Perturbative pseudospin symmetry limit with linear spin-orbit potential*

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Abstract: The pseudospin symmetry (PSS) limits which conserve substantial spin-orbit splitting are investigated. It is found that while the strength of the spin-orbit potential as well as the spin-orbit splitting increase, the pseudospin doublets, e.g., $2p_{3/2}$ and $1f_{5/2}$ states, are always degenerate. Furthermore, by examining the perturbation corrections to the single-particle energies, the perturbative nature of the proposed PSS limits is also discussed.

Key words: pseudospin symmetry, spin-orbit splitting, perturbation theory

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1 Introduction

It is well known that the spin symmetry (SS) breaking, i.e., the remarkable spin-orbit splitting for the spin doublets ($n,\ l,\ j=l\pm1/2$), is one of the most important concepts for understanding the traditional magic numbers in atomic nuclei [1, 2]. Meanwhile, a new kind of symmetry, the so-called pseudospin symmetry (PSS) [3, 4], is introduced to explain the near degeneracy between two single-particle states with the quantum numbers ($n-1,\ l+2,\ j=l+3/2$) and ($n,\ l,\ j=l+1/2$) by defining the pseudospin doublets ($\tilde{n}=n-1,\ \tilde{l}=l+1,\ j=\tilde{l}\pm1/2$). The splitting of both spin and pseudospin doublets play important roles in the evolutions of shell structure. Thus, it is a fundamental task to explore the origin of SS and PSS, as well as the mechanism of their breaking.

Since the suggestion of PSS in atomic nuclei, there have been comprehensive efforts to understand its origin. As a substantial progress, the PSS was shown to be a symmetry of the Dirac Hamiltonian, where the pseudo-orbital angular momentum $\tilde{l}$ is nothing but the orbital angular momentum of the lower component of the Dirac spinor and the equality in magnitude but difference in sign of the scalar potential $S(\boldsymbol{r})$ and vector potential $V(\boldsymbol{r})$ was suggested as the exact PSS limit by reducing the Dirac equation to a Schrödinger-like equation [5]. As a more general condition, $\mathrm{d}(S+V)/\mathrm{d}r=0$, can be approximately satisfied in exotic nuclei with highly diffuse potentials [6–8]. Based on this limit, the pseudospin $SU(2)$ algebra was established [9]. With the same origin, the spin symmetry in the Dirac negative-energy spectrum, i.e., the single anti-nucleon spectrum, was proposed and investigated [10, 11]. Since there exist no bound nuclei within $S+V=\mathrm{Const}$, whether it corresponds to the exact PSS limit in realistic nuclei has become a hot topic during the last decade.

On the other hand, the relativistic harmonic oscillator (RHO) potentials were used to understand the origin of PSS [12–14]. Furthermore, the $U(3)$ algebra was established in the Dirac Hamiltonian with RHO potentials [15]. Recently, Typel pointed out that such a Hamiltonian is one of the simplest cases where the symmetry breaking potential derived in the supersymmetry framework vanishes [16]. Meanwhile, Marcos et al. commented that the quasi-degeneracy of the pseudospin doublets in realistic nuclei can be considered as the breaking of their degeneracy in the Dirac Hamiltonian with RHO potentials [17].

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Recently, the above two PSS limits are evaluated quantitatively by using perturbation theory [18]. It is found that, from the perturbative point of view, the PSS limit in realistic nuclei should correspond to the Dirac Hamiltonian with RHO potentials rather than that with $S+V=\text{Const}$. Nevertheless, at such a $U(3)$ symmetry limit, the spin-orbit splitting vanishes as well, which is in contradiction with the traditional magic numbers in realistic nuclei.

In this work, the PSS limits which conserve substantial spin-orbit splitting will be investigated. As a first attempt, the difference between the scalar and vector potentials, whose derivative provides the spin-orbit potential, is assumed to have a linear form. The perturbative nature of the proposed PSS limits will also be discussed.

2 Theoretical framework

Assuming the spherical symmetry, the single-particle radial Dirac equations can be cast in the form of

$$H\Psi = E\Psi, \tag{1}$$

with

$$H = \begin{pmatrix} \Sigma(r)+M & -\dfrac{\mathrm{d}}{\mathrm{d}r}+\dfrac{\kappa}{r} \\ \dfrac{\mathrm{d}}{\mathrm{d}r}+\dfrac{\kappa}{r} & -\Delta(r)-M \end{pmatrix}, \text{ and } \Psi = \begin{pmatrix} G(r) \\ F(r) \end{pmatrix}, \tag{2}$$

where E is the single-particle energy with nucleon mass, $\Sigma(r)=S(r)+V(r)$ and $\Delta(r)=S(r)-V(r)$ denote the combinations of the scalar and vector potentials and κ is defined as $\kappa=(l-j)(2j+1)$. Taking the nucleus ^{132}Sn as an example, as illustrated with solid lines in Fig. 1, the $\Sigma(r)$ and $\Delta(r)$ potentials for neutrons are described as the Woods-Saxon shape potentials [19], which reproduce more or less the same single-particle spectra as self-consistent relativistic mean field calculations. It is generally found that $\Sigma_{r=0}\sim -70$ MeV and $-\Delta_{r=0}\sim 700$ MeV in the realistic nuclei.

In recent investigations [15–18], it is shown that the Dirac Hamiltonian with RHO potentials, i.e.,

$$\Sigma(r)=\Sigma_0+d_2r^2, \quad \text{and} \quad \Delta(r)=\Delta_0, \tag{3}$$

is one of the limiting cases where the pseudospin doublets are degenerate. For example, as illustrated with the dashed lines in Fig. 1, when the coefficients Σ_0, d_2 and Δ_0 in Eq. (3) are chosen as $\Sigma_0+M=868.68$ MeV, $d_2=1.00$ MeV/fm^2 and $-\Delta_0-M=-587.05$ MeV (the half-depth of the Woods-Saxon potential), the pseudospin doublets $1\tilde{d}$, i.e., $2p_{3/2}$ and $1f_{5/2}$ states, are exactly degenerate at $E_{1\tilde{d}}=914.52$ MeV. However, it is also found that in such a limiting case the spin doublets, e.g., $2p$ and $1f$ states, are degenerate as well, because the spin-orbit potential proportional to $\mathrm{d}\Delta/\mathrm{d}r$ vanishes.

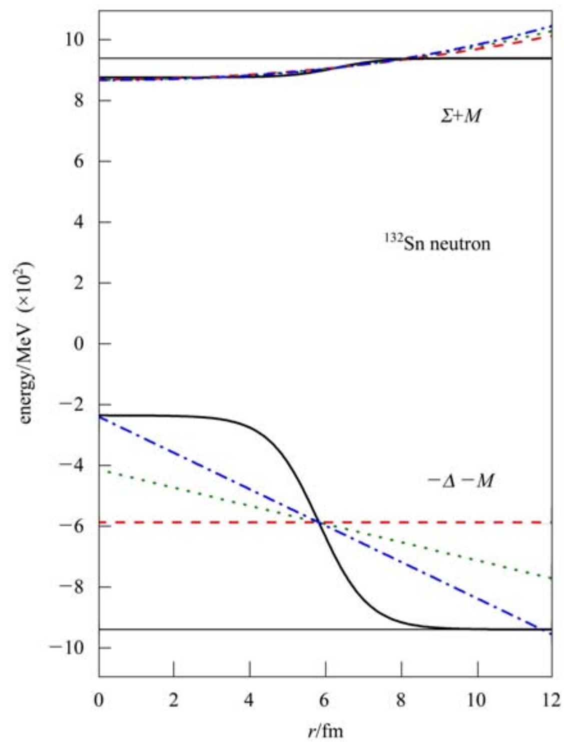

Fig. 1. (color online) Single-particle potentials for neutrons in the nucleus ^{132}Sn. The Woods-Saxon potentials [19] are shown as solid lines and the potentials corresponding to the pseudospin symmetry (PSS) limits with s_1=0, 30, 60 MeV/fm are shown as dashed, dotted and dash-dotted lines, respectively.

In order to conserve the spin-orbit splitting, the potential $\Delta(r)$ should be r dependent. One of the simplest forms reads

$$\Delta(r)=\Delta_0+s_1(r-r_{\rm s}), \tag{4}$$

where the slope s_1 determines the strength of the spin-orbit potential. In the present calculations, the constants Δ_0 and $r_{\rm s}$ are respectively chosen as the half-depth energy and the corresponding radial coordinate for the Woods-Saxon potential. Following the derivations within the supersymmetry framework in Ref. [16], one can obtain the corresponding $\Sigma(r)$ potential for the exact PSS condition, which reads

$$\Sigma(r)=\Sigma_0+d_2\left[1+\frac{s_1(r-r_{\rm s})}{E_0+\Delta_0+M}\right]r^2, \tag{5}$$

where E_0 is the mean value of the single-particle energies of the pseudospin doublets. Eqs. (4) and (5) show that for such PSS conditions the linear term in $\Delta(r)$ leads to the cubic term in $\Sigma(r)$.

Since the potential $\Sigma(r)$ shown in Eq. (5) contains the mean energy E_0, in principle, the Dirac equation should be solved iteratively. Nevertheless, focusing on the pseudospin doublets $1\tilde{d}$, the mean energy E_0 is nothing but $E_0=E_{2p_{3/2}}=E_{1f_{5/2}}$ when these pseudospin doublets are degenerate. Thus, an equivalent but efficient way for determining the PSS limit and getting the single-particle spectrum is the following: for a given slope s_1, the constant Σ_0 is calculated by taking $E_0 = E_{1f_{5/2}} = 914.52$ MeV, thus the potentials of the PSS limit are determined. Finally, the whole single-particle spectrum for this PSS limit can be obtained by solving the Dirac Eq. (1).

3 Results and discussion

For the slopes s_1= 0, 30, 60 MeV/fm, the potentials $-\Delta(r)-M$ and $\Sigma(r)+M$ of the corresponding PSS limits are shown in Fig. 1 with dashed, dotted and dash-dotted lines, respectively. Moreover, in Fig. 2 the single-particle energies of $2p$ and $1f$ states thus obtained are shown as functions of the slope s_1. First of all, it is found that the pseudospin doublets $2p_{3/2}$ and $1f_{5/2}$ are always degenerate, which verifies the PSS limits. Second, in contrast to the spin-orbit splitting, it vanishes when $\Delta(r)$ is a constant, the splitting becomes larger as the slope s_1 increases, i.e., the strength of the spin-orbit potential becomes larger. When $s_1=60$ MeV/fm, the splittings of the $2p$ and $1f$ doublets are around 1.1 MeV and 1.8 MeV, respectively.

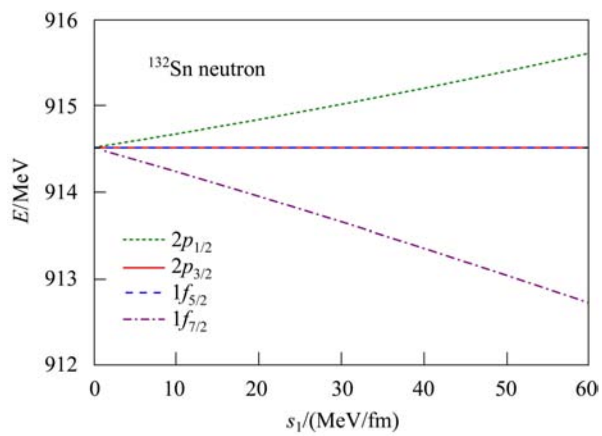

Fig. 2. (color online) Single-particle energies of $2p$ and $1f$ states as functions of the slope s_1 for potential $\Delta(r)$ shown in Eq. (4).

In order to investigate the perturbative nature of the above PSS limits, the Dirac Hamiltonian with PSS limits and that with potentials of Woods-Saxon form are connected by

$$H_{\rm WS}=H_{\rm PSS}+W, \tag{6}$$

or

$$H_{\rm PSS}=H_{\rm WS}-W, \tag{7}$$

where W representing the difference between $H_{\rm WS}$ and $H_{\rm PSS}$ is regarded as the symmetry breaking potential. Following the idea of perturbation theory, the eigenvalues and wave functions of $H_{\rm WS}$ can be expanded on the complete basis of eigenstates of $H_{\rm PSS}$ in the former case, whereas the eigenvalues and wave functions of $H_{\rm PSS}$ can be expanded on the complete basis of eigenstates of $H_{\rm WS}$ in the latter case. Furthermore, using the Rayleigh-Schrödinger perturbation theory, the perturbation corrections to the single-particle energies can be calculated order-by-order.

Taking the $H_{\rm PSS}$ with the slope $s_1=60$ MeV/fm as an example, as shown in Fig. 3, the single-particle energies of $2p$ and $1f$ states obtained at the PSS limit and the corresponding first-, second- and third-order perturbation calculations are compared with those obtained within the Woods-Saxon potential. It can be clearly seen that the single-particle spectrum of $H_{\rm WS}$ is well reproduced by the third-order perturbation calculations on the basis of $H_{\rm PSS}$.

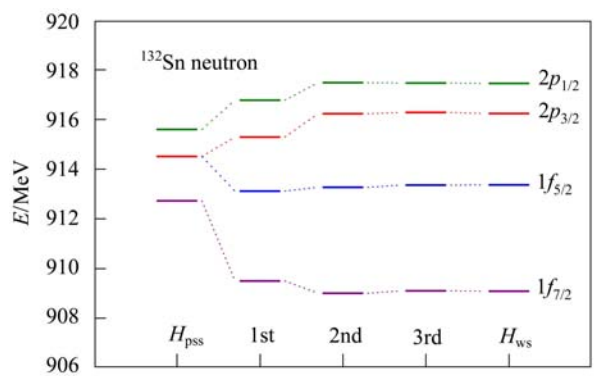

Fig. 3. (color online) Single-particle energies of $2p$ and $1f$ states obtained at the PSS limit and the corresponding first-, second- and third-order perturbation calculations in comparison with those obtained within the Woods-Saxon potential.

In Fig. 4, the single-particle energies of $2p$ and $1f$ states obtained within the Woods-Saxon potential and the corresponding first-, second- and third-order perturbation calculations are compared with those obtained at the PSS limit. It can be clearly seen that the energy degeneracy of the pseudospin doublets is well restored by the third-order perturbation calculations.

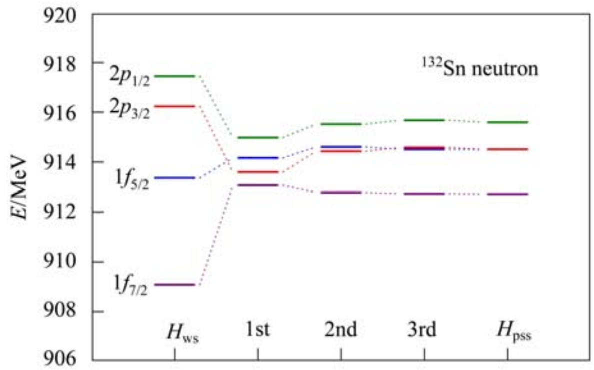

Fig. 4. (color online) Single-particle energies of $2p$ and $1f$ states obtained within the Woods-Saxon potential and the corresponding first-, second- and third-order perturbation calculations in comparison with those obtained at the PSS limit.

This demonstrates that for studying the relationship between the eigenstates of H_{WS} and H_{PSS} by perturbation theory, it is equivalent to use the relations shown in Eq. (6) and Eq. (7). Moreover, it is interesting to conclude from Fig. 3 and Fig. 4 that the proposed PSS limits with substantial spin-orbit splitting can be connected to the Dirac Hamiltonian in realistic nuclei from the perturbative point of view.

4 Summary

In summary, inspired by the exact PSS conditions derived within the supersymmetry framework, the PSS limits which conserve substantial spin-orbit splitting are investigated. The potential $\Delta(r)$ is assumed to be linearly dependent on the radial coordinate r, which correspondingly leads to a cubic term in the potential $\Sigma(r)$ for such PSS conditions. It is found that while the spin-orbit splitting increases as the coefficient s_1 of the linear term increases, the pseudospin doublets $2p_{3/2}$ and $1f_{5/2}$ are always degenerate. Finally, by examining the perturbation corrections to the single-particle energies, it is shown that the proposed PSS limits with non-vanishing spin-orbit potential can be connected to the Dirac Hamiltonian in realistic nuclei.

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