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Half-life of bismuth isotopes predicted by the Coulomb and proximity potential model; a proposition for the spherical nuclei

E. Javadimanesh[1;1)] H. Hassanabadi[1] A. A. Rajabi[1] H. Rahimov[2] S. Zarrinkamar[3]

[1] Physics Department, Shahrood University of Technology, Shahrood, Iran
[2] Computer Engineering Department, Shahrood University of Technology, Shahrood, Iran P.O.Box 3619995161-316 Shahrood, Iran
[3] Department of Basic Sciences, Garmsar Branch, Islamic Azad University, Garmsar, Iran

Abstract: We know that the ground state energy, half-life, spin and parity of the heavy nuclei can be determined via the study of alpha decay. Bearing this in mind, we have calculated the penetration probability in the barrier, the decay constant and thereby the half-lives of 21 isotopes of Bi by using the proximity potential model. The comparison with the existing data is motivating.

Key words: Alpha decay, bismuth, half-life

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1 Introduction

The alpha emission is amongst the dominant decay modes for heavy nuclei, particularly those with $A>150$, and can determine the ground state energy, spin and the parity of the nucleus [1, 2]. The underlying mechanism was first explained by Gamow [3], Gurney and Condon [4] in 1928 under the title of quantum tunneling effect which can be chronologically considered as the first successful quantum description of the nuclear phenomenon. Recently, there has been an increasing motivation to investigate the half-life via alpha-decay studies in different nuclear models including the shell and collective models [5–11].

The alpha emission is a quantum tunneling effect of penetration into the potential barrier of the parent nuclide. In such a phenomenon, the decay energy Q is the key term from which the half-life can be determined. Within the present research, 21 isotopes of Bi with $l=5$ last energy level are considered. Using the Coulomb potential model as well as the potential approximation, we have reported the half-lives and compared the results with Ref. [12].

2 The alpha decay theory

The potential barrier of the parent nucleus is considered as

$$V(r)=\begin{cases} a_0+a_1r+a_2r^2 & R_{\mathrm{p}}\leqslant r\leqslant C_t, \\ \dfrac{Z_1Z_2e^2}{r}+V_{\mathrm{prox}}(z)+\dfrac{\hbar^2 l(l+1)}{2\mu r^2} & r\geqslant C_t, \end{cases} \tag{1}$$

where Z_1, Z_2 respectively represent the atomic number of daughter nuclei and α particle r is the distance between fragment centers, z is the near surfaces distance of the fragments, l is the angular momentum and μ is the reduced mass of the disintegrated system. C_t is the touching configuration of two nuclei defined via [13]

$$C_t=C_1+C_2\ (\mathrm{fm}), \tag{2}$$

where C_i, the Süssmann central radii of fragments is related to sharp radii defined via [13]

$$C_i=R_i-\frac{b^2}{R_i}\ (\mathrm{fm}), \tag{3}$$

$b\approx 1$ is the width of the nuclear surface and [13]

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1) E-mail: elham_javadimanesh89@yahoo.com

$$R_i = 1.28\,A_i^{\frac{1}{3}} - 0.76 + 0.8\,A_i^{-\frac{1}{3}}\ \text{(fm)}, \tag{4}$$

$i = 1,2$ represent the daughter nucleus and the alpha particle, respectively. $V_{\text{prox}}(z)$ is the proximity potential given by [13]

$$V_{\text{prox}}(z) = 4\pi\gamma b \left(\frac{C_1 C_2}{C_1 + C_2}\right) \Phi\left(\frac{z}{b}\right)\ \text{(MeV)}, \tag{5}$$

γ is the nuclear surface tension coefficient calculated from [13]

$$\gamma = 0.9517\left[1 - 1.7826\frac{(N-Z)^2}{A^2}\right]\ (\text{MeV/fm}^2), \tag{6}$$

N, Z and A respectively represent neutron, proton and mass number of the parent, $\Phi\left(\frac{z}{b}\right)$ is the general proximity potential considered as [14]

$$\begin{aligned} &\Phi(\varepsilon) = -1.7817 + 0.9270\varepsilon + 0.0169\varepsilon^2 - 0.05148\varepsilon^3 \\ &0 \leqslant \varepsilon \leqslant 1.9475, \\ &\Phi(\varepsilon) = -4.41\,\mathrm{e}^{\frac{-\varepsilon}{0.7176}} \qquad \varepsilon \geqslant 1.9475, \end{aligned} \tag{7}$$

with $\varepsilon = \dfrac{z}{b}$. The constants a_0, a_1 and a_2 in Eq. (1) are determined from [9]

$$\begin{aligned} &(i)\,R = R_{\text{a}} = R_{\text{P}} \rightarrow V(R) = Q, \\ &(ii)\,R = C_t \rightarrow\ V(R) = V(C_t)\,,\ V'(R) = V'(C_t). \end{aligned} \tag{8}$$

where $R_{\text{P}} = R_{\text{a}}$ is the radius of the parent nuclei. The potential for ^{208}Bi with decay equations $^{208}\text{Bi} \rightarrow {}^{204}\text{Tl} + \alpha$, is plotted in Fig. 1.

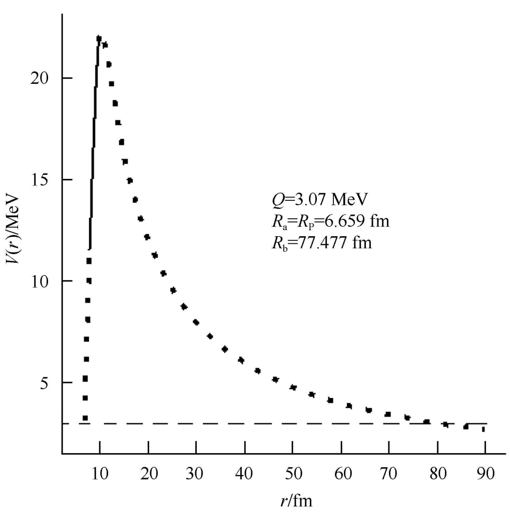

Fig. 1. Potential vs. r.

According to the WKB approximation, the penetration probability is obtained via [15]

$$P = \exp\left\{-\frac{2}{\hbar}\int_{R_{\text{a}}}^{R_{\text{b}}} \sqrt{2\mu(V(r) - Q)}\,\mathrm{d}r\right\}, \tag{9}$$

where $\mu = m\dfrac{A_1 A_2}{A_1 + A_2}$ and m, A_1 and A_2 respectively represent the nucleon mass, mass number of daughter nuclei and alpha particle. The decay energy Q is obtained via [16, 17]

$$\begin{aligned} Q &= B(Z-2, A-4) + 28.3 \\ &\quad - B(Z,A)(\text{MeV}), \end{aligned} \tag{10}$$

$$\begin{aligned} B(Z,A) &= 7.298Z + 8.071(A-Z) \\ &\quad - M(A,Z)(\text{MeV}), \end{aligned} \tag{11}$$

where $B(A,Z)$ and $M(A,Z)$ respectively denote the binding energy and mass excess [16, 17]. The turning boundaries are obtained via $V(R_{\text{a}}) = V(R_{\text{b}}) = Q$ [16, 17]. Now, having defined the prerequisites, the half-life can be calculated as following [18–20]

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\nu P}\ \text{(s)}, \tag{12}$$

with $\nu = (\omega/2\pi) = (2E_\nu/h)$ and λ being the frequency of collision with the barrier per second and decay constant, respectively. E_ν is the empirical zero point vibration energy given by [21, 22]

$$E_\nu = Q\left\{0.056 + 0.039\exp\left[\frac{4 - A_2}{2.5}\right]\right\}. \tag{13}$$

Substitution of E_ν and P in (12) determines the half-life. The alpha particle emission from a nucleus obeys the spin-parity selection rule [23]

$$|I_{\text{i}} - I_{\text{f}}| \leqslant l \leqslant I_{\text{i}} + I_{\text{f}}\ \text{ and }\ \frac{\pi_{\text{i}}}{\pi_{\text{f}}} = (-1)^l, \tag{14}$$

where I_{i}, π_{i} and I_{f}, π_{f} are the spin and parity of the parent and daughter nuclei, respectively. We have calculated the half-lives and considered the ground state to ground state $\left(\dfrac{9}{2}^{-} \rightarrow \dfrac{1}{2}^{+}\right)$ transition of Bismuth isotopes with l=5. Our results are reported in Table 1 with a comparison with Ref. [12] and the experimental data. When we consider the simplicity of the approach, the results look acceptable.

3 Conclusion

We have calculated the penetration probability in the barrier, decay constant and thereby the half-lives of 21 isotopes of Bi by the proximity potential model. In Fig. 2 we have plotted Q vs. the neutron number of the daughter nucleus which indicates a decrease in the decay energy for increasing N_{d}. For $N_{\text{d}} = 126$, however, a jump is observed which is expected because of the magic number of 126. As we expect, there is a sudden decrease in $\lg T_{1/2}$ curve for the magic number $N_{\text{d}} = 126$ (Fig. 3). As shown in Table 1, the results are generally smaller than the available experimental

Table 1. The calculated half-lives.

A	Q/MeV	$T_{1/2}$, Exp./s	$T_{1/2}$, Ref. [12]/s	our model/s
187	7.77	–	1.87×10^{-1}	0.31×10^{-1}
189	7.26	–	0.75×10^{1}	0.14×10^{1}
191	6.76	7.07×10^{2}	4.04×10^{2}	0.879×10^{2}
193	6.29	3.19×10^{4}	2.87×10^{4}	0.730×10^{4}
195	5.81	6.10×10^{6}	3.38×10^{6}	0.97×10^{6}
197	5.19	–	5.27×10^{9}	1.9×10^{9}
199	4.91	–	2.00×10^{11}	0.77×10^{11}
200	4.68	–	5.55×10^{12}	2.32×10^{12}
201	4.48	–	1.26×10^{14}	0.57×10^{14}
202	4.31	–	2.00×10^{15}	0.96×10^{15}
203	4.08	–	1.36×10^{17}	0.72×10^{17}
204	3.97	–	1.85×10^{18}	0.5×10^{18}
205	3.68	–	3.98×10^{20}	2.53×10^{20}
206	3.51	–	1.59×10^{22}	1.1×10^{22}
207	3.26	–	0.70×10^{25}	0.56×10^{25}
208	3.04	–	3.91×10^{27}	3.65×10^{27}
209	3.12	6.0×10^{26}	3.22×10^{26}	2.81×10^{26}
211	6.74	1.54×10^{2}	2.27×10^{2}	0.43×10^{2}
212	6.19	3.73×10^{4}	3.40×10^{4}	0.78×10^{4}
213	5.97	1.41×10^{5}	3.25×10^{5}	0.80×10^{5}
214	5.60	1.45×10^{7}	1.66×10^{7}	0.46×10^{7}

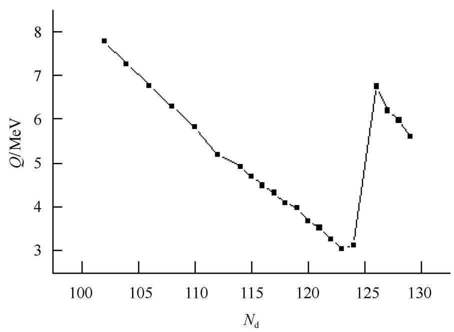

Fig. 2. Decay energy vs. neutron number of daughter nucleus.

data and the theoretical results of Ref. [12]. Although for the full levels, i.e. spherical nuclei, the results are encouraging, for other levels, however, the difference is obvious which is due to the nonspherical shape of the deformed nucleus, but the results show an acceptable agreement with the nuclear shell model. We hope to extend the present study to the latter class as well.

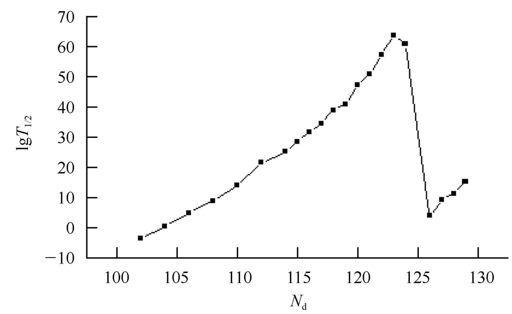

Fig. 3. $\lg T_{1/2}$ vs. neutron number of daughter nucleus.

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