CPC(HEP & NP), 2011, **35**(9): 817–820 Chinese Physics C Vol. 35, No. 9, Sep., 2011

Isospin and mixed symmetry structure in ^{26}Mg*

DONG Hong-Fei(董鸿飞)[1,2;1)] BAI Hong-Bo(白洪波)[1] LÜ Li-Jun(吕立君)[1]

[1] Department of Physics, Chifeng University, Chifeng 024001, China
[2] Physics Department, Tsinghua University, Beijing 100084, China

Abstract: The isospin excitation states and electromagnetic transitions of the ^{26}Mg nucleus are studied with the isospin-dependent interacting boson model (IBM-3). The mixed symmetry states at low spin and the main components of the wave function for some states are also analyzed. The results show good agreement with the available experimental data. From the IBM-3 Hamiltonian expressed in Casimir operator form, the ^{26}Mg is also proved to be a transition nuclei from $U(5)$ to $SU(3)$.

Key words: IBM-3, isospin, energy level, mixed symmetry states, electromagnetic transitions

PACS: 21.10.Re, 21.60.Fw, 27.30.+t **DOI:** 10.1088/1674-1137/35/9/005

1 Introduction

The studies of nuclei deformation, structure, phase transition and mass measurement have great significance in nuclear physics and astrophysics [1, 2]. Great progress had been made by scientists in the research field of normal deformation, super deformation and giant deformation [3].

Nuclei that near the beta stability line ($Z \approx N$) have been an interesting research field during the last few years [4–8]. The structure of these nuclei provides a sensitive test for the isospin symmetry of nuclear force, which is very important.

The Interacting boson model (IBM) is an algebraic model that can be used in the study of nuclear collective motions [9–11]. In the original version (IBM-1), only one kind of boson is considered and it has been successful in describing various properties of medium and heavy even-even nuclei [12–16]. In the second version of the interacting boson model (IBM-2), the bosons are further classified into proton-boson and neutron-boson and mixed symmetry in the proton and neutron degrees of freedom has been predicted [17]. The valence protons and neutrons in light nuclei are filling the same major shell and the isospin effect should also be considered, so the IBM has been extended to the isospin-dependent interacting boson model (IBM-3) [18]. In the IBM-3, three types of bosons including proton-proton (π), neutron-neutron (ν) and proton-neutron (δ) are considered and they form the isospin T=1 triplet. The microscopic foundation of IBM-3 is based on the shell model [19]. Because the protons and neutrons in the lighter nuclei region are in the same major shell, the IBM-3 can describe the low-energy levels of some nuclei well and explain their isospin and F-spin symmetry structure [6–8, 20–21]. And with the IBM, we have studied many nuclei such as ^{36}Ar, ^{28}Si, ^{44}Ti, ^{52}Fe, ^{68}Ge, $^{140-162}$Gd, $^{164-182}$Hf, etc. [22–28].

The dynamical symmetry group for IBM-3 is $U(18)$, which starts with $U_{\rm sd}(6)\times U_{\rm c}(3)$ and must contain $SU_{\rm T}(2)$ and $O_{\rm d}(3)$ as subgroups because both the isospin and the angular momentum are good quantum numbers. The natural chains of IBM-3 group $U(18)$ are the following [29]:

$$U(18) \supset (U_{\rm c}(3)\supset SU_{\rm T}(2)) \times(U_{\rm sd}(6)\supset U_{\rm d}(5)\supset O_{\rm d}(5)\supset O_{\rm d}(3)), \quad (1)$$

$$U(18) \supset (U_{\rm c}(3)\supset SU_{\rm T}(2)) \times(U_{\rm sd}(6)\supset O_{\rm sd}(6)\supset O_{\rm d}(5)\supset O_{\rm d}(3)), \quad (2)$$

$$U(18) \supset (U_{\rm c}(3)\supset SU_{\rm T}(2)) \times(U_{\rm sd}(6)\supset SU_{\rm sd}(3)\supset O_{\rm d}(3)), \quad (3)$$

The subgroups $U_{\rm d}(5)$, $O_{\rm sd}(6)$ and $SU_{\rm sd}(3)$ are used to describe vibrational, γ-unstable and rotational

Received 9 December 2010
* Supported by National Natural Science Foundation of China (10547003)
1) E-mail: hongfeidong@126.com

nuclei respectively [30, 31]. Our cooperaters have studied the ^{24}Mg nucleus recently, calculated the energy level and analyzed the mixed symmetry states at low spin and achieved good results [32]. ^{26}Mg is an even-even nucleus which belongs to the lighter nuclei region. We study the isospin excitation states, electromagnetic transitions and mixed symmetry states at low spin for ^{26}Mg nucleus within the framework of the interacting boson model (IBM-3). The main components of the wave function for some states are also analyzed respectively.

2 The IBM-3 Hamiltonian and the parameter

The isospin-invariant IBM-3 Hamiltonian can be written as [19]

$$H=\varepsilon_{\rm s}\hat{n}_{\rm s}+\varepsilon_{\rm d}\hat{n}_{\rm d}+H_2, \tag{4}$$

where

$$\begin{aligned}H_2 =& \frac{1}{2}\sum_{L_2T_2} C_{L_2T_2}[(\mathrm{d}^\dagger\mathrm{d}^\dagger)^{L_2T_2}\cdot(\tilde{\mathrm{d}}\tilde{\mathrm{d}})^{L_2T_2}]\\ &+\frac{1}{2}\sum_{T_2} B_{0T_2}[(\mathrm{s}^\dagger\mathrm{s}^\dagger)^{0T_2}\cdot(\tilde{\mathrm{s}}\tilde{\mathrm{s}})^{0T_2}]\\ &+\sum_{T_2} A_{2T_2}[(\mathrm{s}^\dagger\mathrm{d}^\dagger)^{2T_2}\cdot(\tilde{\mathrm{d}}\tilde{\mathrm{s}})^{2T_2}]\\ &+\frac{1}{\sqrt{2}}\sum_{T_2} D_{2T_2}[(\mathrm{s}^\dagger\mathrm{d}^\dagger)^{2T_2}\cdot(\tilde{\mathrm{d}}\tilde{\mathrm{d}})^{2T_2}]\\ &+\frac{1}{2}\sum_{T_2} G_{0T_2}[(\mathrm{s}^\dagger\mathrm{s}^\dagger)^{0T_2}\cdot(\tilde{\mathrm{d}}\tilde{\mathrm{d}})^{0T_2}],\end{aligned} \tag{5}$$

and

$$\begin{aligned}&(\mathrm{b}_1^\dagger\mathrm{b}_2^\dagger)^{L_2T_2}\cdot(\tilde{\mathrm{b}}_3\tilde{\mathrm{b}}_4)^{L_2T_2}\\ &= (-1)^{(L_2+T_2)}\sqrt{(2L_2+1)(2T_2+1)}\\ &\quad\times[(\mathrm{b}_1^\dagger\mathrm{b}_2^\dagger)^{L_2T_2}\cdot(\tilde{\mathrm{b}}_3\tilde{\mathrm{b}}_4)^{L_2T_2}]^{00},\end{aligned} \tag{6}$$

$$\tilde{\mathrm{b}}_{lm,m_z} = (-1)^{(l+m+1+m_z)}b_{l-m-m_z}, \tag{7}$$

where T_2 and L_2 represent the two-boson system isospin and angular momentum.The parameters A, B, C, D and G are the two-body matrix elements. $A_{T_2}=\langle sd20\mid H_2\mid sd20\rangle$, $T_2=0,1,2$; $B_{T_2}=\langle s^20T_2\mid H_2\mid s^20T_2\rangle$, $G_{T_2}=\langle s^20T_2\mid H_2\mid d^20T_2\rangle$, $D_{T_2}=\langle sd2T_2\mid H_2\mid d^22T_2\rangle$ and $C_{L_2T_2}=\langle d^2L_2T_2\mid H_2\mid d^2L_2T_2\rangle$, with $T_2=0,2$, $L_2=0,2,4$; $C_{L_21}=\langle d^2L_21\mid H_2\mid d^2L_21\rangle$ with $L_2=1,3$. The parameters A_1, C_{11}, C_{31} are the Majorana parameters which are similar to the ones in IBM-2. These interactions are important for shifting the states with mixed symmetry with respect to the total symmetric ones.

IBM-3 Hamiltonian can be expressed in Casimir operator form, i.e.,

$$\begin{aligned}H_{\rm Casimir} =& \lambda C_{2U_{\rm sd}(6)}+a_{\rm T}T(T+1)\\ &+a_1C_{1U_{\rm d}(5)}+a_2C_{2U_{\rm d}(5)}\\ &+a_3C_{2SU_{\rm sd}(3)}+a_4C_{2O_{\rm d}(5)}\\ &+a_5C_{2O_{\rm d}(3)}+a_6C_{2O_{\rm d}(6)},\end{aligned} \tag{8}$$

where λ parameter is used to determine the position of the mixed symmetry states. By fitting to the experimental spectra, the parameters in the Hamiltonian can be determined. The low-lying levels of ^{26}Mg can be described as follows,

$$\begin{aligned}H_{\rm Casimir} =& 0.359C_{2U_{\rm sd}(6)}+0.361T(T+1)\\ &+0.093C_{1U_{\rm d}(5)}+0.611C_{2U_{\rm d}(5)}\\ &-0.175C_{2SU_{\rm sd}(3)}+0.126C_{2O_{\rm d}(5)}\\ &-0.01C_{2O_{\rm d}(3)}+0.009C_{2O_{\rm d}(6)}.\end{aligned} \tag{9}$$

From the IBM-3 Hamiltonian expressed in Casimir operator form, we can see that the interaction strength of $C_{1U_{\rm d}(5)}$ is 0.093 and that of $C_{2SU_{\rm sd}(3)}$ is 0.175, so ^{26}Mg is in transition from $U(5)$ to $SU(3)$.

3 Energy levels

By the computation program written by Van Isacker [33], the energy levels and wave function are given. The parameters of the calculation are listed in Table 1.

Table 1. The parameters of the IBM-3 Hamiltonian of the ^{26}Mg nucleus.

$\varepsilon_{\rm d\rho}(\rho=\pi,\nu,\delta)$	4.763		
$\varepsilon_{\rm s\rho}(\rho=\pi,\nu,\delta)$	1.171		
$A_i(i=0,1,2)$	−1.408	−0.758	0.758
$C_{i0}(i=0,2,4)$	−0.114	1.876	−0.714
$C_{i2}(i=0,2,4)$	2.052	4.042	1.452
$C_{i1}(i=1,3)$	−0.832	−2.232	
$B_i(i=0,2)$	−0.726	1.440	
$D_i(i=0,2)$	1.310	1.310	
$G_i(i=0,2)$	−1.525	−1.525	

The calculated and experimental energy levels are exhibited in Fig. 1. The theoretical calculations are in agreement with the experimental data when the spin value is below 8^+.

We have analyzed the wave function of the 0_1^+, 2_1^+, 4_1^+, 6_1^+, 1_1^+ and 3_2^+ states, they are:

$|\,0_1^+\rangle = -0.5723\,|\,s_\nu^3 s_\pi^2\rangle - 0.4215\,|\,s_\nu^2 s_\pi d_\nu d_\pi\rangle + 0.3304\,|\,s_\nu^2 s_\pi s_\delta^2\rangle - 0.2980\,|\,s_\nu s_\pi^2 d_\nu^2\rangle - 0.2023\,|\,s_\nu s_\pi^4\rangle + \cdots,$

$|\,2_1^+\rangle = 0.4929\,|\,s_\nu^2 s_\pi^2 d_\nu\rangle - 0.4025\,|\,s_\nu^3 s_\pi d_\pi\rangle - 0.2649\,|\,s_\nu^2 s_\pi d_\nu d_\pi\rangle - 0.2324\,|\,s_\nu^2 s_\pi s_\delta d_\delta\rangle - 0.2324\,|\,s_\nu s_\pi s_\delta^2 d_\nu\rangle + \cdots,$

$|\,4_1^+\rangle = 0.5441\,|\,s_\nu^2 s_\pi d_\nu d_\pi\rangle + 0.3848\,|\,s_\nu s_\pi^2 d_\nu^2\rangle + 0.2565\,|\,s_\nu s_\pi s_\delta d_\nu d_\delta\rangle + 0.2221\,|\,s_\nu^3 d_\pi^2\rangle + \cdots,$

$|\,6_1^+\rangle = -0.5893\,|\,s_\nu s_\pi d_\nu^2 d_\pi\rangle - 0.4129\,|\,s_\delta^2 d_\nu d_\pi^2\rangle + 0.2752\,|\,s_\nu s_\delta d_\nu d_\pi d_\delta\rangle - 0.2384\,|\,s_\pi^2 d_\nu^3\rangle + \cdots,$

$|\,1_1^+\rangle = 0.5799\,|\,s_\nu^2 s_\pi d_\nu d_\pi\rangle - 0.5022\,|\,s_\delta^3 d_\nu d_\delta\rangle + 0.2574\,|\,s_\delta^2 d_\nu d_\delta^2\rangle + 0.2460\,|\,s_\nu s_\pi s_\delta d_\nu d_\delta\rangle + \cdots,$

$|\,3_2^+\rangle = -0.6097\,|\,s_\nu^2 s_\pi d_\nu d_\pi\rangle + 0.5280\,|\,s_\delta^3 d_\nu d_\delta\rangle - 0.2587\,|\,s_\nu s_\pi s_\delta d_\nu d_\delta\rangle - 0.2439\,|\,s_\nu^2 s_\delta d_\pi d_\delta\rangle + \cdots,$

We found that the main components of the wave function for the states above are s^N, $s^{N-1}d$, $s^{N-2}d^2$, $s^{N-3}d^3$, etc. configurations. The wave function of these states contains a significant amount of δ boson component, which shows that it is necessary to consider the isospin effect for the light nuclei. The parameters C_{11} and C_{31} are the Majorana parameter, which have a very large effect on the energy levels of the mixed symmetry state. From Figs. 2 and Fig. 3, we can see that the 1_1^+ and 3_2^+ states have a large change with the parameters C_{11} and C_{31} respectively, which shows that the 1_1^+ and 3_2^+ states are mixed symmetry states.

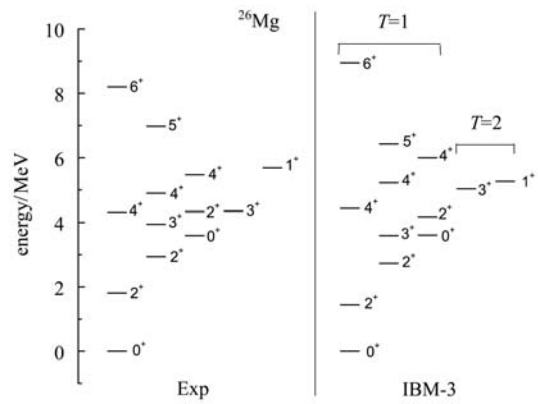

Fig. 1. Comparison between the lowest excitation energy bands of the IBM-3 calculation and the experimental excitation energies of ^{26}Mg .

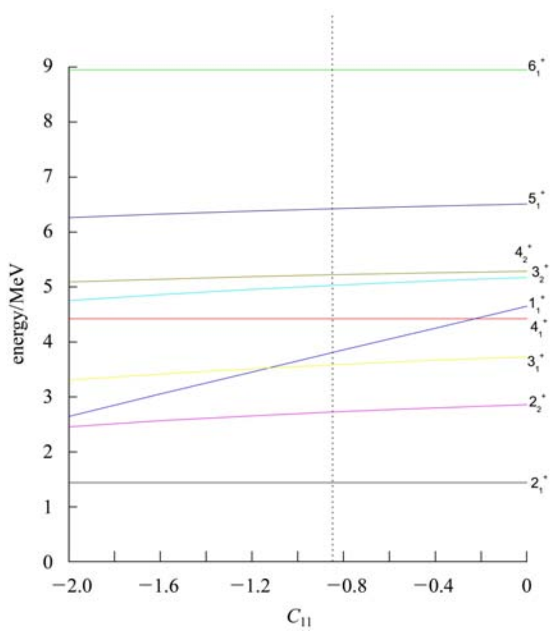

Fig. 2. Variation in level energy of ^{26} Mg as a function of C_{11} respectively.

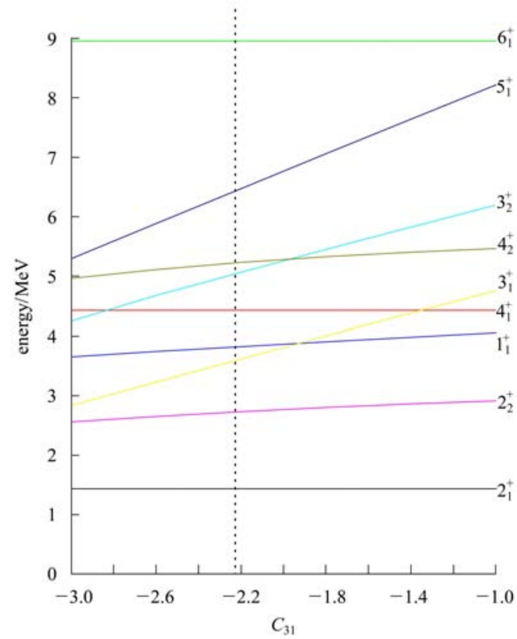

Fig. 3. Variation in level energy of ^{26}Mg as a function of and C_{31} respectively.

4 Electromagnetic transition

In the IBM-3 model, the quadrupole operator is expressed as [29]:

$$Q = Q^0 + Q^1, \tag{10}$$

where

$$Q^0 = \alpha_0\sqrt{3}[(\mathrm{s}^\dagger\tilde{\mathrm{d}})^{20} + (\mathrm{d}^\dagger\tilde{\mathrm{s}})^{20}] + \beta_0\sqrt{3}(\mathrm{d}^\dagger\tilde{\mathrm{d}})^{20}, \tag{11}$$

$$Q^1 = \alpha_1\sqrt{2}[(\mathrm{s}^\dagger\tilde{\mathrm{d}})^{21} + (\mathrm{d}^\dagger\tilde{\mathrm{s}})^{21}] + \beta_1\sqrt{2}(\mathrm{d}^\dagger\tilde{\mathrm{d}})^{21}. \tag{12}$$

The M1 transition is also a one-boson operator with an isoscalar part and an isovector part

$$M = M^0 + M^1, \tag{13}$$

where

$$M^0 = g_0\sqrt{3}(\mathrm{d}^\dagger\tilde{\mathrm{d}})^{10} = g_0 L/\sqrt{10}, \tag{14}$$

$$M^1 = g_1\sqrt{2}(\mathrm{d}^\dagger\tilde{\mathrm{d}})^{11}. \tag{15}$$

where g_0 and g_1 are the isoscalar and isovector g-factors respectively and L is the angular momentum operator. For the ^{26} Mg, the parameters in the elect-

Table 2. The experimental and calculated $B(\mathrm{E2})(e^2\mathrm{fm}^4)$ and $B(\mathrm{M1})(\mu_\mathrm{N}^2)$ for ^{26}Mg.

	B(E2)		B(M1)	
	Exp.	Cal.	Exp.	Cal.
$2_1^+ \to 0_1^+$	0.0061(2)	0.0060		
$2_2^+ \to 2_1^+$		0.13522	0.172(11)	0.008
$2_2^+ \to 0_1^+$	0.00018(2)	0.00021		
$2_2^+ \to 0_2^+$		0.00000		
$2_3^+ \to 0_2^+$		0.00010		
$2_3^+ \to 0_1^+$	0.00011(4)	0.00011		
$2_3^+ \to 2_1^+$		0.05817		0.00340
$2_3^+ \to 2_2^+$		0.06405		0.00189
$0_2^+ \to 2_1^+$	0.00049(1)	0.00053		
$0_2^+ \to 2_2^+$		0.00001		
$1_1^+ \to 0_1^+$				0.00354
$1_1^+ \to 0_2^+$				0.00000
$1_1^+ \to 2_1^+$		0.01363		0.00276
$1_1^+ \to 2_2^+$		0.02337		0.00000
$1_1^+ \to 2_3^+$		0.00237		0.00007
$3_1^+ \to 2_1^+$		0.04049	0.00183(27)	0.00182
$3_1^+ \to 2_2^+$		0.03153	0.0285(41)	0.0000
$3_1^+ \to 2_3^+$		0.00196		0.00001
$3_1^+ \to 4_1^+$		0.17917		0.00497
$4_1^+ \to 2_1^+$	0.0021(1)	0.0023		
$4_1^+ \to 2_2^+$		0.00029		
$4_2^+ \to 2_1^+$	0.0064(14)	0.0045		
$4_2^+ \to 2_2^+$		0.00022		
$4_2^+ \to 2_3^+$		0.00010		
$4_2^+ \to 4_1^+$		0.17705		0.00915

romagnetic transitions are determined by fitting the experimental data, where $\alpha_0 = 0.119$, $\beta_0 = 0.672$, $\alpha_1 = 0.037$, $\beta_1 = 0.581$, $g_0 = 0.000$, $g_1 = 0.301$ respectively. Table 2 gives the electromagnetic transition rate calculated by IBM-3 [34].

Table 2 shows that the calculated B(E2) values are quite close to the experimental ones.The calculated quadrupole moment of the 2_1^+ state is $Q(2_1^+)$= 0.59418 eb, that of the 2_2^+ state is $Q(2_2^+)$=1.12365 eb and that of the 4_1^+ state is $Q(4_1^+)$= 1.41749 eb.

5 Conclusion

By using the isospin-dependent interacting boson model (IBM-3), we have calculated the isospin excitation bands at low spin, electromagnetic transitions and mixed symmetry structure of ^{26}Mg. The results calculated by the IBM-3 show good agreement with the experimental data available in a low energy level. The results conclude that the IBM-3 is good in the description of the low-lying levels in ^{26}Mg nucleus. The present calculations also give the structure information of the isospin and mixed symmetry states of ^{26}Mg nucleus. The 1_1^+ and 3_2^+ states are also proved to be mixed symmetry states. The ^{26}Mg nucleus is in transition from $U(5)$ to $SU(3)$.

References

1 SUN Y. Chin. Sci. Bull., 2009, **54**(24): 4594

2 HE Shao-Rong, YANG Tong-Suo, LI Tian-Xiang et al. Chin. Sci. Bull., 2009, **54**(20): 3772

3 SUN Yang, ZHANG Jing-Ye, LONG Gui-Lu, WU Chang-Li. Chin. Sci. Bull., 2009, **54**(3): 358

4 Sahu R, Kota VKB. Phys. Rev. C, 2003, **67**(5): 054323

5 Bender M, Flocard H, Heenen P H. Phys. Rev. C, 2003, **68**(4): 044321

6 LONG Gui-Lu, SUN Yang. Phys. Rev. C, 2001, **65**(2): 0712

7 Caurier E, Nowacki F, Poves A, Phys. Rev. Lett., 2005, **95**(4): 042502

8 Falih HAK, LI Yan-Son, LONG Gui-Lu. J. Phys. G: Nucl. Part. Phys., 2004, **30**: 1287

9 LUO Yan-An, PAN Feng, NING Ping-Zhi. HEP & NP, 2004, **28**(S1): 87 (in Chinese)

10 ZHANG Yu, HOU Zhan-Feng, LIU Yu-Xin et al. HEP & NP, 2006, **30**(S2): 90 (in Chinese)

11 LEI Yang, XU Zheng-Yu, ZHAO Yu-Min, LU Da-Hai. Sci. Chin. Phys., 2010, **53**(8): 1460

12 Arima A, Iachello F. Ann. Phys. (N. Y.), 1976, **99**: 253

13 Arima A, Iachello F. Ann. Phys. (N.Y.), 1978, **111**: 201

14 Arima A, Iachello F. Ann. Phys. (N.Y.), 1979, **123**: 468

15 LIU Yu-Xin, SONG Jian-Gang, SUN Hong-Zhou, ZHAO En-Guang. Phys. Rev. C, 1997, **56**(3): 1370

16 PAN Feng, DAI Lian-Rong, LUO Yan-An, Draayer J P. Phys. Rev. C, 2003, **68**(1): 014308

17 Iachello F, Arima A. The Interacting Boson Model. Cambridge University Press, 1987

18 Elliott J P, White A P. Phys. Lett. B, 1980, **97**(2): 169

19 Evans J A, LONG Gui-Lu, Elliott J P. Nucl. Phys. A, 1993, **561**: 201

20 Falih HAK, LI Yan-Song, LONG Gui-Lu. HEP & NP, 2004, **28**(4): 370 (in Chinese)

21 Falih HAK, LONG Gui-Lu. Chin. Phys., 2004, **13**(8): 1230

22 BAI Hong-Bo, ZHANG Jin-Fu, CAO Wan-Cang. Commun. Theor. Phys., 2007, **48**(6): 1067

23 LÜ Li-Jun, Falih HAK, ZHANG Jin-Fu, BAI Hong-Bo. Chin. Phys. C (HEP & NP), 2009, **33**(S1): 46

24 ZHANG Jin-Fu, BAI Hong-Bo. Chin. Phys., 2004, **13**(11): 1843

25 ZHANG Jin-Fu, LÜ Li-Jun, BAI Hong-Bo. Science in Chin. G, 2008, **51**(12): 1845

26 BAI Hong-Bo, DONG Hong-Fei, ZHANG Jin-Fu. Chin. Phys. C (HEP & NP), 2009, **33**(S1): 40

27 LÜ Li-Jun, ZHANG Jin-Fu. HEP & NP, 2006, **30**(2): 128 (in Chinese)

28 FAN Ti-Gui, LÜ Li-Jun, ZHANG Jin-Fu. HEP & NP, 2004, **28**(S1): 119 (in Chinese)

29 LONG Gui-Lu. Chinese J. Nucl. Phys., 1994, **16**: 331

30 BAI Hong-Bo, ZHANG Jin-Fu, ZHOU Xian-Rong. HEP & NP, 2005, **29**(8): 752 (in Chinese)

31 ZHANG Jin-Fu, BAI Hong-Bo. Chin. Sci. Bull., 2007, **52**(2): 165

32 LÜ Li-Jun, BAI Hong-Bo, ZHANG Jin-Fu. Chin. Phys. C, 2008, **32**(3): 177

33 Van Isacker P et al. Ann. Phys. (N. Y.), 1986, **171**: 253

34 Firestone R B, Shirley V S. Table of Isotopes. New York: J. Wiley. 1996