] experiments. Besides Ref. [2], there exist several earlier analyses indicating that the smallest neutrino mixing angle θ_{13} might not be very small. For example, $\theta_{13}\approx7.3^{+2.0^\circ}_{-2.9^\circ}$ (1σ) by Fogli et al [5], $\theta_{13}\approx5.1^{+3.0^\circ}_{-3.3^\circ}$ (1σ) by Gonzalez-Garcia et al

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[6], and $\theta_{13} \approx 8.1^{+2.8^\circ}_{-4.5^\circ}$ as the best-fit value by the KamLAND Collaboration [7]. Although the statistical significance of these results remains quite low, they do imply that θ_{13} is possible to lie in the range $5^\circ \lesssim \theta_{13} \lesssim 11^\circ$.

A more robust indication of relatively large θ_{13} comes from the latest T2K measurement:

$$\begin{aligned} &0.03 < \sin^2 2\theta_{13} < 0.28 \text{ or } 5.0^\circ \lesssim \theta_{13} \lesssim 16.0^\circ \text{ (NH)} \\ &0.04 < \sin^2 2\theta_{13} < 0.34 \text{ or } 5.8^\circ \lesssim \theta_{13} \lesssim 17.8^\circ \text{ (IH)}, \end{aligned} \quad (2)$$

for $\delta = 0^\circ$ and at the 90% confidence level [8]. The best-fit points are $\sin^2 2\theta_{13} = 0.11$ (NH) or 0.14 (IH), corresponding to $\theta_{13} = 9.7^\circ$ (NH) or 11.0° (IH). If such a value of θ_{13} is finally established, it will rule out a large number of neutrino mass models on the market and provide us with a great hope to observe leptonic CP violation in the long-baseline neutrino oscillation experiments in the foreseeable future.

In this paper we propose a phenomenologically simple way to generate a sufficiently large value of θ_{13}. The point is to introduce a natural perturbation to the democratic neutrino mixing pattern $\boldsymbol{U}$ [9], such that all three mixing angles of $\boldsymbol{U}$ receive comparable corrections which can be as large as about 10°. We focus on a specific perturbation matrix $\boldsymbol{X}$ and determine its structure by using current experimental data on the full neutrino mixing matrix $\boldsymbol{V} = \boldsymbol{U}\boldsymbol{X}$. This ansatz predicts an interesting correlation between θ_{13} and θ_{23}, which leads to $\theta_{13} \approx 9.6^\circ$ for $\theta_{23} = 45^\circ$, a result in good agreement with the T2K indication. A Wolfenstein-like parametrization of $\boldsymbol{V}$ and leptonic CP violation are also discussed. Finally, we comment on a few other possibilities of obtaining appreciable θ_{13} at the electroweak scale.

2 The ansatz

Given a specific phase convention which will be convenient for our subsequent discussions, the democratic mixing pattern reads as follows [9]:

$$\boldsymbol{U} = \begin{pmatrix} \sqrt{\frac{1}{2}} & \sqrt{\frac{1}{2}} & 0 \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{6}} & -\sqrt{\frac{2}{3}} \\ -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{3}} \end{pmatrix}, \quad (3)$$

whose three mixing angles are $\theta_{12}^{(0)} = 45^\circ$, $\theta_{13}^{(0)} = 0^\circ$ and $\theta_{23}^{(0)} = \arctan(\sqrt{2}) \approx 54.7^\circ$ in the standard parametrization as given in Eq. (1). It has been pointed out that the tri-bimaximal mixing pattern [10], which is simply a "twisted" form of the democratic mixing pattern, can be directly obtained from $\boldsymbol{U}$ by making an equal shift of its two nonzero mixing angles [11]:

$$\theta_* \equiv \theta_{12}^{(0)} - \vartheta_{12}^{(0)} = \theta_{23}^{(0)} - \vartheta_{23}^{(0)} \approx 9.7^\circ, \quad (4)$$

where $\vartheta_{12}^{(0)} = \arctan(1/\sqrt{2}) \approx 35.3^\circ$ and $\vartheta_{23}^{(0)} = 45^\circ$ are the nonzero mixing angles of the tri-bimaximal mixing pattern. Note that the value of θ_* is quite suggestive because it is so close to the best-fit value of θ_{13} given by the present T2K data. Indeed, a novel and viable neutrino mixing ansatz with $\theta_{13} \approx \theta_* \approx 9.7^\circ$ has recently been proposed in Ref. [11].

Note also that $\boldsymbol{U}$ was originally obtained, as the leading term of the lepton flavor mixing matrix $\boldsymbol{V}$, from breaking the $S(3)_{\rm L} \times S(3)_{\rm R}$ flavor symmetry of the charged lepton mass matrix $\boldsymbol{M}_{\rm l}$ in the basis where the neutrino mass matrix $\boldsymbol{M}_\nu$ is diagonal [9]. Here we assume $\boldsymbol{V} = \boldsymbol{U}\boldsymbol{X}$, where $\boldsymbol{X}$ denotes a generic perturbation matrix which can absorb small contributions from the flavor symmetry breaking terms of both $\boldsymbol{M}_{\rm l}$ and $\boldsymbol{M}_\nu$ [9, 12, 13]. In general, of course, $\boldsymbol{U}$ itself might come from either the charged lepton sector or the neutrino sector, or both of them. The details are certainly model-dependent.

To be explicit, we assume that $\boldsymbol{X}$ has a simple pattern parallel to that of $\boldsymbol{U}$:

$$\boldsymbol{X} = \begin{pmatrix} c'_{12} & -s'_{12} & 0 \\ s'_{12}c'_{23} & c'_{12}c'_{23} & s'_{23} \\ s'_{12}s'_{23} & c'_{12}s'_{23} & -c'_{23} \end{pmatrix}, \quad (5)$$

where $c'_{ij} \equiv \cos\theta'_{ij}$ and $s'_{ij} \equiv \sin\theta'_{ij}$ (for $ij = 12, 23$). The phase convention of $\boldsymbol{X}$ is taken in such a way that all three mixing angles of the full flavor mixing matrix $\boldsymbol{V} = \boldsymbol{U}\boldsymbol{X}$ lie in the first quadrant when CP is invariant. For simplicity, we tentatively ignore possible CP-violating phases in $\boldsymbol{U}$ and $\boldsymbol{X}$. In this case we obtain

$$\begin{aligned} V_{\rm e1} &= \sqrt{\frac{1}{2}}\,(c'_{12} + s'_{12}c'_{23}), \\ V_{\rm e2} &= \sqrt{\frac{1}{2}}\,(c'_{12}c'_{23} - s'_{12}), \\ V_{\rm e3} &= \sqrt{\frac{1}{2}}\; s'_{23}, \\ V_{\mu3} &= \sqrt{\frac{1}{6}}\,(2c'_{23} - s'_{23}), \\ V_{\tau3} &= \sqrt{\frac{1}{3}}\,(c'_{23} + s'_{23}), \end{aligned} \quad (6)$$

in which θ'_{12} and θ'_{23} are also assumed to lie in the first quadrant. Comparing this result with the standard parametrization of $\boldsymbol{V}$ in Eq. (1), we immediately arrive at

$$t_{12} = \left|\frac{V_{e2}}{V_{e1}}\right| = \frac{c'_{12}c'_{23} - s'_{12}}{c'_{12} + s'_{12}c'_{23}} , \quad s_{13} = |V_{e3}| = \sqrt{\frac{1}{2}}\ s'_{23} , \quad t_{23} = \left|\frac{V_{\mu 3}}{V_{\tau 3}}\right| = \frac{2c'_{23} - s'_{23}}{\sqrt{2}\,(c'_{23} + s'_{23})} , \tag{7}$$

where $t^{(\prime)}_{ij} \equiv \tan\theta^{(\prime)}_{ij}$ (for $ij = 12,\ 23$). Therefore,

$$t'_{23} = \frac{\sqrt{2}\left(\sqrt{2} - t_{23}\right)}{1 + \sqrt{2}\ t_{23}} , \quad t'_{12} = \frac{1 + \sqrt{2}\ t_{23} - t_{12}\sqrt{5 - 2\sqrt{2}\ t_{23} + 4t^2_{23}}}{t_{12}\left(1 + \sqrt{2}\ t_{23}\right) + \sqrt{5 - 2\sqrt{2}\ t_{23} + 4t^2_{23}}} . \tag{8}$$

Since both θ_{13} and θ_{23} depend on a single parameter θ'_{23}, they have the following correlation:

$$s_{13} = \frac{\sqrt{2} - t_{23}}{\sqrt{5 - 2\sqrt{2}\ t_{23} + 4t^2_{23}}} . \tag{9}$$

This expression can be regarded as the analytical prediction of our ansatz. Some discussions about the above results are in order.

(1) Given $\theta_{23} = 45^\circ$, Eq. (9) leads us to a numerical prediction of the smallest neutrino mixing angle θ_{13}:

$$\theta_{13} = \arcsin\left[\frac{\sqrt{2} - 1}{\sqrt{9 - 2\sqrt{2}}}\right] \approx 9.6^\circ . \tag{10}$$

This result is in good agreement with the best-fit value of θ_{13} extracted from the T2K data. If $\theta_{23} \approx 46^\circ$ is taken [2], one then arrives at $\theta_{13} \approx 8.6^\circ$.

(2) Fixing $\theta_{23} = 45^\circ$, we obtain $\theta'_{23} \approx 13.6^\circ$ from Eq. (8). This value is very close to the Cabibbo angle $\theta_{\rm C} \approx 13^\circ$ of quark flavor mixing [1], whose sine function $\sin\theta_{\rm C} \approx 0.22$ can be treated as a perturbation to the identity matrix to get the realistic Cabibbo-Kobayashi-Maskawa matrix [14]. Taking $\theta_{12} \approx 34^\circ$ together with $\theta_{23} = 45^\circ$, we can also obtain $\theta'_{12} \approx 10.2^\circ$. It is interesting to see that θ'_{12} and θ'_{23} are comparable in magnitude, and they are also comparable with θ_{13}. In this sense, we argue that the perturbation to $\boldsymbol{U}$ is quite natural.

(3) If one simply assumes $\theta'_{12} \approx \theta'_{23} \approx \theta_{\rm C}$ from a model-building point of view at the electroweak scale, then Eq. (7) gives the predictions

$$\theta_{12} = \arctan\left[\frac{\cos^2\theta_{\rm C} - \sin\theta_{\rm C}}{\cos\theta_{\rm C}\,(1 + \sin\theta_{\rm C})}\right] \approx 31.3^\circ , \quad \theta_{13} = \arcsin\left[\sqrt{\frac{1}{2}}\ \sin\theta_{\rm C}\right] \approx 9.2^\circ , \quad \theta_{23} = \arctan\left[\frac{2\cos\theta_{\rm C} - \sin\theta_{\rm C}}{\sqrt{2}\,(\cos\theta_{\rm C} + \sin\theta_{\rm C})}\right] \approx 45.5^\circ , \tag{11}$$

which are also consistent with current experimental data. An explicit neutrino mass model of this nature will be explored elsewhere.

(4) The above hypothesis is interesting in the sense that it suggests a Wolfenstein-like parametrization of the neutrino mixing matrix [15]. Setting $s'_{12} = s'_{23} = \sin\theta_{\rm C} \equiv \lambda \approx 0.22$, we approximately obtain

$$\boldsymbol{V} = \begin{pmatrix} \sqrt{\frac{1}{2}}(1+\lambda) & \sqrt{\frac{1}{2}}(1-\lambda) & \sqrt{\frac{1}{2}}\ \lambda \\ \sqrt{\frac{1}{6}}(1-\lambda) & -\sqrt{\frac{1}{6}}(1+3\lambda) & \sqrt{\frac{2}{3}}\left(1 - \frac{1}{2}\lambda\right) \\ -\sqrt{\frac{1}{3}}(1-\lambda) & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{3}}(1+\lambda) \end{pmatrix} + \mathcal{O}(\lambda^2) + \cdots . \tag{12}$$

It becomes transparent that eight of the nine matrix elements of $\boldsymbol{U}$ receive the $\mathcal{O}(\lambda)$ corrections. In other words, all three mixing angles of $\boldsymbol{U}$ get corrected in a quite similar way and with a quite similar strength.

As pointed out in Ref. [11], it is difficult to generate relatively large θ_{13} from natural perturbations to the tri-bimaximal mixing pattern, unless the perturbations are adjusted in such a way that its two nonzero mixing angles are slightly modified but its vanishing mixing angle is significantly modified. This kind of perturbations seem to be strange.

3 *CP* violation

Now let us look at the possibility of introducing leptonic CP violation into the neutrino mixing matrix $\boldsymbol{V}$. For this purpose, one of the simplest ways is to make the transformation $s'_{12} \to s'_{12}{\rm e}^{{\rm i}\phi}$ with ϕ being a real phase parameter. In this case $\boldsymbol{X}$ becomes complex and thus $\boldsymbol{V} = \boldsymbol{U}\boldsymbol{X}$ contains a nontrivial CP-violating phase. Then

$$V_{e1} = \sqrt{\frac{1}{2}}\left(c'_{12} + s'_{12}c'_{23}{\rm e}^{{\rm i}\phi}\right) , \quad V_{e2} = \sqrt{\frac{1}{2}}\left(c'_{12}c'_{23} - s'_{12}{\rm e}^{{\rm i}\phi}\right) ; \tag{13}$$

and $V_{\mu1}$, $V_{\mu2}$, $V_{\tau1}$ and $V_{\tau2}$ are also complex. We get

$$t_{12} = \left|\frac{V_{e2}}{V_{e1}}\right| = \frac{\sqrt{(c'_{23})^2+(t'_{12})^2-2t'_{12}c'_{23}\cos\phi}}{\sqrt{1+(t'_{12}c'_{23})^2+2t'_{12}c'_{23}\cos\phi}} ,$$

$$J_V \equiv \mathrm{Im}\left(V_{e2}V_{\mu3}V^*_{e3}V^*_{\mu2}\right) = \frac{1}{6}c'_{12}(s'_{12})^2(2c'_{23}-s'_{12})(c'_{23}+s'_{23})\sin\phi , \quad (14)$$

where J_V is the Jarlskog invariant of leptonic CP violation. Note that the results for s_{13} and t_{23} are the same as those in Eq. (7), and thus Eq. (9) also holds in the present ansatz. Typically taking $\theta_{12}\approx 34^\circ$ and $\theta_{23}=45^\circ$, we first obtain $\theta'_{23}\approx 13.6^\circ$ from Eq. (8) and then the constraint equation

$$(t'_{12})^2-4.96t'_{12}\cos\phi+0.86\approx 0 \quad (15)$$

from Eq. (14). In the assumption of $\cos\phi\approx 0.9$, for instance, we arrive at $\theta'_{12}\approx 11.4^\circ$. The leptonic Jarlskog invariant turns out to be $J_V\approx 4.8\times10^{-3}$, about two orders of magnitude larger than the corresponding Jarlskog parameter in the quark sector [14]. Larger CP-violating effects are possible in this ansatz if one assumes ϕ to be reasonably large, but $\phi\approx 90^\circ$ is forbidden as one can easily see from Eq. (15). Because $J_V = c_{12}s_{12}c^2_{13}s_{13}c_{23}s_{23}\sin\delta$ holds in the standard parametrization of V, it is straightforward to establish the relationship between δ and ϕ with the help of Eq. (14).

In the presence of CP violation as introduced above, the Wolfenstein-like parametrization of $\boldsymbol{V}$ in Eq. (12) becomes

$$\boldsymbol{V} = \begin{pmatrix} \sqrt{\frac{1}{2}}(1+\lambda e^{i\phi}) & \sqrt{\frac{1}{2}}(1-\lambda e^{i\phi}) & \sqrt{\frac{1}{2}}\,\lambda \\ \sqrt{\frac{1}{6}}(1-\lambda e^{i\phi}) & -\sqrt{\frac{1}{6}}(1+2\lambda+\lambda e^{i\phi}) & \sqrt{\frac{2}{3}}\left(1-\frac{1}{2}\lambda\right) \\ -\sqrt{\frac{1}{3}}(1-\lambda e^{i\phi}) & \sqrt{\frac{1}{3}}(1-\lambda+\lambda e^{i\phi}) & \sqrt{\frac{1}{3}}(1+\lambda) \end{pmatrix} + \mathcal{O}(\lambda^2)+\cdots . \quad (16)$$

An appreciable value of θ_{13} is also a good news to the leptonic unitarity triangles [14], which can be used to geometrically describe CP violation in the lepton sector. The area of each unitarity triangle is equal to $|J_V|/2\approx\lambda^2|\sin\phi|/6$. If the T2K experiment is finally able to probe the CP-violating asymmetry between the probabilities of $\nu_\mu\to\nu_e$ and $\overline{\nu}_\mu\to\overline{\nu}_e$ oscillations, then it will be possible to determine J_V itself through

$$P(\overline{\nu}_\mu\to\overline{\nu}_e)-P(\nu_\mu\to\nu_e) = 16J_V\sin\frac{\Delta m^2_{21}L}{4E}\sin\frac{\Delta m^2_{31}L}{4E}\sin\frac{\Delta m^2_{32}L}{4E} \quad (17)$$

in the neglect of terrestrial matter effects. Even the matter effects are non-negligible or significant, it is likely to reconstruct the leptonic unitarity triangles in vacuum from those effective ones in matter and then pin down the genuine effect of CP violation [16].

4 Discussions

In summary, we have taken account of the robust T2K indication of a relatively large value of θ_{13} and paid particular attention to how to confront a constant neutrino mixing pattern, which may be motivated by a certain flavor symmetry and can predict $\theta_{13}=0^\circ$ in the symmetry limit, with $\theta_{13}\sim 10^\circ$. We have shown that a natural perturbation to the democratic mixing pattern $\boldsymbol{U}$ can easily produce the realistic neutrino mixing matrix $\boldsymbol{V}$ with sufficiently large θ_{13}. An interesting relationship between θ_{13} and θ_{23} has been predicted in this ansatz, and a Wolfenstein-like parametrization of $\boldsymbol{V}$ has been discussed. We have also shown that it is possible for such an ansatz to accommodate leptonic CP violation, and its phenomenological consequences will soon be tested in a variety of more accurate neutrino oscillation experiments.

Generating $\theta_{13}\sim 10^\circ$ from $\theta_{13}=0^\circ$ is certainly a very nontrivial job. Besides an explicit perturbation to a given constant flavor mixing pattern like $\boldsymbol{U}$, one may also consider finite quantum corrections to θ_{13} at the electroweak scale [17] or renormalization-group running effects on θ_{13} from a superhigh-energy scale down to the electroweak scale [18]. However, it is in general difficult (if not impossible) for both approaches to generate a sufficiently large value of θ_{13}, and in particular θ_{12} is usually most sensitive to radiative corrections.

Of course, one may not necessarily start from $\theta_{13}\sim 0^\circ$ for model building. For example, the so-called tetra-maximal neutrino mixing pattern [19] yields $\theta_{12}=\arctan(2-\sqrt{2})\approx 30.4^\circ$, $\theta_{13}=\arcsin[(\sqrt{2}-1)/(2\sqrt{2})]\approx 8.4^\circ$, $\theta_{23}=45^\circ$ and $\delta=90^\circ$ in the symmetry limit. Hence this pattern can easily fit current experimental data if one introduces slight corrections to it. The open question is how to incorporate such a constant mixing scenario with a natural neutrino mass model, and a possible answer to this question will be explored elsewhere.

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